Research Article

Nonsingular Fast Terminal Adaptive Neuro-sliding Mode Control for Spacecraft Formation Flying Systems

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1.Introduction

With the increasing applications of the spacecraft formation flying system, the importance of improving system performance of complicated spacecraft with practical control design has gained much attention over recent years [1–5]. In the spacecraft formation flying system, stability is the most essential problem to be solved [6]. Many control methods have been applied to solve these problems in the literature, such as the adaptive control [7], sliding mode control [8], backstepping technique control [9], fuzzy control [10], optimal control [11], and $H_{\infty}$ control [12]. However, along with the goal of stabilization for the spacecraft, the disturbances and uncertainties should also be considered, which is often neglected in some research [13].

Sliding mode control is widely used due to its advantages of robustness against external disturbances [14–16] and parameter uncertainties [17], especially in spacecraft formation flying system. In sliding mode control, chattering is a common phenomenon that often degrades system performance and even causes instability. To deal with this problem, the terminal sliding mode control is developed, in which some nonlinear terms are introduced in the sliding surface. In [18], the case of spacecraft final approach, combining with the continuously differential collision avoidance state constraint function is studied and an improved terminal sliding surface is presented. In [19], based on the conditions of external disturbances and inertial uncertain parameters, Ran et al. have proposed the adaptive fuzzy terminal sliding mode control for the second-order spacecraft attitude manoeuvre system and the convergence time of the sliding surface has been reduced. In [20], a new nonsingular terminal sliding mode (NTSM) surface has been introduced to eliminate singularity within the finite time. An adaptive nonsingular terminal sliding mode control also has been presented for an attitude tracking of spacecraft with actuator faults to avoid singularity [21]. Although several drawbacks of the sliding mode control have been compensated by the modified control laws, due to the high-frequency reaching control term in the control input [22], the chattering problem still needs to be solved. In [23], a fractional order control law based on the nonsingular terminal super-twisting sliding mode control is proposed to achieve system stability and accurately estimate the unknown model. Consequently,
the nonsingular fast terminal sliding mode surface has the ability to reduce the chattering and solve the singularity problem.

In practical spacecraft formation flying systems, internal and external disturbances are inevitable. Many control methods have been used to solve this problem [24, 25]. The disturbance observers have been widely considered to estimate the disturbances [26]. In [27], an extended state observer is introduced to estimate disturbances for the multiagent systems with unknown nonlinear dynamics and disturbances. Furthermore, by simultaneously taking the output measurement and delayed control input into account, the extended observer has been used to estimate the unmeasurable system states and the additive disturbances [28]. To have a smaller estimation error of the disturbance, high-order disturbance observers have been investigated in [29]. Among all types of high-order observer, the super-twisting disturbance observer stands out due to its ability of avoiding the excessively high order observer gains that can cause sensor noise within fixed bandwidth [30].

Despite the external disturbances, uncertainties in practical formation flying systems are also inevitable; however, their impacts on the system’s performance have not been studied in most of the aforementioned results. In practice, parameter uncertainties may have direct impact on system performance [31]; hence, it is critical to deal with the problem of uncertainties. Due to the advantage of the neural networks in approximating nonlinear functions, it can be applied in dealing with the nonlinearity in the spacecraft formation systems as well [32–34]. Based on neural network and backstepping techniques, a distributed adaptive coordinated control algorithm has been proposed in [35], which effectively solves the chattering problem. To achieve robustness, in [36], a robust adaptive fuzzy PID-type sliding mode control has been designed with the neural network, with which the system stability is ensured and the transient performance is improved. In [37], the global adaptive neural control for a general class of nonlinear robot manipulators has been proposed, in which the transient performance and robustness are both improved. Compared with other neural networks [38–40], the Radial Basis Function (RBF) neural network stands out given its strong robustness and excellent performance of nonlinear approximation [41]. Therefore, it is desirable to use the RBF neural network to estimate and compensate for the nonlinear functions in the spacecraft formation flying system.

Motivated by the above observations, in this paper, a novel nonsingular fast terminal adaptive neurosliding mode control for the spacecraft formation flying system is proposed. The proposed control law guarantees that the tracking errors converge to zero in finite time. By using an adaptive RBF neural network, the uncertainties in the spacecraft formation flying system is compensated. The contributions of the paper can be summarized as follows:

(1) External disturbances are estimated by a super-twisting disturbance observer in finite time. The super-twisting disturbance observer can avoid high observer gains that may amplify the noises of the sensors.

(2) Compared with [42], our novel fast nonsingular terminal sliding mode control law achieves the stability of spacecraft formation flying system within finite time. Moreover, the singularity is avoided and the chattering problem is also alleviated.

(3) If some of the nonlinear dynamics in the spacecraft formation system are unknown, an adaptive RBF neural network control law is proposed to approximate those nonlinear dynamics.

The remaining parts of the paper are arranged as follows. The background of the spacecraft formation flying system is introduced in Section 2. In Section 3, the proposed control laws and stability analysis are presented. Simulation results are given to verify the effectiveness of the proposed control laws in Section 4. Finally, some conclusions are given in Section 5.
where \( m_t \) and \( m_c \) denote the masses of the target spacecraft and the chaser spacecraft and \( d_{ci} \in \mathbb{R}^3 \) and \( d_{cc} \in \mathbb{R}^3 \) denote the external perturbation forces of the target spacecraft and the chaser spacecraft, respectively. \( \tau_{cc} \in \mathbb{R}^3 \) is the control force of the chaser spacecraft. \( r_t = \|r_t\|_2 \) and \( r_c = \|r_c\|_2 \), where \( r_t \) and \( r_c \) denote the position vectors of the target spacecraft and chaser spacecraft to the geocentric, respectively.

