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Research Article

Projection Synchronization of a Class of Complex Chaotic Systems with Both Uncertainty and Disturbance

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This paper mainly investigates the projection synchronization of complex chaotic systems with both uncertainty and disturbance. Using the linear feedback method and the uncertainty and disturbance estimation- (UDE-) based control method, the projection synchronization of such systems is realized by two steps. In the first step, a linear feedback controller is designed to control the nominal complex chaotic systems to achieve projection synchronization. An UDE-based controller is proposed to estimate the whole of uncertainty and disturbance in the second step. Finally, numerical simulations verify the feasibility and effectiveness of the control method.

1. Introduction

The chaotic synchronization phenomenon that caused a great sensation in academia was firstly proposed by Pecora and Carroll in early 1990 [1]. They achieved chaotic synchronization of two identical systems with different initial conditions in electronic experiments. Until now, many types of chaotic synchronization have been discovered, such as complete synchronization, phase synchronization, lag synchronization, antisynchronization, and projection synchronization, and many other important results have been obtained (see references [2–8]). Especially, projective synchronization has received much attention due to its faster communication and proportionality between the dynamical systems. In case of projective synchronization, the master and the slave system can be synchronized up to a scaling factor and the scaling factor is a constant transformation between the driving and the response variables that can further increase the security of secure communication and the transmission speed of communication. It has potential application prospects in the field of chaotic secure communication.

Many control methods about chaotic projection synchronization have been reported [9–29]. However, most controllers are complicated in structure and difficult in design. Due to the complexity of structure, many control methods are not suitable for projective synchronization control of complex chaotic systems. Among these, the linear feedback controller, because of its simple structure, easy design, and good control effect, was used to realize the projection synchronization of given complex chaotic system. Moreover, in the simulation experiment, it is also proved that the linear feedback controller has a good experimental effect.

We note that most of the literature on solving the control problems of chaotic systems with external perturbations is generally complex and difficult to implement. Moreover, when designing the controller, the method to deal with the external disturbance is just simply to cancel the disturbance term from the formula of the controller, and it is not rigorous in nonlinear system control theory. In fact, in the field of nonlinear system control, the UDE-based controller can deal with many structured and unstructured robust control problems and has been applied to the engineering field in

some literatures [30–32]. In the simulation experiment, we have noticed that the UDE control method, which is composed of filters with appropriate bandwidth, has an ideal processing effect on the external disturbance of the system which is finally used by us.

The main contribution of this paper is to design a physical controller, which is simple in form, to realize the projection synchronization of a complex chaotic system. A linear feedback UDE-based control method is proposed by combining the linear feedback controller and the UED-based controller in two steps. A linear feedback control controller is designed for the nominal complex chaotic system in the first step. In the second step, an UDE-based controller is proposed to estimate the whole of uncertainty and disturbance. In the end, two complex chaotic systems with numerical simulations are used to verify the validity and effectiveness of the proposed theoretical results.

2. Preliminary

Consider the following controlled chaotic system:

$$\dot{X} = F(X) + B^* U^*, \tag{1}$$

where $X \in R^n$ is the state, $F(X) = (F_1(X), \cdots, F_n(X))^T$ is a continuous vector function, $B^* \in R^{n\times l}$, and $U^* = (U_1^*, \cdots, U_l^*)^T$ is the controller to be designed, $l \geq 1$.

Let system (1) be the master system; then, the slave system is given as follows:

$$\dot{Y} = F(Y), \tag{2}$$

where $Y \in R^n$ is the state and $F(Y) = (F_1(Y), \cdots, F_n(Y))^T$ is a continuous vector function.

Let $e = X - \alpha Y$, where $\alpha = \mathrm{Diag}(\alpha_1, \cdots, \alpha_n)$, and the error system is shown as follows:

$$\dot{e} = F(X) - \alpha F(X) + B^* U^*, \tag{3}$$

where $e \in R^n$ is the state vector.

Definition 1. Consider the controlled error system (3). If $\lim_{t\longrightarrow\infty}\|e(t)\| = 0$, then the master system (1) and the slave system (2) are called to achieve projection synchronization.

According to the results in [17], a lemma is introduced as follows.

Remark 1. The projection synchronization of system (1) is achieved if and only it is divided into the following two subsystems:

$$\dot{W}_m = A(Z)W_m, \tag{4}$$

$$\dot{Z} = H(Z, W_m), \tag{5}$$

where $W_m \in R^s$, $Z \in R^{n-s}$, $s \geq 1$, $A(Z) \in R^{s\times s}$ is a matrix with constants and variable Z, and $H(Z, W_m)$ is nonlinear continuous function.

An algorithm was also proposed in [17], by which we can solve the solutions of the projection synchronization and choose the variables W_m and Z.

2.1. Linear Feedback Control-Like Method for Chaos Projection Synchronization. Note that the subsystem $\dot{W}_m = A(Z)W_m$ is a linear system with respect to variable W_m if the variable Z is considered a constant. Thus, the linear feedback control method is very suitable to be adopted to solve the projective synchronization problem of a given nominal complex chaotic system (i.e., there is no both uncertainty and disturbance). We briefly introduce the linear control method next.

Lemma 1. *Consider the following controlled system:*

$$\dot{W}_m = A(Z)W_m + B_1 U, \tag{6}$$

where W_m, Z, $A(Z)$ are given in equations (4) and (5) and $B_1 \in R^{s\times r}$; then, the linear feedback controller is designed as follows:

$$U = K(Z)W_m, \tag{7}$$

where $K(Z)$ satisfies the matrix $(A(Z) + B_1 K(Z))$ which is Hurwitz no matter what Z is.

2.2. UDE-Based Control Method. It is well known that model uncertainty and external disturbance are inevitable in actual control problem, and the UDE-based control method [32] is an effective tool to deal with that problem.

