Research Article

Almost Disturbance Decoupling for a Class of Fractional-Order Nonlinear Systems with Zero Dynamics

Xiaoping Liu ¹ **Yajing Zhao** ² **Caiyun Wang** ³ **Huanqing Wang** ^{2,4}
and **Yucheng Zhou** ²

¹Faculty of Engineering, Lakehead University, Thunder Bay, ON P7B 5E1, Canada

²School of Information and Electrical Engineering, Shandong Jianzhu University, Jinan, China

³School of Science, Shandong Jianzhu University, Jinan, China

⁴The Department of Mathematics, Bohai University, Jinzhou 12100, China

Correspondence should be addressed to Xiaoping Liu; xliu2@lakeheadu.ca

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The problem of almost disturbance decoupling is addressed for fractional-order nonlinear systems. A new definition for the norm is proposed to describe the effect of disturbances on the output tracking error for fractional-order systems. Based on the Lyapunov stability theory and the backstepping design method, a tracking controller is constructed to make the output tracking error converge to zero without external disturbances and to attenuate the effect of disturbances on the tracking error at zero initial conditions. In order to validate these theoretical results, a numerical example and two practical examples are given.

1. Introduction

Recently, there has been a continuous growth in the research of fractional-order systems (FOSs). Thanks to the great efforts contributed by researchers, a large number of research results have been published on FOSs, see [1, 2]. It has been proved that fractional-order calculus can model systems more concisely than integer-order calculus [3]. Therefore, FOSs have been applied in numerous areas.

The output tracking control problem is one of the most important research topics. It is worth mentioning that the output tracking control has been widely applied in engineering systems including industry and economics [4–6]. As for FOSs, few research outcomes are published. An output tracking controller for a class of fractional-order positive switched systems is developed via an observer-based controller method [7]. In [8], a novel adaptive iterative learning sliding mode controller is proposed to control FOSs. What is more, the output tracking controllers can also be developed with the help of backstepping technique for FOSs. For instance, Wang et al. addressed an adaptive fuzzy output tracking control law for a class of uncertain FOSs with

backstepping technique [9]. An output feedback neural network tracking controller for fractional-order nonlinear systems is constructed with the backstepping method [10].

Over the past decades, backstepping has become a widely used technique for adaptive nonlinear control. The technique was comprehensively addressed [11], and the basic idea of the backstepping scheme is to design a controller recursively by using a set of virtual controllers. The integer-order backstepping technique has been widely applied in many fields [12, 13]. An adaptive finite-time fuzzy funnel controller is presented for nonaffine nonlinear systems via backstepping technique [14]. An event-triggered adaptive tracking controller based on a new funnel function for uncertain nonlinear systems is proposed by using backstepping technique [15]. Few research outcomes applying backstepping technique to FOSs are published. In [16], synchronization of two fractional-order chaotic Coulet systems is investigated based on the adaptive backstepping. A novel adaptive control technique named adaptive fractional-order backstepping is proposed for fractional-order nonlinear systems (FONSs) with uncertain constant parameters [17]. In [18], Ding et al. extended the classical backstepping to fractional-order backstepping for stabilising FONSs. Then, the pseudo-state

stabilization problem of commensurate FONSs is constructed via the fractional-order backstepping scheme [19]. Wei et al. proposed a new technique to construct an adaptive backstepping controller for noncommensurate FOSs with the triangular form [20], and Wei et al. originally presented an adaptive output feedback control law for fractional-order systems by using the backstepping method [21]. Owing to most of the results which construct Lyapunov functions limited to the use of the frequency distribution model or linear matrix inequality (LMI), and the fractional-order backstepping techniques are still to be further studied.

It is well known that disturbances exist widely in industrial processes. The issue of disturbance decoupling has drawn great attention. The exact disturbance decoupling problem is to design a feedback controller so that the disturbance does not affect the output of the closed-loop system, which can be solved by the state or output feedback under some strict conditions [22–24]. However, for many practical systems, the exact disturbance decoupling problem is not solvable because the strict conditions are not satisfied. Therefore, the almost disturbance decoupling (ADD) problem is proposed by Weiland and Willems [25], which is to design a feedback controller such that the closed-loop system is stable and the L_2 gain from the disturbance to the output is less than or equal to an arbitrary positive constant. Up to now, there have been many research outcomes for the ADD problem for both linear and nonlinear control systems. For example, the ADD problem for SISO nonlinear systems is proposed [26]. The ADD problem of MIMO nonlinear systems and its application to chemical processes are addressed in [27]. In [28], Lin et al. discussed the problem of ADD for nonlinear systems with unstable zero dynamics. Also, in [29], Lin et al. developed a global result about the ADD problem for nonlinear systems with unstable zero dynamics. In [30], the authors investigated the problem of nonlinear output feedback tracking with almost disturbance decoupling. What is more, the ADD problem for high-order nonlinear systems is addressed by [31]. Tracking and almost disturbance decoupling for nonlinear systems with uncertainties are solved by Chen et al. [32]. Chen et al. [33] developed a direct adaptive fuzzy almost disturbance decoupling control method for nonlinear MIMO systems with unknown nonlinearities. For more research results on the ADD problem, see [34–38]. However, the research studies mentioned above are conducted only for integer-order systems. To the best of our knowledge, there are no reports on the ADD problem for FOSs.

