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Research Article

Modeling Repayment Behavior of Consumer Loan in Portfolio across Business Cycle: A Triplet Markov Model Approach

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With a view to develop a more realistic model for credit risk analysis in consumer loan, our paper addresses the problem of how to incorporate business cycles into a repayment behavior model of consumer loan in portfolio. A particular Triplet Markov Model (TMM) is presented and introduced to describe the dynamic repayment behavior of consumers. The particular TMM can simultaneously capture the phases of business cycles, transition of systematic credit risk of a loan portfolio, and Markov repayment behavior of consumers. The corresponding Markov chain Monte Carlo algorithms of the particular TMM are also developed for estimating the model parameters. We show how the transition of consumers' repayment states and systematic credit risk of a loan portfolio are affected by the phases of business cycles through simulations.

1. Introduction

The widespread use of consumer loan has not only alleviated the financial distress of consumers but also benefited credit companies. However, with the increasing consumer loan and credit limit, default risk has risen rapidly and even triggered systemic risk such as the subprime crisis in 2007. Dynamic management of consumer loan risk becomes more and more important in the credit business.

It is an important and practically relevant issue to assess and measure credit risk of a consumer loan in portfolio. In order to model the behavior of credit accounts, a stationary Markov chain approach was introduced [1]. Since then, the line of research has been developed [2–4]. In a previous study [5], a Markov chain model based on behavioral scores was developed for establishing the credit risk of consumer loans in portfolio. Moreover, a Markov model (MM) to measure transition of loan accounts is presented [6], in which the Markov transition probability is a function of account states, collection actions applied and borrower characteristics.

For linking the risk states of a debtor with loan default, Hidden Markov Model (HMM) or Double Chain Markov Model (DCMM), as important extension to the simple MM, has been popular in credit modeling in recent years. Giamperi et al. [7] introduced an HMM to model the occurrence of defaults within a bond portfolio. They assume that the default probability of each bond in a portfolio depends on the hidden states of an HMM, which are interpreted as risk states of the bonds. In Banachewicz et al. [8], an HMM is developed to model and predict corporate default frequencies; the hidden states in their paper correspond to the industry credit cycles. In Fitzpatrick and Marchev [9], a multivariate DCMM is applied to credit rating dynamics of financial companies where a hidden process is regarded as a broader economy indicator affecting the transition of credit rating of companies. Quirini and Vannucci [10] link HMM with the analysis of credit risk of consumer loans in portfolio; the hidden states, which are regarded as credit market conditions, are endogenously extracted from repayment behavior records to measure loans' creditworthiness.

The hidden states of the model can be interpreted as systematic credit risk states of the loan portfolio when HMM or DCMM is used to measure the credit risk of a portfolio of consumer loan. Based on HMM or DCMM, one can link the systematic credit risk of a loan portfolio with the repayment behavior of consumers. However, the literature mentioned above leaves a fundamental question unanswered: What influences the transition of both repayment behavior of consumers and systematic credit risk of a loan portfolio?

Numerous studies have found a significant relationship between nonperforming loans (NPLs) and business cycles [11–13]. It is common to observe high NPL ratios during business cycle contraction, owing to the contraction in economic activity and the consequent decline in consumers' ability to repay their debts. In contrast, economic growth allows consumers to keep their finances buoyant and pay back their installments on time, resulting in low NPL ratios [14]. The evolution of consumers' repayment behavior and systematic credit risk of a loan portfolio are affected by business cycle, and different portfolios may be affected differently. A realistic model should account for this.

This paper aims to develop a credit risk model for consumer loan portfolio across business cycle. A particular TMM is presented and introduced to describe the dynamic Markov repayment behavior of consumers in the field of consumer loan. The particular TMM can test for and model two paths of impact of business cycle on the transition of consumers' repayment behavior. One of the paths is that the transition consumers' repayment behavior is directly affected by the business cycle. The other path is the business cycle affects the transition of systematic credit risk state of the loan portfolio and then affects the transition of consumers' repayment behavior. Simulation studies are provided to illustrate how the model functions.

TMM, also known as Triplet Markov Chains, was first proposed in Pieczynski et al. [15]. TMM consists of three processes: a hidden process X, an observed process Y and a third process Z. The model is called a TMM if there exists a stochastic process Z, where Z takes its value in a finite set, such that the triplet (X, Y, Z) is a Markov chain. TMM has been applied in the context of signal processing image processing and others [16–19]. To the authors' knowledge, there is little research that links credit risk measurement with TMM. In view of this, the present paper is the first study to introduce TMM into the credit risk analysis of consumer loan in portfolio.

By assessing the consumers' repayment behavior across business cycles, our proposed approach provides more information about the creditworthiness of consumer loans in portfolio, especially at extreme phases of the business cycle, which is very useful for credit risk management. Scenario analysis is more realistic and persuasive since our approach simultaneously captures the phases of a business cycle, transition of hidden risk states of loan portfolio, and Markov repayment behavior of consumers.

The remainder of this paper proceeds as follows. Section 2 describes the architecture of the particular TMM. Section 3 proposes a specific TMM for consumer loan. Section 4 gives simulation studies to illustrate how the model works in consumer loan. Section 5 provides a conclusion.

2. The Particular Triplet Markov Model Architecture

In this section, we present the structure of our particular TMM. For ease of comprehension and comparison, we first introduce two simpler models: HMM and DCMM.

