Research Article

A Mode Selected Mixed Logic Dynamic Model and Model Predictive Control of Buck Converter

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A Mixed Logic Dynamic (MLD) model and control method based on mode selection are proposed for the Buck converter. In establishing the hybrid system model, the factors such as the inductor current are neglected, and the Model Predictive Control (MPC) is used to switch the most favorable working state of the control target. Since the modeling process ignores the inductance and current, it is necessary to convert the optimization prediction control resulting to avoid the problem that the model is inconsistent with the control object. The method proposed in this paper uses fewer auxiliary logic variables and mixed logic variables in the modeling process, simplifying the model and improving the solution speed. This method can not only make the Buck converter work in the Continuous Current Mode (CCM) but also work in the Discontinuous Current Mode (DCM), extending the adjustment range of the Buck converter. The simulation results show that the proposed method has a better control performance than the traditional MLD model.

1. Introduction

DC-DC converters have been greatly used in daily life. The traditional DC-DC converter modeling methods mostly use the average and approximate ones to obtain the linear model of the converter, such as the state-space small signal averaged model. The linear models and theories are used to design the pulse width modulation- (PMW-) based controllers, whether the controller is MPC, neural networks, fuzzy control, or others [1–4]. For the traditional method, its accuracy depends on the working point. If the working point changes over a large range, the performance of the controller will degrade. In addition, the dynamics within the switching period are ignored in the average model, which may lead to fast-scale instability [5].

DC-DC converters belong to a typical hybrid system with a continuous part and discrete part. The continuous part is the linear system determined by the switching state, and the discrete part is the switching action of the switch. The hybrid modeling of the converter is accurate in the sense of no approximation or linearization used; its performance does not depend on the working point. In the traditional modeling method, it is difficult to fully reflect the nonlinear characteristics of the converter. For hybrid system modeling of the converters, methods such as Piecewise Affine, Hybrid Automata [6, 7], and MLD model are proposed [8, 9]. Among them, the MLD model provides a framework for modeling DC-DC converter considering different working modes, with all constraints within an optimal problem [5, 10].

At present, the hybrid modeling theory has been gradually applied to the modeling and control of DC-DC converters [9, 11, 12]. As the converters have the discrete characteristics, the mixed logic dynamic model is built by unifying the two parts in one system without approximation and averaging [13]. Thus, an accurate model for the switching converter is established with the MLD model reflecting the dynamic characteristics of the system. For the application of the switching converter, Mihaela Sbarciog proposed a MLD model of the boost converter [2], determination of the specific implementation method of the MLD model applied to the converter. However, this method uses too many logical variables and the model is more complicated. The converter cannot be operated in the DCM mode.
Hejri and Giua [14] proposed the concept of forward MLD (FMLD) model and backward MLD (BMLD) model; he classified the traditional MLD model as BMLD and solved the one-step delay of the BMLD model when working mode switch in the FMLD model. In the subsequent work, the proposed model is improved and a voltage PI controller is added externally for output compensation [15]. This method allows the system to work in the DCM mode and also uses more logic variables. Hejri and Mokhtari proposed a boost converter control model including CCM and DCM modes, dividing the control problem into several regions for the optimal solution of predictive control. The control rate of the corresponding region is obtained by looking up the table. So, the operation speed is enhanced by this method [16]. However, this method does not improve the model simplification but chooses to optimize the control method. Ren et al. proposed a simplified MLD modeling method in which he uses fewer logic variables to reduce the solution time and also uses the PI compensator to adjust the current reference value in the outer loop to reduce the steady-state error of the output voltage [5]. However, this method limits the operating range of the DC-DC converter due to the constraints of the MLD model when the converter is controlled, so that the system cannot switch from the CCM mode to the DCM mode.

Aiming at the modeling control problem by the Buck converter, this paper proposes a mode selected MLD (MSMLD) modeling and control method. The control of the object system is realized through the establishment of the MLD model, the predictive control, and the transformation of predictive control results under constraints. Experimental results show that the system can not only work in CCM mode and DCM mode but also use two discrete variables by which to improve the speed of the algorithm. Experimental results prove that the proposed method has a better control performance.

2. Buck Converter MSMLD Model and Its Predictive Control Method

In this paper, the inductor current is ignored in the modeling of Buck converter. The modeling process only considers the working state of the converter to establish the MLD model. It converts the general control problem into the optimal state selection according to the control target and by the MLD model. The most favorable working state for the control target is selected by the MPC method, which the switch switched. Since the MLD modeling ignores the inductor current factor, it is necessary to convert the optimization result of prediction control under the constraint of the inductor current to avoid model mismatch.

