

Research Article

Multi-Innovation Stochastic Gradient Parameter and State Estimation Algorithm for Dual-Rate State-Space Systems with d -Step Time Delay

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Received 22 May 2020; Revised 16 June 2020; Accepted 22 June 2020; Published 13 July 2020

Guest Editor: Qiang Chen

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This paper presents a multi-innovation stochastic gradient parameter estimation algorithm for dual-rate sampled state-space systems with d -step time delay by the multi-innovation identification theory. Considering the stochastic disturbance in industrial process and using the gradient search, a multi-innovation stochastic gradient algorithm is proposed through expanding the scalar innovation into an innovation vector in order to obtain more accurate parameter estimates. The difficulty of identification is that the information vector in the identification model contains the unknown states. The proposed algorithm uses the state estimates of the observer instead of the state variables to realize the parameter estimation. The simulation results indicate that the proposed algorithm works well.

1. Introduction

The mathematical model can represent the basic features of the system, and system identification applies the statistical methods to set up the mathematical models of dynamic systems from available data [1–4]. There exist some identification methods for state-space models with and without state-delay [5, 6], such as the recursive least squares (RLS) algorithm and the stochastic gradient (SG) algorithm. The SG algorithm is used in adaptive control because of its small computation. Recently, Chen et al. presented an Aitken based the modified Kalman filtering stochastic gradient algorithm for dual-rate nonlinear models [7]. Many identification methods have been developed for linear stochastic systems [8–11], bilinear stochastic systems [12–14], and nonlinear systems with colored noises [15–17].

In system identification, some algorithms concentrate on reducing the calculation amount and improving the recognition accuracy [18–20]. Since the gradient optimization method only requires calculating the first-order derivative,

its computation is small [21]. However, the calculation accuracy of the gradient algorithm is low [22]. In order to improve the calculation accuracy, many improved gradient algorithms have been proposed. Although these improved algorithms can improve the parameter estimation accuracy, the computational complexity is large. The innovation is effective information to improve the parameter estimation accuracy [23, 24]. It can promote the convergence of the algorithm in the recursive process. In order to improve the estimation accuracy through using more innovation, the multi-innovation theory has been used in system recognition.

The stabilities and identification of time-delay systems have drawn a great deal of attention of many researchers in system control and system analysis [25]. In industrial processes and control systems, time delays are difficult to avoid due to material transmission and signal interruptions. The time delay makes it difficult for the control system to respond to the input changes in time [26]. In addition, the time delay can cause instability and unsatisfactory

performance of the controlled process. The recognition of time-delay systems has been a hot topic [27]. For example, Sanz et al. studied an observation and stabilization of LTV systems with time-varying measurement delay [28]. Li et al. discussed the local discontinuous Galerkin method for reaction-diffusion dynamical systems with time delays [29].

This paper studies identification problems of a dual-rate state-space model with d -step time delay. The main contributions of this paper are as follows. The input-output representation is derived from a canonical state-space model of the state-delay system for the identification through eliminating the state variables in the systems, to derive a joint parameter and state estimation algorithm by means of the multi-innovation identification theory and the state observer for reducing the computational burden and improving the parameter estimation accuracy and the convergence speed.

This paper is organized as follows: Section 2 gives the canonical state-space model for state-delay systems; Section 3 introduces the identification model; Section 4 presents a combined multi-innovation stochastic gradient (MISG) parameter and state estimation algorithm; Section 5 provides an illustrative example; and finally, we offer some concluding remarks in Section 6.

2. The Canonical State-Space Model for State-Delay Systems

Let us introduce some symbols. The relation $A := X$ or $X := A$ means that A is defined as X ; \mathbf{I} (\mathbf{I}_n) stands for an identity matrix of appropriate size ($n \times n$); z denotes a unit forward shift operator: $zx(k) = x(k+1)$; T represents the matrix/vector transpose; $\hat{\theta}(k)$ is the estimate of θ at time k ; $\mathbf{1}_n$ means that an $n \times 1$ vector whose elements are all unity; E denotes the expectation operator; $\text{adj}[\mathbf{X}]$ stands for the adjoint matrix of the square matrix \mathbf{X} : $\text{adj}[\mathbf{X}] = \det[\mathbf{X}]\mathbf{X}^{-1}$; $\det[\mathbf{X}] = |\mathbf{X}|$ represents the determinant of the square matrix \mathbf{X} .

Consider the following state-space system with d -step state-delay:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{x}(k-d) + \mathbf{f}u(k), \quad (1)$$

$$y(k) = \mathbf{c}\mathbf{x}(k) + w(k), \quad (2)$$

$$w(k) = \sum_{r=1}^{n_g} g_r v(k-r) + v(k), \quad (3)$$

$$\begin{aligned} \mathbf{A} &:= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_1 \end{bmatrix} \in \mathbb{R}^{n \times n}, \\ \mathbf{B} &:= \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \in \mathbb{R}^{n \times n}, \\ \mathbf{b}_i &\in \mathbb{R}^{1 \times n}, \\ \mathbf{f} &:= [f_1, f_2, \dots, f_n]^T \in \mathbb{R}^n, \\ \mathbf{c} &:= [1, 0, 0, \dots, 0] \in \mathbb{R}^{1 \times n}, \end{aligned} \quad (4)$$

where $x(k) \in \mathbb{R}^n$ is the system state vector, $u(k) \in \mathbb{R}$ is the system input, $y(k) \in \mathbb{R}$ is the system output, $v(k) \in \mathbb{R}$ is a random noise with zero mean, and $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times n}$, $\mathbf{f} \in \mathbb{R}^n$, and $\mathbf{c} \in \mathbb{R}^{1 \times n}$ are the system parameter matrices/vectors. Assume that (\mathbf{c}, \mathbf{A}) is observable and $u(k) = 0$ and $y(k) = 0$ for $k \leq 0$. The system matrices/vectors \mathbf{A} , \mathbf{B} , and \mathbf{f} are the unknown parameters to be estimated from the input-output data $\{u(k), y(k)\}$. If we remove $\mathbf{B}\mathbf{x}(k-d)$ in equation (1), then it becomes the conventional standard state-space model.