HMM proposed in Baum and Petrie [20] has been widely used in various problems [21–23]. It consists of two stochastic processes X_t and Y_t, where X_t is a Markov chain and not directly visible (hidden), but the output of another variable Y_t whose distribution depends on the hidden process X_t, is visible. A specific value of output variable is usually called observation. This is shown in Figure 1. A drawback of HMM is that the outputs are assumed to be conditionally independent. Considering that the conditional independence between outputs of an HMM is not always justified, the literature [24] proposes a DCMM as an extension of HMM to overcome this drawback. Figure 2 gives the Bayesian network representation of DCMM.

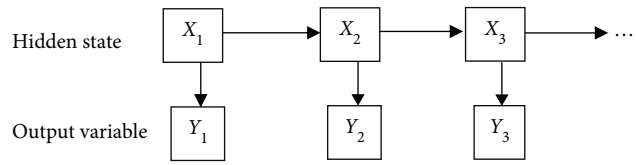

Figure 1: Bayesian network representation of HMM.

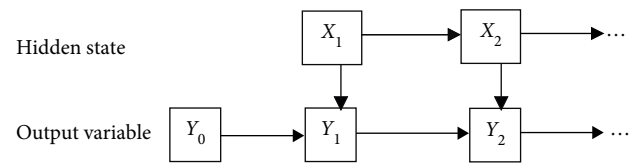

Figure 2: Bayesian network representation of DCMM.

The TMM extends DCMM by adding a discrete value process Z_t, such that the triplet (X_t, Y_t, Z_t) is a Markov chain. The particular TMM proposed in our paper consists of three variables: an additional input variable Z_t, an underlying process X_t which is hidden from the output variable Y_t and depends on the input variable, the output variable Y_t which depends on both the input variable and the underlying process. The model is characterized by the following elements:

(i) $S(Z) = \{1, 2, \ldots, U\}$, the set of input variables.
(ii) $S(X) = \{1, 2, \ldots, M\}$, the set of hidden states.
(iii) $S(Y) = \{1, 2, \ldots, L\}$, the set of output variables.
(iv) $\pi_1 = (\pi_{11}, \pi_{12}, \ldots, \pi_{1M})$, the initial probability distribution of a hidden state.
(v) $A = \{a_{ij}\} = \{P(Z_t = j | Z_{t-1} = i)\}, i, j \in S(Z)$, transition probabilities between successive inputs.
(vi) $B^{(u)} = \{b_{ij}^{(u)}\} = \{P(X_t = j | X_{t-1} = i, Z_t = u)\}, i, j \in S(X)$, $u \in S(Z)$, transition probabilities between successive hidden states given a specific value of input variable.
(vii) $C^{(u,s)} = \{c_{ij}^{(u,s)}\} = \{P(Y_t = j | Y_{t-1} = i, X_t = s, Z_t = u)\}$, $i, j \in S(Y), s \in S(X), u \in S(Z)$, transition probabilities between successive outputs given specific input value and hidden state.

Figure 3 shows the Bayesian network of our particular TMM. We can see that the transition process of X_t and Y_t depends on the specific value of input variable and are all non-homogeneous.

3. Triplet Markov Model for Consumer Loan

In this section, we set up the particular TMM for consumer loan. As described previously, the model consists of three Markov chains. We next construct each of them in the context of consumer loan.

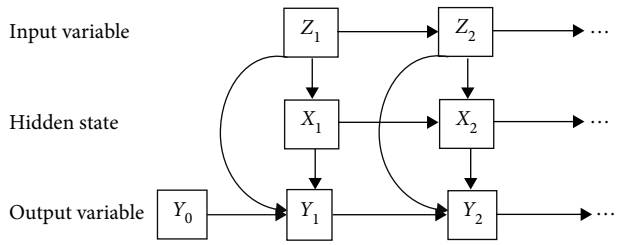

Figure 3: Bayesian network representation of our particular TMM.

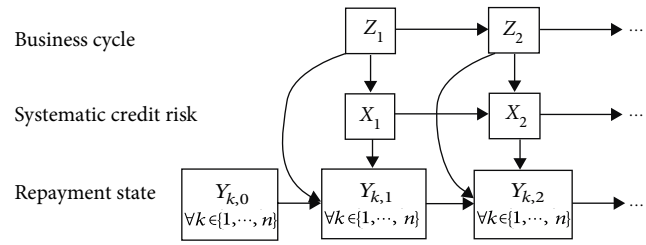

Figure 4: Bayesian network representation of TMM for consumer loan.

We take the business cycle as the input variable of the particular TMM. In facing a real case, practitioners can use the business cycle chronology compiled by the National Bureau of Economic Research (NBER) as input variable. Scholars have divided the business cycle into separate phases or regimes, in particular treating expansions separately from contractions [25, 26]. Following this tradition, we present the business cycle in using a very simple two-states model where $S(Z) = \{1, 2\}$ and "1" for expansion, "2" for contraction. The process of two phases of business cycle is governed by the following homogenous Markov transition matrix:

$$A = \{a_{ij}\} = \{P(Z_t = j | Z_{t-1} = i)\}, \quad \text{for } i, j \in \{1, 2\}, \quad (1)$$

In our paper, the hidden states of TMM are regard as systematic credit risk states of a loan portfolio. We assume the number of hidden risk states is m. Taking the input variable Z_t into account, the process of hidden risk states evolves according to a non-homogeneous Markov process:

$$X_t | X_{t-1} \sim \text{Markov}\left(B^{(u)}, \pi_1\right), \quad (2)$$

where $B^{(u)} = \{b_{ij}^{(u)}\}$ is the one-step transition probability matrix of the chain with $Z_t = u$, i.e., $b_{ij}^{(u)} = P(X_t = j | X_{t-1} = i, Z_t = u)$ for $i, j \in \{1, 2, \ldots, m\}$, and π_1 is the probability distribution at $t = 1$. From (2), it can be seen that the transition of consumer' hidden risk states switches between two Markov regimes according to the input value of the business cycle $u \in \{1, 2\}$.

