Hindawi
Complexity
Volume 2020, Article ID 6514236, 10 pages
https://doi.org/10.1155/2020/6514236

Research Article

Modeling Method of the Grey GM(1,1) Model with Interval Grey Action Quantity and Its Application

Bo Zeng,[1] **Xin Ma**,[2] **and Juanjuan Shi**[3]

[1]*Chongqing Key Laboratory of Spatial Data Mining and Big Data Integration for Ecology and Environment, Rongzhi College of Chongqing Technology and Business University, Chongqing 401320, China*
[2]*School of Science, Southwest University of Science and Technology, Mianyang 621010, China*
[3]*School of Rail Transportation, Soochow University, Suzhou 215137, China*

Correspondence should be addressed to Bo Zeng; bozeng@ctbu.edu.cn

Received 25 September 2019; Accepted 2 January 2020; Published 31 January 2020

Academic Editor: Eulalia Martínez

GM(1,1) is a univariate grey prediction model with incomplete structural information, in which the real number form of the simulation or prediction data does not conform to the Nonuniqueness Principle of Grey theoretical solution. In light of the network model of GM(1,1), the connotation of grey action quantity is systematically analyzed and the interval grey number form of grey action quantity is restored under uncertain influencing factors. A novel GM(1,1) model is then constructed. The new model has the basic characteristics of the grey model under incomplete information. Moreover, it can be fully compatible with the traditional GM(1,1) model. The developed model is employed to the natural gas consumption prediction in China, showing that its predicting rationality is much better than that of the traditional GM(1,1) model. It is worth mentioning that, for the first time, the grey property of GM(1,1) has been restored in structure, which is of significance for both academia and industry.

1. Introduction

In 1982, Professor Deng proposed the GM(1,1) model [1] with predictive function based on cybernetics. GM(1,1) is a single-variable grey prediction model with a first-order difference equation [2]. Its greatest feature is that GM(1,1) has only a dependent variable but no independent variables [3, 4]. Grey theory holds that the development and evolution of a system are influenced by many uncertain external environments and internal factors (Grey causes) [5]. Under such circumstances, it is difficult to establish a definite functional relationship between dependent variables and independent variables to analyze and predict the future development trend of the system [6, 7]. However, under the influence and restriction of many factors, the operation results of the system are determined (White results) [8]. In other words, the results of system operation are the final manifestation of the system under the influence of many factors, which can comprehensively reflect the evolution trend and development law of the system under the combined action of these factors [9, 10].

GM(1,1) has many advantages [5, 11], such as small amount of data needed, simple modeling process, and easy to learn and use. It has been widely used to solve various prediction problems in production and life [12]. With the deepening of application, the theoretical system of GM(1,1) has been enriched and improved, and a lot of research results have been produced. Generally speaking, these achievements mainly include the following four aspects:

(a) Optimization of GM(1,1) parameters: such as initial condition optimization [13, 14], background value optimization [15, 16], and accumulation order optimization [17–19]

(b) Optimization of GM(1,1) structure: realizing the optimization of model structure from the single exponential form to intelligent variable structure [20–22]

(c) Extension of GM(1,1) modeling object: to achieve the expansion of modeling objects from real data to grey uncertain data [23–25]

(d) GM(1,1) combined forecasting model: the combination prediction technologies of GM(1,1) and other methods are studied, such as Grey neural network model [26–28], Grey Markov model [29, 30], Grey support vector machine [31, 32], and Grey deep learning [33, 34]

The above research results play an important role in improving the simulation and prediction performance and expanding the application scope of GM(1,1). However, GM(1,1) is a grey model with incomplete structural information (the absence of independent variables). According to the "Nonuniqueness Principle" of grey theory [35], the solution with incomplete and uncertain information is not unique. Therefore, the simulation or prediction results of GM(1,1) should be nonunique. On the contrary, the current GM(1,1) model's simulation or prediction results are unique [36]. This is mainly because the GM(1,1) model does not consider the "grey" uncertainty of grey action quantity and simplifies it to a real number. However, the real number means that the GM(1,1) model is a time sequence prediction model with deterministic structure, so its simulation and prediction results are unique.

It can be seen that the existing GM(1,1) model is a simplified model, its research process ignores the uncertainty characteristics of grey action quantity, and its prediction results also violate the "nonuniqueness principle of solution" of grey theory. For this reason, starting from the network model of GM(1,1), the interval uncertainty form of grey action quantity is restored. On this basis, a new GM(1,1) model is established. The simulation and prediction results of the new model are both interval grey numbers with known probability functions.

The remainder of this paper is organized as follows. In Section 2, we analyzed the essence and connotation of the grey action quantity "b" of GM(1,1). In Section 3, we proposed and deduced the new GM(1,1) model with an interval grey action quantity. In Section 4, we employed the new model to simulate and predict the nature gas total consumption in China and compared and analyzed the reasonableness of the results. Our conclusions are presented in Section 5.

2. Essence and Connotation of Grey Action Quantity

In the univariate grey system, system characteristic variables describe the evolution law of the system, which is the result of the interaction of many complex external factors. They are all real numbers. The influencing factors of system development are "cause." The result of change embodied in the system is "result." In cybernetics, the former is called input, and the latter is called output. In a single-variable grey system, because the independent variables are unknown, the comprehensive effect of many uncertain and complex factors on the development of the system is expressed by parameter "b." Therefore, parameter "b" is called the grey action quantity and represents all grey uncertainty information (Grey Information Coverage) [37].