The active disturbance rejection control (ADRC) problem, which utilizes an estimation and cancellation method to attenuate the negative impact of internal unmodelled dynamics and external disturbances, has been paid extensive attention recently. The ADRC problem has been addressed for nonlinear systems [39–44].

Motivated by the aforementioned reviews, this article develops an adaptive backstepping control algorithm to solve the problem of ADD for a class of fractional-order nonlinear systems. To this end, the following two challenges have to be overcome:

- (1) Aghababa [45] has proved that Lin et al. [46] mistakenly applied the integer-order integral to deal with the fractional-order differential, which causes that the proposed controller in [46] cannot guarantee the H_∞ performance of the closed-loop system, which implies that the L_2 -norm cannot be directly used to solve the ADD problem. To solve this problem, a new form of norm based on the definition of Caputo fractional integral is proposed.
- (2) When the backstepping design method is used, the fractional-order derivative of the virtual controller is required, which is a function of state variables. Therefore, the fractional-order derivative of a composite function is necessary. Unlike the integer-order derivative, the fractional-order derivative of a composite function is so complicated that it cannot be used to the fractional-order derivative of the virtual controller. To avoid such a challenge, the nonlinear functions in all the subsystems but last one are assumed to be linear.

Compared with the existing results, the main contributions are listed as follows:

- (1) The problem of ADD is first addressed for a class of fractional-order nonlinear systems with external disturbances and zero dynamics.
- (2) A new form of the norm based on the definition of Caputo fractional integral is proposed in order to prove the H_∞ tracking performance for FOSs.
- (3) With the help of backstepping technique, a tracking feedback control law is constructed, which ensures that the effect of external disturbances on the output tracking error is attenuated. A new form of Lyapunov function is chosen without the use of the frequency distribution model or LMI.

The remaining parts are arranged as follows. In Section 2, several definitions and useful lemmas relevant to the fractional-order calculus are shown. Section 3 proposes the problem formulation and gives the aim of this paper. In Section 4, the main results of this paper are presented. Section 5 gives a numerical example and two practical examples to further support the theoretical results. In the end, the conclusions are summarized in Section 6.

2. Preliminaries

2.1. Fractional Calculus

Definition 1 (Caputo fractional derivative [3]). Let $f(t)$ have continuous derivatives up to order $n - 1$, i.e., $f^{(n-1)}(t)$ is a real continuously differentiable function. The Caputo fractional-order derivative of order α of $f(t)$ is defined by

$${}_{t_0}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t f^{(n)}(\sigma)(t-\sigma)^{n-\alpha-1} d\sigma, \quad (1)$$

where $0 \leq n - 1 < \alpha \leq n$, $n \in \mathbb{N}$, and $\Gamma(\cdot)$ is the gamma function.

For simplicity in notations, ${}_{t_0}D_t^\alpha$ is denoted as D^α for $t_0 = 0$.

Definition 2 (Caputo fractional integral [3]). Assume that $f(t)$ is an integrable function. Its Caputo fractional integral of order α is defined by

$${}_{t_0}D_t^{-\alpha}f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t f(\sigma)(t-\sigma)^{\alpha-1}d\sigma, \quad (2)$$

where $\alpha > 0$ and t_0 is the lower terminal.

For convenience in notations, ${}_{t_0}D_t^{-\alpha}$ is denoted by $D^{-\alpha}$ for $t_0 = 0$.

Properties of the Caputo fractional calculus, which are used in the paper, are shown as Lemmas 1 and 2.

Lemma 1 (an inequality of the Caputo fractional derivative [2]). For a differentiable function $x(t) \in \mathbb{R}^n$, a positive-definite matrix $P \in \mathbb{R}^{n \times n}$, and $0 < \alpha < 1$, the inequality

$$\frac{1}{2}D^\alpha(x^T(t)Px(t)) \leq x^T(t)PD^\alpha x(t), \quad (3)$$

is true for any $t \geq 0$.

Lemma 2 (see [47]). Assume that $0 \leq n-1 < \alpha \leq n$ and $f(t)$ is differentiable up to order $n-1$, that is, $f^{(n-1)}(t)$ is a differentiable function. Then,

$${}_{t_0}D_t^{-\alpha}({}_{t_0}D_t^\alpha f(t)) = f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(t_0)}{k!}(t-t_0)^k. \quad (4)$$

In particular, for a continuous function $f(t)$ and $0 < \alpha \leq 1$, the following holds:

$${}_{t_0}D_t^{-\alpha}({}_{t_0}D_t^\alpha f(t)) = f(t) - f(t_0). \quad (5)$$

2.2. Stability of Fractional-Order Systems. Consider the following FOS:

$${}_{t_0}D_t^\alpha x(t) = f(x, t), \quad (6)$$

where t stands for the time and $0 < \alpha \leq 1$.

Definition 3. A function $\beta: [0, t) \rightarrow [0, \infty)$ is said to be a class- $\mathcal{K}$ function if it is continuous and strictly increasing with $\beta(0) = 0$.

Li et al. [48] extended the Lyapunov direct method to FOSs, which is shown as Lemma 3.

Lemma 3 (see [48]). If there exist class- $\mathcal{K}$ functions β_i ($i = 1, 2, 3$) and a function $V(x, t)$ so that

$$\beta_1(\|x\|) \leq V(x, t) \leq \beta_2(\|x\|), \quad (7)$$

and

$${}_{t_0}D_t^\alpha V(x, t) \leq -\beta_3(\|x\|), \quad (8)$$

with $0 < \alpha < 1$, then the equilibrium point $x = 0$ of (6) is asymptotically stable.