Define \( \rho = r_c - r_t \); then, from (2) and (3) the relative acceleration vector is obtained as

\[
\ddot{\rho} = \frac{\mu_c}{r_c^3} \rho + \frac{\mu_e}{r_t^3} \rho + \frac{d_{cc}}{m_c} + \frac{f_{cc}}{m_c}. \tag{4}
\]

Furthermore, let \( \rho_c \) denote the projection vector in the chaser spacecraft body coordinate frame \( F_c \), which denotes

\[
f_c = \frac{\mu_c}{r_c^3} \rho + \frac{\mu_e}{r_t^3} \rho,
\]

\[
d_c = \frac{d_{cc}}{m_c} + \frac{f_{cc}}{m_c}, \tag{5}
\]

\[
u_c = f_{cc}.
\]

Then,

\[
\ddot{\rho}_c + \omega_c^\times (\omega_c^\times \rho_c) + 2\omega_c^\times \dot{\rho}_c + \dot{\omega}_c^\times \rho_c = f_c + \frac{d_c}{m_c} + \frac{u_c}{m_c}, \tag{6}
\]

where \( \omega_c = [\omega_{c1}, \omega_{c2}, \omega_{c3}]^T \) is the angular velocity of \( F_c \) relative to the inertial coordinate frame \( F_I \) under \( F_c \) frame. In this paper, the desired relative position and velocity under \( F_c \) frame are assumed to be \( \rho_{cd} \) and \( 0 \), respectively. Then, the error vector can be expressed as \( e_c = \rho_c - \rho_{cd} \); then, (6) can be written as the Euler–Lagrange equation form to express the orbit relative motion:

\[
m_c \ddot{e}_c + A_c \dot{e}_c + B_c e_c + g_c = u_c + d_c,
\]

where

\[
A_c = 2m_e \omega_c^\times,
\]

\[
B_c = m_e \begin{bmatrix} -\omega_{c2}^2 - \omega_{c3} \omega_{c2} - \omega_{c3} \omega_{c1} + \dot{\omega}_{c2} \\ \omega_{c1} \omega_{c2} + \dot{\omega}_{c2} \\ \omega_{c1} \omega_{c3} - \omega_{c2}^2 - \omega_{c3} \omega_{c1} - \dot{\omega}_{c1} \end{bmatrix}, \tag{8}
\]

\[
g_c = -m_e f_c + m_c \ddot{d}_c + A_c \dot{d}_c + B_c d_c.
\]

The Modified Rodrigues Parameters (MRPs) are used to describe the attitude dynamics of the chaser spacecraft which can be defined as

\[
\sigma = \frac{\vec{q}}{1 + \vec{q}} = k_c \tan \frac{\phi}{4}, \tag{9}
\]

where \( \vec{q} = [q_1, q_2, q_3]^T \) denotes attitude quaternion and \( k_c \) and \( \phi \in (-2\pi, 2\pi) \) denote Euler axis and Euler angular of the attitude rotation, respectively.

Then, the attitude kinematics and the dynamics of the chaser spacecraft are given as

\[
\sigma = G(\sigma) \omega_c,
\]

\[
G(\sigma) = \frac{1}{4} \left[ (1 - \sigma^T \sigma) I_{3 \times 3} + 2\sigma \sigma^T + 2\sigma^2 \sigma^T \right], \tag{10}
\]

\[
J \ddot{\omega}_c = -\omega_c^\times J \omega_c + u_c + d_c,
\]

where \( J \in \mathbb{R}^{3 \times 3} \) denotes the inertia matrix of the chaser spacecraft and \( u_c \in \mathbb{R}^3 \) and \( d_c \in \mathbb{R}^3 \) denote the control torque and the external disturbance torque of the chaser spacecraft, respectively.

Defining the error attitude \( \sigma_c \) and the error angular velocity \( \omega_c \) as
\[
\sigma_c = \frac{(1 - \sigma_d^T \sigma_d) \sigma - (1 - \sigma^T \sigma) \sigma_d + 2 \sigma^T \sigma_d}{1 + \sigma^T \sigma_d \sigma_d + 2 \sigma_d^T \sigma},
\]
\[
\omega_c = \omega_c - R(\sigma_c) \omega_d,
\]
\[
R(\sigma) = I_{3 \times 3} - \frac{4(1 - \sigma^T \sigma)}{(1 + \sigma^T \sigma)^2} \sigma \sigma^T + \frac{8 \sigma^T \sigma}{(1 + \sigma^T \sigma)^2} \sigma^T \sigma^T,
\]
where \( \sigma_d \in \mathbb{R}^3 \) denotes the desired attitude and \( \omega_d \in \mathbb{R}^3 \) denotes the desired angular velocity. Furthermore, the attitude model of the chaser spacecraft can be expressed as the Euler–Lagrange form:
\[
M_c \ddot{\alpha}_c + A_c \dot{\alpha}_c + g_t = G^{-T}(\sigma_c) d_c + G^{-T}(\sigma_c) u_t,
\]
where
\[
M_c = G^{-T}(\sigma_c) G^{-1}(\sigma_c),
\]
\[
G(\sigma) = \frac{1}{4} [(1 - \sigma^T \sigma) I_{3 \times 3} + 2 \sigma \sigma^T + 2 \sigma \sigma^T],
\]
\[
A_c = -G^{-T}(\sigma_c) \left[ (f G^{-1}(\sigma_c) \sigma \sigma^T G^{-1}(\sigma_c) + f G^{-1}(\sigma_c) \sigma \sigma^T) + G^{-T}(\sigma_c) \right] J R(\sigma_c) \omega_d \times (R(\sigma_c) \omega_d \times J R(\sigma_c) \omega_d + J R(\sigma_c) \omega_d).
\]

Remark 1. It can be seen in (12), the attitude and orbit in the spacecraft formation flying system are coupled. Therefore, it is necessary to build the spacecraft coupling motion model (15) and (16) to enhance the control accuracy and efficiency. From (15) and (16), it can be seen that the coupling of attitude and orbit is reflected by the matrix \( B_c \) and the vector \( d_c \). Simultaneously, \( B_c \) represents the attitude effect on orbit. In addition, the gravity gradient torque contains orbit information, and the disturbance torque can reflect the influence of the orbit on the attitude, thus \( d_c \) can reflect the influence of the orbit on the attitude.