2.1. Model Establishment. The principle of the Buck converter and the equivalent circuit is shown in Figure 1, where \( L \) is the energy storage inductor, \( C \) the output filter capacitor, \( R \) the load resistor, \( V_g \) the voltage input, \( S \) the ideal switch tube, and \( D \) the freewheeling diode. The circuit state is switched by the on/off of the switch tube, with the circuit entering the first switching state when the switch tube conduction is shown in Figure 1(b). In this state, the power supplies energy into the energy storage element in the circuit; the circuit enters into the second switching state when the switch tube turns off and is shown in Figure 1(c). The energy storage component gradually bleeds. At this time, the inductor current decreases linearly. The inductor current drops to zero and the switching transistor not turned on, and the circuit enters the third switching state, as shown in Figure 1(d).

The independent states of the Buck converter are the inductor current \( i_L(t) \) and the output voltage \( v_o(t) \) across the output capacitor. \( x(t) = [i_L(t), v_o(t)] \) is the status vector. The equation to state for each case can be written as

\[
\dot{x}(t) = A_n x(t) + B_n, \quad n = 1, 2, 3,
\]

where

\[
A_1 = \begin{bmatrix}
0 & -\frac{1}{L} \\
-\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
V_g \\
0
\end{bmatrix}
\]

\[\text{state 1},\]

\[
A_2 = \begin{bmatrix}
0 & -\frac{1}{L} \\
-\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[\text{state 2},\]

\[
A_3 = \begin{bmatrix}
0 & 0 \\
0 & -\frac{1}{RC}
\end{bmatrix}, \quad B_3 = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[\text{state 3},\]

\[
C = \begin{bmatrix}
1 & 0
\end{bmatrix},
\]

where \( n = 1, 2, \) and 3, which, respectively, corresponds to the state in which the switch tube is turned on, the switch tube is turned off, and the inductor current is greater than zero. The switch tube turnoff inductor current is equal to zero. The three logic variables are defined corresponding to the above three state equations:

\[
\delta_1 = 1 \iff \dot{x}(t) = A_1 x(t) + B_1,
\]

\[
\delta_2 = 1 \iff \dot{x}(t) = A_2 x(t) + B_2,
\]

\[
\delta_3 = 1 \iff \dot{x}(t) = A_3 x(t) + B_3.
\]

Since the converter can only be in one state at the same time, the logical variables have the following constraints:

\[
\delta_1 + \delta_2 + \delta_3 = 1.
\]
In the above modeling process, the resulting logical variables can be used to represent the state equation to the system.

\[ x(t) = \delta_1 (A_1 x(t) + B_1) + \delta_2 (A_2 x(t) + B_2) + \delta_3 (A_3 x(t) + B_3), \]

\[ y(t) = C x(t). \]

Further,

\[ \dot{x}(t) = \begin{bmatrix} \delta_1 = 1 & \Rightarrow & \delta_2 = 0 \\ \delta_2 = 1 & \Rightarrow & \delta_1 + \delta_2 \geq 0, \\ \delta_1 + \delta_2 \leq 1. \end{bmatrix} \]

According to \( \delta_1 + \delta_2 + \delta_3 = 1 \), the state variables \( \delta_1 \) and \( \delta_2 \) can be used to represent the state equation to the system. In the above modeling process, the resulting logical variables are constrained as follows:

\[ \begin{cases} \delta_1 = 1 \iff \delta_2 = 0 \\ \delta_2 = 1 \iff \delta_1 + \delta_3 \geq 0, \\ \delta_1 + \delta_3 \leq 1. \end{cases} \]

State 1: \( \delta_1 = 1, \delta_2 = 0 \); state 2: \( \delta_1 = 0, \delta_2 = 1 \); state 3: \( \delta_1 = 0, \delta_2 = 0 \). Discretize the system as follows:

\[ k_1 = \frac{1}{L} T, \]

\[ k_2 = \frac{1}{C} T, \]

\[ k_3 = 1 - \frac{1}{RC} T, \]

\[ k_4 = \frac{V}{L} T, \]

\[ x(k + 1) = \begin{bmatrix} 1 & k_1 (\delta_1 + \delta_2) \\ k_2 (\delta_1 + \delta_2) \\ k_3 \end{bmatrix} x(k) + \begin{bmatrix} k_4 \delta_1 \\ 0 \end{bmatrix} u(k), \]

where \( T \) is the sampling period. The equations of state are written separately:

\[ x_1(k + 1) = x_1(k) + k_1 (\delta_1 + \delta_2) x_2(k) + k_4 \delta_1, \]

\[ x_2(k + 1) = 0 \]

