Research Article

Model-Free Adaptive Control of pH Value of Wet Desulfurization Slurry under Switching of Multiple Working Conditions

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In limestone-gypsum wet flue gas desulfurization process, the change process of pH value of slurry in absorption tower is a typical nonlinear system with time delay and various uncertainties, so it is difficult to establish an accurate mathematical model of slurry pH control process. According to the pH control process of the slurry of wet flue gas desulfurization process, a model-free adaptive control algorithm based on compact form dynamic linearization (CFDL-MFAC) is designed to realize the tracking control of the pH value of the slurry. Due to various interference factors in the pH control process of slurry in absorption tower, it is easy to cause jump change of control system parameters and even structure. Therefore, a model-free adaptive control algorithm based on switching strategy is proposed in this paper. According to different working conditions, several model-free adaptive controllers are established. The stability of the algorithm is analyzed for the two cases of fixed system parameters and jumping system parameters. It was found that the model-free adaptive controller based on the switching strategy can switch multiple controllers under the condition of system parameter jump, so as to realize the fast tracking control of the slurry pH value of the system absorption tower under different working conditions. Through this method, the overshoot can be reduced and the control quality can be improved.

1. Introduction

Sulfur dioxide is one of the main air pollutants. The pollution caused by sulfur dioxide not only has a great impact on the economy, but also has a great threat to human health. In recent years, the emission of sulfur dioxide in China has been high. More than 55% of sulfur dioxide comes from coal combustion in coal-fired power plants. China’s growing demand for electricity leads to the continuous growth of coal consumption, which leads to the increase of sulfur dioxide emissions from coal combustion. Therefore, the reduction of sulfur dioxide emission from coal-fired power plants will improve the atmospheric environment quality. At present, limestone-gypsum wet flue gas desulfurization (WFGD) process [1, 2] is most widely used in coal-fired power plant desulfurization system. In this process, the pH value of absorber slurry is a key factor that directly affects the absorption efficiency of sulfur dioxide in flue gas and the quality of final product (gypsum) [3, 4]. The operation experience of most desulfurization projects shows that, under the condition that other parameters are basically stable, increasing the pH value can improve the desulfurization efficiency to a great extent, but if the high pH value is maintained for a long time, the quality of gypsum will be reduced. On the other hand, low pH value will inhibit the absorption of sulfur dioxide. Therefore, it is of great significance to accurately control the pH value of absorption tower slurry in WFGD process.
At present, the commonly used methods for controlling the slurry pH in coal-fired power plants are manual control and PID control. The manual control requires the power plant staff to have good working experience, and the control requirements of the slurry valve opening are high, so large pH fluctuation is easy to be caused in the control process. Although PID control has improved the automation level of slurry pH control to a certain extent, for the complex control object such as the slurry pH of absorption tower, when PID control is used, the controller parameters are difficult to adjust, and the adaptive ability is poor. In recent years, many experts and scholars use advanced control methods to study the pH control of absorption tower slurry in WFGD process. In [5], the internal model control algorithm was introduced into the pH control by analyzing the control process of the slurry pH value in the absorption tower in the current WFGD process, which improved the control accuracy of the slurry pH value and the real-time strain capacity. In [6], aiming at the defects of the existing single loop control system for slurry pH value in the absorption tower and the composite control system commonly used in current desulfurization projects, a cascade-given mole difference control scheme was proposed. The parameters of the controller were optimized by particle swarm optimization algorithm. Finally, good control effect is obtained. The change process of pH value of slurry in absorption tower is complex, and it is difficult to achieve ideal control effect by using traditional PID control. In the process of field adjustment, the control parameters are often adjusted manually. In order to overcome these shortcomings, a RBF-PID controller was designed in [7] to adjust the three parameters of PID controller online, which solved the problem that the pH value control accuracy of absorption tower slurry was not high due to the difficulty in parameter setting and poor adaptive ability of traditional PID controller. After analyzing the process of slurry pH control in absorption tower, an internal model control based on improved RBF neural network is proposed to control the pH value in [8]. The simulation results show that the scheme has good control performance and adaptive ability for slurry pH control in desulfurization system. The traditional method takes the pH value of slurry as the feedback signal to adjust the slurry feeding, but the dynamic characteristics of the slurry cannot be obtained, so the control effect is not good. In [9], a control method for pH value of desulfurization slurry in coal-fired power plant based on fuzzy algorithm is proposed. According to the principle of fuzzy controller, this scheme combines fuzzy controller with PI controller to realize high precision control of PH value of desulfurization slurry. In [10], in order to realize accurate control of pH value of slurry in absorption tower during desulfurization process and solve the problems of slow speed, low precision, and easiness to fall into local minimum of BP neural network PID control in wet flue gas desulfurization system, chaos simulated annealing particle swarm optimization was used to optimize the weight and threshold value of neural network, and good control effect was achieved. In [11], a configuration predictive control method was proposed by algorithm simplification and function fitting. Based on the algorithm, the slurry pH control system was designed through DCS configuration. The control effect is obviously better than the current mainstream cascade PID control method, and the adjustment accuracy of pH value is significantly improved. Although some achievements have been made in the study of pH value control of absorption tower slurry in WFGD process, the change process of slurry pH value in absorption tower has the characteristics of nonlinear, time-varying, and large lag, so it is impossible to establish its accurate mathematical model. In the process of pH control of absorber slurry, a lot of process data will be generated every moment. For such a system, model-free adaptive control (MFAC) [12–15] is an effective control scheme. In this method, the concept of pseudo-partial-derivative (PPD) is introduced, and the nonlinear system is linearized by using the input and output data of the controlled system. Then a weighted one-step forward controller is designed to realize the data-driven model-free adaptive control of the nonlinear system. The whole process does not need to establish the process model of the system. Model-free adaptive control has the advantages of small amount of calculation, simple controller structure, and easy implementation in practical systems. It has been widely used in the field of automatic control such as chemical process, spacecraft, and multiagent control. In this paper, the model-free adaptive control scheme is applied to the process control of the slurry pH value of the absorption tower, and a CFDL-MFAC controller is designed for the slurry pH control process. In the process of slurry pH value control, there are many disturbances that can lead to the jump change of control system parameters and even structure. The common factors that may cause the parameter jump of pH value control system include the coal combustion degree in the boiler, the change of the sulfur content of the coal, and the change of the generator unit load. In order to solve this problem, a model-free adaptive control algorithm based on switching strategy is proposed in this paper. The simulation results show that the model-free adaptive controller based on the switching strategy can switch multiple controllers according to different working conditions, effectively reduce the overshoot, and quickly realize the tracking control of the slurry pH value of the absorption tower.