In GM(1,1), the relationship between grey action quantity "b" and system output $x^{(0)}(k)$ (system characteristic variable) [36] is shown in Figure 1.

In Figure 1, the input variable "b" represents all the uncertain factors (Grey factors) affecting the system development and the output variable $x^{(0)}(k)$ is the characteristic variable (White result) of the system. $x^{(0)}(k)$ adjusts the size of parameter "b" by AGO (Accumulation Generation Operator, weakening randomness) and MEAN (MEAN generation of consecutive neighbors sequence, improving smoothness). The main purpose of AGO [35] and MEAN [35] is to weaken the influence of extreme values in raw data on input variable "b." In Figure 1, the feedback coefficient "a" is called the development coefficient and its size and symbols reflect the development trend of $x^{(0)}(k)$.

According to the relationship between input, output, and feedback of the system in Figure 1, $b - a \cdot \mathrm{MEAN} = x^{(0)}(k) \Longrightarrow x^{(0)}(k) + az^{(1)}(k) = b$ can be obtained, which is the basic form of the classical GM(1,1) model. The parameters "a" and "b" are estimated by the least square method, which are all real numbers. Because grey action quantity "b" represents the influence of all external factors on the development trend of the system, it is essentially uncertain (Grey factors), and its form should be grey number. However, in the modeling process of the GM(1,1), the grey attribute of "b" is not taken into account which is estimated and modeled with a real number. This obviously does not agree with the actual meaning of "b," which leads to the poor reliability of the prediction results of the GM(1,1) model.

The GM(1,1) model is a grey model with incomplete structural information. The uncertainty and complexity of the influencing factors are caused by incomplete structural information. However, the simulation and prediction results of the current GM(1,1) model are determined as real numbers, which is totally inconsistent with the nonuniqueness principle of the grey theory solution. Therefore, it is necessary to restore the "grey" uncertainty characteristics of grey action quantity "b" and build a new GM(1,1) model on this basis.

3. New GM(1,1) Model

In this section, the interval grey number form of grey action quantity "b" will be restored under the uncertainty of influencing factors. On this basis, a new GM(1,1) model is constructed. Because the grey action quantity "b" is an interval grey number, the simulation and prediction results of GM(1,1) are also interval grey numbers, which satisfies the nonuniqueness of GM(1,1) prediction results under uncertain conditions.

3.1. Basic Concepts of the GM(1,1) Model

Definition 1 (see [35]). Assume that $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ is a nonnegative sequence, where $x^{(0)}(k) \geq 0$, $k = 1, 2, \ldots, n$. Then, $X^{(1)} = (x^{(1)}(1), x^{(1)}(2),$

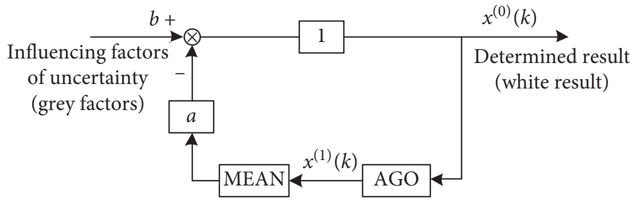

Figure 1: The network model diagram of GM(1,1) [37].

$\ldots, x^{(1)}(n))$ is called the 1-AGO (Accumulating Generation Operator) sequence of $X^{(0)}$, where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k), \quad k = 1, 2, \ldots, n, \tag{1}$$

and $Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n))$ is called the mean generation of consecutive neighbors sequence of $X^{(1)}$, where

$$z^{(1)}(k) = 0.5 \times \left(x^{(1)}(k) + x^{(1)}(k-1)\right), \quad k = 2, 3, \ldots, n. \tag{2}$$

Definition 2 (see [1]). Let $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ be the same as in Definition 1; then,

$$x^{(0)}(k) + az^{(1)}(k) = b, \tag{3}$$

is called the basic form of GM(1,1), which is derived from Figure 1, that is,

$$\begin{aligned} b - a \cdot \text{MEAN} = x^{(0)}(k) &\Longrightarrow b - az^{(1)}(k) = x^{(0)}(k) \\ &\Longrightarrow x^{(0)}(k) + az^{(1)}(k) = b. \end{aligned} \tag{4}$$

Theorem 1 (see [1]). *Let $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ be the same as in Definition 1, $\widehat{a} = (a, b)^T$ be a sequence of parameters, and*

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}. \tag{5}$$

Then, the least square estimate sequence of grey differential equation $x^{(0)}(k) + az^{(1)}(k) = b$ satisfies

$$\widehat{a} = \left(B^T B\right)^{-1} B^T Y. \tag{6}$$

Detailed proof can be found in reference [38].

3.2. Interval Grey Number Form of Grey Action Quantity. In cybernetics, there is a corresponding relationship between each input and output. Grey action quantity covers all unascertained information and has different sizes at different time points (Figure 2). Usually, $b_2, b_3, \ldots, b_n$ are not equal, that is, $b_2 \neq b_3 \neq \cdots \neq b_n$.

According to Theorem 1, the parameters $\widehat{a} = (a, b)^T$ are estimated by the least square method under the condition of minimizing the sum of squares of simulation errors of $x^{(0)}(k), k = 2, 3, \ldots, n$. In other words, the parameters "b" in Theorem 2 is an approximate value, which is used to represent all the grey action quantities $b_2, b_3, \ldots$, and b_n of each input. Then, the information difference between grey action quantities is completely ignored. Therefore, the simulated and predicted data based on parameter "b" in Theorem 2 are only an approximate solution. It can be seen that the traditional GM(1,1) model violates the nonuniqueness principle of the solution of grey theory under incomplete information.