The auxiliary mixed variable \( z(k) = \delta \cdot x(k) \) is defined. There is the product of the logical variable and the continuous variable, shown as

\[ z_1(k) = \delta_1 x_1(k), \]

\[ z_2(k) = \delta_1 x_2(k), \]

\[ z_3(k) = \delta_1 x_3(k), \]

\[ z_4(k) = \delta_1 x_4(k). \]

The resulting auxiliary mixed variable constraints are as follows:

\[ z_n \leq M \delta_n, \]

\[ z_n \geq m \delta_n, \]

\[ z_n \leq f(x) - m (1 - \delta_n), \]

\[ z_n \geq f(x) - M (1 - \delta_n), \]

where \( M \) and \( m \) are the maximum and minimum or upper and lower bounds of \( f(x) \).

The equation of state is organized as follows:

\[ x_1(k + 1) = x_1(k) + k_1 \delta_1 + k_2 \delta_2 + k_3 \delta_3 + z_1(k), \]

\[ x_2(k + 1) = k_4 \delta_1 + k_2 \delta_2 + k_3 \delta_3. \]

The MLD model of the Buck converter is available:

\[ x(k + 1) = \begin{bmatrix} 1 & 0 & k_1 \\ 0 & k_3 \\ k_2 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \end{bmatrix} u(k), \]

\[ y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k), \]

where the auxiliary logical variable \( \delta(k) = \delta_1(k) \) and the auxiliary mixed variable \( z(k) = [z_1 \ z_2 \ z_3 \ z_4]^T \). Let \( u(k) = \delta_1(k) \) be the input. The logical variable \( \delta_1(k) \) in the modeling process is also included in \( z(k) \). The model contains only two auxiliary logic variables and four mixed logic variables, greatly reducing the complexity of the model. This MLD is simpler than that in the literature [2, 5, 14–16].

### 2.2. Mixed Logic Dynamic Model Predictive Control

Since the mixed logic dynamic model contains information such as continuous dynamics, discrete states and physical constraints, the model predictive control is used to optimize the continuous and discrete parts of the model, which are significant control developments [17, 18]. The optimized control problem of this paper can be expressed: given the current system state \( x(k) \) and the control target \( y^+(k) \), the optimal control sequence \( T_p \) is solved within the prediction step \( \{u_0, u_1, \ldots, u_{T-1}\} \), and the first control amount in the control process.
sequence is applied to the system. Use the following objective function:

$$\min J = \|x(k) - x_0\|_{Q_1}^2 + \|y(k) - y_0\|_{Q_2}^2,$$ \hspace{1cm} (14)

where $Q_1$ and $Q_2$ are the weights matrices, $x$ and $y$ are the current state variable and the output value, and $x_0$, $y_0$ each represents the state variable and the expected value of the output.

The MLD model is included in the constraint, i.e., the state equation of the model is used as the equality constraint. The model’s inequality constraint matrix is used as the inequality constraint of the function to be optimized:

$$\begin{cases}
    \text{MLD model}, \\
x(0 \mid t) = x(t), \\
x_{\min} \leq x(1 \mid t), x(2 \mid t), \ldots, x(T_p \mid t) \leq x_{\max}.
\end{cases} \hspace{1cm} (15)$$

Since the model contains both integer and noninteger variables, the above formula is written as a form of mixed integer quadratic programming (MIQP) and solved by the branch and bound (B&B) algorithm. The principle is to divide the total solution space into smaller subsets and to calculate the target lower bound for each subset (for the minimum problem), gradually reducing the optimal solution range by continuously discarding the subsets that do not meet the requirements. Determining the optimal solution is achieved by continuously dividing the feasible solution range and gradually reducing the upper and lower limits of the solution.

The hybrid system controller based on the MPC and MLD models has the properties of stability, traceability, and constraint fulfillment, and its theoretical basis can be found in [22, 23].