Lemma 4 (Young's inequality). The inequality

$$x^T y \leq \frac{\varepsilon^p}{p} \|x\|^p + \frac{1}{q\varepsilon^q} \|y\|^q, \quad (9)$$

holds for any $x, y \in \mathbb{R}^r$, $\varepsilon > 0$, $p > 1$, and $q > 1$ with $(p-1)(q-1) = 1$.

3. Problem Formulation

Consider a fractional-order strict-feedback nonlinear system with unknown disturbances and zero dynamics defined by

$$\begin{cases} D^\alpha z(t) = f_0(z(t)), \\ D^\alpha x_1(t) = x_2(t) + f_1(\bar{x}_1(t)) + P_1 \omega(t), \\ D^\alpha x_2(t) = x_3(t) + f_2(\bar{x}_2(t)) + P_2 \omega(t), \\ \vdots \\ D^\alpha x_{n-1}(t) = x_n(t) + f_{n-1}(\bar{x}_{n-1}(t)) + P_{n-1} \omega(t), \\ D^\alpha x_n(t) = f(x(t)) + g(x(t))u(t) + f_n(\bar{x}_n(t)) + P_n \omega(t), \\ y(t) = x_1(t), \end{cases} \quad (10)$$

where $0 < \alpha < 1$, $z(t)$ denotes the zero dynamics of (10), and $f_0(z(t))$ is a known nonlinear function with respect to $z(t)$. $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ and $\bar{x}_i(t) = [x_1(t), \dots, x_i(t)]^T \in \mathbb{R}^i$ with $x_i(t)$ being the state of the system, while $y(t) \in \mathbb{R}$ and $u(t) \in \mathbb{R}$ are the system output and input, respectively. $\omega(t) \in \mathbb{R}^r$ represents the disturbance signal. $f_i(\bar{x}_i(t)) = a_i \bar{x}_i(t) \in \mathbb{R}$ ($i = 1, \dots, n$) with a_i being a known vector. $f(\cdot)$ and $g(\cdot)$ are assumed to be nonlinear and known functions. $P_i \in \mathbb{R}^{1 \times r}$ is a known vector for $i = 1, \dots, n$. For convenience, denote $f(x(t))$, $g(x(t))$, $f_0(z(t))$, $f_i(\bar{x}_i(t))$, $\bar{x}_i(t)$, $x_i(t)$, and $z(t)$ as f , g , f_0 , f_i , $\bar{x}_i$, x_i , and z , respectively.

The aim is to construct a state feedback controller to drive the output $y(t)$ to track a desired reference signal $y_r(t)$, i.e., $\lim_{t \rightarrow \infty} (y(t) - y_r(t)) = 0$ when $\omega(t) = 0$ and to have an H_∞ tracking performance for the zero initial state, i.e.,

$$\|(y(t) - y_r(t))\|_\alpha \leq \gamma \|\omega(t)\|_\alpha, \quad (11)$$

where $\|\cdot\|_\alpha$ is defined later and $\gamma > 0$.

4. Main Results

4.1. The L_α Norm. In this section, a new form of norm named L_α norm based on the definition of Caputo fractional integral is presented.

Definition 4. The L_α norm of a function $\omega(t)$ is defined by

$$\|\omega(t)\|_\alpha = \lim_{t \rightarrow \infty} \sqrt{\int_0^t \omega^T(\sigma)(t-\sigma)^{\alpha-1} \omega(\sigma) d\sigma}, \quad (12)$$

where $0 < \alpha < 1$.

This L_α norm satisfies three conditions of the norm, i.e., nonnegative (equation (13)), homogeneity (equation (14)), and triangle inequality (equation (15)).

Lemma 5. The L_α norm satisfies the following properties:

(i) Nonnegative:

$$\begin{aligned} \|\omega(t)\|_\alpha &\geq 0; \\ \|\omega(t)\|_\alpha &= 0, \quad \text{if and only if } \omega(t) = 0. \end{aligned} \quad (13)$$

(ii) Homogeneity:

$$\|k\omega(t)\|_\alpha = |k| \|\omega(t)\|_\alpha. \quad (14)$$

(iii) Triangle inequality:

$$\|\omega_1(t) + \omega_2(t)\|_\alpha \leq \|\omega_1(t)\|_\alpha + \|\omega_2(t)\|_\alpha. \quad (15)$$

Proof

(i) According to Definition (12), it can be easily verified that (13) is true.