Assumption 1. The external disturbance vector \( d = [d_1, \ldots, d_6] \) in the spacecraft formation flying system (15) and (16) is assumed to satisfy \( \|d\| \leq d_m \) and \( |d| \leq \varsigma_0 \), where \( d_m > 0 \) and \( \varsigma_0 > 0 \).

Assumption 2. All the state variables of the spacecraft formation flying system are assumed to be measurable.

2.2. Preliminary Knowledge. In this section, some related lemmas and definitions in this paper are given below.

Lemma 1 (see [43]). For \( \hat{x} = f(x, t) \), \( f(0, t) = 0 \) and \( x \in U_0 \subset R^n \), where \( f : U_0 \times R^+ \rightarrow R^n \) is continuous with respect to \( x \) on an open neighborhood \( U_0 \) of the origin \( x = 0 \). Suppose that there is a positive-definite function \( V(x, t) \) (defined on \( \mathbb{U} \times R^+ \), where \( \mathbb{U} \subset U_0 \subset R^n \) is a neighborhood of the origin), real numbers \( c > 0 \) and \( 0 < \alpha < 1 \), such that \( V(x, t) + c V^\alpha(x, t) \) is negative semidefinite on \( \mathbb{U} \). Then, \( V(x, t) \) is locally finite-time convergent, or equivalently, and becomes 0 locally in finite time, with its setting time \( T \leq V(x(t_0), t_0)^{1-\alpha}/c(1-\alpha) \) for a given initial condition \( x(t_0) \) in a neighborhood of the origin in \( \mathbb{U} \).

Lemma 2 (see [44]). Suppose Lyapunov function \( V(\tilde{z}) \) is defined on a neighborhood \( U \subset R^n \) of the origin, and \( V(\tilde{z}) + \lambda_0 V^\alpha \leq 0 \) in which \( \lambda_0 > 0, 0 < \alpha < 1 \). Then, there exists an area \( U_0 \subset R^n \) such that any \( V(\tilde{z}) \) that start from \( U_0 \subset R^n \) can reach \( V(\tilde{z}) = 0 \) within finite time \( T \leq V^{1-\alpha}/(\lambda_0 (1-\alpha)) \) in which \( V(0) \) is the initial time of \( V(\tilde{z}) \).

Definition 1 (see [45]). For the autonomous system,
\[
\tilde{z} = f(\tilde{z}),
\]
where \( \tilde{z} \in R^n \) and \( f(0) = 0 \), if the Lyapunov function \( V(\tilde{z}) \) satisfies the following conditions:

(a) \( V(\tilde{z}) \) is a positive continuous differentiable function
(b) There exist \( a_1 > 0, a_2 > 0, p \in (0, 1) \), and an open neighborhood \( U \subset U_0 \) containing the origin, such that the inequality \( V(\tilde{z}) \leq -a_1 V(\tilde{z}) - a_2 V^p(\tilde{z}) \) holds; then, it can be concluded that the origin is the finite time stable equilibrium of system (20)
3. Control Law Design

In this section, first, we employ a super twisting observer to estimate the external disturbances in the system. Second, we propose a fast nonsingular terminal sliding mode control law along with the super twisting disturbance observer for the spacecraft formation flying system. In the last section, the nonlinear matrices $A$ and $g$ in (18) and (19), respectively, are assumed to be unknown, and an adaptive neurosliding model control law for the spacecraft formation flying system is proposed.

3.1. The Fast Nonsingular Terminal Sliding Mode Control Law with a Super Twisting Disturbance Observer. The block diagram of the fast nonsingular terminal sliding mode control law with the super twisting disturbance observer is shown in Figure 2. In Figure 2, it can be seen that the overall control law consists of two parts: a fast nonsingular terminal sliding mode control law along with the super twisting disturbance observer. The super twisting disturbance observer is applied to estimate the disturbances for the spacecraft formation flying system if the system dynamics are known.

3.1.1. Super Twisting Disturbance Observer. In practical applications, the disturbances in the spacecraft formation flying system are fast time-varying disturbances. Hence, it is difficult for conventional observers to estimate those disturbances. In this section, a super twisting disturbance observer is employed to estimate those disturbances. The super twisting disturbance observer can estimate the disturbance in finite time and also it avoids high-order observer gains with fixed bandwidth.