Remark 1. For the system in (1) and (2), if the state vector $x(k)$ is known, the system matrix/vector (\mathbf{A}, \mathbf{b}) is easy to identify. This paper considers the case that the state $x(k)$ is completely unavailable. The objective is to propose new methods for jointly estimating the unknown states and parameters from the measurement data $\{u(k), y(k): k = 1, 2, \dots\}$ and to study the performance of the proposed methods.

3. The Identification Model

This section derives the identification model of the canonical state-space model in (1) and (2). From (1), we have

$$\begin{aligned} x_i(k+1) &= x_{i+1}(k) + b_i x(k-d) + f_i u(k), \\ & \quad i = 1, 2, \dots, n-1, \end{aligned} \quad (5)$$

$$\begin{aligned} x_n(k+1) &= a_n x_1(k) + a_{n-1} x_2(k) + \cdots + a_1 x_n(k) \\ & \quad + b_n x(k-d) + f_n u(k). \end{aligned} \quad (6)$$

Let $\mathbf{a} := [a_n, a_{n-1}, \dots, a_1]^T \in \mathbb{R}^n$, using the properties of the shift operator z , multiplying (5) by z^{-i} and (6) by z^{-n} , and adding all expressions give

$$\begin{aligned} x_1(k) &= \mathbf{a}\mathbf{x}(k-n) + \mathbf{b}_1\mathbf{x}(k-d-1) + \mathbf{b}_2\mathbf{x}(k-d-2) + \dots + \mathbf{b}_{n-1}\mathbf{x}(k-d-n+1) \\ &\quad + \mathbf{b}_n\mathbf{x}(k-d-n) + f_1u(k-1) + f_2u(k-2) + \dots + f_nu(k-n) \\ &= \mathbf{a}\mathbf{x}(k-n) + \sum_{i=1}^n \mathbf{b}_i\mathbf{x}(k-d-i) + \sum_{i=1}^n f_iu(k-i). \end{aligned} \quad (7)$$

When $d \leq n-1$, define the information vector $\varphi(k)$ and the parameter vector θ :

$$\begin{aligned} \boldsymbol{\varphi}(k) &:= [\boldsymbol{\varphi}_1^T(k), \boldsymbol{\varphi}_n^T(k)]^T, \\ \boldsymbol{\varphi}_1(k) &:= \begin{bmatrix} \mathbf{x}^T(k-d-1), \mathbf{x}^T(k-d-2), \dots, \mathbf{x}^T(k-n), \dots, \mathbf{x}^T(k-n-d), \\ u(k-1), u(k-2), \dots, u(k-n) \end{bmatrix}^T \in \mathbb{R}^{n^2+n}, \\ \boldsymbol{\varphi}_n(k) &:= [v(k-1), v(k-2), \dots, v(k-n_g)]^T \in \mathbb{R}^{n_g}, \\ \boldsymbol{\theta} &:= [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_n^T]^T, \\ \boldsymbol{\theta}_1 &:= [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{a} + \mathbf{b}_{n-d}, \dots, \mathbf{b}_n, \mathbf{f}^T]^T \in \mathbb{R}^{n^2+n}, \\ \boldsymbol{\theta}_n &:= [g_1, g_2, \dots, g_{n_g}]^T \in \mathbb{R}^{n_g}. \end{aligned} \quad (8)$$

When $d \geq n$, define the information vector $\varphi(k)$ and the parameter vector θ :

$$\begin{aligned} \boldsymbol{\varphi}(k) &:= [\boldsymbol{\varphi}_1^T(k), \boldsymbol{\varphi}_n^T(k)]^T, \\ \boldsymbol{\varphi}_1(k) &:= \begin{bmatrix} \mathbf{x}^T(k-n), \mathbf{x}^T(k-d-1), \mathbf{x}^T(k-d-2), \dots, \mathbf{x}^T(k-d-n+1), \\ \mathbf{x}^T(k-d-n), u(k-1), u(k-2), \dots, u(k-n) \end{bmatrix}^T \in \mathbb{R}^{n^2+2n}, \\ \boldsymbol{\varphi}_n(k) &:= [v(k-1), v(k-2), \dots, v(k-n_g)]^T \in \mathbb{R}^{n_g}, \\ \boldsymbol{\theta} &:= [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_n^T]^T, \\ \boldsymbol{\theta}_1 &:= [\mathbf{a}, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_{n-1}, \mathbf{b}_n, \mathbf{f}^T]^T \in \mathbb{R}^{n^2+2n}, \\ \boldsymbol{\theta}_n &:= [g_1, g_2, \dots, g_{n_g}]^T \in \mathbb{R}^{n_g}. \end{aligned} \quad (9)$$