(ii) Clearly,

$$\begin{aligned} \|k\omega(t)\|_\alpha &= \lim_{t \rightarrow \infty} \sqrt{\int_0^t k^2 \omega^T(\sigma)(t-\sigma)^{\alpha-1} \omega(\sigma) d\sigma} \\ &= |k| \|\omega(t)\|_\alpha. \end{aligned} \quad (16)$$

(iii) For convenience, denote $\omega(t)$ as ω . It can be seen that the proof of condition (15) is equivalent to verify

$$\|\omega_1 + \omega_2\|_\alpha^2 \leq (\|\omega_1\|_\alpha + \|\omega_2\|_\alpha)^2. \quad (17)$$

On the one hand,

$$\begin{aligned} \|\omega_1 + \omega_2\|_\alpha^2 &= \lim_{t \rightarrow \infty} \int_0^t (\omega_1 + \omega_2)^T (t-\sigma)^{\alpha-1} (\omega_1 + \omega_2) d\sigma \\ &= \lim_{t \rightarrow \infty} \int_0^t (\omega_1^T \omega_1 + 2\omega_1^T \omega_2 + \omega_2^T \omega_2) (t-\sigma)^{\alpha-1} d\sigma \\ &= \|\omega_1\|_\alpha^2 + \|\omega_2\|_\alpha^2 + 2 \lim_{t \rightarrow \infty} \int_0^t \omega_1^T (t-\sigma)^{\alpha-1} \omega_2 d\sigma. \end{aligned} \quad (18)$$

On the other hand,

$$\begin{aligned} (\|\omega_1\|_\alpha + \|\omega_2\|_\alpha)^2 &= \|\omega_1\|_\alpha^2 + 2\|\omega_1\|_\alpha \|\omega_2\|_\alpha + \|\omega_2\|_\alpha^2 \\ &= \|\omega_1\|_\alpha^2 + \|\omega_2\|_\alpha^2 \\ &\quad + 2 \lim_{t \rightarrow \infty} \left[\sqrt{\left(\int_0^t \omega_1^T (t-\sigma)^{\alpha-1} \omega_1 d\sigma \right)} \right] \\ &\quad \cdot \left[\sqrt{\left(\int_0^t \omega_2^T (t-\sigma)^{\alpha-1} \omega_2 d\sigma \right)} \right]. \end{aligned} \quad (19)$$

It is a simple exercise to show that the proof of (17) is equivalent to prove

$$\begin{aligned} &\left[\int_0^t \omega_1^T (t-\sigma)^{\alpha-1} \omega_2 d\sigma \right]^2 \\ &\leq \left[\int_0^t \omega_1^T (t-\sigma)^{\alpha-1} \omega_1 d\sigma \right] \cdot \left[\int_0^t \omega_2^T (t-\sigma)^{\alpha-1} \omega_2 d\sigma \right]. \end{aligned} \quad (20)$$

In order to verify (20), we construct a quadratic function of u as follows:

$$\begin{aligned} \Phi(u) &= u^2 \int_0^t \omega_1^T(\sigma)(t-\sigma)^{\alpha-1} \omega_1(\sigma) d\sigma \\ &\quad + 2u \int_0^t \omega_1^T(\sigma)(t-\sigma)^{\alpha-1} \omega_2(\sigma) d\sigma \\ &\quad + \int_0^t \omega_2^T(\sigma)(t-\sigma)^{\alpha-1} \omega_2(\sigma) d\sigma. \end{aligned} \quad (21)$$

After a little manipulation, one can get the following result:

$$\begin{aligned} \Phi(u) &= \int_0^t \left[u^2 \omega_1^T(\sigma)(t-\sigma)^{\alpha-1} \omega_1(\sigma) \right. \\ &\quad + 2u \omega_1^T(\sigma)(t-\sigma)^{\alpha-1} \omega_2(\sigma) \\ &\quad + \left. \omega_2^T(\sigma)(t-\sigma)^{\alpha-1} \omega_2(\sigma) \right] d\sigma \\ &= \int_0^t (t-\sigma)^{(\alpha-1)/2} [u\omega_1(\sigma) + \omega_2(\sigma)]^T \\ &\quad \cdot [u\omega_1(\sigma) + \omega_2(\sigma)] d\sigma \\ &\geq 0. \end{aligned} \quad (22)$$

It is easily seen from (21) and (22) that the quadratic function intersects the u -axis at most one point, which implies that

$$\begin{aligned} &\left[2 \int_0^t \omega_1^T(\sigma)(t-\sigma)^{\alpha-1} \omega_2(\sigma) d\sigma \right]^2 \\ &\quad - 4 \left[\int_0^t \omega_1^T(\sigma)(t-\sigma)^{\alpha-1} \omega_1(\sigma) d\sigma \right] \\ &\quad \cdot \left[\int_0^t \omega_2^T(\sigma)(t-\sigma)^{\alpha-1} \omega_2(\sigma) d\sigma \right] \leq 0. \end{aligned} \quad (23)$$

which is equivalent to (20). $\square$

4.2. Control Laws for H_∞ Tracking Performance

Assumption 1. For the zero dynamics $D^\alpha z = f_0(z)$ in system (10), there exists a positive-definite differentiable function V_0 so that its Caputo fractional derivative with order $0 < \alpha < 1$ satisfies the following inequality:

$$D^\alpha V_0 < 0. \quad (24)$$

Assumption 2. The known nonlinear function $g(\cdot)$ satisfies the following inequality:

$$|g(\cdot)| > 0. \quad (25)$$

Assumption 3. The desired reference signal $y_r(t)$ and $\bar{D}_i$ are bounded and piecewise continuous, where $\bar{D}_i = \underbrace{D^\alpha(D^\alpha(\dots D^\alpha(y_r)))}_i, i = 0, 1, 2, \dots, n$.