Consider the following system:

$$\dot{x} = f(x) + u_d + bu, \tag{8}$$

where $x \in R^n$ is the state, $u_d = \Delta f(x) + d(t)$ is the whole of model uncertainty and external disturbance, $b \in R^{n\times k}$ is a constant matrix, $k \geq 1$, and $u \in R^k$ is the controller to be designed.

The stable linear reference model is given as

$$\dot{x}_m = A_m x_m + B_m C, \tag{9}$$

where $x_m \in R^n$ is the reference state, $A_m \in R^{n\times n}$ is the Hurwitz matrix, $B_m \in R^{n\times k}$, and $C \in R^{k\times 1}$ is a piecewise continuous and uniformly bounded command to the system.

Lemma 2 (see [32]). *Consider system (8). If the designed filter $g_f(t)$ satisfies the following condition:*

$$\tilde{u}_d = \hat{u}_d - u_d \longrightarrow 0, \tag{10}$$

where $\hat{u}_d = (\dot{x} - f(x) - bu) * g_f(t)$, then the UDE-based controller u is designed as

$$u = b^+\left\{-f(x) + \ell^{-1}\left[\frac{1}{1 - G_f(s)}\right] * (A_m x + B_m C - Ke)\right\} - b^+\left\{\ell^{-1}\left[\frac{sG_f(s)}{1 - G_f(s)}\right] * x(t)\right\}, \tag{11}$$

where ℓ^{-1} denotes the inverse Laplace transform operator, $b^+ = (b^T b)^{-1} b^T$, $*$ is the convolution operator, and $G_f(s) = \ell[g_f(t)]$.

Remark 2. According to the existing result in [32], the following two filters are often used. One is the first-order low-pass filter:

$$G_f(s) = \frac{1}{\tau s + 1}. \tag{12}$$

The other is the secondary filter:

$$G_f(s) = \frac{as + c - w_0^2}{s^2 + as + c}, \tag{13}$$

where $w_0 = 4\pi$, $a = 10w_0$, and $c = 100w_0$.

3. Main Results

In this section, the UDE-based linear feedback control method is proposed in two steps. In the first step, the linear feedback control method is proposed for the nominal system. The UDE-based control method is given in the second step.

3.1. Linear Feedback Control Method for Projection Synchronization. Consider the following nominal system:

$$\dot{X} = F(X) + BU, \tag{14}$$

where $X \in R^n$ is the state vector, $F(X) = (F_1(X), \cdots, F_n(X))^T$ is a continuous function, $B \in R^{n\times r}, r \geq 1$, $U = (U_1, \cdots, U_r)^T$ is the linear feedback controller to be designed, and $(F(X), B)$ is assumed to be controllable.

If the projection synchronization of system (14) exists, then it can be divided into the following two subsystems:

$$\dot{W}_m = A(Z)W_m + B_1U, \tag{15}$$

$$\dot{Z} = H(Z, W_m), \tag{16}$$

where W_m, Z, $A(Z)$, $H(Z, W_m)$ are given in equations (4) and (5), respectively, $B_1 \in R^{s\times r}$ is given in equation (6), and $(A(Z), B_1)$ is also controllable.

The corresponding slave system is presented as follows:

$$\dot{W}_s = A(Z)W_s, \tag{17}$$

where $H(Z, W_m)$ is given in equation (15), $W_m \in R^r$, $Z \in R^{n-r}$, and $A(Z)$ is a constant matrix.

Let $e = W_m - \beta W_S$ be the error state, where the scalar $|\beta| \neq 0, 1$, and the error system is obtained as follows:

$$\dot{e} = A(Z)e + B_1U. \tag{18}$$

Theorem 1 Consider error system (18). *If $(A(Z), B_1)$ is controllable no matter what Z is, then the linear feedback controller U is designed as follows:*

$$U = K(Z)e, \tag{19}$$

where $K(Z)$ satisfies the matrix $(A(Z) + B_1K(Z))$ which is Hurwitz no matter what Z is; then, error system (18) is globally asymptotically stable. That is, the master system (15) and the slave system (17) achieve the projection synchronization.

Proof. Since the matrix $(A(Z) + B_1K(Z))$ is Hurwitz no matter what Z is, error system (18) is globally asymptotically stable; therefore, the master system (15) and the slave system (17) achieve the projection synchronization. □

3.2. UDE-Based Control Method for Projection Synchronization. In this section, the UDE controller is proposed to cancel the uncertainty and disturbance of the complex chaotic system.

Consider the following controlled master system:

$$\dot{W}_m = A(Z)W_m + B_1V + U_d, \tag{20}$$

where W_m, Z, $A(Z)$, $H(Z, W_m)$ are given in equations (4) and (5), respectively, $B_1 \in R^{s\times r}$ is given in equation (6), $(A(Z), B_1)$ is controllable, $U_d = \Delta A(Z) + D(t)$, $\Delta A(Z)$ represents the uncertainty and $D(t)$ represents the disturbance, and V is the controller to be designed, in which

$$V = U + u_{ude}. \tag{21}$$

The corresponding salve system is

$$\dot{W}_s = A(Z)W_s. \tag{22}$$

Let $e = W_m - \beta W_s$ be the error state vector, where $|\beta| \neq 0, 1$; then, the corresponding error system is shown as follows:

$$\dot{e} = A(Z)e + u_d + B_1V. \tag{23}$$

The controller V is designed in two steps:

Step one: according to Theorem 1, the linear feedback controller U is designed for the nominal system.

Step two: the controller u_{ude} is proposed according to the following theorem.