According to the attitude-orbit coupling system (15) and (16), the disturbance vector $d$ can be rewritten as follows:

$$d = C_2^{-1}(M \dot{x}_c + Ax_c + g - C_1u).$$  

(22)

Define the state variables as follows:

$$x_1 = \int_0^t d\,dt$$

$$= C_2^{-1}M \dot{x}_c - \int_0^t \frac{d}{dr} (C_2^{-1}M)x_c + C_2^{-1}(Ax_c + g - C_1u)dr,$$

$$x_2 = d.$$  

(23)

Then, the disturbance dynamic system for the spacecraft formation flying is given as

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = d.$$  

(24)

For each disturbance $d_i$ in the spacecraft formation flying system, the disturbance’s model is given as follows:

$$\dot{x}_{1i} = x_{2i},$$

$$\dot{x}_{2i} = d_i, \quad i = 1, 2, \ldots, 6.$$  

(25)

The super twisting disturbance observer is given as follows:

$$\dot{x}_{1i} = -\lambda_1|e_{1i}|^{1/2}\text{sign}(e_{1i}) + \bar{x}_{2i},$$

$$\dot{x}_{2i} = -\lambda_2\text{sign}(e_{1i}),$$  

(26)

where $\lambda_1$ and $\lambda_2$ denote the positive observer gains, $\bar{x}_{1i}$ and $\bar{x}_{2i}$ are the estimations of the state vectors $x_{1i}$ and $x_{2i}$, and $e_{1i} = x_{1i} - \bar{x}_{1i}$ and $e_{2i} = x_{2i} - \bar{x}_{2i}$ are the estimation errors, respectively.

According to the attitude-orbit coupling system (15) and (16) and the super twisting state observer (26), the estimation errors can be rewritten as

$$\dot{e}_{1i} = -\lambda_1|e_{1i}|^{1/2}\text{sign}(e_{1i}) + e_{2i},$$

$$\dot{e}_{2i} = -\lambda_2\text{sign}(e_{1i}) - \hat{d}_i.$$  

(27)

Define the state vectors $\kappa = [\kappa_1, \kappa_2]^T$, where $\kappa_1 = |e_{1i}|^{1/2}\text{sign}(e_{1i}), \kappa_2 = e_{2i}$.

Then, the derivatives of the state vectors $\kappa_1$ and $\kappa_2$ can be written as

$$\dot{\kappa}_1 = -\frac{\lambda_1}{2}|e_{1i}|^{-1}e_{1i} - \frac{1}{2}|e_{1i}|^{-1/2}e_{2i},$$

$$\dot{\kappa}_2 = -\frac{\lambda_2}{2}|e_{1i}|^{-1}e_{1i} - \dot{d}_i.$$  

(28)

Similar to [46], the following Lyapunov function for (28) is constructed:

$$V_{st} = \kappa^T \theta_1 \kappa$$

$$= \frac{1}{2}\left(\lambda_1^2 + 4\lambda_2^2\right)\kappa_1^T - \lambda_1\kappa_1\kappa_2 + \kappa_2^2,$$

(29)

where $\theta_1 = 1/2 \left[ \begin{array}{c} \lambda_1^2 + 4\lambda_2^2 \\ -\lambda_1 \\ 2 \end{array} \right]$.

Then, the derivative of Lyapunov function $V_{st}$ with respect to the time is given as

$$\dot{V}_{st} = \left(\lambda_1^2 + 4\lambda_2^2\right)\kappa_1\kappa_1 - \lambda_1\kappa_1\kappa_2 - \lambda_1\kappa_1\kappa_2 + 2\kappa_2\dot{\kappa}_2$$

$$= \left(\lambda_1^2 + 4\lambda_2^2\right)|e_{1i}|^{(1/2)}e_{1i}\left(-\frac{\lambda_1}{2}|e_{1i}|^{-1}e_{1i} + \frac{1}{2}|e_{1i}|^{-1/2}e_{2i}\right)$$

$$+ \frac{\lambda_1^2}{2}|e_{1i}|^{-1}e_{1i}e_{2i} - \lambda_1|e_{1i}|^{(1/2)}e_{2i}$$

$$- \lambda_1|e_{1i}|^{(1/2)}e_{1i}\left(-\frac{\lambda_1}{2}|e_{1i}|^{-1}e_{1i} - \dot{d}_i\right)$$

$$- 2\lambda_2|e_{1i}|^{-1}e_{1i}e_{2i} - 2\kappa_1\kappa_2$$

$$= \left(\lambda_1^2 + 4\lambda_2^2\right)\left(-\frac{\lambda_1}{2}|e_{1i}|^{(1/2)} + \frac{1}{2}|e_{1i}|^{-1}e_{1i}e_{2i}\right)$$

$$+ \frac{\lambda_1^2}{2}|e_{1i}|^{-1}e_{1i}e_{2i} - \lambda_1|e_{1i}|^{(1/2)}e_{2i}$$

$$+ \lambda_1\lambda_2|e_{1i}|^{(1/2)} + \lambda_1|e_{1i}|^{(1/2)}e_{1i}\dot{d}_i - 2\lambda_2|e_{1i}|^{-1}e_{1i}e_{2i} - 2\kappa_1\kappa_2.$$  

(30)

Furthermore, (30) can be written as follows:
Desired states $\sigma_d, \omega_d, \rho_{c,d}$

Spacecraft formation flying system model

$x_{e}, x_{de}$

Supertwisting disturbance observer

Fast nonsingular terminal sliding mode control law based on the supertwisting disturbance observer

\begin{align*}
V_{st} &= -\left(\frac{\lambda_1^2}{2} + \lambda_1 \lambda_2\right)\|e_{i1}\|^{1/2} + \lambda_2^2 \|e_{i2}\|^{1/2} \text{sign}(e_{i1}) - \frac{\lambda_1}{2} \|e_{i1}\|^{1/2} \\
&+ \lambda_1 \|e_{i1}\|^{1/2} \text{sign}(e_{i1}) d_i - 2e_{i2}d_i \\
&= -\frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(\left(\lambda_1^2 + 2\lambda_2\right)\|e_{i1}\|^{1/2} - 2\lambda_1 e_{i2}\|e_{i1}\|^{1/2} \text{sign}(e_{i1}) + e_{i2}^2\right) \\
&+ \frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(2\|e_{i1}\| \text{sign}(e_{i1}) - \frac{4}{\lambda_1^2}\|e_{i1}\|^{1/2} e_{i2}\right) d_i \\
&\leq -\frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(\left(\lambda_1^2 + 2\lambda_2\right)\|e_{i1}\|^{1/2} - 2\lambda_1 e_{i2}\|e_{i1}\|^{1/2} \text{sign}(e_{i1}) + e_{i2}^2\right) \\
&+ \frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(2\|e_{i1}\| + \frac{4}{\lambda_1^2}\|e_{i1}\|^{1/2} e_{i2}\right) \|d_i\|.
\end{align*}