Define the control law by

$$u = \frac{1}{g} \left[-f - k_n z_n - z_{n-1} - f_n + \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (x_{s+1} + f_s) + \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \bar{D}_s} \bar{D}_{s+1} - \frac{\hat{P}_n \hat{P}_n^T}{4\gamma_n} z_n \right], \quad (26)$$

and the virtual control laws by

$$\begin{cases} \alpha_0 = y_r(t), \\ \alpha_1 = -k_1 z_1 - f_1 + D^\alpha \alpha_0 - \frac{P_1 P_1^T}{4\gamma_1} z_1, \\ \dots \\ \alpha_i = -k_i z_i - z_{i-1} - f_i + \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (x_{s+1} + f_s) \\ + \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \bar{D}_s} \bar{D}_{s+1} - \frac{\hat{P}_i \hat{P}_i^T}{4\gamma_i} z_i, \quad i = 2, \dots, n-1, \end{cases} \quad (27)$$

where

$$\begin{cases} z_1 = x_1 - \alpha_0, \\ z_i = x_i - \alpha_{i-1}, \\ i = 2, 3, \dots, n, \end{cases} \quad (28)$$

$$\hat{P}_i = P_i - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} P_s, \quad i = 2, 3, \dots, n, \quad (29)$$

and

$$\bar{D}_i = \underbrace{D^\alpha(D^\alpha(\dots D^\alpha(\alpha_0)))}_i, \quad i = 0, 1, 2, \dots, n. \quad (30)$$

Theorem 1. With Assumptions 1–3, virtual control laws (26), and feedback controller (25), the closed-loop system corresponding to (10) has the following properties:

- (1) $y(t)$ asymptotically tracks the desired reference signal $y_r(t)$, i.e.,

$$\lim_{t \rightarrow \infty} (y(t) - y_r(t)) = 0, \quad \text{when } \omega(t) = 0. \quad (31)$$

- (2) $y(t)$ achieves the H_∞ tracking performance, i.e.,

$$\|(y(t) - y_r(t))\|_\alpha \leq \gamma \|\omega(t)\|_\alpha, \quad (32)$$

for the zero initial condition when $\omega(t) \neq 0$, where $\gamma > 0$.

Proof. The proof is given by constructing virtual control laws and a control law based on backstepping design approach.

Step 1: select the following Lyapunov function:

$$V_1 = \frac{1}{2} z_1^2. \quad (33)$$

Then, the fractional derivative of V_1 is given by

$$\begin{aligned} D^\alpha V_1 &\leq z_1 D^\alpha z_1 \\ &= z_1 [D^\alpha x_1 - D^\alpha \alpha_0] \\ &= z_1 (x_2 + f_1 + P_1 \omega - D^\alpha \alpha_0) \\ &= z_1 (z_2 + \alpha_1 + f_1 + P_1 \omega - D^\alpha \alpha_0) \\ &= z_1 z_2 + z_1 (\alpha_1 + f_1 - D^\alpha \alpha_0) + z_1 P_1 \omega, \end{aligned} \quad (34)$$

where the first inequality is generated from (3).

According to Lemma 4, let $p = q = 2$ and $\varepsilon = 1/\sqrt{2\gamma}$; then, (34) can be estimated by

$$\begin{aligned} D^\alpha V_1 &\leq z_1 z_2 + \gamma_1 \omega^T \omega + z_1 (\alpha_1 + f_1 - D^\alpha \alpha_0) + z_1 \frac{P_1 P_1^T}{4\gamma_1} z_1 \\ &= z_1 z_2 + \gamma_1 \omega^T \omega + z_1 \left(\alpha_1 + f_1 - D^\alpha \alpha_0 + \frac{P_1 P_1^T}{4\gamma_1} z_1 \right). \end{aligned} \quad (35)$$

Then, choose a virtual control law as

$$\alpha_1 = -k_1 z_1 - f_1 + D^\alpha \alpha_0 - \frac{P_1 P_1^T}{4\gamma_1} z_1, \quad (36)$$

where k_1 and γ_1 are positive constants and can be designed by us.

Furthermore, substituting (36) into (35) results in

$$D^\alpha V_1 \leq -k_1 z_1^2 + z_1 z_2 + \gamma_1 \omega^T \omega. \quad (37)$$

Step 2:

Define a Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}z_2^2. \quad (38)$$

It follows from (28) and (27) that

$$\begin{aligned} D^\alpha z_2 &= D^\alpha x_2 - D^\alpha \alpha_1 \\ &= x_3 + f_2 + P_2 \omega - D^\alpha \alpha_1, \end{aligned} \quad (39)$$

with

$$\begin{aligned} D^\alpha \alpha_1 &= \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1 + P_1 \omega) + \frac{\partial \alpha_1}{\partial \alpha_0} D^\alpha \alpha_0 \\ &\quad + \frac{\partial \alpha_1}{\partial D^\alpha \alpha_0} D^\alpha (D^\alpha \alpha_0). \end{aligned} \quad (40)$$

Substituting (40) into (39) yields

$$z_2 D^\alpha z_2 = z_2 \left(x_3 + f_2 + \hat{P}_2 \omega - \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1) - \sum_{s=0}^1 \frac{\partial \alpha_1}{\partial \hat{D}_s} \hat{D}_{s+1} \right). \quad (41)$$