The paper is organized as follows. In Section 2, the WFGD process and Hammerstein model of slurry pH control system are formulated. In Section 3, The CFDLMFAC controller and the CFDL-MFAC controller based on switching strategy are established, respectively, and the convergence of the algorithm is analyzed. In Section 4, the effectiveness of the model-free adaptive control algorithm based on switching strategy is verified by MATLAB simulation. Finally, conclusions are given in Section 5.

2. Analysis of pH Value Control in WFGD Process

2.1. WFGD Process. As the most widely used desulfurization technology, WFGD process flow is shown in Figure 1. The WFGD system is mainly composed of limestone slurry preparation system, flue gas heat exchange system, absorber...
desulfurization system, gypsum dehydration system, wastewater treatment system, etc. Firstly, limestone and water are mixed and stirred in limestone slurry preparation system to prepare desulfurization slurry. The prepared slurry enters the bottom of absorption tower through valves and pipes. The raw flue gas discharged from the boiler comes to the heat exchanger through the induced draft fan and booster fan to cool down, so as to protect various facilities in the tower. The raw flue gas after cooling treatment enters the absorption tower and flows from the bottom of the tower to the top of the tower. At the same time, the desulfurization slurry is sprayed out from the spray layer at the top of the absorption tower under the action of the circulating pump. The raw flue gas and the droplets of the desulfurization slurry fully contact to cause a chemical reaction, and calcium sulfite is formed after the reaction. During the entire reaction process, the oxidation fan continuously blows air into the absorption tower, and calcium sulfite is oxidized into calcium sulfate by the oxygen in the air. Calcium sulfate becomes wet gypsum after crystallization, and the wet gypsum is dehydrated to become solid gypsum used in building materials. The desulfurized flue gas first passes through the demister at the top of the absorption tower to remove moisture. Then the dried flue gas is heated to 85°C through a heat exchanger and discharged into the atmosphere through the flue and chimney.