In this section, according to the relationship between each input and output of the system, the uncertain information contained in grey action quantity is fully excavated, and the interval grey number form of grey action quantity is restored. On this basis, a new GM(1,1) model is constructed.

According to equation (3), the grey action quantity with different values of $k (k = 2, 3, \ldots, n)$ can be calculated, as follows:

$$\begin{cases} k = 2 \longrightarrow b_2 = x^{(0)}(2) + az^{(1)}(2), \\ k = 3 \longrightarrow b_3 = x^{(0)}(3) + az^{(1)}(3), \\ \vdots \\ k = n \longrightarrow b_n = x^{(0)}(n) + az^{(1)}(n). \end{cases} \tag{7}$$

Then, we call $Bs = \{b_2, b_3, \ldots, b_n\}$ is the sequence of grey action quantity of GM(1,1). The maximum value $b_{\max}$ and minimum value $b_{\min}$ of $Bs = \{b_2, b_3, \ldots, b_n\}$ can be obtained, as follows:

$$\begin{aligned} b_{\max} &= \max\{b_2, b_3, \ldots, b_n\}, \\ b_{\min} &= \min\{b_2, b_3, \ldots, b_n\}. \end{aligned} \tag{8}$$

After this, grey action quantity of GM(1,1) can be expressed as the interval grey number form, that is, $\otimes_b \in [b_{\min}, b_{\max}]$.

According to equation (3), the grey action quantity b_k is positively correlated with $x^{(0)}(k)$. That is, the bigger the b_k is, the bigger the $x^{(0)}(k)$ is. The parameter "b" in GM(1,1) is estimated by the least square method, which is a compromise value between $b_{\min}$ and $b_{\max}$. Obviously, $b_{\min} \leq b \leq b_{\max}$, that is, $b \in [b_{\min}, b_{\max}]$. On the other hand, under the existing conditions, the maximum possible value of interval grey number $\otimes_b$ is neither $b_{\min}$ or not $b_{\max}$, but "b." The parameter "b" is the real number most likely to represent the whitening value of interval grey number $\otimes_b \in [b_{\min}, b_{\max}]$, that is, $\widetilde{\otimes}_b = b$. According to the definition of probability function [35], $\otimes_b \in [b_{\min}, b_{\max}]$ can be expressed as in Figure 3.

3.3. New GM(1,1) Model with Interval Grey Action Quantity

Definition 3. Let $X^{(0)}$, $X^{(1)}$, $Z^{(1)}$, and a be the same as in Definition 1 and Theorem 1. Then, $P = (a, \otimes_b)^T$ is called the sequence of grey parameters, and a is named as the

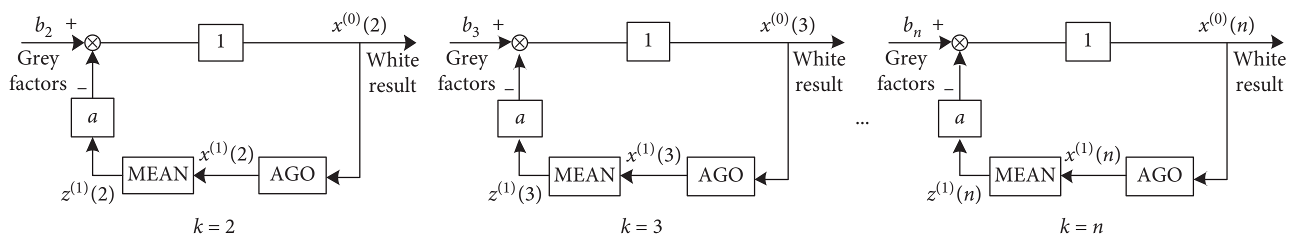

Figure 2: Corresponding relationship between input and output of GM(1,1).

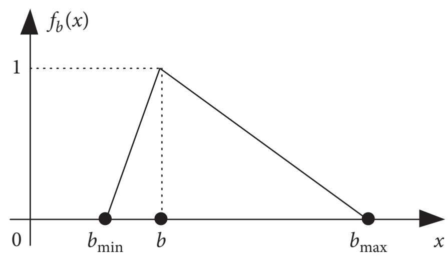

Figure 3: Interval grey number form of Grey action quantity and its possibility function.

development coefficient, and $\otimes_b \in [b_{\min}, b_{\max}]$ is called the interval grey action quantity.

Definition 4. Let $X^{(0)}$, $X^{(1)}$, $Z^{(1)}$, and P be the same as in Definitions 1 and 3; then,

$$x^{(0)}(k) + az^{(1)}(k) = \otimes_b \in [b_{\min}, b_{\max}], \tag{9}$$

is called the GM(1,1) model in which grey action quantity is the interval grey number $\otimes_b$, $\mathrm{GM}(1,1,\otimes_b)$ for short. And

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = \otimes_b \in [b_{\min}, b_{\max}], \tag{10}$$

is called the whitinization (or image) equation of $x^{(0)}(k) + az^{(1)}(k) = \otimes_b \in [b_{\min}, b_{\max}]$.