From (2) and (7), we have

$$y(k) = x_1(k) + w(k) = \boldsymbol{\varphi}^T(k)\boldsymbol{\theta} + w(k). \quad (10)$$

Multirate systems include dual-rate systems [30, 31] and nonuniformly sampled systems [32, 33] as a special cases. In the dual-rate scheme, the observed output is sampled by the sampler and the sampling period is multiple of the input retention period. Assume that the sampling interval is τ

($\tau \geq 2$ is an integer), and thus, the measured input-output data are $\{u(k): k=0, 1, 2, \dots\}$ at the fast rate and $\{y(k\tau): k=0, 1, 2, \dots\}$ at the slow rate. Replacing k in (10) with $k\tau$ gives

$$y(k\tau) = x_1(k\tau) + w(k\tau) = \boldsymbol{\varphi}^T(k\tau)\boldsymbol{\theta} + w(k\tau). \quad (11)$$

The identification model is important for parameter estimation, and many estimation algorithms have been

proposed based on the identification models from observation data [34, 35] such as the gradient algorithms, the least squares algorithms, and the Newton algorithms [36].

Remark 2. This is the identification model of the dual-rate state-space system with d -step state-delay. The information vector $\varphi(k\tau)$ consists of the state vector $x(k\tau - i)$, the input $u(k\tau - i)$, and the correlated noise $w(k\tau - i)$, and the parameter vector θ consists of the parameters a_i , b_i , and f_i of the state-space model in (1) and (2).

Remark 3. In what follows, a SG algorithm is derived for the state-space system with colored noise. Furthermore, a MISG algorithm is presented to reduce the computational burden and enhance the parameter estimation accuracy. A simulation example is provided to evaluate the estimation accuracy and the computational efficiency of the proposed algorithms.

4. The Parameter and State Estimation Algorithm

This section derives a multi-innovation stochastic gradient algorithm to estimate the parameter vector θ in (11) and uses the observer to estimate the state vector $x(k\tau)$ of the system.

4.1. The SG Algorithm. Defining and minimizing the cost function,

$$J(\theta) := [y(k\tau) - \boldsymbol{\varphi}^T(k\tau)\theta]^2, \quad (12)$$

$$\begin{aligned} \widehat{\boldsymbol{\varphi}}(k\tau) &= [\widehat{\boldsymbol{\varphi}}_1^T(k\tau), \widehat{\boldsymbol{\varphi}}_n^T(k\tau)]^T, \\ \widehat{\boldsymbol{\varphi}}_1(k\tau) &= \begin{bmatrix} \widehat{\boldsymbol{x}}^T(k\tau - d - 1), \widehat{\boldsymbol{x}}^T(k\tau - d - 2), \dots, \widehat{\boldsymbol{x}}^T(k\tau - n), \dots, \widehat{\boldsymbol{x}}^T(k\tau - n - d), \\ u(k\tau - 1), u(k\tau - 2), \dots, u(k\tau - n) \end{bmatrix}^T, \\ \widehat{\boldsymbol{\varphi}}_n(k\tau) &= [\widehat{\boldsymbol{v}}(k\tau - 1), \widehat{\boldsymbol{v}}(k\tau - 2), \dots, \widehat{\boldsymbol{v}}(k\tau - n_g)]^T \in \mathbb{R}^{n_g}. \end{aligned} \quad (17)$$

When $d \geq n$, we have

$$\begin{aligned} \widehat{\boldsymbol{\varphi}}(k\tau) &= [\widehat{\boldsymbol{\varphi}}_1^T(k\tau), \widehat{\boldsymbol{\varphi}}_n^T(k\tau)]^T, \\ \widehat{\boldsymbol{\varphi}}_1(k\tau) &= \begin{bmatrix} \widehat{\boldsymbol{x}}^T(k\tau - n), \widehat{\boldsymbol{x}}^T(k\tau - d - 1), \dots, \widehat{\boldsymbol{x}}^T(k\tau - d - n + 1), \widehat{\boldsymbol{x}}^T(k\tau - d - n), \\ u(k\tau - 1), u(k\tau - 2), \dots, u(k\tau - n) \end{bmatrix}^T, \\ \widehat{\boldsymbol{\varphi}}_n(k\tau) &= [\widehat{\boldsymbol{v}}(k\tau - 1), \widehat{\boldsymbol{v}}(k\tau - 2), \dots, \widehat{\boldsymbol{v}}(k\tau - n_g)]^T \in \mathbb{R}^{n_g}. \end{aligned} \quad (18)$$

4.2. The MISG Algorithm. In order to improve the accuracy of the SG algorithm, we extend the SG algorithm and derive a multi-innovation stochastic gradient algorithm by expanding the innovation length.