It is worth emphasizing that the particular TMM in our paper has only one common hidden sequence, and multiple output sequences are all driven by this common hidden sequence. That is to say, the hidden systemic credit risk is common to all consumers within a loan portfolio, even if each consumer's repayment behavior is different. This design of hidden sequence in our particular TMM is consistent with that in DCMM represented in Fitzpatrick and Marchev [9].

We now consider the repayment behavior of consumers affected by both the business cycle and the systemic credit risk of the loan portfolio. For a consumer k, let $Y_{k,t}$ denote his/her random repayment state expressed by the number of unpaid installments at time t. In practice, a default state is usually considered and regarded as an absorbing state which means any consumer who has reached this state can never return back. Here, we adopt the assumption in [10, 27] a consumer is assumed to become a defaulter when three accumulative installments are unpaid. Hence, we have:

$$Y_{k,t} \in \{0, 1, 2, 3(\text{default})\}. \quad (3)$$

The process of each consumer's repayment behavior is a nonhomogeneous Markov chain with the following transition matrices:

$$C^{(u,s)} = \{c_{ij}^{(u,s)}\} = \{P(Y_{k,t} = j | Y_{k,t-1} = i, X_t = s, Z_t = u)\}, \quad (4)$$

for $i, j \in \{0, 1, 2, 3\}, s \in \{1, 2, \ldots, m\}, u \in \{1, 2\}, k \in \{1, 2, \ldots, n\}$, where n is the number of consumers in portfolio.

Form (3) shows that affected by both business cycle and hidden systemic credit risk, the transition of consumers' repayment states switches among $2m$ Markov regimes.

The relationship among the business cycles, systematic credit risk of a loan portfolio, and consumers' repayment state is plotted in Figure 4. We can see that the particular TMM can test for and model two paths of impact of business cycle on the transition of consumers' repayment state. One of the paths is that the transition of consumers' repayment state is directly affected by the business cycle. The other path is indirect impact, where the business cycle affects the transition of systematic credit risk state of the loan portfolio, and then affects the transition of consumers' repayment state.

The impacts of business cycle on consumers' repayment behavior are reflected in the difference of the transition probabilities in the transition matrix $B^{(u)}$ and $C^{(u,s)}$, respectively. In practical terms, the probability of occurrence of a low/high systematic credit risk state increases when the external business cycle is in the expansion/contraction stage. At the same time, the repayment behavior of consumers in loan portfolio is more likely to ameliorate/deteriorate because of the expansive/contractive business cycle and low/high systemic credit risk.

The particular TMM provides an alternative approach to measure the credit risk of consumer loans in portfolio across business cycles. In practice, credit managers can focus on a portfolio of borrowers from a particular type or market, for example, borrowers from the energy sector. The systematic credit risk of the loan portfolio can then be seen as a proxy variable of credit market condition of the energy sector. Obviously, the credit market condition of the energy sector is affected by external business cycles. TMM provides us an approach to measure the performance of loans in portfolio under different business cycles and credit market conditions of the energy sector, while also considering the impact of a business cycle on the credit market condition of the energy sector.

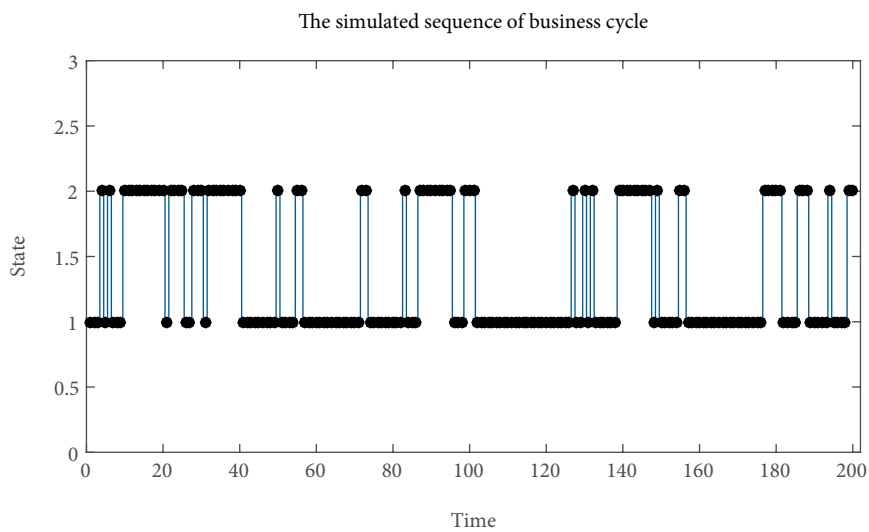

Figure 5: Simulated sequence of business cycle.

4. Simulation Studies

4.1. Generation of Simulated Data. For the sake of simplicity and without loss of generality, we consider there are only two hidden risk states since it is a well-accepted theory that risk states fluctuate between two Markov regimes: normal risk and enhanced risk. We denote: "N" for normal risk and "E" for enhanced risk.