Theorem 2 Consider error system (23). *If the designed filter $g_f(t)$ satisfies the following condition:*

$$\tilde{u}_d = \hat{u}_d - u_d \longrightarrow 0, \tag{24}$$

where $\hat{u}_d = (\dot{e} - A(Z, e) - B_1u_{ude}) * g_f(t)$, then the UDE-based controller u is designed as

$$u_{ude} = B_1^+\left\{\ell^{-1}\left[\frac{G_f}{1 - G_f(s)}\right] * A(Z, e) - \ell^{-1}\left[\frac{sG_f(s)}{1 - G_f(s)}\right] * e(t)\right\}, \tag{25}$$

where $A(Z, e) = (A(Z) + B_1K(Z))e$, $B_1^+ = (B_1^TB)^{-1}B_1^T$, ℓ^{-1} is the inverse Laplace transform, $*$ is the convolution sign, and $G_f(s) = \ell[g_f(t)]$.

Proof. Substituting V in (21) into system (23) results in

$$\dot{e} = A(Z)e + u_d + B_1V = \big(A(Z) + B_1K(Z)\big)e + Bu_{ude} - u_d. \tag{26}$$

According to condition (24), it leads to

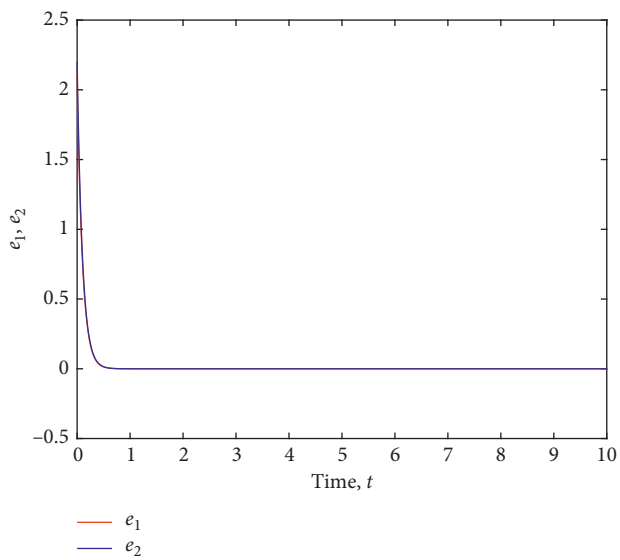


Figure 1: e_1, e_2 are asymptotically stable.

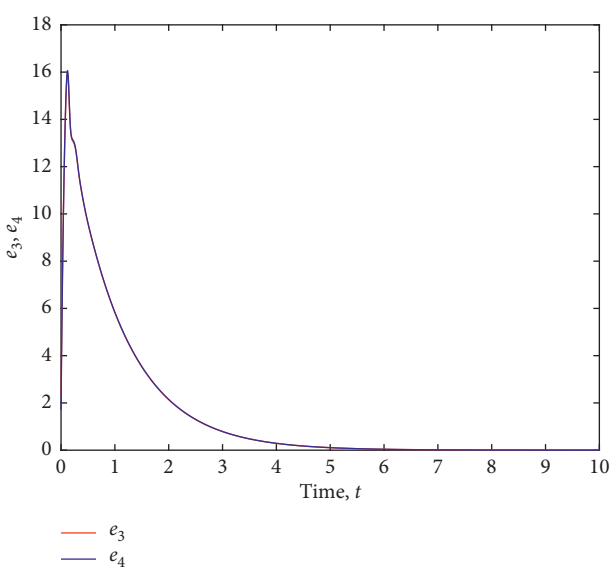


Figure 2: e_3, e_4 are asymptotically stable.

$$B_1 u_{ude} = -\widehat{u}_d. \tag{27}$$

Thus,

$$\dot{e} = A(z) + BV + u_d = A(Z, e) + \widetilde{u}_d \tag{28}$$

is globally asymptotically stable, which completes the proof. □

4. Illustrative Example with Numerical Simulation

In this section, one example with numerical simulations is used to demonstrate the effectiveness and validity of the proposed results.

Consider the following complex Lorenz system:

$$\begin{aligned} \dot{x}_1 &= 10(x_1 - x_2), \\ \dot{x}_2 &= 110x_1 - x_1x_3 - x_2, \\ \dot{x}_3 &= -2x_3 + \frac{1}{2}(\overline{x}_1 x_2 + x_1 \overline{x}_2), \end{aligned} \tag{29}$$

where $x_1 = x_1^r + jx_1^i, x_2 = x_2^r + jx_2^i$ are complex variables, x_3 is a real variable, $j^2 = -1$ represents imaginary unit, and $\overline{x}_1$ and $\overline{x}_2$ are complex conjugate variables of x_1, x_2, respectively.

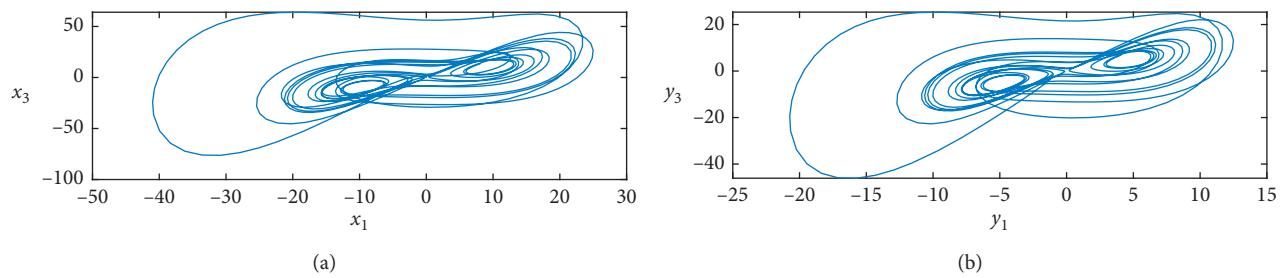
Separating the real and imaginary parts of complex variables x_1, x_2 in system (29), i.e., setting $X_1 = x_1^r, X_2 = x_1^i, X_3 = x_2^r, X_4 = x_2^i$, and $X_5 = x_3$ representing x_5, a new real-variable system is shown as follows:

$$\begin{aligned} \dot{X}_1 &= 10(X_3 - X_1), \\ \dot{X}_2 &= 10(X_4 - X_2), \\ \dot{X}_3 &= 110X_1 - X_1X_5 - X_3, \\ \dot{X}_4 &= 110X_2 - X_2X_5 - X_4, \\ \dot{X}_5 &= -2X_5 + X_1X_3 + X_2X_4. \end{aligned} \tag{30}$$