Define an innovation vector:

and using the gradient search principle, we may obtain a stochastic gradient algorithm:

$$\widehat{\boldsymbol{\theta}}(k\tau) = \widehat{\boldsymbol{\theta}}(k\tau - \tau) + \frac{\boldsymbol{\varphi}(k\tau)}{r(k\tau)} [y(k\tau) - \boldsymbol{\varphi}^T(k\tau)\widehat{\boldsymbol{\theta}}(k\tau - \tau)], \quad (13)$$

$$\begin{aligned} r(k\tau) &= r(k\tau - \tau) + \|\boldsymbol{\varphi}(k\tau)\|^2, \\ r(0) &= 1, \end{aligned} \quad (14)$$

where $1/r(k\tau)$ is the step-size or convergence factor. The choice of $r(k\tau)$ guarantees that the parameter estimation error converges to zero. However, difficulties arise in that the information vector $\boldsymbol{\varphi}(k\tau)$ contains the unknown state vector $x(k\tau - i)$ ($i = 1 + d, 2 + d, \dots, n + d$) and the SG algorithm in (13) and (14) cannot compute the estimate of θ in (11). The approach here is to replace the unknown $x(k\tau - i)$ in $\boldsymbol{\varphi}(k\tau)$ with its $\widehat{\boldsymbol{x}}(k\tau - i)$. Based on the identification model in (11), we can obtain the following stochastic gradient parameter estimation algorithm for estimating θ :

$$\widehat{\boldsymbol{\theta}}(k\tau) = \widehat{\boldsymbol{\theta}}(k\tau - \tau) + \frac{\widehat{\boldsymbol{\varphi}}(k\tau)}{r(k\tau)} [y(k\tau) - \widehat{\boldsymbol{\varphi}}^T(k\tau)\widehat{\boldsymbol{\theta}}(k\tau - \tau)], \quad (15)$$

$$\begin{aligned} r(k\tau) &= r(k\tau - \tau) + \|\widehat{\boldsymbol{\varphi}}(k\tau)\|^2, \\ r(0) &= 1. \end{aligned} \quad (16)$$

When $d \leq n - 1$, we have

$$E(p, k\tau) = \begin{bmatrix} e(k\tau) \\ e(k\tau - 1) \\ \vdots \\ e(k\tau - p + 1) \end{bmatrix} \in \mathbb{R}^p, \quad (19)$$

where the positive integer p represents the innovation length, and

$$e(k\tau - i) = y(k\tau - i) - \hat{\varphi}^T(k\tau - i)\hat{\theta}(k\tau - i - \tau). \quad (20)$$

In general, one may think that the estimate $\hat{\theta}(k\tau - \tau)$ is closer to θ than $\hat{\theta}(k\tau - \tau)$ at time $k\tau - i$ ($i = 2, 3, 4, \dots, p - 1$). Thus, the innovation vector is taken more reasonably to be

$$E(p, k\tau) = \begin{bmatrix} y(k\tau) - \hat{\varphi}^T(k\tau)\hat{\theta}(k\tau - \tau) \\ y(k\tau - 1) - \hat{\varphi}^T(k\tau - 1)\hat{\theta}(k\tau - \tau) \\ \vdots \\ y(k\tau - p + 1) - \hat{\varphi}^T(k\tau - p + 1)\hat{\theta}(k\tau - \tau) \end{bmatrix} \in \mathbb{R}^p. \quad (21)$$

Defining the information matrix $\hat{\Phi}(p, k\tau)$ and stacking output vector $Y(p, k\tau)$ as

$$\begin{aligned} \hat{\Phi}(p, k\tau) &:= [\hat{\varphi}(k\tau), \hat{\varphi}(k\tau - 1), \dots, \hat{\varphi}(k\tau - p + 1)] \\ &\in \mathbb{R}^{(n^2 + n + n_g) \times p}, \\ Y(p, k\tau) &:= [y(k\tau), y(k\tau - 1), \dots, y(k\tau - p + 1)]^T \\ &\in \mathbb{R}^p, \end{aligned} \quad (22)$$

the innovation vector $E(p, k\tau)$ can be equivalently expressed as

$$E(p, k\tau) = Y(p, k\tau) - \hat{\Phi}^T(p, k\tau)\hat{\theta}(k\tau - \tau). \quad (23)$$

Furthermore, we can obtain the following multi-innovation stochastic gradient algorithm with the innovation length p :

$$\hat{\theta}(k\tau) = \hat{\theta}(k\tau - \tau) + \frac{\hat{\Phi}(p, k\tau)}{r(k\tau)} E(p, k\tau), \quad (24)$$

$$E(p, k\tau) = Y(p, k\tau) - \hat{\Phi}^T(p, k\tau)\hat{\theta}(k\tau - \tau), \quad (25)$$

$$\begin{aligned} r(k\tau) &= r(k\tau - \tau) + \|\hat{\varphi}(p, k\tau)\|^2, \\ r(0) &= 1, \end{aligned} \quad (26)$$

$$Y(p, k\tau) = [y(k\tau), y(k\tau - 1), \dots, y(k\tau - p + 1)]^T, \quad (27)$$

$$\hat{\Phi}(p, k\tau) = [\hat{\varphi}(k\tau), \hat{\varphi}(k\tau - 1), \dots, \hat{\varphi}(k\tau - p + 1)]. \quad (28)$$

When $d \leq n - 1$, we have

$$\hat{\varphi}(k\tau) = [\hat{\varphi}_1^T(k\tau), \hat{\varphi}_n^T(k\tau)]^T, \quad (29)$$

$$\hat{\varphi}_n(k\tau) = \begin{bmatrix} \hat{x}^T(k\tau - d - 1), \hat{x}^T(k\tau - d - 2), \dots, \hat{x}^T(k\tau - n), \dots, \hat{x}^T(k\tau - n - d) \\ u(k\tau - 1), u(k\tau - 2), \dots, u(k\tau - n) \end{bmatrix}^T, \quad (30)$$

$$\hat{\varphi}_n(k\tau) = [\hat{v}(k\tau - 1), \hat{v}(k\tau - 2), \dots, \hat{v}(k\tau - n_g)]^T \in \mathbb{R}^{n_g}. \quad (31)$$