2.2. Hammerstein Model of Slurry pH Control System. The pH value of the slurry in the absorption tower is a key factor affecting not only the desulfurization efficiency, but also the limestone utilization rate and the purity of gypsum. The pH value of the slurry is usually maintained between 5.0 and 6.0. In the desulfurization process, the control of the pH value is realized by controlling the flow of the limestone slurry, and the size of limestone slurry flow is controlled by the opening of the slurry valve, so the control of the pH value of the slurry can be regarded as the control of limestone slurry valve opening. In this paper, the slurry pH value control process takes limestone slurry valve opening as control input and slurry pH value as system output.

Hammerstein model [21] of slurry pH control system is as follows:

\[
x(k) = f[u(k)] = u(k) + y_1u^2(k) + y_2u^3(k),
\]

\[
G(z) = \frac{y(k)}{x(k)} = \frac{a_1z^{-3} + a_2z^{-4} + a_3z^{-5}}{1 + b_1z^{-1} + b_2z^{-2}},
\]

where \(y(k)\) is the system output, and it denotes the pH value of absorption tower slurry; \(u(k)\) is the system input, and it denotes the opening of limestone slurry valve; and \(x(k)\) is the output of the nonlinear part, and it is also the input of the linear system. As an intermediate variable, \(x(k)\) cannot be measured. Equation (1), which is a typical Hammerstein model, contains a nonlinear gain polynomial and a discrete transfer function.

3. Controller Design and Algorithm

3.1. Design of CFDL-MAFC Controller. It is assumed that the discrete nonlinear input-output system of the slurry pH control process is as follows:

\[
y(k + 1) = f(y(k), \ldots, y(k - n_y), u(k), \ldots, u(k - n_u)),
\]

where \(y(k) \in R\) is the output of the control system at \(k\) time; \(u(k) \in R\) is the input of the control system at \(k\) time; \(n_y\) is the output order of the system; and \(n_u\) is the input order of the system.

Assumption 1. In system (2), the partial derivative of \(f(\cdots)\) with respect to \(u(k)\) is continuous.
Assumption 2. System (2) satisfies the generalized Lipschitz condition; that is, for any time \( k_1 \neq k_2, k_1, k_2 \geq 0 \) and \( u(k_1) \neq u(k_2) \), the following inequalities can be obtained:

\[
|y(k_1 + 1) - y(k_2 + 1)| \leq b|u(k_1) - u(k_2)|,
\]

where \( y(k_1 + 1) = f(y(k_1), \ldots, y(k_1 - n_y), u(k_1), \ldots, u(k_1 - n_y)), \quad i = 1, 2; b > 0 \) is a constant.

From the practical point of view, the above assumption of control object is reasonable and acceptable. Assumption 1 is a typical constraint condition for general nonlinear systems in control system design. Assumption 2 is a restriction on the upper bound of the change rate of system input and output. From the energy point of view, the change of bounded slurry valve opening can only cause the change of bounded slurry pH in the system.

**Theorem 1.** For the nonlinear system (2) that satisfies Assumption 1 and Assumption 2, when \( |\Delta u(k)| \neq 0 \), there must be a partial derivative (PPD) \( \varphi_c(k) \in R \), so that system (2) can be transformed into the following compact format dynamic linearization (CFDL) data model:

\[
\Delta y(k + 1) = \varphi_c(k) \Delta u(k),
\]

where \( \Delta y(k + 1) = u(k + 1) - u(k) \) is the input change of two adjacent moments; \( \Delta y(k + 1) = y(k + 1) - y(k) \) is the output change of two adjacent moments; and \( \varphi_c(k) \) is a time-varying parameter and bounded to any time \( k \).

According to Theorem 1, when the nonlinear system (2) satisfies Assumption 1 and Assumption 2, and it satisfies the condition \( \Delta u(k) \neq 0 \) for all \( k \) time, its CFDL data model can be expressed as

\[
y(k + 1) = y(k) + \varphi_c(k) \Delta u(k).
\]

If the control increment of two adjacent moments is too large, the input of the system will jump change sharply, and the burden of the actuator will be increased. Therefore, the following control input criterion functions are considered:

\[
J(u(k)) = |y^*(k + 1) - y(k + 1)|^2 + \lambda |\Delta u(k)|^2,
\]

where \( \lambda > 0 \) is the weight factor, which is used to limit the change of control input, and \( y^*(k + 1) \) is the expected output of the system.