Theorem 2. *Let $X^{(0)}$, $X^{(1)}$, $Z^{(1)}$, and P be the same as in Definitions 1 and 3; then,*

(i) The solution (or called a time response function) of $(dx^{(1)}/dt) + ax^{(1)} = \otimes_b \in [b_{\min}, b_{\max}]$ is given by

$$\begin{aligned} x_{\min}^{(1)}(t) &= \left(x^{(1)}(1) - \frac{b_{\min}}{a}\right)e^{-at} + \frac{b_{\min}}{a}, \\ x_{\mathrm{mid}}^{(1)}(t) &= \left(x^{(1)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a}, \\ x_{\max}^{(1)}(t) &= \left(x^{(1)}(1) - \frac{b_{\max}}{a}\right)e^{-at} + \frac{b_{\max}}{a}. \end{aligned} \tag{11}$$

(ii) The time response sequence of $(dx^{(1)}/dt) + ax^{(1)} = \otimes_b \in [b_{\min}, b_{\max}]$ is given by

$$\begin{aligned} \hat{x}_{\min}^{(1)}(k+1) &= \left(x^{(0)}(1) - \frac{b_{\min}}{a}\right)e^{-ak} + \frac{b_{\min}}{a}, \quad k = 1, 2, \ldots, n; \\ \hat{x}_{\mathrm{mid}}^{(1)}(k+1) &= \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, \quad k = 1, 2, \ldots, n; \\ \hat{x}_{\max}^{(1)}(k+1) &= \left(x^{(0)}(1) - \frac{b_{\max}}{a}\right)e^{-ak} + \frac{b_{\max}}{a}, \quad k = 1, 2, \ldots, n. \end{aligned} \tag{12}$$

(iii) The restored values can be given by

$$\begin{aligned} \hat{x}_{\min}^{(0)}(k+1) &= \hat{x}_{\min}^{(1)}(k+1) - \hat{x}_{\min}^{(1)}(k) \\ &= \left(1 - e^{a}\right)\left(x^{(0)}(1) - \frac{b_{\min}}{a}\right)e^{-ak}, \quad k = 1, 2, \ldots, n, \\ \hat{x}_{\mathrm{mid}}^{(0)}(k+1) &= \hat{x}_{\mathrm{mid}}^{(1)}(k+1) - \hat{x}_{\mathrm{mid}}^{(1)}(k) \\ &= \left(1 - e^{a}\right)\left(x^{(0)}(1) - \frac{b_{\mathrm{mid}}}{a}\right)e^{-ak}, \quad k = 1, 2, \ldots, n, \\ \hat{x}_{\max}^{(0)}(k+1) &= \hat{x}_{\max}^{(1)}(k+1) - \hat{x}_{\max}^{(1)}(k) \\ &= \left(1 - e^{a}\right)\left(x^{(0)}(1) - \frac{b_{\max}}{a}\right)e^{-ak}, \quad k = 1, 2, \ldots, n. \end{aligned} \tag{13}$$

According to Theorem 2, when the grey action quantity of GM(1,1) is expanded from real number b to interval grey number $\otimes_b$, the GM (1,1) model evolves into the new $\mathrm{GM}(1,1,\otimes_b)$ model, and the simulation or predicted results of $\mathrm{GM}(1,1,\otimes_b)$ have the following characteristics:

(1) The simulated or predicted result of $\mathrm{GM}(1,1,\otimes_b)$ is an interval grey number $\otimes(k)$
(2) The interval grey number $\otimes(k)$ has the definite lower $\hat{x}_{\min}^{(0)}(k)$ and upper bounds $\hat{x}_{\max}^{(0)}(k)$, that is, $\otimes(k) \in [\hat{x}_{\min}^{(0)}(k), \hat{x}_{\max}^{(0)}(k)]$
(3) The possibility function of the interval grey number $\otimes(k)$ is a triangle, and its maximum possible value $\tilde{\otimes}(k)$ is $\hat{x}_{\mathrm{mid}}^{(0)}(k)$, that is $\tilde{\otimes}(k) = \hat{x}_{\mathrm{mid}}^{(0)}(k)$

The schematic diagram of the interval grey number $\otimes(k)$ and its probability function is shown in Figure 4.

It can be seen that when the grey action quantity "b" is restored to an interval grey number $\otimes_b \in [b_{\min}, b_{\max}]$, the simulation and prediction data of the $\mathrm{GM}(1,1,\otimes_b)$ model are also interval grey numbers. In the case of uncertain system

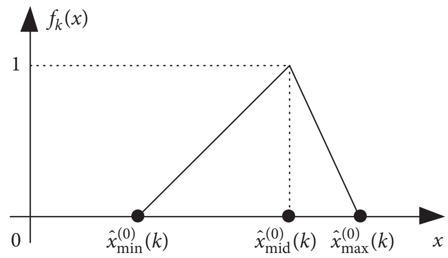

FIGURE 4: The possibility function of the simulated or predicted result $\otimes(k)$ of $\mathrm{GM}(1,1,\otimes_b)$.

structure information, the grey number form of simulation or prediction results conforms to the Nonuniqueness Principle of Grey theory solution. Meanwhile, the real number form of simulation and prediction results of the traditional GM(1,1) model is retained in the results. Compared with the traditional GM(1,1) model, which simplifies the grey action quantity b excessively and the reasonable prediction results may be lost, the proposed new $\mathrm{GM}(1,1,\otimes_b)$ model extends the effective range of the simulation and prediction results of GM(1,1) to the greatest extent.

4. Model Application and Rationality Analysis

With the increasing demand for natural gas in China's civil and industrial sectors, China has surpassed Japan to become the world's largest importer of natural gas and also the world's most heavily dependent importer of natural gas. In 2018 alone, China imported 125.4 billion cubic meters of natural gas, a growth rate of 31.7%. Under the background of the international trade rule of "take or pay" of natural gas and the rapid increase of China's demand for natural gas, the stable and orderly supply of natural gas has become an important factor threatening China's energy security.