Considering the empirical characteristics of transition matrices of credit card loan accounts in Leow and Crook [27], the parameter values in our paper are given as follows:

The transition matrix of business cycle is

$$A = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{pmatrix} \end{array}, \tag{5}$$

transition matrices of systematic credit risk states are

$$B^{(1)} = \begin{array}{c} \\ \mathrm{N} \\ \mathrm{E} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{N} & \mathrm{E} \end{array} \\ \begin{pmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{pmatrix} \end{array}, \quad B^{(2)} = \begin{array}{c} \\ \mathrm{N} \\ \mathrm{E} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{N} & \mathrm{E} \end{array} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} \end{array} \tag{6}$$

transition matrices of consumers' repayment states are

$$\begin{aligned} C^{(1,N)} &= \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.98 & 0.02 & 0 & 0 \\ 0.9 & 0.08 & 0.02 & 0 \\ 0.8 & 0.08 & 0.02 & 0.10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \quad C^{(2,N)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.84 & 0.11 & 0.05 & 0 \\ 0.71 & 0.11 & 0.05 & 0.13 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \\ C^{(1,E)} &= \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.92 & 0.08 & 0 & 0 \\ 0.78 & 0.14 & 0.08 & 0 \\ 0.62 & 0.14 & 0.08 & 0.16 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \quad C^{(2,E)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.86 & 0.14 & 0 & 0 \\ 0.66 & 0.20 & 0.14 & 0 \\ 0.44 & 0.20 & 0.14 & 0.22 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \end{aligned} \tag{7}$$

We can see that the systematic credit risk states transfer according to whether $B^{(1)}$ or $B^{(2)}$ depend on the value of the input business cycle. The systematic credit risk state is more likely to remain or move to normal risk state when the phase of the external business cycle is expansion. On the contrary, the probability of turning into the enhance risk state increases during the contractive phase of the business cycle. The elements in matrices $C^{(u,s)}, u \in \{1,2\}, s \in \{N,E\}$ show that the probability of recovering overdue installments is larger than the probability of paying nothing, and the assumption is diminished if systematic credit risk state is "E" or the phase of business cycle is "2."

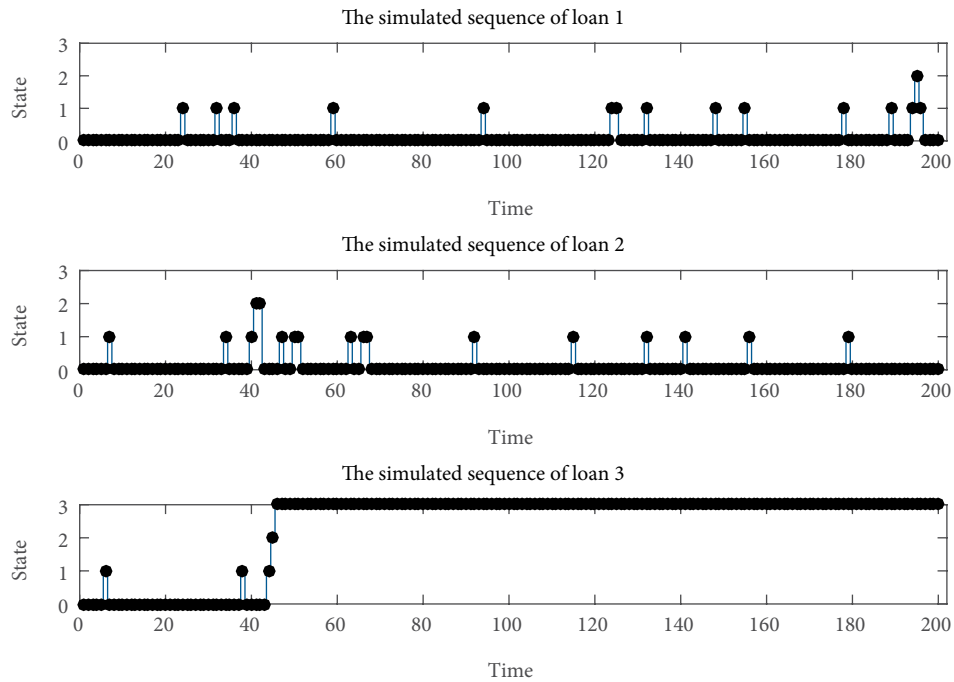

Figure 6: Simulated repayment state sequences of three randomly selected loans.

Table 1: Posterior inference for parameters of our specific TMM.

	$u = 1$	$u = 2$
$\widetilde{B}^{(u)}$	$\begin{pmatrix} 0.9205 & 0.0795 \\ 0.4376 & 0.5624 \end{pmatrix}$	$\begin{pmatrix} 0.5933 & 0.4067 \\ 0.1689 & 0.8311 \end{pmatrix}$
$\widetilde{C}^{(u,N)}$	$\begin{pmatrix} 0.9800 & 0.0200 & 0.0000 & 0.0000 \\ 0.8996 & 0.843 & 0.0161 & 0.0000 \\ 0.7882 & 0.0936 & 0.0249 & 0.0933 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$	$\begin{pmatrix} 0.9515 & 0.0485 & 0.0000 & 0.0000 \\ 0.8340 & 0.1182 & 0.0478 & 0.0000 \\ 0.6255 & 0.1331 & 0.0645 & 0.1769 \\ 0.0001 & 0.0001 & 0.0001 & 0.9997 \end{pmatrix}$
$\widetilde{C}^{(u,E)}$	$\begin{pmatrix} 0.9190 & 0.0810 & 0.0000 & 0.0000 \\ 0.7857 & 0.1376 & 0.0767 & 0.0000 \\ 0.6357 & 0.1567 & 0.0735 & 0.1341 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$	$\begin{pmatrix} 0.8594 & 0.1406 & 0.0000 & 0.0000 \\ 0.6674 & 0.1960 & 0.1366 & 0.0000 \\ 0.4293 & 0.2024 & 0.1491 & 0.2192 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$

We use the TMM with the given parameter values to generate our simulated data. In addition to containing an input sequence of business cycle, the simulated data also contain repayment states of $N = 1000$ consumers, a total of $T = 200$ time intervals, that is to say, there are 1000 sequences, each containing 200 repayment histories. Figures 5 and 6 show the simulated business cycle and three randomly selected loans, respectively.