Substituting model (4) into the criterion function (6), finding the partial derivative of \( u(k) \), and making its partial derivative 0, we can get the weighted one-step controller of the system:

\[
u(k) = u(k - 1) + \frac{\rho \varphi_c(k)}{\lambda + \varphi_c(k)} (y^*(k + 1) - y(k)),
\]

where \( \rho \in (0, 1) \) is the step factor.

It can be known from the controller (7) that the implementation of CFDL-MFAC algorithm depends on PPD \( \varphi_c(k) \). \( \varphi_c(k) \) is an unknown time-varying parameter, so it is necessary to use the input and output data of slurry pH control system to estimate it in real time. The following \( \varphi_c(k) \) estimation criterion function is considered:

\[
J(\varphi_c(k)) = |y(k) - y(k - 1) - \varphi_c(k) \Delta u(k - 1)|^2 + \mu |\varphi_c(k) - \bar{\varphi}_c(k - 1)|^2,
\]

where \( \mu > 0 \) is the step factor.

Calculating the extremum of (8), the estimation algorithm of \( \varphi_c(k) \) can be obtained as follows:

\[
\bar{\varphi}_c(k) = \bar{\varphi}_c(k - 1) + \frac{\eta \Delta u(k - 1)}{\mu + \Delta u(k - 1)^2} (\Delta y(k) - \bar{\varphi}_c(k - 1) \Delta u(k - 1)),
\]

where \( \eta \in (0, 1] \) is the step factor and \( \bar{\varphi}_c(k) \) is the estimated value of PPD \( \varphi_c(k) \).

The slurry pH control is a time-varying system. In order to make the estimation algorithm (9) of PPD track the time-varying parameters of the slurry pH control system well, the following PPD reset algorithm is introduced:

If \( |\bar{\varphi}_c(k)| \leq \varepsilon, |\Delta u(k - 1)| \not\leq \varepsilon \), or \( \text{sign}(\bar{\varphi}_c(k)) \neq \text{sign}(\bar{\varphi}_c(1)), \bar{\varphi}_c(k) \) is reset to

\[
|\bar{\varphi}_c(k)| = |\bar{\varphi}_c(1)|,
\]

where \( \varepsilon \) is a sufficiently small positive number.

Equations (7), (9), and (10) constitute the CFDL-MFAC algorithm of slurry pH control system.

3.2. Stability Analysis of CFDL-MFAC Algorithm

**Assumption 3.** For a given bounded expected output signal \( y^*(k + 1) \), there is always a bounded \( u^*(k) \), so that the system is driven by this control input signal and its output is equal to \( y^*(k + 1) \).

**Assumption 4.** For any time \( k \) and \( \Delta u(k) \neq 0 \), the sign of the system PPD remains unchanged; that is, \( \varphi_c(k) > \varepsilon > 0 \) or \( u^*(k) \), where \( \varepsilon \) is a small positive number.

**Theorem 2.** For the nonlinear system (2), under the condition that Assumption 1, Assumption 2, Assumption 3, and Assumption 4 are satisfied, when \( y^*(k + 1) = y^* = \text{const} \) and the CFDL-MFAC scheme is adopted, there is always a positive number \( \lambda_{\text{in}} > 0 \) so that when \( \lambda > \lambda_{\text{in}} \), the following conclusions are obtained:

(1) The output tracking error of the system converges monotonically, and \( \lim_{k \to \infty} |y^* - y(k + 1)| = 0 \) is satisfied

(2) The closed-loop system is bounded-input bounded-output (BIBO) stable; that is, the output sequence \( \{y(k)\} \) and input sequence \( \{u(k)\} \) are bounded

**Proof.** If the condition \( |\bar{\varphi}_c(k)| \leq \varepsilon, |\Delta u(k - 1)| \not\leq \varepsilon \), or \( \text{sign}(\bar{\varphi}_c(k)) \neq \text{sign}(\bar{\varphi}_c(1)) \) is satisfied, \( \bar{\varphi}_c(k) \) is obviously bounded.