\begin{align*}
\text{Since } \kappa_1^2 - 2\|\kappa_1\| \kappa_2 + \kappa_2^2 \geq 0, \text{ (31) can be rewritten as}
\end{align*}

\begin{align*}
V_{st} &\leq -\frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(\left(\lambda_1^2 + 2\lambda_2\right)\kappa_1^2 - 2\lambda_1 \kappa_1 \kappa_2 + \kappa_2^2\right) \\
&+ \frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(2\kappa_1^2 + \frac{4}{\lambda_1^2} \|\kappa_1\| \|\kappa_2\|\right) \xi_i \\
&\leq -\frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(\left(\lambda_1^2 + 2\lambda_2\right)\kappa_1^2 - 2\lambda_1 \kappa_1 \kappa_2 + \kappa_2^2\right) \\
&+ \frac{\lambda_1}{2\|e_{i1}\|^{1/2}} \left(2\kappa_1^2 + \frac{2\kappa_1^2}{\lambda_1^2} + \frac{2\kappa_2^2}{\lambda_1^2}\right) \xi_i.
\end{align*}

Define the following matrix as

\begin{align*}
\theta_2 =
\begin{bmatrix}
\lambda_1^2 + 2\lambda_2 - 2(1 + \frac{1}{\lambda_1})\xi_i & -\lambda_1 \\
-\lambda_1 & 1 - 2\xi_i \lambda_1
\end{bmatrix}
\end{align*}

Remark 1. To guarantee that the trajectory errors converge to zero within finite time, the supertwisting disturbance observer gains $\lambda_1$ and $\lambda_2$ should be selected appropriately so that $\theta_1$ and $\theta_2$ are positive definite matrices.

\subsection{Fast Nonsingular Terminal Sliding Mode Control Law with Supertwisting Disturbance Observer}

To design a fast
nonsingular terminal sliding mode control law, the following sliding mode surface is chosen:

\[ s = x_c + \varepsilon_1|x_c|^{c_1}\text{sign}(x_c) + \varepsilon_2|x_{de}|^{m/m}\text{sign}(x_{de}), \quad (39) \]

where \(\varepsilon_1, \varepsilon_2 > 0, c_1 > n/m,\) and \(1 < n/m < 2\) are all positive constants.

If the spacecraft formation flying system state errors reach the sliding mode \(s = 0\), then

\[ x_c + \varepsilon_1|x_c|^{c_1}\text{sign}(x_c) + \varepsilon_2|x_{de}|^{m/m}\text{sign}(x_{de}) = 0. \quad (40) \]

From (15), substituting the equation \(\dot{x}_c = x_{de}\) into (40), we have

\[ \dot{x}_c = -\left( \frac{1}{\varepsilon_2} \right)^{m/m}\left(x_c + \varepsilon_1|x_c|^{c_1}\text{sign}(x_c)\right)^{m/m} \]

In order to analyze the fast convergence of the nonsingular fast terminal sliding mode surface, assume that the integration time of the state variable \(x_c\) denotes as \(x_c(t_i)\) and \(x_c(t_i + \tau_i)\), respectively. Then, integrating the time along both sides of (41),

\[ \int_{x_c(t_i)}^{x_c(t_i + \tau_i)} \left( \frac{1}{x_c} \right)^{m/m} dx_c = \int_{t_i}^{t_i + \tau_i} \left( \frac{1}{\varepsilon_2} \right)^{m/m}\left(1 + \varepsilon_1|x_c|^{c_1-1}\right)^{m/m} dr \]

Furthermore, it can be seen from (42) that the tracking errors of the spacecraft formation flying system can converge to zero in a limited time \(t_s \leq (\varepsilon_2^{m/m})/(n-m)x_c(t_i)\).

**Remark 3.** The proposed nonsingular fast terminal sliding mode surface (39) indicates that when the system state variable \(x_c\) is far from the equilibrium point, the term \(x_c\) plays the major role in making the system trajectory converge at a fast rate. When \(x_c\) approaches to the equilibrium point, the term \(\varepsilon_1|x_c|^{c_1}\text{sign}(x_c)\) plays the dominant role to achieve rapid convergence of the system trajectory.

A design procedure for a fast nonsingular terminal sliding mode control law with a supertwisting disturbance observer is given in the following theorem.

**Theorem 1.** Consider the spacecraft formation flying system described by (15) and (16) with

\[ u = C_i^{-1}\left(AX_{de} + g - C_2\bar{x}_2\right) + C_i^{-1}M\left(u_0 + u_i + u_2\right), \quad (43) \]

where

\[ u_0 = \frac{1}{\varepsilon_2} \frac{m}{n}x_{del}^{2-(n/m)}\text{sign}(x_{de}), \]

\[ u_1 = c_1\left( \frac{\varepsilon_1}{\varepsilon_2} \right) \frac{m}{n}x_{de}^{2-(n/m)}x_{de}^{c_1-1}\text{sign}(x_{de}), \]

\[ u_2 = \alpha_i\text{sign}(s) + \alpha_2|s|^\eta\text{sign}(s), \]

and \(-\alpha_i\text{sign}(s) - \alpha_2|s|^\eta\text{sign}(s)\) is the reaching law with \(\alpha_1 > 0, \alpha_2 > 0,\) and \(\eta > 1.\) Then, the proposed control law along with the supertwisting disturbance observer (26) can guarantee the tracking errors of the spacecraft formation flying system converge to zero in finite time.