When $d \geq n$, we have

$$\hat{\varphi}(k\tau) = [\hat{\varphi}_1^T(k\tau), \hat{\varphi}_n^T(k\tau)]^T, \quad (32)$$

$$\hat{\varphi}_1(k\tau) = \begin{bmatrix} \hat{x}^T(k\tau - n), \hat{x}^T(k\tau - d - 1), \dots, \hat{x}^T(k\tau - d - n + 1), \\ \hat{x}^T(k\tau - d - n), u(k\tau - 1), u(k\tau - 2), \dots, u(k\tau - n) \end{bmatrix}^T, \quad (33)$$

$$\hat{\varphi}_n(k\tau) = [\hat{v}(k\tau - 1), \hat{v}(k\tau - 2), \dots, \hat{v}(k\tau - n_g)]^T \in \mathbb{R}^{n_g}. \quad (34)$$

When the innovation length $p = 1$, the MISG algorithm degrades to the SG algorithm.

Theorem 1. For the system in (11) and the MISG algorithm in (24)–(34), assume that the system noise $v(k\tau)$ is random noise with zero mean and variance σ^2 that is uncorrelated with the input $u(k\tau)$, and the mean square is bounded, that is, (A1) $E[v(k\tau)] = 0$, $E[\|v(k\tau)\|^2] = \sigma^2(k\tau) \leq \sigma^2$; the existence of constant $0 < \alpha \leq \beta < \infty$ and integer $p \geq n$ make the following

condition: (A2) $\alpha I \leq (1/p) \sum_{i=1}^p \varphi(k\tau - i + 1)\varphi^T(k\tau - i + 1) \leq \beta I$, a.s., $k\tau \geq p$. Then, the mean square error of the parameter estimation error given by MISG algorithm is bounded, that is, $\limsup_{k\tau \rightarrow \infty} E[\|\hat{\theta}(k\tau) - \theta\|^2] \leq 4\beta^3\sigma^2/\alpha^4$.

4.3. The State Estimation Algorithm. Using the parameter estimation vector $\hat{\theta}(k\tau)$ to form the system matrices/vectors $\hat{A}(k\tau)$, $\hat{B}(k\tau)$, and $\hat{f}(k\tau)$ and based on the canonical state-

space model in (1)-(2), we can use the following observer to estimate the state vector $x(k\tau)$:

$$\hat{x}(k\tau + \tau) = \hat{A}(k\tau)\hat{x}(k\tau) + \hat{B}(k\tau)\hat{x}(k\tau - d\tau) + \hat{f}(k\tau)u(k\tau), \quad (35)$$

$$\hat{A}(k\tau) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1 \\ \hat{a}_n(k\tau) & \hat{a}_{n-1}(k\tau) & \cdots & \hat{a}_1(k\tau) \end{bmatrix}, \quad (36)$$

$$\hat{B}(k\tau) = \begin{bmatrix} \hat{b}_1(k\tau) \\ \hat{b}_2(k\tau) \\ \vdots \\ \hat{b}_n(k\tau) \end{bmatrix}, \quad (37)$$

$$\hat{f}(k\tau) = \begin{bmatrix} \hat{f}_1(k\tau) \\ \hat{f}_2(k\tau) \\ \vdots \\ \hat{f}_n(k\tau) \end{bmatrix},$$

$$\hat{\theta}(k\tau) = \begin{bmatrix} \hat{b}_1(k\tau), \hat{b}_2(k\tau), \dots, \hat{a}(k\tau) + \hat{b}_{n-d}(k\tau), \\ \dots, \hat{b}_n(k\tau), \hat{f}^T(k\tau) \end{bmatrix}^T, \quad d \leq n-1, \quad (38)$$

$$\hat{\theta}(k\tau) = \begin{bmatrix} \hat{a}(k\tau), \hat{b}_1(k\tau), \hat{b}_2(k\tau), \dots, \hat{b}_{n-1}(k\tau), \\ \hat{b}_n(k\tau), \hat{f}^T(k\tau) \end{bmatrix}^T, \quad d \geq n. \quad (39)$$

The proposed algorithms in this paper can combine some mathematical tools [37] and other identification methods [38–40] to investigate the parameter identification methods of other linear and nonlinear systems [41–49] and can be applied to other literature studies [50–53] such as engineering application systems.

The steps of computing the parameter estimate $\hat{\theta}(k\tau)$ in (23)–(28) and the state estimate $\hat{x}(k\tau + \tau)$ in (34)–(39) are listed in the following:

- (1) Let $k = 1$ and set the initial values $\hat{\theta}(0) = 1_{n^2+2n}/p_0$, $r(0) = 1$, and $p_0 = 10^6$.
- (2) Collect the input-output data $u(k\tau)$ and $y(k\tau)$ and form $\hat{\varphi}(k\tau)$ by (29) or (32), $\hat{\Phi}(p, k\tau)$ by (28), and $Y(p, k\tau)$ by (27), respectively.
- (3) Compute $E(p, k\tau)$ by (25) and $r(k\tau)$ by (26).
- (4) Update the parameter estimation vector $\hat{\theta}(k\tau)$ by (24).
- (5) Read $\hat{a}_i(k\tau)$, $\hat{b}_i(k\tau)$, and $\hat{f}(k\tau)$ from $\hat{\theta}(k\tau)$ according to the definition of $\hat{\theta}(k\tau)$.
- (6) Form $\hat{A}(k\tau)$ and $\hat{B}(k\tau)$ by (36) and (37).
- (7) Compute the state estimation vector $\hat{x}(k\tau + \tau)$ by (35).