According to China's Statistical Yearbook (data.-stats.gov.cn/easyquery.htm?cn=C01), China's total natural gas consumption (ten thousand tons of standard coal) in 2009–2018 is shown in Table 1.

In order to test the comprehensive performance of the $\mathrm{GM}(1,1,\otimes_b)$ model, it is necessary to test the simulation and prediction results of the model at the same time. In this paper, the first seven data in Table 1 are used as the raw data to build the $\mathrm{GM}(1,1,\otimes_b)$ model and the last three data are used as the reserved data to test the prediction performance of the $\mathrm{GM}(1,1,\otimes_b)$ model.

Then, the modeling data $X^{(0)}$ is as follows:

$$\begin{aligned} X^{(0)} &= \left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5),\right. \\ &\quad \left. x^{(0)}(6), x^{(0)}(7)\right) \\ &= (11764.41, 14425.92, 17803.98, 19302.62, 22096.39, \\ &\quad 24270.94, 25364.40). \end{aligned} \tag{14}$$

Step 1. Generating new sequences $X^{(1)}$ and $Z^{(1)}$:

According to Definition 1, $X^{(1)}$ and $Z^{(1)}$ are be obtained, as follows:

$$\begin{aligned} X^{(1)} &= \left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), x^{(1)}(4), x^{(1)}(5), x^{(1)}(6), x^{(1)}(7)\right) \\ &= (11764.41, 26190.33, 43994.31, 63296.93, 85393.32, \\ &\quad 109664.26, 135028.66), Z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3),\right. \\ &\quad \left. z^{(1)}(4), z^{(1)}(5), z^{(1)}(6), z^{(1)}(7)\right) = (18977.37, 35092.32, \\ &\quad 53645.62, 74345.125, 97528.79, 122346.46). \end{aligned} \tag{15}$$

Step 2. Constructing Matrices Y and B and computing parameters a and b:

According to Theorem 1, Matrices Y and B can be constructed, as follows:

$$\begin{aligned} Y &= \begin{bmatrix} 14425.92 \\ 17803.98 \\ 19302.62 \\ 22096.39 \\ 24270.94 \\ 25364.40 \end{bmatrix}; \\ B &= \begin{bmatrix} -18977.37 & 1 \\ -35092.32 & 1 \\ -53645.62 & 1 \\ -74345.125 & 1 \\ -97528.79 & 1 \\ -122346.46 & 1 \end{bmatrix}. \end{aligned} \tag{16}$$

Then,

$$\hat{a} = (a,b)^T = \left(B^T B\right)^{-1} B^T Y = \begin{bmatrix} -0.1046 \\ 13538.1421 \end{bmatrix}. \tag{17}$$

Step 3. Constructing the interval grey action quantity $\otimes_b \in [b_{\min}, b_{\max}]$:

According to Definition 1 and the development coefficient a, the known data $x^{(0)}(k)$ and $z^{(1)}(k)$, $(k = 2, 3, \ldots, 7)$, the grey action quantity b_k at time point k can be computed, as follows:

$$\begin{aligned} Bs &= \{b_2, b_3, b_4, b_5, b_6, b_7\} \\ &= \{12441.2212, 14133.9410, 13692.2325, 14321.1986, \\ &\quad 14071.1454, 12569.1140\}. \end{aligned} \tag{18}$$

Then,

$$\begin{aligned} b_{\max} &= \max\{b_2, b_3, b_4, b_5, b_6, b_7\} = 14321.1986, \\ b_{\min} &= \max\{b_2, b_3, b_4, b_5, b_6, b_7\} = 12441.2212. \end{aligned} \tag{19}$$

So, the interval grey action quantity $\otimes_b \in [b_{\min}, b_{\max}]$ is as follows:

Table 1: China's total natural gas consumption (TC) in 2009–2018 (unit: ten thousand tons standard coal).

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
TC	11764.41	14425.92	17803.98	19302.62	22096.39	24270.94	25364.40	27904.00	31397.04	36192.00

$$\otimes_b \in [12441.2212, 14321.1986], \quad \tilde{\otimes}_b = b = 13538.1421, \tag{20}$$

and the possibility function of $\otimes_b \in [b_{\min}, b_{\max}]$ is shown in Figure 5.

The relationship between the grey action quantity at different time points and the grey action quantity b of the traditional GM (1,1) model is shown in Figure 6.

According to Figure 6, we can see that the grey action quantity b of the traditional GM(1,1) model is a compromise value and the size of b is estimated under the condition of minimizing the sum of squares of residual errors of the simulated data. Therefore, the process conceals the difference of grey action quantity at different points and loses some known information, which is the main reason why the simulation and prediction results of the traditional GM(1,1) model are unstable.

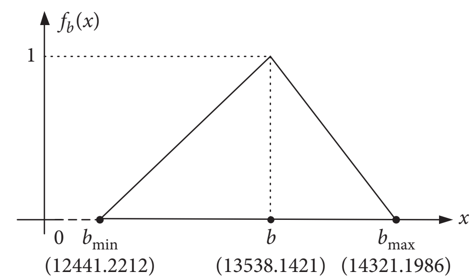

Figure 5: The possibility function of $\otimes_b \in [b_{\min}, b_{\max}]$.