4.2. Estimation of Model Parameters. Our paper focuses on estimating the value of transition matrices $B^{(u)}, C^{(u,s)}$ from the simulated data set. From a Bayesian perspective, we use Markov chain Monte Carlo (MCMC) algorithms to estimate the parameters of the particular TMM. Since the classic Expectation-Maximization algorithm is sensitive to the starting values and is easy to fall into a local optimal solution. However, MCMC makes posterior risk minimization, and make full use of the experience, history information, and other information of samples. Employing the great computational power of MCMC, the model parameters can be quickly extracted. The MCMC algorithms of our particular TMM are similar to the MCMC algorithms of DCMM [9]. For the sake of brevity, all MCMC algorithms are consigned to Appendix A.

Table 2: SSE of the full sample and three subsamples.

	SSE1	SSE2	Total SSE
$N = 1000, T = 200$	0.0106	0.0122	0.0228
$N = 1000, T = 190$	0.0157	0.0121	0.02788
$N = 950, T = 200$	0.0107	0.0155	0.0262
$N = 950, T = 190$	0.0157	0.0157	0.0314

Note: SSE is the sum of squared differences of true parameter values used to generate the simulated data and their estimates.

We run our MCMC algorithms of TMM 10000 iterations on software R with the first 5000 iterations being discarded as burn-in. The Dirichlet prior parameters in MCMC algorithms

Table 3: SSE of four cases with different parameters.

	SSE1	SSE2	Total SSE
Case 1	0.0165	0.0108	0.0273
Case 2	0.0226	0.0062	0.0288
Case 3	0.0231	0.0100	0.0331
Case 4	0.0145	0.0149	0.0294

Table 4: Posterior inference for parameters of DCMM

	Estimated parameter values
$\widetilde{B}$	$\begin{pmatrix} 0.8247 & 0.1753 \\ 0.3046 & 0.6954 \end{pmatrix}$
$\widetilde{C}^{(N)}$	$\begin{pmatrix} 0.9736 & 0.0264 & 0.0000 & 0.0000 \\ 0.8772 & 0.0959 & 0.0269 & 0.0000 \\ 0.7179 & 0.1106 & 0.0418 & 0.1297 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$
$\widetilde{C}^{(E)}$	$\begin{pmatrix} 0.8826 & 0.1174 & 0.0000 & 0.0000 \\ 0.7103 & 0.1747 & 0.1150 & 0.0000 \\ 0.5006 & 0.1868 & 0.1226 & 0.1900 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$

are all set to 0.1. The estimated values of parameters are obtained from the posterior means of the last 5000 iterations and the results are given in Table 1.

The estimated matrices in Table 1 are close to their true values. The estimated matrices $\widetilde{B}^{(u)}, u \in \{1, 2\}$ give the transition probabilities between the two systematic credit risk states at different phases of the business cycle. Furthermore, by comparing the matrices $\widetilde{C}^{(u,s)}, u \in \{1, 2\}, s \in \{N, E\}$, the different transition probabilities among repayment states of consumers in different systematic credit risk states and business cycles can be obtained. The results will assist practitioners for assessing, managing, and monitoring credit risk in their loan portfolio.

As a robustness check, Table 2 lists the sum of squared errors (SSE) of full sample and three subsamples. The SSE of two transition matrices of systematic credit risk states and four transition matrices of consumers' repayment states are donated by SSE1 and SSE2 respectively.

Table 5: Posterior inference for parameters of MM.

	Estimated parameter values
$\widetilde{C}$	$\begin{pmatrix} 0.9412 & 0.0588 & 0.0000 & 0.0000 \\ 0.7813 & 0.1412 & 0.0775 & 0.0000 \\ 0.5800 & 0.1589 & 0.0931 & 0.1680 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$

The results of SSE show that a reduction in the number of consumers and the length of time interval has a negative impact on the estimation accuracy of transition matrices. Whereas, on average, the estimated deviations of each element in the transition matrix $\widetilde{B}^{(u)}$ and $\widetilde{C}^{(u,s)}$ are only $\sqrt{0.0157/8} = 0.0443$ and $\sqrt{0.0157/64} = 0.0157$, respectively. These small deviations are not enough to have a significant impact on the estimation accuracy of transition matrices. That is to say, a 5% reduction in the number of borrowers and a 5% reduction in the length of time interval have no significant effect on the robustness of TMM.

We now test the robustness of our model with different parameter values. The following four cases are taken into consideration.

Case 1. The transition matrix of business cycle is reset to

$$A = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix} \end{array}, \tag{8}$$

Case 2. Transition matrices of systematic credit risk states are changed to

$$B^{(1)} = \begin{array}{c} \\ \mathrm{N} \\ \mathrm{E} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{N} & \mathrm{E} \end{array} \\ \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix} \end{array}, \quad B^{(2)} = \begin{array}{c} \\ \mathrm{N} \\ \mathrm{E} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{N} & \mathrm{E} \end{array} \\ \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix} \end{array}, \tag{9}$$

Case 3. Transition matrices of consumers' repayment states are given as

$$C^{(1,N)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.98 & 0.02 & 0 & 0 \\ 0.9 & 0.08 & 0.02 & 0 \\ 0.8 & 0.08 & 0.02 & 0.10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \quad C^{(2,N)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.96 & 0.04 & 0 & 0 \\ 0.86 & 0.10 & 0.04 & 0 \\ 0.74 & 0.10 & 0.04 & 0.12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array},$$

$$C^{(1,E)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.94 & 0.06 & 0 & 0 \\ 0.82 & 0.12 & 0.06 & 0 \\ 0.68 & 0.12 & 0.06 & 0.14 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \quad C^{(2,E)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.90 & 0.10 & 0 & 0 \\ 0.74 & 0.16 & 0.10 & 0 \\ 0.56 & 0.16 & 0.10 & 0.18 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \tag{10}$$

Table 6: Predicted values of expected default rate and deviation rate (in parentheses) after 12 time periods.