2.3. Predictive Control Result Conversion. In the process shown above, the predictive control only selects the most favorable working state for the control target according to the object model, and the modeling process does not consider inductor current. If the above optimization results are directly used for control, the prediction model will be inconsistent with the actual object, causing a control deviation. Therefore, the result of the optimization solution needs to be converted according to the inductor current. In this process, the normal state and the “morbid state” (the state in which the system state is indicated by the optimized switching state and the inductor current is unreasonable) will appear, which is described in detail below.

Let $\delta' = [\delta_1, \delta_2]$ be the logic variable. For the inductor current $i_i(t) > 0$, the optimization results are shown in Table 1.

The switching state obtained by the predictive control optimization solution is $\delta' = [0 \ 0]^T$, which means that the system entering state3 at the next time is more favorable to the control target. However, this is inconsistent with the inductor current greater than zero, which is called the “morbid” state. It means that the inductor current is greater than that in the case of 0, and that the system cannot enter the state3, so when this happens, the state should be converted to $\delta' = [0 \ 1]^T$ into state2. When the inductor current is greater than zero, the other predictive control optimization results are normal.

Similarly, the state of the inductor current $i_i(t) = 0$ also has the optimization results in Table 2.

When the switching state obtained by the predictive control optimization solution is $\delta' = [0 \ 1]^T$, it means that at the next time the system entering state2 is more favorable to the control target, but that the inductance current is equal to zero, which is also called as a “morbid” state. In this case, the state should be converted to $\delta' = [0 \ 0]^T$ and enter state3.

According to the above state switching process, the state transition of the Buck converter is shown in Figure 2.

The state in the “[ ]” sign in Figure 2 is the “morbid state” obtained by the optimal control. This “morbid state” is impossible for the real system; the state without the “[ ]” bracket is the normal state, and the system state transitions can be directly performed based on predictive control optimization results. In the mixed logic dynamic model of a general Buck converter, these “morbid” states are limited by constraints and therefore do not occur in the model. However, the modeling method proposed in this paper will have these morbidities. It is necessary to adjust these “morbid” optimization results according to the inductor current to complete the prediction model state transition, which is represented by the symbol “≥.”

The model prediction control flow chart based on the MSMLD model is shown in Figure 3.

3. Experimental Result

The simulation results of the Buck converter given in this section are shown in Figure 4. The system parameters are given as follows: input voltage 15 V, energy storage inductance 1 mH, filter capacitor 300 µF, load resistance 13 Ω, target output voltage 4 V, and sampling frequency 20 kHz (the higher the frequency, the smaller the voltage fluctuation, but the control period will be shorter; shorter control cycles are detrimental to slower predictive control that translates into MIQP problems), directly with the load voltage as the control target. It can be seen from Figure 4 that the proposed method allows the Buck converter to work not only in the CCM state but also in the DCM state. Only two discrete variables are used in the model.

For the MLD modeling method proposed in this paper, the MLD model does not contain the constraint on the inductor current. Therefore, it is necessary for the model prediction control optimization result to be converted according to the inductor current. Otherwise, the prediction model and the control model will be inconsistent. The control results are shown in Figure 5. In the control process that does not include the conversion of the predictive control optimization results, the lack of information on the inductor current makes the control deviation of the model and the controlled system inconsistent. When the deviation is large, the system may get out of control.

The SMLD method proposed in [5] cannot switch to DCM mode due to the constraint limitation in controlling.
Table 1: Predictive control optimization result conversion.

<table>
<thead>
<tr>
<th>Inductor current</th>
<th>Predictive control optimization result</th>
<th>Result conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 &gt; 0 )</td>
<td>( \delta' = [1 0]^T )</td>
<td>( \delta = [0 1]^T )</td>
</tr>
<tr>
<td>( x_2 &gt; 0 )</td>
<td>( \delta' = [0 1]^T )</td>
<td></td>
</tr>
<tr>
<td>( x_3 &gt; 0 )</td>
<td>( \delta' = [0 0]^T )</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Optimization result conversion of the model predictive control.

<table>
<thead>
<tr>
<th>Inductor current</th>
<th>Predictive control optimization result</th>
<th>Result conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 = 0 )</td>
<td>( \delta' = [1 0]^T )</td>
<td>( \delta = [0 0]^T )</td>
</tr>
<tr>
<td>( x_2 = 0 )</td>
<td>( \delta' = [0 1]^T )</td>
<td></td>
</tr>
<tr>
<td>( x_3 = 0 )</td>
<td>( \delta' = [0 0]^T )</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Buck converter state transition diagram.