**Proof.** From (39), the derivative of the sliding surface \(s\) is given as follows:

\[ s = x_c + \varepsilon_1c_1|x_c|^{c_1-1}x_{de} + \varepsilon_2|x_{de}|^{n/m-1}x_{de} \]

\[ = x_c + \varepsilon_1c_1|x_c|^{c_1-1}x_{de} + \varepsilon_2|x_{de}|^{n/m-1}\left(M^{-1}(-Ax_{de} - g + C_1u + C_2d)\right). \]

Consider the following Lyapunov function:

\[ V_1(t) = \frac{1}{2}s^Ts + \sum_{i=1}^{6}V_{ii} \]

Taking the time derivative of (48), we have

\[ \dot{V}_1(t) = s^Ts + \sum_{i=1}^{6}\dot{V}_{ii} \]

Substituting the proposed control law (44)–(46) into (49),

\[ \dot{V}_1(t) = -s^T\varepsilon_2\frac{n}{m}x_{de}^{(n/m)-1}(\alpha_1\text{sign}(s) + \alpha_2|s|^\eta\text{sign}(s)) \]

\[ + s^T\varepsilon_2\frac{n}{m}x_{de}^{(n/m)-1}M^{-1}C_2(x_2 - d) \]

\[ - \alpha_0 \sum_{i=1}^{6}V_{ii}^{1/2}. \]

Define \(x_{de,\min} = \min\{x_{de}^{(n/m)-1}, \ldots, x_{de}^{(n/m)-1}\}, \bar{a}_1 = x_{de,\min} \varepsilon_2(n/m)a_1,\) and \(\bar{a}_2 = x_{de,\min} \varepsilon_2(n/m)a_2,\) then
\[ \dot{V}_1(t) \leq -\alpha_1 \|s\| - \alpha_2 \|s\|^2 + \varepsilon_1 \frac{n}{m} \|x_{de}\| \|M^{-1} \| x_2 \]
\[ C_2(x_2 - d) - \alpha_0 \sum_{i=1}^{6} V_{1i}^{1/2} \]
\[ \leq -\alpha_1 \|s\| - \alpha_2 \|s\|^2 + \varepsilon_2 \frac{n}{m} \|s\| \|x_{de}\| \|M^{-1} C_2\| \|x_2\| - d\| - \alpha_0 \sum_{i=1}^{6} V_{1i}^{1/2}. \]

(51)

Define \( \alpha_\varepsilon = \varepsilon_2 \frac{n}{m} \|x_{de}\| \|M^{-1} C_2\| \|x_2\| - d\| \) and select \( \alpha_e = \alpha_\varepsilon \geq 0 \); we have
\[ \dot{V}_1(t) \leq -\alpha_\varepsilon \|s\| - \alpha_0 \sum_{i=1}^{6} V_{1i}^{1/2}. \]

(52)

Finally, define \( \alpha_m = \min \{ \sqrt{2} \alpha_\varepsilon, \alpha_0 \} \), then (52) can be rewritten as
\[ \dot{V}_1(t) \leq -\alpha_m \left( \frac{1}{\sqrt{2}} \|s\| + \sum_{i=1}^{6} V_{1i}^{1/2} \right)^{1/2} \]
\[ \leq -\alpha_m \left( \frac{1}{2} \|s\|^2 + \sum_{i=1}^{6} V_{1i} \right)^{1/2} \]
\[ = -\alpha_m V_1(t)^{1/2}. \]

Remark 4. When the system state variable is far from the fast nonsingular terminal sliding mode surface, the term \( \alpha_\varepsilon \|s\|^2 \) in the reaching law plays the major role. On the contrary, the term \( \alpha_\varepsilon \|s\|^2 \) in the reaching law dominates. By choosing different \( \alpha_\varepsilon \) and \( \alpha_0 \) and selecting appropriate parameters in the fast terminal sliding mode surface, the convergence speed can be controlled. Moreover, compared with the conventional reaching law in [47], the proposed reaching law has faster convergence and better sliding performance. When \( s = 0 \) and \( \dot{s} = 0 \), the speed of the state variable approaching the sliding surface is reduced to zero, which can alleviate the chattering phenomenon.

3.2. Fast Nonsingular Terminal Sliding Mode Control Law Based on the Adaptive Neural Network. In this section, an adaptive RBF neural network is employed to approximate the unknown nonlinear \( A \) and \( g \) in the spacecraft formation flying system (15) and (16), which is not easy to be precisely estimated in practical situations. The block diagram of a fast nonsingular terminal sliding mode control law based on the adaptive neural network is depicted in Figure 3.

The structure of RBF neural network with multiple inputs and multiple outputs is shown in Figure 4.

It can be seen that the RBF neural network has three layers including input layer, output layer, and hidden layer.