In other cases, we define \( \bar{\varphi}_c(k) = \bar{\varphi}_c(k) - \varphi_c(k) \) as PPD estimation error and subtract \( \varphi_c(k) \) from both sides of the parameter estimation algorithm (9) to obtain
Taking absolute values on both sides of (11), we get

$$
|\phi_e(k)| = \left| 1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2} \right| |\phi_e(k)| + |\phi_e(k)| - |\phi_e(k)|.
$$

(12)

It is noted that the function $$(\eta |\Delta u(k-1)|^2)/(\mu + |\Delta u(k-1)|^2)$$ is monotonically increasing with respect to the variable $|\Delta u(k-1)|^2$, and its minimum value is $\eta \epsilon^2/\mu + \epsilon^2$. When $0 < \eta \leq 1$ and $\mu > 0$, there must be $d_1$ satisfying

$$
0 \leq \left| 1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2} \right| \leq \frac{1 - \eta \epsilon^2}{\mu + \epsilon^2} = d_1 < 1.
$$

(13)

According to the conclusion $|\phi_e(k)| \leq \overline{B}$ in Theorem 1, $|\phi_e(k-1) - \phi_e(k)| \leq 2\overline{B}$ can be obtained. Using (12) and (13), we have the following inequality:

$$
|\phi_e(k)| \leq d_1|\phi_e(k-1)| + 2\overline{B} \leq d_1|\phi_e(k-2)| + 2d_1\overline{B} + 2\overline{B}
$$

$$
\leq \cdots \leq d_1^{k-1}|\phi_e(1)| + \frac{2\overline{B}(1 - d_1^{-1})}{1 - d_1}
$$

(14)

Equation (14) means that $\phi_e(k)$ is bounded, and because $\phi_e(k)$ is bounded, $\phi_e(k)$ is bounded.

The system tracking error is defined as

$$
e(k + 1) = y^* - y(k + 1).
$$

(15)

Substituting the CFDL data model (5) into (15) and taking absolute values of both sides of (15), we get

$$|e(k + 1)| = |y^* - y(k + 1)| = |y^* - y(k) - \phi_e(k)\Delta u(k)|
$$

$$
\leq 1 - \frac{\rho \phi_e(k)|\phi_e(k)|}{\lambda + |\phi_e(k)|^2} |e(k)|.
$$

(16)

According to (17), $0 < \rho \leq 1$, and $\lambda > \lambda_{\min}$, there must be a constant $d_2 < 1$ such that the following formula holds:

$$
1 - \frac{\rho \phi_e(k)|\phi_e(k)|}{\lambda + |\phi_e(k)|^2} \leq 1 - \rho M_1 = d_2 < 1.
$$

(18)

Combining (16) and (18), we have

$$|e(k + 1)| \leq d_2|e(k)| \leq d_2^2|e(k-1)| \leq \cdots \leq d_2^k|e(1)|.
$$

(19)

Formula (19) means that conclusion (2) of Theorem 2 holds.

Since $y^*$ is constant, the convergence of the output tracking error $e(k)$ means that $y(k)$ is bounded.

Using inequalities $(\sqrt{\lambda} \lambda^2 + |\phi_e(k)|^2) \geq 2\sqrt{\lambda} \phi_e(k)$ and $\lambda > \lambda_{\min}$, the following formula is obtained from (7):

$$|\Delta u(k)| = \frac{|\rho \phi_e(k)| |y^* - y(k)|}{|\phi_e(k)|^2} \leq \frac{|\rho \phi_e(k)|}{2\sqrt{\lambda} \phi_e(k)} |e(k)|
$$

$$
\leq \frac{\rho \phi_e(k)}{2\sqrt{\lambda} \phi_e(k)} |e(k)| \leq \frac{\rho}{2\sqrt{\lambda_{\min}}} |e(k)| = M_2 |e(k)|,
$$

(20)

where $M_2 = \rho/\sqrt{\lambda_{\min}}$ is a bounded constant.

Using (19) and (20), we obtain

$$|u(k)| \leq |u(k) - u(k-1)| + |u(k-1)|
$$

$$
\leq |u(k) - u(k-1)| + |u(k-1) - u(k-2)| + |u(k-2)|
$$

$$
\leq |u(k)| + |\Delta u(k-1)| + \cdots + |\Delta u(2)| + |\Delta u(1)|
$$

$$
\leq M_2 (|e(k)| + |e(k-1)| + \cdots + |e(2)|) + |u(1)|
$$

$$
\leq M_2 (d_2^{-k-1}|e(1)| + \cdots + d_2^{-k-2}|e(1)| + \cdots + d_2^{-k-1}|e(1)|) + |u(1)|
$$

$$
< M_2 \frac{d_2}{1 - d_2} |e(1)| + |u(1)|.
$$

(21)

Therefore, conclusion (2) of Theorem 2 is proved.