Similar to (35), applying Lemma 4 to $\hat{P}_2 \omega$, (41) can be estimated by

$$\begin{aligned} z_2 D^\alpha z_2 &\leq z_2 \left(x_3 + f_2 - \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1) - \sum_{s=0}^1 \frac{\partial \alpha_1}{\partial \hat{D}_s} \hat{D}_{s+1} + \frac{\hat{P}_2 \hat{P}_2^T}{4\gamma_2} z_2 \right) \\ &\quad + \gamma_2 \omega^T \omega. \end{aligned} \quad (42)$$

Then, the fractional derivative of V_2 is given by

$$\begin{aligned} D^\alpha V_2 &\leq D^\alpha V_1 + z_2 D^\alpha z_2 \\ &\leq -k_1 z_1^2 + \sum_{s=1}^2 \gamma_s \omega^T \omega + z_2 z_3 \\ &\quad + z_2 \left[\alpha_2 + z_1 + f_2 + \frac{\hat{P}_2 \hat{P}_2^T}{4\gamma_2} z_2 - \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1) \right. \\ &\quad \left. - \sum_{s=0}^1 \frac{\partial \alpha_1}{\partial \hat{D}_s} \hat{D}_{s+1} \right], \end{aligned} \quad (43)$$

where the first inequality is based on (37) and (3), respectively.

Similar to Step 1, the virtual control law is chosen as

$$\begin{aligned} \alpha_2 &= -k_2 z_2 - z_1 - f_2 + \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1) + \sum_{s=0}^1 \frac{\partial \alpha_1}{\partial \hat{D}_s} \hat{D}_{s+1} \\ &\quad - \frac{\hat{P}_2 \hat{P}_2^T}{4\gamma_2} z_2, \end{aligned} \quad (44)$$

where k_2 is a positive constant. With virtual controller (43), (44) can be expressed as

$$D^\alpha V_2 \leq -k_1 z_1^2 - k_2 z_2^2 + z_2 z_3 + \sum_{s=1}^2 \gamma_s \omega^T \omega. \quad (45)$$

Step i ($i = 3, \dots, n-1$):

In this step, we repeat the procedure in a recursive way. It follows from (27) and (28) that

$$\begin{aligned} D^\alpha z_i &= D^\alpha x_i - D^\alpha \alpha_{i-1} \\ &= x_{i+1} + f_i + P_i \omega - D^\alpha \alpha_{i-1}, \end{aligned} \quad (46)$$

with

$$D^\alpha \alpha_{i-1} = \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (x_{s+1} + f_s + P_s \omega) + \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{D}_s} \hat{D}_{s+1}. \quad (47)$$

Substituting (47) into (46) yields

$$\begin{aligned} z_i D^\alpha z_i &= z_i \left(x_{i+1} + f_i + \hat{P}_i \omega - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (x_{s+1} + f_s) \right. \\ &\quad \left. - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{D}_s} \hat{D}_{s+1} \right) \\ &\leq z_i \left(x_{i+1} + f_i - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (x_{s+1} + f_s) \right. \\ &\quad \left. - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{D}_s} \hat{D}_{s+1} + \frac{\hat{P}_i \hat{P}_i^T}{4\gamma_i} z_i \right) + \gamma_i \omega^T \omega, \end{aligned} \quad (48)$$

where the inequality is based on Lemma 4 with $p = q = 2$ and $\varepsilon = 1/\sqrt{2\gamma}$.

With (48), it can be derived that the Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \quad (49)$$

has the following fractional derivative:

$$\begin{aligned}
D^\alpha V_i &\leq D^\alpha V_{i-1} + z_i D^\alpha z_i \\
&\leq -\sum_{s=1}^{i-1} k_s z_s^2 + z_i z_{i+1} + \sum_{s=1}^i \gamma_s \omega^T \omega + z_i \\
&\quad \cdot \left[\alpha_i + z_{i-1} + f_i - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (x_{s+1} + f_s) \right. \\
&\quad \left. - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{D}_s} \hat{D}_{s+1} + \frac{\hat{P}_i \hat{P}_i^T}{4\gamma_i} z_i \right]. \tag{50}
\end{aligned}$$

The virtual control law is chosen as

$$\begin{aligned}
\alpha_i &= -k_i z_i - z_{i-1} - f_i + \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (x_{s+1} + f_s) \\
&\quad + \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{D}_s} \hat{D}_{s+1} - \frac{\hat{P}_i \hat{P}_i^T}{4\gamma_i} z_i, \tag{51}
\end{aligned}$$

where k_i is a positive constant. Next, substituting (51) into (50) results in

$$D^\alpha V_i \leq -\sum_{s=1}^i k_s z_s^2 + z_i z_{i+1} + \sum_{s=1}^i \gamma_s \omega^T \omega. \tag{52}$$

Step n :

In the final step, select a Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2. \tag{53}$$

It follows from (27) and (28) that

$$\begin{aligned}
D^\alpha z_n &= D^\alpha x_n - D^\alpha \alpha_{n-1} \\
&= f + gu + f_n + P_n \omega - D^\alpha \alpha_{n-1}, \tag{54}
\end{aligned}$$

with

$$D^\alpha \alpha_{n-1} = \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (x_{s+1} + f_s + P_s \omega) + \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{D}_s} \hat{D}_{s+1}. \tag{55}$$

Substituting (55) into (54) yields

$$\begin{aligned}
z_n D^\alpha z_n &= z_n \left(f + gu + f_n + \hat{P}_n \omega - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (x_{s+1} + f_s) \right. \\
&\quad \left. - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{D}_s} \hat{D}_{s+1} \right) \\
&\leq z_n \left(f + gu + f_n - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (x_{s+1} + f_s) \right. \\
&\quad \left. - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{D}_s} \hat{D}_{s+1} + \frac{\hat{P}_n \hat{P}_n^T}{4\gamma_n} z_n \right) + \gamma_n \omega^T \omega, \tag{56}
\end{aligned}$$

where the inequality is based on Lemma 4 with $p = q = 2$ and $\varepsilon = 1/\sqrt{2\gamma}$.