	Scenario 1			Scenario 2			Scenario 3		
	TMM	DCMM	MM	TMM	DCMM	MM	TMM	DCMM	MM
Repayment state 0	0.0003 (24.25%)	0.0010 (137.29%)	0.0091 (2003.89%)	0.0509 (18.4%)	0.0304 (41.40%)	0.0091 (82.51%)	0.0052 (4.94%)	0.0085 (57.37%)	0.0091 (67.11%)
Repayment state 1	0.0020 (24.19%)	0.0049 (89.79%)	0.0247 (850.37%)	0.0871 (1.96%)	0.0564 (36.46%)	0.0247 (72.13%)	0.0166 (8.71%)	0.0312 (71.73%)	0.0247 (36.16%)
Repayment state 2	0.0961 (6.30%)	0.1366 (33.18%)	0.1946 (89.69%)	0.2996 (0.35%)	0.2432 (18.54%)	0.1946 (34.83%)	0.1498 (15.56%)	0.2216 (24.89%)	0.1946 (9.65%)

Note: The rates of deviation are obtained from the formula $\left|\tilde{\theta} - \theta\right|/\theta$, where $\tilde{\theta}$ is the expected default rate based on the estimated parameter values of each model, θ is the expected default rate based on the true parameter values of TMM in Section 4.1.

Case 4. The parameter values are simultaneously changed to the values given in the above three cases.

We regenerate the simulated data according to the parameters given in above four cases. Again, using MCMC algorithms in Appendix A, we can obtain the estimated parameter values. SSE1, SSE2, and total SSE can then be obtained by comparing them with the true parameter values. The results are presented in Table 3. We can see that the SSE1, SSE2, and total SSE with different parameters are very close to those previously listed in Table 2. This shows that the accuracy of estimation does not seem to decrease with the above parameter values and TMM is robust to these parameter values.

4.3. Comparative Analysis. Now we study other cases for comparison. We estimate the parameter values of DCMM and the MM based on the same simulated data. The MCMC algorithms of DCMM are presented in Fitzpatrick and Marchev [9]. The algorithms of MM can be found in the R package "MCTM" (https://cran.r-project.org/web/packages/MCTM/index.html). All codes were run on software R. The estimated parameter values of DCMM and MM are obtained and listed in Tables 4 and 5, respectively.

By comparing the results of Tables 1, 4 and 5, we can see that the estimated matrices $\widetilde{C}^{(s)}, s \in (N, E)$ in DCMM can be regarded as a complex combination of $\widetilde{C}^{(1,s)}$ and $\widetilde{C}^{(2,s)}$ in TMM. Further, the estimated matrix $\widetilde{C}$ in MM can be obtained by a complex combination of $\widetilde{C}^{(N)}$ and $\widetilde{C}^{(E)}$ in DCMM. Both DCMM and MM cannot explicitly analyze the influence of business cycle on consumers' repayment behavior. Whereas, the transition process of consumers' repayment states across business cycle can be modeled and estimated by TMM. Based on TMM, the analysis of consumer repayment behavior in loan portfolio is more meticulous and persuasive.

Once the estimated parameter values of TMM, DCMM, and MM are obtained, the prediction of future default probability of consumers can be made. We predict the expected default probability of current non-defaulting consumers after 12 time periods based on estimated parameter values. For the sake of simplicity, but without losing generality, we focus on the following three scenarios.

Scenarios 1. All phases of a business cycle are expansion and all systematic credit risk states are normal risk over the next 12 time periods.

Scenarios 2. All phases of a business cycle are contraction and all systematic credit risk states are enhanced risk over the next 12 time periods.

Scenarios 3. The current phase of the business cycle is contraction, but the current systematic credit risk state is normal risk.

Obviously, Scenarios 1 and 2 are the best and worst case of TMM respectively. In Scenario 3, the occurrences of business cycle and systematic risk state in the next 12 time periods are stochastic, and are governed by the estimated transition matrices. Before the process of prediction, we adjust the last row of the estimated matrices $\widetilde{C}^{(u,s)}, u \in \{1, 2\}, s \in \{N, E\}$ to $(0, 0, 0, 1)$ since the repayment state "3" is an absorbing state. The predicted values of expected default rate (deviation rates are listed in parentheses) of current nondefaulting consumers after the 12 time periods of TMM, DCMM, and MM are given in Table 6.

The results in Table 6 show that the probability of becoming a defaulter increases with the increase in the number of unpaid instalments. We can also see that the deviation rate of TMM is the smallest among the three models, especially in Scenarios 1 and 2. This suggests that the TMM proposed in this paper can be used to accurately predict and assess the credit risk of consumers in loan portfolio, especially when business cycle stays at one stage for a long time.

Notice that the transition matrices, which characterize the expected changes in credit quality of consumers, are cardinal inputs to portfolio risk assessment. The particular TMM in this paper can integrate business cycle into the analysis of consumers' repayment behavior and reveal more information about transition matrices of a loan portfolio. We believe our analysis provides a useful method to stress testing a loan portfolio and could assist practitioners in managing credit risk in loan portfolio.