4.1. The Existence of Projection Synchronization of the Complex Lorenz System. According to the results in [17], for system (30), the results are obtained as follows:

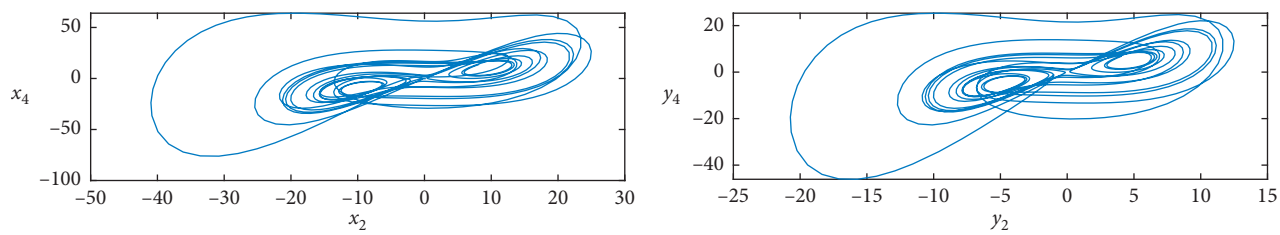
$$\begin{aligned} &F_1(\alpha X) - \alpha_1 F_1(\alpha X) = 10(\alpha_3 - \alpha_1)X_3 \equiv 0, \\ &F_2(\alpha X) - \alpha_2 F_2(\alpha X) = 10(\alpha_4 - \alpha_2)X_4 \equiv 0, \\ &F_3(\alpha X) - \alpha_3 F_3(\alpha X) = (\alpha_1 - \alpha_1\alpha_5)X_1 - (\alpha_3 - \alpha_1\alpha_5)X_3 \equiv 0, \\ &F_4(\alpha X) - \alpha_4 F_4(\alpha X) = (\alpha_2 - \alpha_2\alpha_5)X_2 - (\alpha_4 - \alpha_2\alpha_5)X_4 \equiv 0, \\ &F_5(\alpha X) - \alpha_5 F_5(\alpha X) = (\alpha_5 - \alpha_1\alpha_3)X_5 - (\alpha_1\alpha_3 - \alpha_2\alpha_4)X_1X_3 \equiv 0. \end{aligned} \tag{31}$$

It results in



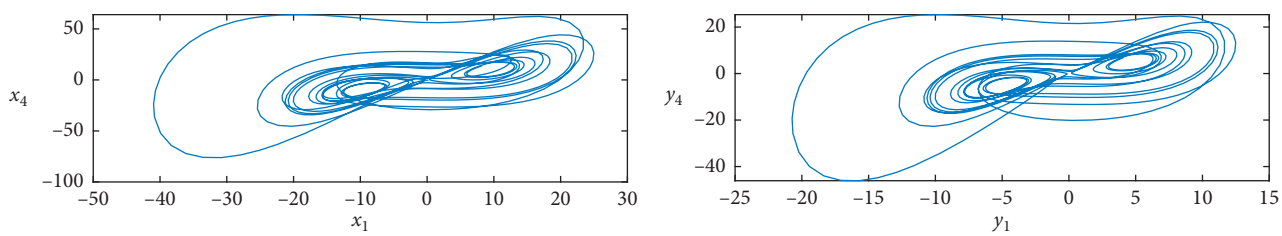
(a) (b)

Figure 3: The phase portrait of master subsystem and the slave subsystem.



(a) (b)

Figure 4: The phase portrait of master subsystem and the slave subsystem.



(a) (b)

Figure 5: The phase portrait of master subsystem and the slave subsystem.

$$\alpha_1 = \alpha_3, \tag{32}$$

$$\alpha_2 = \alpha_4, \tag{33}$$

$$\begin{aligned} \alpha_1 &= \alpha_1\alpha_5, \\ \alpha_3 &= \alpha_1\alpha_5, \end{aligned} \tag{34}$$

$$\begin{aligned} \alpha_2 &= \alpha_2\alpha_5, \\ \alpha_4 &= \alpha_2\alpha_5, \end{aligned} \tag{35}$$

$$\begin{aligned} \alpha_5 &= \alpha_1\alpha_3, \\ \alpha_1\alpha_3 &= \alpha_2\alpha_4. \end{aligned} \tag{36}$$

It is easy to obtain that $\alpha = \mathrm{Diag}(\beta, \beta, \beta, \beta, 1)$ is the one solution of equations (32)–(35), where $|\beta| \neq 1$ is a nonzero scalar.