Then, the fractional-order derivative of V_n can be estimated by

$$\begin{aligned}
D^\alpha V_n &\leq D^\alpha V_{n-1} + z_n D^\alpha z_n \\
&\leq -\sum_{s=1}^{n-1} k_s z_s^2 + \sum_{s=1}^n \gamma_s \omega^T \omega + z_n \left[f + gu + f_n + z_{n-1} \right. \\
&\quad \left. + \frac{\hat{P}_n \hat{P}_n^T}{4\gamma_n} z_n - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (x_{s+1} + f_s) - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{D}_s} \hat{D}_{s+1} \right]. \tag{57}
\end{aligned}$$

Introduce the control law as follows:

$$u = \frac{1}{g} \left[-f - \left(k_n + \frac{\hat{P}_n \hat{P}_n^T}{4\gamma_n} \right) z_n - z_{n-1} - f_n \right. \tag{58}$$

$$\left. + \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (x_{s+1} + f_s) + \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{D}_s} \hat{D}_{s+1} \right],$$

where k_i is a positive constant. Next, substituting (58) into (57) results in

$$D^\alpha V_n \leq -\sum_{s=1}^n k_s z_s^2 + \sum_{s=1}^n \gamma_s \omega^T \omega. \tag{59}$$

(1) When $\omega(t) = 0$, $D^\alpha V_n \leq 0$ can be guaranteed. Then, the output signal $y(t)$ asymptotically tracks the desired reference signal $y_r(t)$.

(2) When $\omega(t) \neq 0$, (59) can be estimated by

FIGURE 1: The tracking performance in Example 1.

FIGURE 2: The state responses in Example 1.

$$D^\alpha V_n \leq -k_1 z_1^2 + \bar{\gamma} \omega^T \omega, \quad (60)$$

i.e.,

where $\bar{\gamma} = \sum_{s=1}^n \gamma_s$.

To simplify the notation, V_n is abbreviated to be V . It follows from (53) that $V \geq 0$. For the H_∞ performance, assume the initial condition is zero. Then, $V(0) = 0$ is true. Calculate the Caputo fractional-order integral with order α of (60):

$$V = D^{-\alpha}(D^\alpha V) \leq D^{-\alpha}(-k_1 z_1^T z_1 + \bar{\gamma} \omega^T \omega). \quad (61)$$

It means that

$$0 \leq V \leq D^{-\alpha}(-k_1 z_1^T z_1 + \bar{\gamma} \omega^T \omega), \quad (62)$$

$$D^{-\alpha}(k_1 z_1^T z_1) \leq D^{-\alpha}(\bar{\gamma} \omega^T \omega). \quad (63)$$

Then,

$$\begin{aligned} & \frac{k_1}{\Gamma(\alpha)} \int_0^t z_1^T(\sigma) (t-\sigma)^{\alpha-1} z_1(\sigma) d\sigma \\ & \leq \frac{\bar{\gamma}}{\Gamma(\alpha)} \int_0^t \omega^T(\sigma) (t-\sigma)^{\alpha-1} \omega(\sigma) d\sigma. \end{aligned} \quad (64)$$

Let $t \rightarrow \infty$:

FIGURE 3: The control input in Example 1.

FIGURE 4: The tracking performance in Example 2.

$$\|z_1(t)\|_\alpha \leq \sqrt{\frac{\gamma}{k_1}} \|\omega(t)\|_\alpha, \quad (65)$$

i.e.,

$$\|y(t) - y_r(t)\|_\alpha \leq \gamma \|\omega(t)\|_\alpha, \quad (66)$$

with $\gamma = \sqrt{\gamma/k_1}$.

It means that the effect of disturbances on the output tracking error is attenuated to a given degree γ . Therefore, the H_∞ tracking performance can be achieved. $\square$

5. Simulation Results

In order to verify the excellent properties of the proposed tracking controller, a numerical example is presented as Example 1, and two practical examples are given as Examples 2 and 3.

5.1. Example 1. Consider system (10) with $\alpha = 0.93$, $f_0(z) = -z^3$, $f_1(\bar{x}_1) = 0.1x_1(t)$, $f_2(\bar{x}_2) = 0.5x_1(t) + x_2(t)$, $f_3(\bar{x}_3) = 0.2x_1(t) - 0.01x_2(t) + x_3(t)$, $g(x) = x_1^2(t)$, $f(x) = x_2^2(t)$, $P_1 = P_2 = P_3 = 0.1$, and $\omega(t) = \cos(t)e^{-0.5t}$. The desired reference signal is set to $y_r(t) = 0.5 \sin(\pi t/5)$. The initial state $[x_1(0), x_2(0), x_3(0)]^T = [0, 0.3226, -0.2443]^T$ and $z(0) = 0.1$ are considered.

FIGURE 5: The state responses in Example 2.