3.3. Design of CFDL-MAFC Controller Based on Switching Strategy. The pH control process of the absorption tower slurry is a complex object with nonlinear, time-varying parameters and many uncertainties. In this paper, we mainly consider how to improve the control effect of slurry pH value control system when the parameters of absorption tower slurry pH control system change under different working conditions. In view of the characteristics of the time-varying parameters of the slurry pH control system, we consider the jump change of parameters which is a special case of time-varying parameters. Multiple model-free adaptive controllers are designed to cover all jump parameters according to different working conditions to improve the control accuracy of slurry pH control system in the case of parameters’ jump change.
(1) According to the system input and output data under different working conditions, the mathematical models of absorption tower slurry pH control system are established. The model of slurry pH control system under different working conditions is as follows:

\[ M = \{ M_i \mid i = 1, 2, \ldots, n \}, \quad (22) \]

where \( M_i \) denotes \( n \) system models, and it can be written as

\[ x(k) = \frac{y(k)}{f(u(k))} = u(k) + y_i u^2(k) + y_i^2 u^3(k), \]

\[ G(z) = \frac{y(k)}{x(k)} = \frac{a_1 z^{-3} + a_2 z^{-4} + a_3 z^{-5}}{1 + b_1 z^{-1} + b_2 z^{-2}}. \quad (23) \]

(2) The corresponding model-free adaptive controllers are established according to the different working conditions of the slurry pH control process of the absorption tower:

\[ C = \{ C_i \mid i = 1, 2, \ldots, n \}, \quad (24) \]

where \( C_i \) is

\[ u_i(k) = u_i(k-1) + \frac{\rho_i \varphi_i^*(k)}{\lambda_i + \left| \varphi_i^*(k) \right|} \left( y^* (k+1) - y_i(k) \right), \]

\[ \varphi_i^*(k) = \varphi_i^*(k-1) + \frac{\eta_i \Delta u(k-1)}{\mu_i + \Delta u(k-1)} \left( \Delta y(k) - \varphi_i^*(k-1) \cdot \Delta u(k-1) \right). \quad (25) \]

If \[ \left| \varphi_i^*(k) \right| \leq \varepsilon \] or \[ \left| \Delta u(k-1) \right| \leq \varepsilon, \] \( \varphi_i^*(k) \) is reset to

\[ \left| \varphi_i^*(k) \right| = \left| \varphi_i^*(1) \right|. \quad (26) \]

(3) Index switching function is established:

\[ L_i(k) = a e_i^2(k) + \beta \sum_{n=1}^{N} \delta_i e_i^2(k-n), \quad a \geq 0, \beta, \delta > 0, \]

\[ (27) \]

to the system switching index function; i.e., model \( M_l(k) \) is the optimal model. \( l(k) \) is defined by

\[ l(k) = \arg \min_{i \in [1,n]} \left( a e_i^2(k) + \beta \sum_{n=1}^{N} \delta_i e_i^2(k-m) \right). \quad (28) \]

(3) At the next sampling time, the system controller is switched to the optimal CF-DL-MAFC controller \( C_{l(k)} \) based on model \( M_{l(k)} \).

3.4. Stability Analysis of CF-DL-MAFC Algorithm Based on Switching Strategy

**Lemma 1** (see [22]). For \( A \in C^{\infty} \) with its eigenvalues \( \lambda_i, \) \( i \in \{1,2,\ldots,n\} \), if \[ \max_{i \in \{1,2,\ldots,n\}} |\lambda_i| < \delta < 1, \] the following inequality can be guaranteed:

\[ \| A^k \|_2 < \sigma y^k, \quad (29) \]

where \( \sigma \) is a positive constant and \( \delta < \gamma < 1. \)

**Proof.** For an arbitrary positive \( \varepsilon \), there is a compatible norm \( \| \cdot \|_a \) which satisfies

\[ \| A \|_a < \max_{i} |\lambda_i| + \varepsilon < \delta + \varepsilon. \quad (30) \]

Due to \( \delta < 1 \), we can let the \( \varepsilon \) satisfy \( \delta + \varepsilon < 1 \), and let \( y = \delta + \varepsilon \); then,

\[ \| A^k \|_2 < \| A \|_a \cdot \cdots \cdot A \|_a < y^k < 1. \quad (31) \]

From the functional analysis, compatible norm satisfies

\[ \| A^k \|_2 < \sigma \| A^k \|_a, \quad (32) \]

where \( \sigma \) is a positive number which is determined by the relationship between 2-norm and *-norm.