Thus, the master system (30) is divided into the following two subsystems:

$$\dot{W}_m = A(Z)W_m, \tag{37}$$

$$\dot{Z} = H(Z, W_m), \tag{38}$$

where

$$\begin{aligned} W_m &= \begin{pmatrix} X_1 \\ \vdots \\ X_4 \end{pmatrix}, \\ Z &= X_5, \\ A(Z) &= \begin{pmatrix} -10 & 0 & 10 & 0 \\ 0 & -10 & 0 & 10 \\ 110 - Z & 0 & -1 & 0 \\ 0 & 110 - Z & 0 & -1 \end{pmatrix}, \end{aligned} \tag{39}$$

$$H(Z, W_m) = -2Z + W_{m1}W_{m3} + W_{m2}W_{m4}. \tag{40}$$

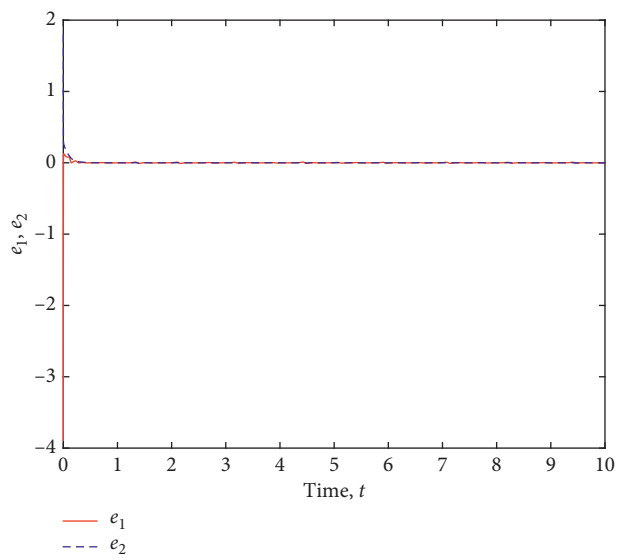


FIGURE 6: e_1, e_2 are asymptotically stable.

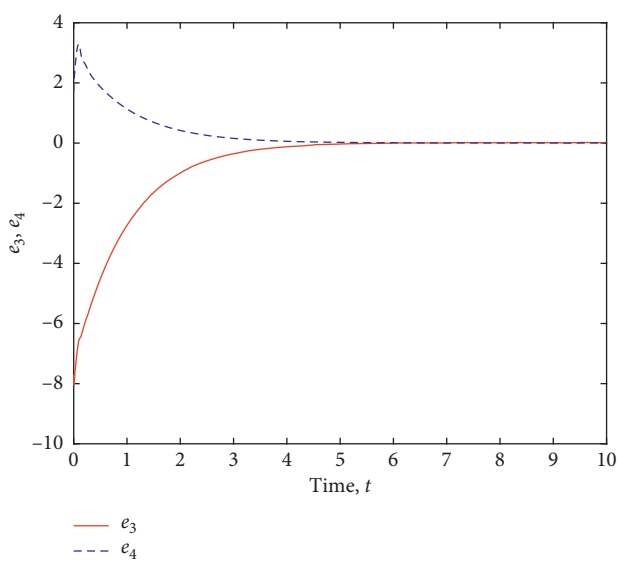


FIGURE 7: e_3, e_4 are asymptotically stable.

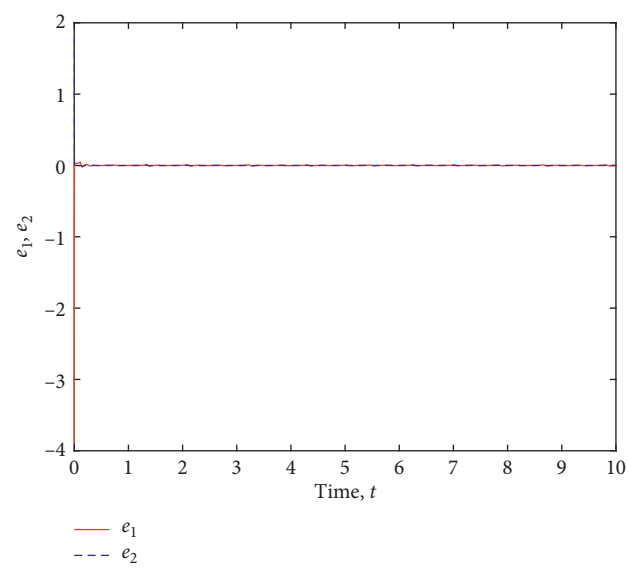


FIGURE 8: The phase portrait of master subsystem and the slave subsystem.

4.2. The UDE-Based Linear Feedback Controller Design. The UDE-based linear feedback controller is designed by the following two steps.

Step one:

$$\dot{W}_m = A(Z)W_m + B_1U. \tag{41}$$

W_m, Z, $A(Z)$ are given in equations (38) and (39), respectively, and

$$B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{42}$$

Then, the corresponding slave system is given as follows:

$$\dot{W}_s = A(Z)W_s. \tag{43}$$

$A(Z)$, Z are given in equations (38) and (39), respectively.

Let $e = W_m - \beta W_s$, where $\beta = 2$, and the uncontrolled error system is given as follows:

$$\begin{aligned} \dot{e}_1 &= 10(e_3 - e_1), \\ \dot{e}_2 &= 10(e_4 - e_2), \\ \dot{e}_3 &= (110 - X_5)e_1 - e_3, \\ \dot{e}_4 &= (110 - X_5)e_2 - e_4. \end{aligned} \tag{44}$$