5. Conclusions

The proposed TMM in this paper extends the credit risk measurement of consumer loan portfolio in HMM and DCMM by taking the impact of a business cycle into consideration. The structure of TMM can incorporate two paths of impact of a business cycle on the transition of consumers' repayment behavior. One is the transition consumers' repayment behavior is directly affected by the business cycle. The other is the business cycle affects the transition of systematic credit risk of the loan portfolio and then affects the transition of consumers' repayment behavior. It is the first time that a TMM is applied to consumer loan. We also develop the corresponding MCMC algorithms of the particular TMM for estimating model parameters. Numerical examples illustrated that the proposed TMM is more accurate in assessing and predicting the credit risk of consumers in loan portfolio than DCMM and MM, especially when the business cycle is stuck in one phase for a long time.

Our research still has some limitations, which may be addressed in future research. The simulation studies confirm that the model fitting process can retrieve the original parameters closely. However, the simulated data do not assess potential practical limits and difficulties under real conditions. Concerning the using of TMM in real credit risk situations, an application to real data would substantially improve the quality of our paper. The early payoff case and recovery from default state can also be incorporated according to the real situation. A more complicated data structure such as default correlations among consumers and the missing data case can be further extended.

Appendix

To simplify the notation, for $k \in \{1, \ldots, n\}, 0 \le t \le T$, we define:

$$Y(t) := \bigcup_k \{Y_{k,t}\}, Y^{(t)} := \bigcup_k \{(Y_{k,0}, \ldots, Y_{k,t})\}, \tag{A.1}$$

$$\begin{aligned} &Z_t^T := (Z_t, \ldots, Z_T), X_t^T := (X_t, \ldots, X_T), \\ &Y_t^T := \bigcup_k \{(Y_{k,t}, \ldots, Y_{k,T})\}, \end{aligned} \tag{A.2}$$

$$\lambda = \left(\pi_0, A^{(u)}, B^{(u,s)}\right),\ s = 1,\ldots,M, u = 1,\ldots,U, \tag{A.3}$$

where n is the number of consumers in portfolio and T is the term of each loan.

The MCMC algorithms of the particular TMM are similar to the MCMC algorithms of DCMM which are given in Fitzpatrick and Marchev [9]. The five main steps of the MCMC algorithms of the particular TMM are as follows.

A. Priors Specification

The priors on λ are Dirichlet as follows:

$$\begin{aligned}\pi_0 &\sim D(\eta_{01},\ldots,\eta_{0M}), a_{m1}^{(u)},\ldots,a_{mM}^{(u)}\\ &\sim D\left(\eta_{m1}^{(u)},\ldots,\eta_{mM}^{(u)}\right), m = 1,\ldots,M, u = 1,\ldots,U,\end{aligned} \tag{A.4}$$

$$\begin{aligned}b_{l1}^{(u,s)},\ldots,b_{lL}^{(u,s)} &\sim D\left(\eta_{l1}^{(u,s)},\ldots,\eta_{lL}^{(u,s)}\right),\\ l = 1,\ldots,L, s &= 1,\ldots,M, u = 1,\ldots,U.\end{aligned} \tag{A.5}$$

B. Sampling from $P\left(X_1^T \middle| Y^{(T)}, Z_1^T, \lambda\right)$

Given the observations $Y^{(T)}$ and the current value of the parameters λ, we wish to simulate a sample path X_1^T of the hidden Markov chain, from its conditional distribution:

$$\begin{aligned}P\left(X_1^T \middle| Y^{(T)}, Z_1^T, \lambda\right) &= P\left(X_T \middle| Y^{(T)}, Z_1^T, \lambda\right)\\ &\cdot \prod_{t=1}^{T-1} P\left(X_t \middle| Y^{(T)}, X_{t+1}^T, Z_1^T, \lambda\right),\end{aligned} \tag{B.1}$$

We draw values for $X_T, X_{T-1},\ldots,X_1$ backward. The "typical term" in (B.1) can be written as:

$$\begin{aligned}&P\left(X_t \middle| Y^{(T)}, X_{t+1}^T, Z_1^T, \lambda\right)\\ &= P\left(X_t \middle| Y^{(t)}, Y_{t+1}^T, X_{t+1}^T, Z_1^T, \lambda\right)\\ &= \frac{P\left(X_{t+1}^T, Y_{t+1}^T \middle| X_t, Y^{(t)}, Z_1^T, \lambda\right) P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right)}{P\left(X_{t+1}^T, Y_{t+1}^T \middle| Y^{(t)}, Z_1^T, \lambda\right)}\\ &= P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right) \frac{P\left(X_{t+1}, X_{t+2}^T, Y_{t+1}^T \middle| X_t, Y^{(t)}, Z_1^T, \lambda\right)}{P\left(X_{t+1}^T, Y_{t+1}^T \middle| Y^{(t)}, Z_1^T, \lambda\right)}\\ &= P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right) P\left(X_{t+1} \middle| X_t, Y^{(t)}, Z_1^T, \lambda\right)\\ &\quad \cdot \frac{P\left(X_{t+2}^T, Y_{t+1}^T \middle| Y^{(t)}, X_t, X_{t+1}, Z_1^T, \lambda\right)}{P\left(X_{t+1}^T, Y_{t+1}^T \middle| Y^{(t)}, Z_1^T, \lambda\right)}\\ &= P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right) P\left(X_{t+1} \middle| X_t, Y^{(t)}, Z_1^T, \lambda\right)\\ &\quad \cdot \frac{P\left(X_{t+2}^T, \middle| Y_{t+1}^T\ Y^{(t)}, X_t, X_{t+1}, Z_1^T, \lambda\right)}{P\left(X_{t+1}^T, \middle| Y_{t+1}^T\ Y^{(t)}, Z_1^T, \lambda\right)}\\ &\propto P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right) P\left(X_{t+1} \middle| X_t, Z_{t+1}, A^{(u)}\right)\end{aligned} \tag{B.2}$$