Hence

\[ \| A^k \|_2 < \sigma y^k. \quad (33) \]

This completes the proof.

**Theorem 3.** For the nonlinear system (2), under the condition that Assumption 1, Assumption 2, Assumption 3, and Assumption 4 are satisfied, when \( y^*(k+1) = y^* = \text{const} \) and the CF-DL-MAFC scheme is adopted, there is always a positive number \( \lambda_{\min} > 0 \) so that when \( \lambda > \lambda_{\min} \), the following conclusions are obtained:

(1) The output tracking error of the system converges monotonically, and \( \lim_{k \to \infty} \| y^* - y(k+1) \| = 0 \) is satisfied.

(2) The closed-loop system is bounded-input bounded-output (BIBO) stable; that is, the output sequence \{y(k)\} and input sequence \{u(k)\} are bounded.

(3) System parameters jump change among a limited number of values.
Proof. The proof of Theorem 3 is similar to that of Theorem 1. Assuming that, during the process of pH control of the absorption tower slurry, the system parameters change among a limited number of values, we can write (19) as

\[ |e(k+1)| \leq d^k_1 e(1) \]

where \( k_0 + k_1 + \cdots + k_{m-1} + k_m = k \).

From Lemma 1, we have

\[ d^k_1 \cdots d^k_2 \cdots d^k_1 < \sigma_1 y^{k_1} \cdot \sigma_2 y^{k_2} \cdots \sigma_m y^{k_m} \]

\[ = (\sigma_1 \sigma_2 \cdots \sigma_{m-1} \sigma_m) \cdot (y^{k_1} y^{k_2} \cdots y^{k_m}) \cdot e(1) \]

Substituting (35) into (34), we obtain

\[ |e(k+1)| \leq d^k_1 |e(1)| \]

\[ \cdot (\sigma_1 \sigma_2 \cdots \sigma_{m-1} \sigma_m) \cdot (y^{k_1} y^{k_2} \cdots y^{k_m}) \cdot e(1) \]

(36)

According to Lemma 1, we can easily know that \( \sigma_1 \sigma_2 \cdots \sigma_{m-1} \sigma_m \) is a positive constant. If \( m \) is a finite value, \( \sigma_1 \sigma_2 \cdots \sigma_{m-1} \sigma_m \) is a bounded positive constant. Therefore, when \( k \rightarrow \infty \), we can obtain \( y^{k_1} y^{k_2} \cdots y^{k_m} \rightarrow 0 \); then, \( |e(k+1)| \rightarrow 0 \). Conclusion (1) and conclusion (3) of Theorem 3 are proved. The proof methods of conclusion (2) of Theorem 3 and conclusion (2) of Theorem 2 are the same, and they will not be repeated here.

4. Simulation Research and Analysis

4.1. Simulation Results of the Slurry pH Control System with Fixed Parameters. The parameters of the Hammerstein model of the pH control system of the absorption tower slurry are as follows:

\[ y_1 = -1.3; \]
\[ y_2 = 1.2; \]
\[ a_1 = 1.331; \]
\[ a_2 = -0.9366; \]
\[ a_3 = 1.617; \]
\[ b_1 = -1.586; \]
\[ b_2 = 0.5872. \]

The desired output of the system is as follows:

\[ y_d = \begin{cases} 5.5, & k < 500, \\ 5.2, & 500 \leq k < 1200, \\ 5.8, & 1200 \leq k < 2000. \end{cases} \]

Using CFDL-MFAC algorithm for MATLAB simulation, we set the simulation parameters to

\[ u(1) = u(2) = u(3) = u(4) = 0; \]
\[ y(1) = y(2) = y(3) = y(4) = 0.1; \]
\[ \bar{y}_c(1) = 2; \]
\[ \epsilon = 0.05; \]
\[ \rho = 0.1; \]
\[ \lambda = 20; \]
\[ \eta = 1; \]
\[ \mu = 0.3. \]