- (8) Increase k by 1 and go to step 2, and continue the recursive calculation.

Remark 4. The flop number is used to measure the calculation efficiency (calculation amount) of a complex algorithm. The total number of four floating-point operations required by an algorithm is defined as its calculation amount. Based on this, the calculation efficiency of the algorithm is evaluated as a benchmark, and an efficient and economical algorithm is sought. The calculation method is necessary to analyze the performance of the proposed algorithm.

The computational efficiency of the MISG and the SG algorithms is shown in Tables 1–3. Total floating-point operation (flop) numbers of the MISG and the SG algorithms are $N_1 = (n^2 + n + n_g)(4p + 3)$ and $N_2 = 7(n^2 + n + n_g)$, respectively. The difference between the MISG algorithm and the SG algorithm is

$$\begin{aligned} N_1 - N_2 &= (n^2 + n + n_g)(4p + 3) - 7(n^2 + n + n_g) \\ &= (4p - 4)(n^2 + n + n_g) > 0. \end{aligned} \quad (40)$$

Thus, the SG algorithm has smaller computational efforts than the MISG algorithm.

5. Example

Consider the following dual-rate time-delay system with $\tau = 2$:

$$\begin{aligned} x(t+1) &= \begin{bmatrix} 0 & 1 \\ -0.01 & -0.22 \end{bmatrix} x(t) + \begin{bmatrix} 0.19 & -0.08 \\ 0.16 & -0.12 \end{bmatrix} x(t-2) \\ &\quad + \begin{bmatrix} 0.8 \\ -0.8 \end{bmatrix} u(t), \\ y(t) &= [1, 0]x(t) + v(t). \end{aligned} \quad (41)$$

The parameter vector to be identified is

$$\begin{aligned} \theta &= [a_2, a_1, b_{11}, b_{12}, b_{21}, b_{22}, f_1, f_2]^T \\ &= [-0.01, -0.22, 0.19, -0.08, 0.16, -0.12, 0.8, -0.8]^T. \end{aligned} \quad (42)$$

In simulation, the input $\{u(t)\}$ is taken as an uncorrelated persistent excitation signal sequence with zero mean and unit variance, and $\{v(t)\}$ as a white noise sequence with zero mean and variances $\sigma^2 = 0.30^2$ and $\sigma^2 = 1.00^2$. The parameter estimation based MISG algorithm in (24)–(34) to estimate the parameter vector θ and the state estimation algorithm in (35)–(39) to estimate the state vector $x(t)$ of this example system are applied. The parameter estimates and their estimation errors are shown in Tables 4 and 5 and the parameter estimation errors δ versus t are shown in Figures 1 and 2 with $p = 1, 2, 5$, respectively, and the state estimates $\hat{x}_1(t)$ and $\hat{x}_2(t)$ versus t are shown in Figures 3 and 4.

From Tables 4 and 5 and Figures 1–4, we can draw the following conclusions:

TABLE 1: The computational burden of the MISG algorithm.

Computational sequences	Number of multiplications	Number of additions
$\hat{\theta}(k\tau) = \hat{\theta}(k\tau - \tau) + (\hat{\Phi}(p, k\tau)/r(k\tau))E(p, k\tau)$	$(n^2 + n + n_g)(p + 1)$	$(n^2 + n + n_g)p$
$E(p, k\tau) = Y(p, k\tau) - \hat{\Phi}^T(p, k\tau)\hat{\theta}(k\tau - \tau)$	$(n^2 + n + n_g)p$	$(n^2 + n + n_g)p$
$r(k\tau) = r(k\tau - \tau) + \ \hat{\varphi}(p, k\tau)\ ^2$	$n^2 + n + n_g$	$n^2 + n + n_g$
Sum	$2(n^2 + n + n)(p + 1)$	$(n^2 + n + n_g)(2p + 1)$
Total flops	$(n^2 + n + n)(4p + 3)$	

TABLE 2: The computational efficiency of the SG algorithm.

Computational sequences	Number of multiplications	Number of additions
$\hat{\theta}(k\tau) = \hat{\theta}(k\tau - \tau) + (\hat{\varphi}(k\tau)/r(k\tau))[y(k\tau) - \hat{\varphi}^T(k\tau)\hat{\theta}(k\tau - \tau)]$	$3(n^2 + n + n_g)$	$2(n^2 + n + n_g)$
$r(k\tau) = r(k\tau - \tau) + \ \hat{\varphi}(k\tau)\ ^2$	$n^2 + n + n$	$n^2 + n + n$
Sum	$4(n^2 + n + n_g)$	$3(n^2 + n + n_g)$
Total flops	$7(n^2 + n + n)$	

TABLE 3: Comparison of the computational efficiency of the MISG and the SG algorithms.

Algorithms	Number of multiplications	Number of additions	Total flops
MISG	$2(n^2 + n + n_g)(p + 1)$	$(n^2 + n + n_g)(2p + 1)$	$(n^2 + n + n_g)(4p + 3)$
SG	$4(n^2 + n + n_g)$	$3(n^2 + n + n_g)$	$7(n^2 + n + n_g)$

TABLE 4: The parameter estimates and errors with $\sigma^2 = 0.30^2$.