By Bayes theorem, the mass function of hidden state given information up to t is:

$$\begin{aligned}P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right) &= P\left(X_t \middle| Y^{(t-1)}, Z_1^T, \lambda\right)\\ &\cdot \frac{P\left(Y(t) \middle| X_t, Y^{(t-1)}, Z_1^T, \lambda\right)}{P\left(Y(t) \middle| Y^{(t-1)}, Z_1^T, \lambda\right)}.\end{aligned} \tag{B.3}$$

By the law of total probability, we have:

$$\begin{aligned}&P\left(X_t \middle| Y^{(t-1)}, Z_1^T, \lambda\right)\\ &= \sum_{l=1}^{m} P\left(X_t \middle| X_{t-1} = l, Z_1^T, \lambda\right) P\left(X_{t-1} = l \middle| Y^{(t-1)}, Z_1^T, \lambda\right)\\ &= \sum_{l=1}^{m} P\left(X_t \middle| X_{t-1} = l, Z_t, A^{(u)}\right) P\left(X_{t-1} = l \middle| Y^{(t-1)}, Z_1^T, \lambda\right).\end{aligned} \tag{B.4}$$

The initial probability distribution of hidden states is:

$$P\left(X_1 \middle| Y^{(0)}, Z_1^T, \lambda\right) = P\left(X_1 \middle| Y(0), Z_1, \lambda\right) = \pi_1. \tag{B.5}$$

Using (B.3)–(B.5), we can obtain $P\left(X_t \middle| Y^{(t)}, Z_1^T, \lambda\right)$ for $t = 1,\ldots,T$.

Combining with (B.1)–(B.2), $X_T, X_{T-1},\ldots,X_1$ can be simulated.

C. Extra Permutation Step

To improve the convergence properties of the algorithm, a random permutation of the labels is applied. See Fitzpatrick and Marchev [9] for a detailed description.

$$g \in \{\text{permutations of } (1, 2,\ldots,M)\}, \tag{C.1}$$

$$g \sim P(g(X_1),\ldots,g(X_T)). \tag{C.2}$$

Then we set $X' = gX$.

D. Sampling from $P\left(\lambda \middle| X_1^T, Y^{(T)}, Z_1^T\right)$

Given the process of hidden state X, it is rather straightforward to determine the posterior distribution of λ. Using Bayes' theorem, the posterior distribution can be simulated separately and independently as follows:

$$\pi_0 \middle| X, Y, Z \sim D(\eta_{0,1} + \omega_{0,1},\ldots,\eta_{0,M} + \omega_{0,M}), \tag{D.1}$$

$$\begin{aligned}&a_{m1}^{(u)},\ldots,a_{mM}^{(u)} \middle| X, Y, Z \sim D\left(\eta_{m1}^{(u)} + \omega_{m1}^{(u)},\ldots,\eta_{mM}^{(u)} + \omega_{mM}^{(u)}\right),\\ &m = 1,\ldots,M,\ u = 1,\ldots,U,\end{aligned} \tag{D.2}$$

$$\begin{aligned}&b_{l1}^{(u,s)},\ldots,b_{lL}^{(u,s)} \middle| X, Y, Z \sim D\left(\eta_{l1}^{(u,s)} + \omega_{lL}^{(u,s)},\ldots,\eta_{lL}^{(u,s)} + \omega_{lL}^{(u,s)}\right),\\ &l = 1,\ldots,L,\ s = 1,\ldots,M,\ u = 1,\ldots,U,\end{aligned} \tag{D.3}$$

where $\omega_{0,i} := I(X_1 = i), \omega_{i,j}^{(u)} := \sum_{t=2}^{T} I(X_{t-1} = i, X_t = j, Z_t = u)$, $\omega_{i,j}^{(u,s)} := \sum_{k=1}^{N}\sum_{t=2}^{T} I(Y_{k,t-1} = i, Y_{k,t} = j, X_t = s, Z_t = u)$, I is indicator function.

The steps B, C and D are repeated until a maximum number of iterations is reached.

E. Post-Processing Algorithm

The posterior inference step should be trivial once X is drawn but the draw is complicated by a nonidentifiability problem called *label switching*. This will mean that ergodic averages of component-specific quantities will be identical and thus useless for inference. In dealing with the problem of *label switching*, a post-processing algorithm is applied to ensure the labels of the hidden states are consistent for all iteration. See Figure 3 of Boys and Henderson [28] for a d etailed description.

If at iteration i the current estimate of $\widehat{X}^*$ is $\widehat{X}^*_{(i-1)}$ then

(E.1) Choose a permutation v_i to minimize $-\sum_{t=1}^{T} I\left(v_i\left(X_{t,(i)}\right) = \widehat{X}^*_{t,(i-1)}\right)$;

(E.2) Apply the permutation v_i to output $X_{(i)}$ and $\lambda_{(i)}$;

(E.3) For $t = 1, 2, \ldots, T$, set $\widehat{X}^*_{t,(i)} = \arg\max_{j \in S(X)} \sum_{o=1}^{i} I\left(v_k\left(X_{t,(o)}\right) = j\right)$;

(E.4) Let Q be the size of sampling. We estimate the posterior probabilities of the hidden states X_t along with the sequence by $P(X_t = j|Y, Z) = (1/Q)\sum_{q=1}^{L} I\left(\widehat{X}^*_{t,q} = j\right)$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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