The simulation results are shown in Figures 2 and 3. Figure 2 shows the tracking control curve of slurry pH value in the process of pH value control of absorption tower slurry. It can be seen from the figure that the actual output pH value of slurry can well track the expected output pH value, and high control accuracy is achieved. Figure 3 shows the tracking error curve of the actual output pH value of the slurry to the expected output pH value in the tracking process. It can be seen that the tracking error of the pH value in the whole tracking process is almost zero except at the time when the expected output pH value changes. This indicates the effectiveness of the CFDL-MFAC algorithm for slurry pH control.

4.2. Simulation Results of the Slurry pH Control System with Jumping Parameters. The simulation results in Section 4.1 are obtained under the condition that the parameters of the slurry pH control system are fixed. In this section, the simulation analysis is carried out for the slurry pH control system with jumping parameters. It is assumed that the model parameter \( b_1 \) of the slurry pH control system jumps during the control process, and the jumping parameters are as follows:

\[ b_1 = \begin{cases} -1.586, & k < 600, \\ -1.645, & 600 \leq k \leq 1300, \\ -1.537, & k > 1300. \end{cases} \]

In modeling process, the model parameters we get are often inaccurate. We randomly take several estimations of \( b_1 \) around the true value for simulation. Three estimated values of the parameters are given as \( b_1^1 = -1.58, b_1^2 = -1.64, b_1^3 = -1.53 \), corresponding to model 1, model 2, and model 3. For system with jumping parameter, we use CFDL-MFAC control algorithm and CFDL-MAF control algorithm based on switching strategy for simulation analysis.

4.2.1. CFDL-MFAC Algorithm. When the system parameters jump during the pH control of the slurry, the simulation results using the CFDL-MFAC algorithm are shown in Figures 4 and 5. Figure 4 shows the tracking control curve of slurry pH value under the condition of system parameters jumping. It can be seen from the figure that, due to the jump of system parameter \( b_1 \) at 600 s and 1300 s sampling times, the control effect of slurry pH value at these two moments is poor, and there is a large overshoot, which cannot meet the
Figure 2: pH tracking curve.

Figure 3: pH tracking error curve.

Figure 4: pH tracking curve.
control requirements. Figure 5 shows the tracking error curve of slurry pH value under the condition of system parameters jumping. It can be seen from the curve that the pH value tracking error is large in a period of time after 600 s and 1300 s sampling time, which is beyond the reasonable range of slurry pH value regulation.

4.2.2. CFDL-MFAC Algorithm Based on Switching Strategy.
For the jumping parameter system, the simulation results of CFDL-MFAC algorithm based on switching strategy are shown in Figures 6–8. As can be seen from Figure 8, when the system parameters jump, the CFDL-MFAC algorithm based on the switching strategy can switch to the optimal CFDL-MFAC controller at the next sampling time according to the index switching function. For example, the parameters jump from −1.586 to −1.645 at the 600th sampling time, and the system switches from controller 1 to controller 2 at the next sampling time. Through this control strategy, the output pH value of the system achieves a good tracking effect on the expected value, and the tracking error is controlled within a reasonable range. The tracking effect and tracking error are shown in Figures 6 and 7. Compared with CFDL-MFAC algorithm, CFDL-MFAC algorithm based on switching strategy can effectively reduce overshoot and improve control effect when system parameters jump.

5. Conclusions
Aiming at the pH control of the absorption tower slurry in limestone-gypsum wet flue gas desulfurization, a model-free adaptive control algorithm is proposed in this paper, and a CFDL-MFAC controller is designed according to the slurry pH control process. The slurry pH value is a complex controlled object with characteristics of nonlinearity, large inertia, hysteresis, time-varying, etc., and the system parameters jump easily due to external interference in the
process of pH control, so a model-free adaptive control algorithm based on switching strategy is proposed. The convergence of the algorithm is proved theoretically. The simulation results show that the model-free adaptive control algorithm based on switching strategy can deal with the jumping system parameters in the process of slurry pH control. Compared with CFDL-MFAC algorithm, it can effectively reduce the overshoot and improve the effect of pH control.

Data Availability

The experimental data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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