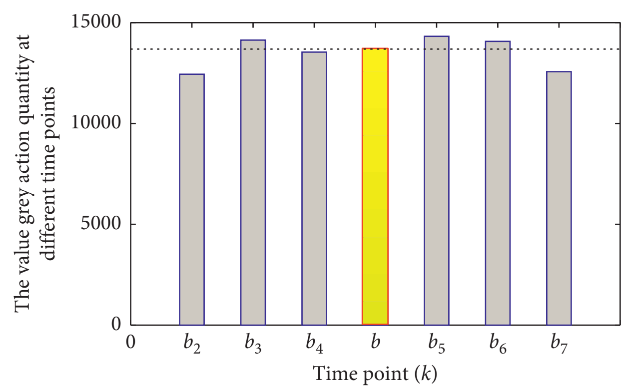

Figure 6: Grey action quantity at different time point.

Step 4. Computing the simulation and prediction data: $\hat{x}^{(0)}_{\min}$, $\hat{x}^{(0)}_{\max}(k)$, and $\hat{x}^{(0)}_{\mathrm{mid}}(k)$:

According to Theorem 2 and $P = (a, \otimes_b)^T$, when $k = 2, 3, 4, 5, 6, 7$, the simulated data $\hat{x}^{(0)}_{\min}(k)$, $\hat{x}^{(0)}_{\max}(k)$, and $\hat{x}^{(0)}_{\mathrm{mid}}(k)$ can be computed, as follows:

$$\begin{aligned}
\hat{x}^{(0)}_{\min}(2) &= 14412.06;\\
\hat{x}^{(0)}_{\min}(3) &= 16000.95;\\
\hat{x}^{(0)}_{\min}(4) &= 17765.00;\\
\hat{x}^{(0)}_{\min}(5) &= 19723.54;\\
\hat{x}^{(0)}_{\min}(6) &= 21897.99;\\
\hat{x}^{(0)}_{\min}(7) &= 24312.18,\\
\hat{x}^{(0)}_{\mathrm{mid}}(2) &= 15568.40;\\
\hat{x}^{(0)}_{\mathrm{mid}}(3) &= 17284.76;\\
\hat{x}^{(0)}_{\mathrm{mid}}(4) &= 19190.35;\\
\hat{x}^{(0)}_{\mathrm{mid}}(5) &= 21306.03;\\
\hat{x}^{(0)}_{\mathrm{mid}}(6) &= 23654.95;\\
\hat{x}^{(0)}_{\mathrm{mid}}(7) &= 26262.84,\\
\hat{x}^{(0)}_{\max}(2) &= 16393.86;\\
\hat{x}^{(0)}_{\max}(3) &= 18201.24;\\
\hat{x}^{(0)}_{\max}(4) &= 20207.87;\\
\hat{x}^{(0)}_{\max}(5) &= 22435.72;\\
\hat{x}^{(0)}_{\max}(6) &= 24909.19;\\
\hat{x}^{(0)}_{\max}(7) &= 27655.35.
\end{aligned} \tag{21}$$

Then,

$$\begin{aligned}
&\otimes(2) \in [14412.06, 16393.86];\\
&\hat{x}^{(0)}_{\mathrm{mid}}(2) = \tilde{\otimes}(2) = 15568.40,\\
&\otimes(3) \in [16000.95, 18201.24];\\
&\hat{x}^{(0)}_{\mathrm{mid}}(3) = \tilde{\otimes}(3) = 17284.76,\\
&\otimes(4) \in [17765.00, 20207.87];\\
&\hat{x}^{(0)}_{\mathrm{mid}}(4) = \tilde{\otimes}(4) = 19190.35,\\
&\otimes(5) \in [19723.54, 22435.72];\\
&\hat{x}^{(0)}_{\mathrm{mid}}(5) = \tilde{\otimes}(5) = 21306.03,\\
&\otimes(6) \in [21897.99, 24909.19];\\
&\hat{x}^{(0)}_{\mathrm{mid}}(6) = \tilde{\otimes}(6) = 23654.95,\\
&\otimes(7) \in [24312.18, 27655.35];\\
&\hat{x}^{(0)}_{\mathrm{mid}}(7) = \tilde{\otimes}(7) = 26262.84.
\end{aligned} \tag{22}$$

Similarly, when $k = 8, 9, 10$, the predicted data $\hat{x}^{(0)}_{\min}(k)$, $\hat{x}^{(0)}_{\max}(k)$, and $\hat{x}^{(0)}_{\mathrm{mid}}(k)$ can be computed, as follows:

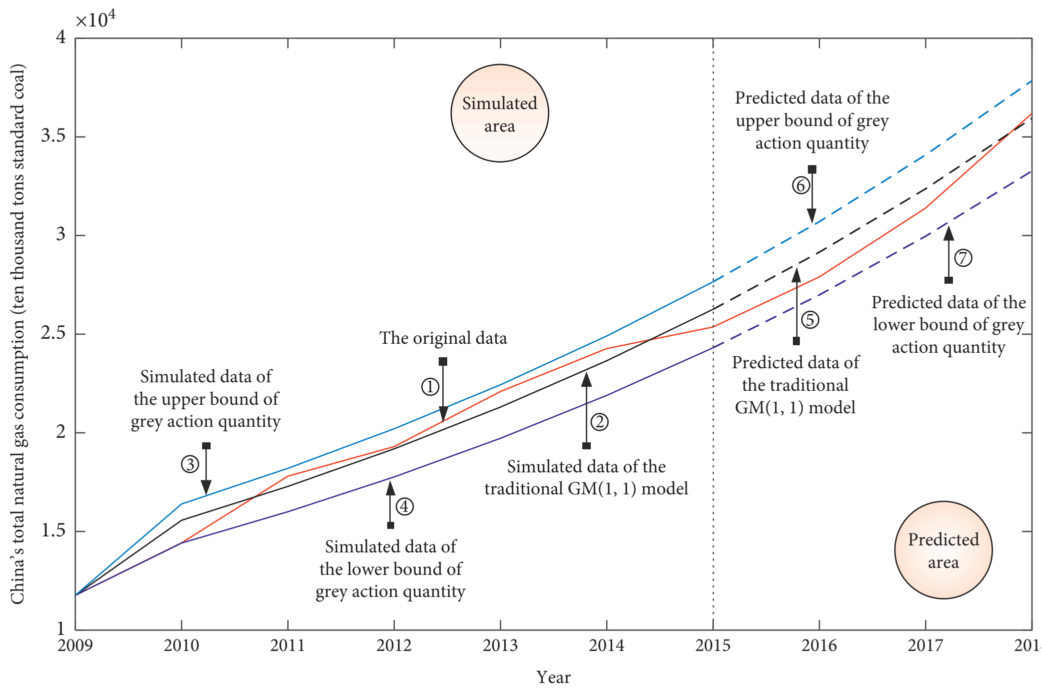

Figure 7: Comparisons of the original data and various simulation and prediction data.

$$
\begin{aligned}
&\hat{x}^{(0)}_{\min}(8) = 26992.52;\\
&\hat{x}^{(0)}_{\min}(9) = 29968.36;\\
&\hat{x}^{(0)}_{\min}(10) = 33272.28,\\
&\hat{x}^{(0)}_{\text{mid}}(8) = 29158.23;\\
&\hat{x}^{(0)}_{\text{mid}}(9) = 32372.84;\\
&\hat{x}^{(0)}_{\text{mid}}(10) = 35941.84,\\
&\hat{x}^{(0)}_{\max}(8) = 30704.26;\\
&\hat{x}^{(0)}_{\max}(9) = 34089.31;\\
&\hat{x}^{(0)}_{\max}(10) = 37847.55,\\
&\otimes(8) \in [26992.52, 30704.26];\\
&\hat{x}^{(0)}_{mid}(8) = \tilde{\otimes}(8) = 29158.23,\\
&\otimes(9) \in [29968.36, 34089.31];\\
&\hat{x}^{(0)}_{mid}(9) = \tilde{\otimes}(9) = 32372.84,\\
&\otimes(10) \in [33272.28, 37847.55];\\
&\hat{x}^{(0)}_{mid}(10) = \tilde{\otimes}(10) = 35941.84.
\end{aligned}
\tag{23}
$$

Step 5. Analyzing the rationality of simulation and prediction data:

Based on the above calculation results, the original data and various simulation and prediction data curves are drawn, as shown in Figure 7.

According to Figure 7, before analyzing the rationality of the proposed $\text{GM}(1,1,\otimes_b)$ model in this paper, we first analyze the irrationality of the traditional GM(1,1) model:

(a) The overall trend of China's total natural gas consumption is increasing year by year, but it is not balanced, such as the rapid growth in 2012–2014 and the slowdown in 2014-2015. However, the traditional GM(1,1) model is an exponential model with a constant growth rate, so it is difficult for the GM(1,1) model to achieve unbiased simulation of China's total natural gas consumption. It can be found from Figure 7 that there are obvious deviations between curves ① and ②.

(b) In the traditional GM(1,1) model, the grey action quantity b represents the influence of all external factors on the development trend of the system. It is essentially uncertain, and its form should be grey number. However, in the modeling process of GM(1,1), the size of b is estimated by the least squares method, which is a real number. This completely ignores the uncertainty characteristics of grey action quantity and leads to the poor reliability of the simulation and prediction results of the traditional GM(1,1) model (see curves ② and ⑤).

(c) The GM(1,1) model is a grey model with incomplete structural information which mainly reflects in the uncertainty and complexity of the influencing factors. According to the "Nonuniqueness Principle" of Grey theory, solutions with incomplete and uncertain information show nonuniqueness. Therefore, the simulation and prediction results of GM(1,1) should be nonunique. However, the GM(1,1) model is a time sequence prediction model with deterministic structure, and its simulation and prediction results are unique (see curves ② and ⑤), which does not conform to the Nonuniqueness Principle of solution of Grey theory.

In order to study the actual meaning of grey action quantity b under uncertain (Grey factors) conditions, the interval grey number form of b is obtained by calculating and comparing the grey action quantity b at different time points. On this basis, a new GM(1,1) model, $\mathrm{GM}(1,1,\otimes_b)$, is constructed. Compared with the traditional GM(1,1) model, the rationality of the $\mathrm{GM}(1,1,\otimes_b)$ model is reflected in the following aspects:

(i) The model structure of $\mathrm{GM}(1,1,\otimes_b)$ satisfies the essential characteristics of uncertainty of the grey prediction model. The grey action quantity of $\mathrm{GM}(1,1,\otimes_b)$ is the interval grey number $\otimes_b \in [b_{\min}, b_{\max}]$ with known probability function, which restores the interval grey number form of grey action quantity under incomplete structural information. After this, the interval structure of grey prediction model is realized, which satisfies the essential characteristics of uncertainty of the grey prediction model.