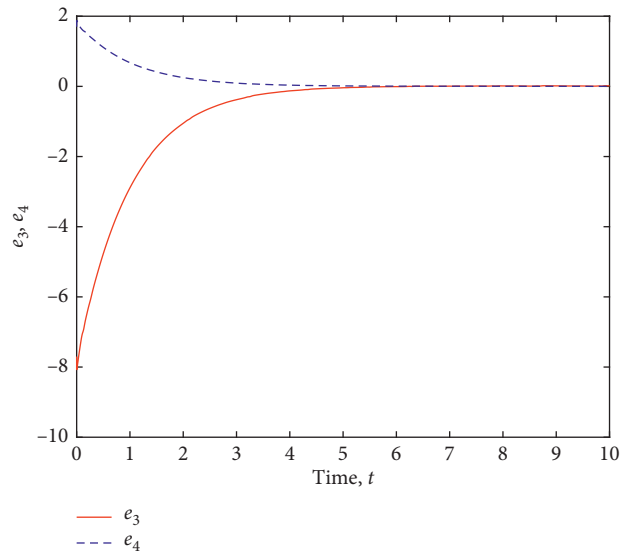
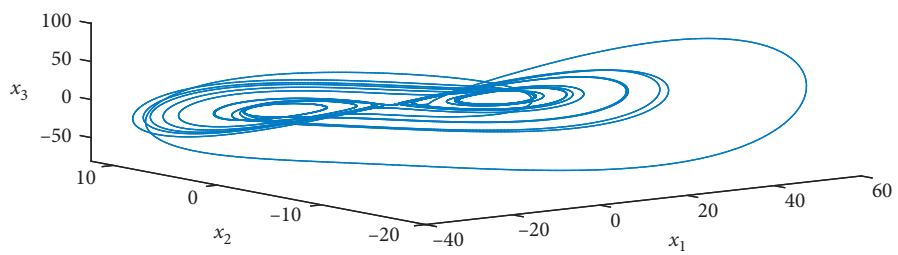
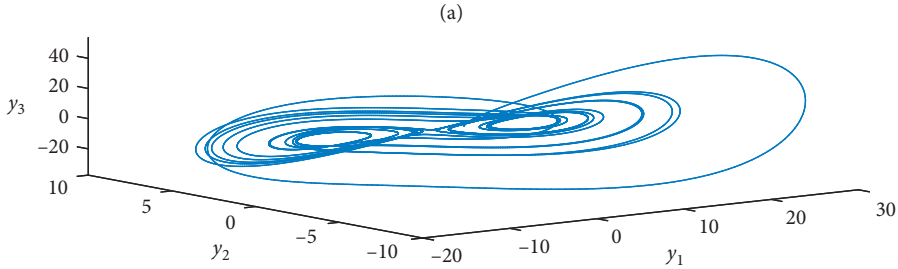


Figure 9: The phase portrait of master subsystem and the slave subsystem.

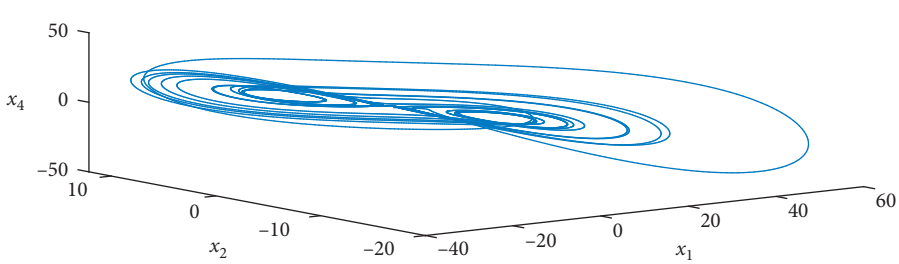


(a)



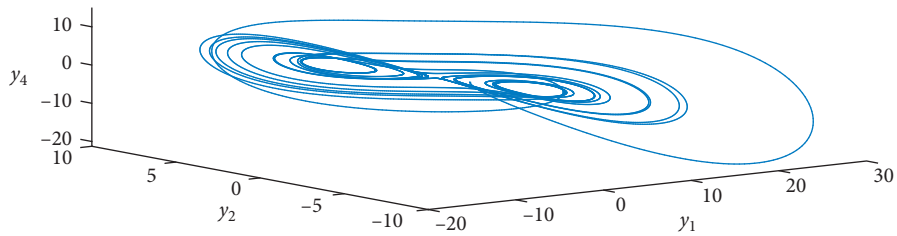
(b)

Figure 10: The phase portrait of master subsystem and the slave subsystem.



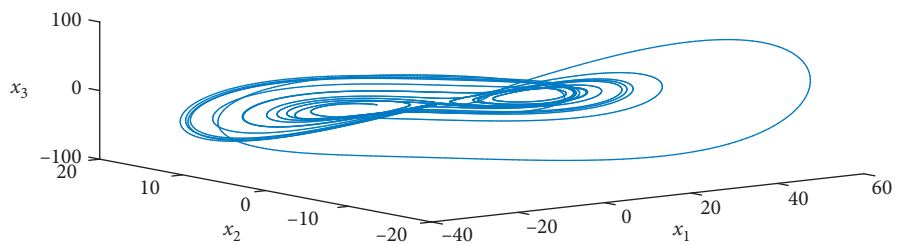
(a)

Figure 11: Continued.

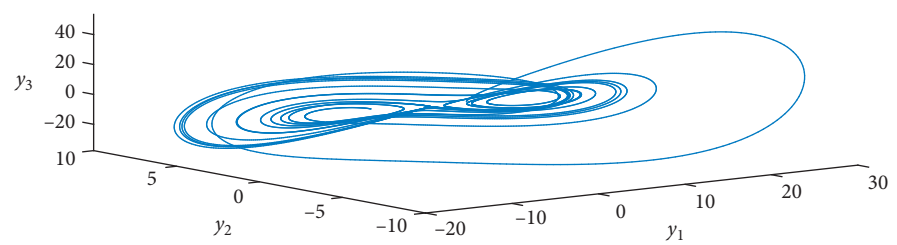


(b)

Figure 11: The phase portrait of master subsystem and the slave subsystem.