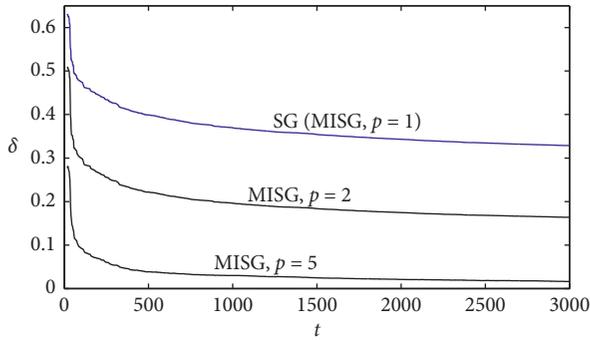
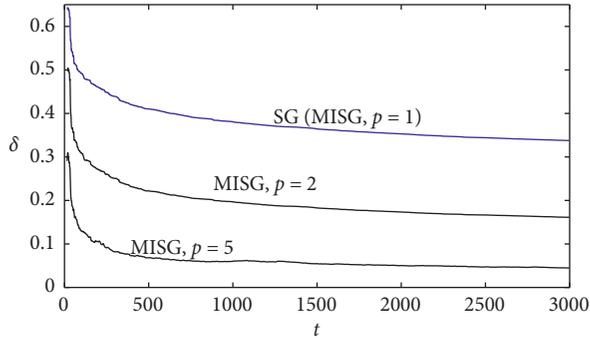
Algorithms	t	a_2	a_1	b_{11}	b_{12}	b_{21}	b_{22}	f_1	f_2	$\delta(\%)$
SG (MISG, $p = 1$)	100	-0.00912	-0.15036	0.04307	-0.06630	0.02505	-0.01004	0.49833	-0.38674	47.52683
	200	-0.00071	-0.15897	0.03430	-0.06226	0.03583	-0.01874	0.51905	-0.41671	44.53744
	500	-0.01158	-0.16218	0.04174	-0.07458	0.04926	-0.02578	0.55028	-0.45958	39.90966
	1000	-0.01300	-0.16742	0.04407	-0.07779	0.05693	-0.02935	0.57120	-0.48809	36.94100
	2000	-0.01608	-0.17243	0.04797	-0.08160	0.06551	-0.03294	0.58943	-0.51204	34.32511
	3000	-0.01644	-0.17616	0.04817	-0.08312	0.07199	-0.03621	0.59949	-0.52557	32.86166
MISG, $p = 2$	100	0.00359	-0.10678	0.07097	-0.11681	0.07671	-0.04656	0.66219	-0.54089	29.99962
	200	0.01599	-0.12109	0.06185	-0.11177	0.09003	-0.05590	0.68106	-0.57759	26.68912
	500	0.00850	-0.13309	0.07485	-0.12515	0.10632	-0.06220	0.70645	-0.62350	22.16532
	1000	0.01018	-0.14260	0.07880	-0.12629	0.11409	-0.06384	0.72115	-0.65182	19.62474
	2000	0.00966	-0.15129	0.08537	-0.12916	0.12141	-0.06584	0.73406	-0.67410	17.47593
	3000	0.01029	-0.15688	0.08764	-0.13042	0.12688	-0.06787	0.74038	-0.68567	16.37969
MISG, $p = 5$	100	-0.01604	-0.16063	0.16648	-0.04942	0.12274	-0.11018	0.77804	-0.72833	9.28037
	200	-0.00941	-0.17261	0.16361	-0.05183	0.13136	-0.11273	0.78161	-0.75681	6.94835
	500	-0.01370	-0.18867	0.17725	-0.06884	0.14888	-0.11953	0.79311	-0.77446	3.86359
	1000	-0.00956	-0.19552	0.17681	-0.06720	0.15297	-0.11685	0.79300	-0.78499	3.00103
	2000	-0.00865	-0.20173	0.18160	-0.07054	0.15557	-0.11647	0.79656	-0.79077	2.10426
	3000	-0.00843	-0.20539	0.18430	-0.07311	0.15762	-0.11675	0.79793	-0.79282	1.61476
True values		-0.01000	-0.22000	0.19000	-0.08000	0.16000	-0.12000	0.80000	-0.80000	

TABLE 5: The parameter estimates and errors with $\sigma^2 = 1.00^2$.