Denote \( x = [x_1, x_2, \ldots, x_n]^T \) as the input, and \( \phi_i(x) \) is the Gaussian function of the \( i \)th neuron in the hidden layer. Then, the Gaussian function can be expressed as below:
\[ \phi_i(x) = \exp \left( \frac{\|x - c_i\|^2}{2g_i^2} \right), \quad i = 1, 2, \ldots, 6, \]
(54)

where \( c_i \) is the \( i \)th neural radial basis function and \( g_i \) is the width of the \( i \)th neural radial basis function. In this section, we choose \( f_{mn} \) to approximate and compensate the unknown nonlinear parts in the spacecraft formation flying system, where \( f_{mn} = -Ax_{de} - g \) is the unknown nonlinear term. We rewrite \( f_{mn} \) as follows:
\[ f_{mn} = W_i^T \phi_i(x_i) + \xi_i, \]

(55)

where \( W_i \) denotes the ideal weighting matrix and \( \xi_i \) denotes the approximation error, \( i = 1, 2, \ldots, 6. \)

The estimation of \( f_{mn} \) can be expressed as follows:
\[ \tilde{f}_{mn} = \tilde{W}_i^T \phi_i(x_i), \]

(56)

where \( \tilde{W}_i \) denotes the estimation of \( W_i \).

Theorem 2. Consider the spacecraft formation flying system described by (15) and (16), with the following control law and the adaptive updating law of the RBF neural network:
\[ u = C_1^{-1}(\tilde{f}_{mn} - u_3) - C_1^{-1} M(u_0 + u_1 + u_2), \]
(57)

where
\[ u_0 = \frac{1}{\varepsilon_2} \frac{m}{n} \|x_{de}\|^2 \|s\| \|x_{de}\| \|s\| \]
\[ u_1 = c_1 \left( \frac{\varepsilon_1}{\varepsilon_2} \right) \frac{m}{n} \|x_{de}\|^2 \|s\| \|x_{de}\| \|s\| \]
\[ u_2 = \alpha_\varepsilon \|s\|^2 + \alpha_\varepsilon \|s\|^2 \|s\| \]
\[ u_3 = M \|M^{-1} C_1\| \|s\| d_m, \]

(58)

(59)

(60)

(61)

\[ \dot{W}_i = \Pi_{w} \phi_i(x_i) q_i, \]

(62)

where \( q_i \) is the \( i \)th element of the term \( s^T \varepsilon_2 (n/m) x_{de} (n/m)^{-1} M^{-1} \), \( i = 1, 2, \ldots, 6 \), and \( \Pi_{w} \) is a positive symmetrical matrix. The tracking errors of the spacecraft formation flying system will converge to zero.

Proof. Define the error of the weighting vector as \( \dot{W}_i = W_i - \tilde{W}_i \). Consider the following Lyapunov function:
\[ V_2(t) = \frac{1}{2} s^T s + \frac{1}{2} \sum_{i=1}^{n} \text{tr}(\tilde{W}_i^T \Pi_{w} \tilde{W}_i). \]

(63)

Taking the derivative of (63) with respect to time and substituting the proposed control law (57)–(62) and the adaptive updating law (62), then (63) can be converted as
\[
V_2(t) = s^T \dot{s} = -s^T \sum_{i=1}^{n} \mathrm{tr} \left( \hat{W}_i^T \Pi_i^{-1} \hat{W}_i \right) \\
= s^T \left( -\varepsilon \sum_{i=1}^{n} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \right) \\
+ \varepsilon \sum_{i=1}^{n} x_{de_d}^{(n-1)-1} \left( \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \right) \\
+ \varepsilon \sum_{i=1}^{n} x_{de_d}^{(n-1)-1} \left( \alpha_1 \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \right) \\
- \sum_{i=1}^{n} \mathrm{tr} \left( \hat{W}_i^T \Pi_i^{-1} \hat{W}_i \right) \\
= s^T \left( -\varepsilon \sum_{i=1}^{n} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \right) \\
+ \varepsilon \sum_{i=1}^{n} x_{de_d}^{(n-1)-1} \left( \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \right) \\
- \sum_{i=1}^{n} \mathrm{tr} \left( \hat{W}_i^T \Pi_i^{-1} \hat{W}_i \right). \\
\]

(64)

Since
\[
s^T \left( -\varepsilon \sum_{i=1}^{n} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \right) \\
= -\varepsilon \sum_{i=1}^{n} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \|
\]

(65)

Therefore,
\[
V_2(t) \leq s^T \sum_{i=1}^{n} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \|
\]

(66)

can be rewritten as follows:
\[
V_2(t) \leq -\varepsilon \sum_{i=1}^{n} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \|
\]

(67)

Define \( \xi_h = \varepsilon \| x_{de_d} \|_{\infty} \| M_1C_2 \|_{\infty} \| \sigma_m \|_{\infty} \| M_1C_2 \|_{\infty} \|
\]

(68)

where \( \xi = \xi_1 - \xi_h\).

Therefore, it can be concluded that the tracking errors of the spacecraft formation flying system converge to zero and the proof is completed. \(\square\)

4. Numerical Simulation

In this section, the simulation result that shows the effectiveness of the super twisting disturbance observer, the nonsingular fast terminal sliding mode control law, and the adaptive neuro sliding mode control law are presented, respectively, in each section.

The parameters in the formation flying system are given as follows: the masses of the target and the chaser spacecraft are \( m_t = m_c = 110 \text{ kg} \), the initial orbit parameters of the target spacecraft are set as the semimajor axis \( a = 7000 \text{ km} \), the eccentricity \( e = 0.01 \), the orbit inclination \( i = 30^\circ \), and the ascending node is \( \Omega = 45^\circ \), and apogee and perigee are both \( 0^\circ \).

The inertial matrix of the chaser spacecraft is \( J = [3, 0.7; 13.5, 1; 0.5, 1] \text{ kg} \cdot \text{m}^2 \), and the disturbance torque \( d_e = 0.1 \text{ sin}(t) - \cos(0.3t) \text{ sin}(0.5t) \) Nm. The desired attitude and position are given as \( \rho_{de} = [2 - 2 - 0.5] \text{ m} \) and \( \sigma_{de} = 0.001 \text{ sin}(10t) - \cos(10t) \text{ sin}(10t) \).