The Matlab tic/toc method is used to obtain the running time by different algorithms. The prediction step size, control step length, and solution time length of the different algorithms are the same. The average running time is calculated 20 times. All are shown in Table 1. It can be seen that the method is simpler and faster than the method proposed in [2, 5] that it greatly improves the controllable range of the system. The MIQP problem is NP-hard [22]. The computational power required for the MIQP problem and B&B algorithm is large, especially when the prediction horizon is not one. Therefore, the simplicity of the method is of importance for the Buck converter [5, 10].

The SMLD proposed in [5] has fewer optimized discrete variables and is combined with the idea of one-step prediction that the inductor current is set to zero [14]. By comparing the model prediction control optimization indicator, the Buck converter can enter the DCM. The combined method is compared with the method proposed in this paper, as shown in Figure 7. It can be seen that the combined method and the method proposed in this paper are basically consistent in control performance. However, the method is worse than the proposed method in terms of the solution speed and the number of discrete variables. The solution speed and number of auxiliary variables are shown in Table 3.

In the case where the control target changes, by comparing the effects of the method with the MSMLD, the SMLD, and the SMLD for setting inductor current, the result is shown in Figure 8. The control target is changed from 8 V to 10 V. It can be seen that the MSMLD proposed in this paper still has a good control effect.

The control flow chart based on the MSMLD model is shown in Figure 3. It can be seen that the control target is changed from 8 V to 10 V. It can be seen that the MSMLD proposed in this paper still has a good control effect.

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Figure 4: Curve of inductor current, output voltage, system input, and model state of MSMLD. (a) System state and model state. (b) Control input. (c) Auxiliary logic variable.

Figure 5: MSMLD control curve without state conversion. (a) System state and model state. (b) Control input. (c) Auxiliary logic variable.
Figure 6: Comparison of SMLD and the proposed method. (a) System states of SMLD and MSMLD. Control input of (b) SMLD and (c) MSMLD.

Figure 7: Comparison of SMLD combined with inductor current zero prediction and proposed method. (a) System state of one step prediction and MSMLD. Control input of (b) one step prediction and (c) MSMLD.
In a noisy environment, the control effect of the proposed method will decrease. However, the effect of noise can be reduced by adjusting the model feedback coefficients ($\alpha$), as shown in Figure 10. The noise is Gaussian noise with a mean of zero.

In practical situation, the variation of load resistance makes the current reference change. Due to the uncertainty of the parameter, the steady state error of output voltage will increase. Some methods for hybrid model control have been proposed to solve this problem [5, 9, 16, 21]. Most methods

<table>
<thead>
<tr>
<th>Model</th>
<th>Computational time(s)</th>
<th>Number of auxiliary variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMLD model in [5]</td>
<td>2.024556</td>
<td>3 auxiliary, 4 mixed</td>
</tr>
<tr>
<td>MLD in [2]</td>
<td>2.345872</td>
<td>7 auxiliary, 7 mixed</td>
</tr>
<tr>
<td>MSMLD in this paper</td>
<td>1.614711</td>
<td>1 auxiliary, 4 mixed</td>
</tr>
<tr>
<td>Combination method in [5] and method in [14]</td>
<td>3.104398</td>
<td>3 auxiliary, 4 mixed</td>
</tr>
</tbody>
</table>

**Figure 8:** Inductor current and output voltage responses of three methods when target voltage changes.

**Figure 9:** Comparison of different control methods.
are more complicated than PI controller [5, 16]. In order to eliminate steady state error, the outer loop uses the PI controller, with the inner loop being MSMLD in this paper. In the case where the load resistance changes, by comparing the effects of the method with the MSMLD and the MSMLD with PI controller, the result is shown in Figure 11. The load resistance is changed from 13 Ω to 26 Ω at $t = 500$. It can be seen that the MSMLD with PI controller can eliminate the steady state error caused by the variation of load resistance.
4. Conclusion

In this paper, a mode selected MLD model is proposed for the Buck converter, which switches the most favorable state of the control target at the current time. Compared with the traditional Buck converter MLD model, fewer auxiliary logic variables and mixed logic variables are used, so it has a faster solution speed. This method considers the state of the “morbid” into the system that was originally unreachable in the model, rather than it being constrained to be outside of the solution domain. The “morbid” state is transformed according to the magnitude of the inductor current to migrate to the correct system state, greatly expanding the feasible domain of the system. The effectiveness of the proposed method is proved by simulation experiments and it has a better system performance than the traditional MLD model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References