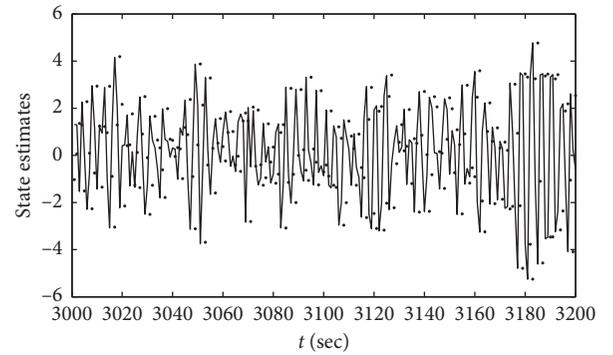
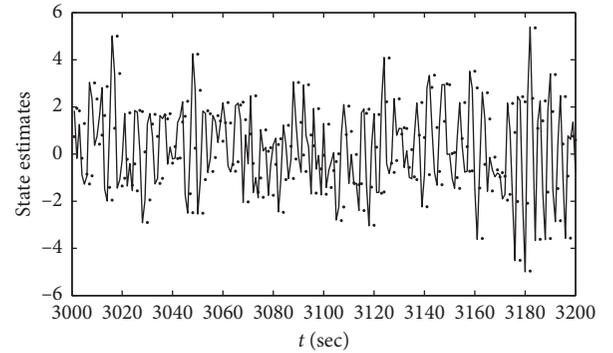
Algorithms	t	a_2	a_1	b_{11}	b_{12}	b_{21}	b_{22}	f_1	f_2	$\delta(\%)$
SG (MISG, $p = 1$)	100	-0.01550	-0.15470	0.04028	-0.06265	0.02094	-0.01630	0.52050	-0.34565	49.20737
	200	-0.00846	-0.16114	0.03142	-0.05900	0.02910	-0.02302	0.53817	-0.38165	46.04580
	500	-0.02283	-0.16425	0.04190	-0.07513	0.04531	-0.03215	0.57022	-0.42744	41.04400
	1000	-0.02246	-0.16947	0.04135	-0.07521	0.05323	-0.03429	0.58828	-0.45950	38.05525
	2000	-0.02481	-0.17522	0.04532	-0.07876	0.06177	-0.03769	0.60570	-0.48594	35.31488
	3000	-0.02492	-0.17967	0.04580	-0.08054	0.06861	-0.04099	0.61533	-0.50079	33.76525

TABLE 5: Continued.

Algorithms	t	a_2	a_1	b_{11}	b_{12}	b_{21}	b_{22}	f_1	f_2	$\delta(\%)$
MISG, $p = 2$	100	-0.00070	-0.09901	0.08115	-0.11549	0.07387	-0.05378	0.68241	-0.51521	30.88244
	200	0.01133	-0.11098	0.06966	-0.10975	0.08403	-0.06043	0.69602	-0.56265	27.17174
	500	0.00040	-0.12474	0.08572	-0.12822	0.10703	-0.07177	0.72361	-0.60977	22.19946
	1000	0.00665	-0.13482	0.08321	-0.12254	0.11560	-0.07075	0.73290	-0.64205	19.67043
	2000	0.00802	-0.14527	0.08971	-0.12475	0.12278	-0.07232	0.74507	-0.66653	17.34685
	3000	0.00917	-0.15233	0.09273	-0.12666	0.12873	-0.07427	0.75100	-0.67898	16.12797
MISG, $p = 5$	100	-0.02959	-0.13914	0.19948	-0.01531	0.08421	-0.15341	0.76481	-0.73279	12.99066
	200	-0.01861	-0.15341	0.18733	-0.01475	0.09415	-0.14951	0.76310	-0.78040	10.55685
	500	-0.02916	-0.17950	0.20785	-0.04599	0.13403	-0.16618	0.79390	-0.78371	6.83573
	1000	-0.01718	-0.18774	0.19516	-0.03222	0.14082	-0.15378	0.78631	-0.79542	6.02358
	2000	-0.01503	-0.19804	0.20265	-0.03826	0.14446	-0.14983	0.79412	-0.79944	5.03453
	3000	-0.01509	-0.20501	0.20834	-0.04462	0.14803	-0.14864	0.79741	-0.80005	4.46176
True values		-0.01000	-0.22000	0.19000	-0.08000	0.16000	-0.12000	0.80000	-0.80000	

FIGURE 1: The parameter estimation errors δ versus t with $\sigma^2 = 0.30^2$.FIGURE 2: The parameter estimation errors δ versus t with $\sigma^2 = 1.00^2$.

- (1) The parameter estimates converge fast to their true values for large p , see Tables 4 and 5
- (2) The MISG algorithm with $p \geq 2$ has higher accuracy than the SG algorithm, see Figures 1 and 2
- (3) The parameter estimation errors given by the MISG algorithm become smaller with the data length t and the innovation length p increasing, see Tables 4 and 5 and Figures 1 and 2
- (4) The state estimates are close to their true values with t increasing, see Figures 3 and 4

FIGURE 3: The state and state estimate $\hat{x}_1(t)$ versus t ; solid line: the true $x_1(t)$; dots: the estimated $\hat{x}_1(t)$.FIGURE 4: The state and state estimate $\hat{x}_2(t)$ versus t ; solid line: the true $x_2(t)$; dots: the estimated $\hat{x}_2(t)$.

6. Conclusions

This study has taken up a category of state-space models with state time delay as the research background and accordingly developed two folds of solutions for the model identification (parameter and state estimation in specific). The theoretical analysis has proved that the estimates converge to the real value under the condition of continuous excitation in modelling. The algorithms used in this paper can be applied to hybrid switched impulsive power networks and uncertain

chaotic nonlinear systems with time delay [54–57] and can be applied to other literature studies [58–64]. The simulation case study has demonstrated that the proposed algorithms/procedures are effective and efficient in design and implementation.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (nos. 61903050 and 61673277), the Natural Science Foundation of Jiangsu Province (no. BK20181033), the Natural Science Fundamental Research Project of Colleges and Universities in Jiangsu Province (no. 18KJB120001), the Science and Technology Development Plan Project of Changshu under grant no. CR201711, and Jiangsu Planned Projects for Postdoctoral Research Funds.

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