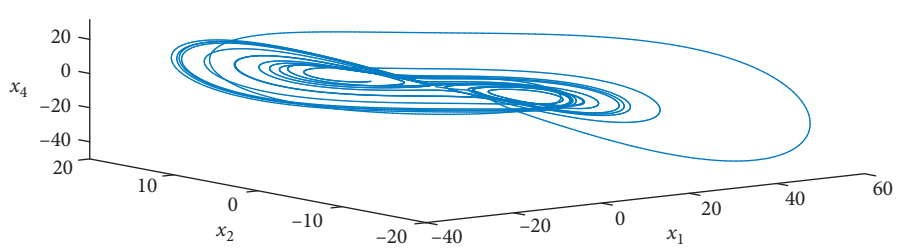


(a)

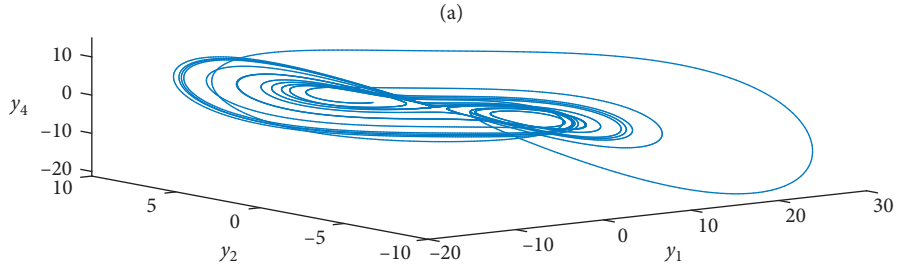


(b)

Figure 12: $\hat{u}_{d1}$ tends to u_{d1}.



(a)



(b)

Figure 13: $\hat{u}_{d2}$ tends to u_{d2}.

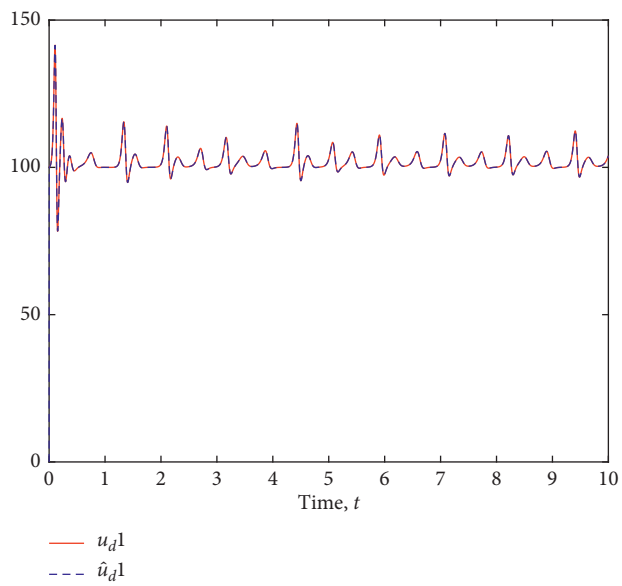


FIGURE 14: e_1, e_2 are asymptotically stable.

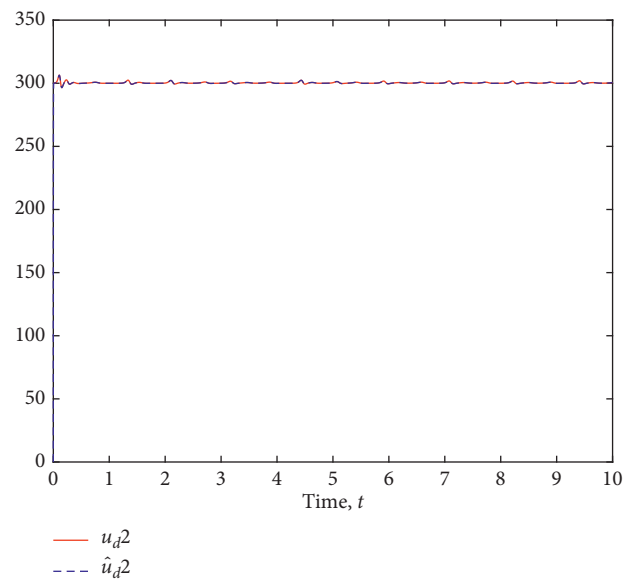


FIGURE 15: e_3, e_4 are asymptotically stable.

Note that if $e_1 = 0$ and $e_2 = 0$, the following system:

$$\begin{aligned} \dot{e}_3 &= -e_3, \\ \dot{e}_4 &= -e_4, \end{aligned} \tag{45}$$

is globally asymptotically stable.

Thus, $(A(Z), B_1)$ is controllable. According to Theorem 1, the linear feedback controller U is obtained as follows:

$$U = K(Z)e = \begin{pmatrix} -10e_3 \\ -10e_4 \end{pmatrix}. \tag{46}$$

Numerical simulation is given, and the initial values of the master-slave systems of given complex Lorenz system are chosen as follows: $X_1(0) = 0.1, X_2(0) = 0.2, X_3(0) = 0.3, X_4(0) = -0.3, X_5(0) = 1.1, Y_1(0) = -1, Y_2(0) = -1, Y_3(0) = -1, Y_4(0) = -1$.

From Figures 1 and 2, we observed that under linear feedback control, the error system between the master system and slave system is globally asymptotically stable. Through the observation of Figures 3–5, it is found that the master system and slave system achieve the projection synchronization. That is, the controlled master system and slave system have the same phase portrait, but the axis is different.

Step two: consider the following master system with both model uncertainty and external disturbance:

$$\dot{W}_m = A(Z)W_m + B_1V + U_d, \tag{47}$$

where $A(Z)$ is given in equation (39), $H(Z, W_m)$ is presented in equation (40), B_1 is given in equation (42), and U_d is the whole of model uncertainty and external disturbance.