4.1. Super twisting Disturbance Observer. The gains of the super twisting disturbance observer are chosen as \( \lambda_1 = 2 \) and
Fast nonsingular terminal sliding mode control law based on the adaptive neural network

**Figure 3:** The block diagram of the control law based on the adaptive neural network.

**Figure 4:** The structure of RBF neural network.

**Figure 5:** Comparisons of disturbance estimation error between (a) the linear disturbance observer and (b) the supertwisting disturbance observer.
Figure 6: The state responses of $x_e$ with the fast nonsingular terminal sliding mode control law (43) based on the supertwisting disturbance observer.

Figure 7: The state responses of $x_{de}$ with the fast nonsingular terminal sliding mode control law (43) based on the supertwisting disturbance observer.

Figure 8: The state responses of $x_e$ with the nonlinear PD control law (72).
Figure 9: The state responses of $x_{de}$ with the nonlinear PD control law (72).

Figure 10: The state responses of $x_e$ with the fast nonsingular terminal sliding mode control law (57) based on the adaptive RBF neural network.

Figure 11: The state responses of $x_{de}$ with the fast nonsingular terminal sliding mode control law (57) based on the adaptive RBF neural network.
\( \lambda_2 = 4 \). In order to demonstrate the advantage of the proposed supertwisting disturbance observer, we compare its performance with the traditional linear observer as below:

\[
\begin{align*}
\dot{x}_1 &= -L_1 (x_1 - \hat{x}_1) + x_2, \\
\dot{x}_2 &= -L_2 (x_2 - \hat{x}_2).
\end{align*}
\] (70)

The \( L_1 \) and \( L_2 \) in (70) are selected as

\[
\begin{align*}
L_1 &= \text{diag}(0.023, 0.011, 0.014, 0.012, 0.010, 0.008) \times 10^4, \\
L_2 &= \text{diag}(1.320, 0.300, 0.400, 0.270, 0.160, 0.070) \times 10^3.
\end{align*}
\] (71)

The disturbance estimation error given by the linear disturbance observer (70) and the proposed supertwisting disturbance observer (26) are depicted in Figures 5(a) and 5(b), respectively.

Remark 5. It can be seen from Figure 5 that the supertwisting disturbance observer estimation error converges to zero in finite time, whereas the estimation error from the linear disturbance observer takes longer time to converge to zero. Therefore, the proposed supertwisting observer can estimate the disturbance in finite time, which demonstrates its advantage over the linear one in the spacecraft formation flying system.

4.2. The Nonsingular Fast Terminal Sliding Mode Control Law with Supertwisting Disturbance Observer. The parameters for the nonsingular fast terminal sliding mode surface are set as \( \epsilon_1 = 0.1 \), \( \epsilon_2 = 0.02 \), \( c_1 = 27/19 \), and \( n/m = 21/19 \). The parameters in the control law are set as \( \alpha_1 = 100 \), \( \alpha_2 = 0.1 \), \( \epsilon = 0.5 \), and \( \eta = 1.5 \).

The state responses of \( x_e \) and \( x_{de} \) under the proposed control sliding model control law (43) are given in Figures 6 and 7, respectively.

Furthermore, to illustrate the effectiveness of the proposed control law, in this part, we compare it with a widely used nonlinear PD control law given as follows:

\[
u = C^{-1}M(-K_p x_e - K_d x_{de}),
\] (72)

where the coefficient \( K_p \) and \( K_d \) are calculated as follows:

\[
K_p = \text{diag}(16.5, 3.75, 50, 13.5, 20, 8.75),
\] (73)

\[
K_d = \text{diag}(5.75, 2.75, 3.5, 3.0, 2.5, 2.0).
\]

The state responses of vectors \( x_e \) and \( x_{de} \) under the nonlinear PD control law are given in Figures 8 and 9.

Remark 6. From Figures 6–9, it can be seen that the proposed control law (43) yields better transient responses as compared to the nonlinear PD control law. Also, it is obvious that the proposed control law (43) guarantees the tracking errors converge to zero in a finite time.

4.3. Adaptive Neurosliding Mode Control Law. The parameters of the RBF neural network, \( c_i \) is evenly selected over the space and \( g_i = 10.200 \) neurons are used to approximate each \( f_{m_1}, f_{w_1} = \text{diag}(100, 100, 100, 100, 100, 100) \).

The state responses of the system under the RBF neural network control law (57) are given in Figures 10 and 11.

Remark 7. Figures 10 and 11 indicate that the tracking errors converge to a small region of zero with a good adaptive performance. This concludes that the RBF neural network control law (57) is able to compensate for the unknown dynamics in the spacecraft formation flying system.

5. Conclusion

This paper has proposed the nonsingular fast terminal sliding mode control laws to solve the tracking control problem for spacecraft formation flying systems. First, a supertwisting disturbance observer has been introduced to estimate the disturbances in finite time. Then, based on a nonsingular fast terminal sliding mode surface, a novel control law has been presented to guarantee the tracking errors of the spacecraft formation converge to zero in finite time. Furthermore, for the unknown nonlinear parts in the spacecraft formation flying system, a RBF neural network has been proposed to approximate them and then compensate them. Finally, via simulations the effectiveness of the proposed control laws has been demonstrated. In our future work, we will investigate (1) the spacecraft formation system subject to the time-delay on terminal adaptive neurosliding mode control [48, 49] and (2) nonlinear observer design for the spacecraft formation system.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

All authors declare that they have no conflicts of interest.

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