The virtual laws are chosen as

$$\begin{aligned}\alpha_1 &= -\left(k_1 + \frac{P_1 P_1^T}{4\gamma_1}\right)z_1 - f_1 + D^\alpha \alpha_0, \\ \alpha_2 &= -\left(k_2 + \frac{\hat{P}_2 \hat{P}_2^T}{4\gamma_2}\right)z_2 - z_1 - f_2 + \frac{\partial \alpha_1}{\partial x_1}(x_2 + f_1) \\ &\quad + \sum_{s=0}^1 \frac{\partial \alpha_1}{\partial \hat{D}_s} \hat{D}_{s+1}.\end{aligned}\quad (67)$$

The adaptive control law is determined by

$$\begin{aligned}u &= \frac{1}{g} \left[-f - \left(k_3 + \frac{\hat{P}_3 \hat{P}_3^T}{4\gamma_3}\right)z_3 - z_2 - f_3 + \sum_{s=1}^2 \frac{\partial \alpha_2}{\partial x_s}(x_{s+1} + f_s) \right. \\ &\quad \left. + \sum_{s=0}^2 \frac{\partial \alpha_{n-1}}{\partial \hat{D}_s} \hat{D}_{s+1} \right],\end{aligned}\quad (68)$$

where the design parameters are set to $k_1 = 12$, $k_2 = 10$, $k_3 = 10$, $\gamma_1 = 0.01$, $\gamma_2 = 0.2$, and $\gamma_3 = 4$.

Simulation results of Example 1 are shown as follows. Figure 1 gives the tracking performance, from which it can be concluded that the asymptotic tracking can be realized. It is obviously seen that the state variables are influenced by external disturbances in Figure 2. Figure 3 indicates the trajectories of the control input.

5.2. Example 2. Consider the fractional-order model of the magnetic levitation (FO-MAGLEV) suspension system in [49]:

FIGURE 6: The control input in Example 2.

$$\begin{cases} D^\alpha x_1(t) = x_2(t) + P_1 \omega(t), \\ D^\alpha x_2(t) = x_3(t) + P_2 \omega(t), \\ D^\alpha x_3(t) = -6233.8x_1(t) - 65.2x_2(t) - 4.8x_3(t) \\ \quad + 0.9212u + P_3 \omega(t), \\ y(t) = x_1(t), \end{cases} \quad (69)$$

with $\alpha = 0.93$, $P_1 = 0.1$, $P_2 = 0$, $P_3 = -65.17 * 0.1$, $y_r(t) = 0.5 \sin(\pi t/5)$, $[x_1(0), x_2(0), x_3(0)]^T = [0, 0.3226, 0.046]^T$, and $\omega(t) = \cos(t)e^{-0.5t}$.

The virtual laws and the adaptive control law are designed as (27) and (26) with the parameters chosen as

FIGURE 7: The tracking performance in Example 3.

FIGURE 8: The state response in Example 3.

FIGURE 9: The control input in Example 3.

$k_1 = 16$, $k_2 = 12$, $k_3 = 10$, $\gamma_1 = 0.01$, $\gamma_2 = 0.1$, and $\gamma_3 = 5$. Figures 4–6 demonstrate the simulation results for Example 2. The tracking performance is illustrated in Figure 4. The state responses of Example 2 are shown in Figure 5. Figure 6 presents the control input of Example 2. It is seen that the tracking controller succeeds to make the output tracking error converge to zero without external disturbances and to attenuate the effect of disturbances on the tracking error.

5.3. Example 3. Consider the fractional-order model of the horizontal platform system (FO-HPS) given in [50]:

$$\begin{cases} D^\alpha x_1(t) = x_2(t) + P_1 \omega(t), \\ D^\alpha x_2(t) = -\bar{a} x_2(t) - b \sin x_1(t) + l \sin x_1(t) \cos x_1(t) \\ \quad + h \cos \tau t + u(t) + P_2 \omega(t), \\ y(t) = x_1(t), \end{cases} \quad (70)$$

with $\alpha = 0.93$, $P_1 = 0$, $P_2 = 1$, $y_r(t) = 0.5 \sin(\pi t/5)$, $\bar{a} = 4/3$, $b = 3.776$, $l = 4.6 \times 10^{-6}$, $h = 3.4/4$, $\tau = 1.8$, $[x_1(0), x_2(0)]^T = [0, 0.3226]^T$, and $\omega(t) = \cos(t)e^{-0.5t}$.

The virtual laws and the adaptive control law are determined by (27) and (26), where the design parameters are chosen as $k_1 = 10$, $k_2 = 10$, $\gamma_1 = 0.01$, and $\gamma_2 = 0.05$.

The simulation results of Example 3 are presented in Figures 7–9. The tracking performance of Example 3 is shown in Figure 7. The state responses of Example 3 are given in Figure 8. Figure 9 presents the control input of Example 3. It is observed that the reference signal can be tracked closely by the output signal and the impact of the external disturbances on the output tracking error can be reduced to a certain extent.

6. Conclusion

The ADD problem has been addressed for a type of nonlinear FOSs with time-varying disturbances and zero dynamics. With the help of the backstepping method, a stabilising state feedback control law has been constructed, which guarantees that asymptotic tracking is achieved without external disturbances and the effect of the external disturbances on the output tracking error can be reduced to a certain extent. In addition, a numerical example and two practical examples are given to validate the theoretical results.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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