The salve system is

$$\dot{W}_s = A(Z)W_s, \tag{48}$$

where $A(Z)$ is given in equation (39).

Let $e = W_m - \beta W_s$; then, the error system is shown as follows:

$$\dot{e} = A(Z)e + U_d + B_1V, \tag{49}$$

where

$$V = U + u_{ude}, \tag{50}$$

where U is given in equation (46).

According to Theorem 2, the UDE-based controller u_{ude} is designed as follows:

$$u_{ude} = \begin{pmatrix} u_{ude1} \\ u_{ude2} \end{pmatrix} = \begin{pmatrix} \ell^{-1}\left\{\left[\dfrac{G_f}{1-G_f(s)}\right] * (-10e_1) - \left[\dfrac{sG_f(s)}{1-G_f(s)}\right] * e_1\right\} \\ \ell^{-1}\left\{\left[\dfrac{G_f}{1-G_f(s)}\right] * (-10e_2) - \left[\dfrac{sG_f(s)}{1-G_f(s)}\right] * e_2\right\} \end{pmatrix}, \tag{51}$$

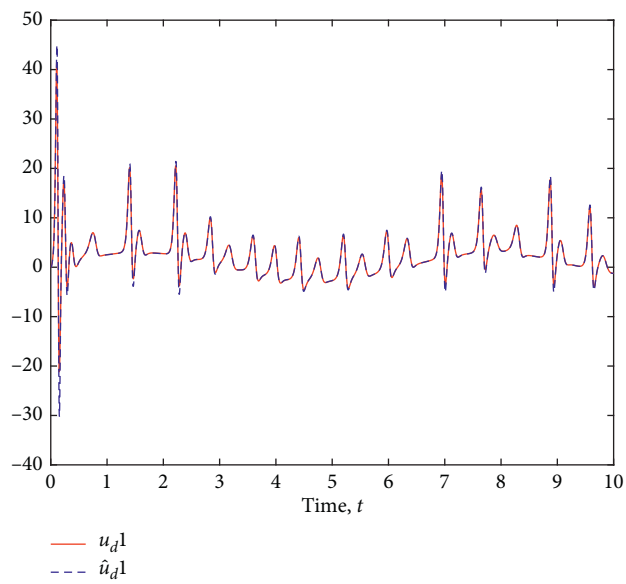


FIGURE 16: $\hat{u}_{d1}$ tends to u_{d1}.

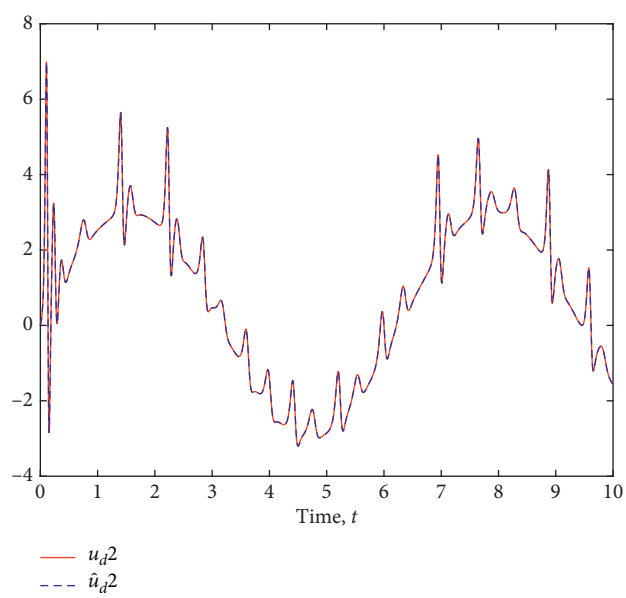


FIGURE 17: $\hat{u}_{d2}$ tends to u_{d2}.

where ℓ^{-1} is the inverse Laplace transform, $*$ is the convolution sign, $G_f(s) = \ell[g_f(t)]$, and the design of the filter $g_f(t)$ is given in Lemma 2.

Numerical simulation results are given with the following conditions: $X_1(0) = 0.1, X_2(0) = 0.2, X_3(0) = 0.3, X_4(0) = -0.3, X_5(0) = 1.1, \quad Y_1(0) = -1, Y_2(0) = -1, \quad Y_3(0) = -1, Y_4(0) = -1, \beta = 2$.

Case 1:

$$U_d = \begin{pmatrix} 0.1X_1X_2 + 100 \\ 0.2X_3X_4 + 400 \end{pmatrix}. \tag{52}$$

Case 2:

$$U_d = \begin{pmatrix} 0.1X_1X_2 + 0.1\sin(t) \\ 0.2X_3X_4 + 0.3\sin(t) \end{pmatrix}. \tag{53}$$

It can be seen from Figures 6–9 that the error system is asymptotically stable. Through the observation of Figures 10–13, it is found that the master system and slave system achieve the projection synchronization. That is, the controlled master system and slave system have the same phase portrait, but the axis is different. Figure 14 shows that $\hat{u}_{d1}$ tends to u_{d1}, and Figure 15 shows that $\hat{u}_{d2}$ tends to u_{d2}. Similarly, we found that $\hat{u}_{d1}$ tends to u_{d1} and $\hat{u}_{d2}$ tends to u_{d2} from Figures 16 and 17.

5. Conclusion

In conclusion, the projective synchronization of a class of complex chaotic systems with both uncertainty and disturbance has been solved. First, the linear feedback control method is proposed for the nominal system (without uncertainty and disturbance), and projection synchronization of such system has been realized. Then, the UDE-based linear feedback control method is presented by two steps, by which the projection synchronization of the complex chaotic systems with both uncertainty and disturbance has been completed. Finally, an experimental simulation example has been used to verify the feasibility and effectiveness of the obtained results.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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