(ii) The simulation and prediction results of the $\mathrm{GM}(1,1,\otimes_b)$ model conform to the Nonuniqueness Principle of solution of Grey theory. The interval grey number sequences with clear lower bound $\widehat{x}^{(0)}_{\min}(k)$ and upper bound $\widehat{x}^{(0)}_{\max}(k)$ with known probability function are obtained based on the new $\mathrm{GM}(1,1,\otimes_b)$ model, rather than real number sequences based on the traditional GM(1,1) model. The $\mathrm{GM}(1,1,\otimes_b)$ model conforms to the Nonuniqueness Principle of solution under incomplete structural information (see curves ③, ④, ⑥, and ⑦).

(iii) The $\mathrm{GM}(1,1,\otimes_b)$ model conforms to the "Minimum Information Principle" of Grey theory. The $\mathrm{GM}(1,1,\otimes_b)$ model makes full use of the all information of grey action quantity at each time point. The traditional GM(1,1) model employs the least square method to estimate the grey action quantity, which is actually a simplified process, and leads to the loss of some known information.

(iv) The predicted results of the $\mathrm{GM}(1,1,\otimes_b)$ model are more valuable than the traditional GM(1,1) model. The prediction result of $\mathrm{GM}(1,1,\otimes_b)$ is an interval grey number (see curves ⑥ and ⑦), which enables the decision maker to clearly understand the future change range of the research object. However, the prediction result of GM(1,1) is a determined real number (see curve ⑤), which usually has some errors; it leads decision makers to question its reliability. In this case, a certain interval is often more valuable than an uncertain real number.

(v) The new $\mathrm{GM}(1,1,\otimes_b)$ model is compatible with the traditional GM(1,1) model. In $\mathrm{GM}(1,1,\otimes_b)$, the grey action quantity b of the traditional GM(1,1) model is just the whitening value of $\otimes_b \in [b_{\min}, b_{\max}]$, namely, $\widetilde{\otimes}_b = b$, and then, the $\widehat{x}^{(0)}_{\mathrm{mid}}(k)$ is calculated based on b accordingly. In fact, $\widehat{x}^{(0)}_{\mathrm{mid}}(k)$ is the simulation or prediction result of the traditional GM(1,1) model. Therefore, $\mathrm{GM}(1,1,\otimes_b)$ is compatible with GM(1,1).

(vi) From the predicted area in Figure 7, it can be found that the actual value of natural gas consumption in China from 2016 to 2018 (see curve ①) is totally smaller than the predicted value of the upper bound grey action quantity $b_{\min}$ (see curve ⑥) of the $\mathrm{GM}(1,1,\otimes_b)$ model but larger than the predicted value of lower bound grey action quantity $b_{\max}$ (see curve ⑦). This shows that the $\mathrm{GM}(1,1,\otimes_b)$ model is effective in predicting the range of natural gas consumption in China in the next three years and proves the rationality of the prediction results of the $\mathrm{GM}(1,1,\otimes_b)$ model again.

5. Conclusions

The single variable grey prediction model represented by GM(1,1) simply uses a real number (grey action quantity) "b" to express the comprehensive effect of many uncertain and complex factors on the system development because the factors affecting the system (independent variables) are unknown. In other words, grey action quantity "b" represents the influence of all external factors on the system development trend. Hence, the parameter "b" is essentially uncertain and should be in the form of grey number. However, in the traditional GM(1,1) modeling process, the grey attribute of "b" is not taken into account, which is estimated and modeled according to the real number, which is obviously inconsistent with the actual meaning of "b".

On the other hand, the GM(1,1) model is a grey model with incomplete structural information (the absence of independent variables). According to the "Nonuniqueness Principle" of Grey theory, the solution with incomplete and uncertain information is not unique. Therefore, the simulation and prediction results of GM(1,1) should be nonunique. However, the current GM(1,1) model is a time sequence prediction model with deterministic structure, so its simulation and prediction results are unique, which obviously violates the "Nonuniqueness Principle" of Grey theory.

Starting from the origin of the grey prediction model, this paper analyses the defects of the traditional GM(1,1) model. Then, according to the Nonuniqueness Principle and Minimum Information Principle of Grey theory, the interval grey number form of grey action quantity b is restored and the new $\mathrm{GM}(1,1,\otimes_b)$ model is put forward. The new $\mathrm{GM}(1,1,\otimes_b)$ model is applied to simulate and forecast China's natural gas consumption, and the rationality of the simulation and prediction results of $\mathrm{GM}(1,1,\otimes_b)$ and GM(1,1) is analyzed. The results show that the prediction results of $\mathrm{GM}(1,1,\otimes_b)$ have more reference values.

Although this paper only extends grey action quantity b from real number to interval grey number $\otimes_b \in [b_{\min}, b_{\max}]$, it is no exaggeration to say that the proposed $\mathrm{GM}(1,1,\otimes_b)$ model makes the classical grey prediction model really to have the "grey" attribute. At present, there are many kinds of grey prediction models, and GM(1,1) is only one of the most primitive grey models. Therefore, how to use GM$(1, 1, \otimes_b)$ as the basis to carry out in-depth research on the "grey" attributes of other grey models, so as to build the new grey prediction model with stronger modeling ability, is the next work of our team.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper

Acknowledgments

In addition, the authors would like to thank Dr. Luo Chengming of Huazhong Agricultural University for his careful revision and editing of the English language. This work was supported by the National Natural Science Foundation of China (71771033) and Chongqing Natural Science Foundation of China (cstc2019jcyj-msxmX0003 and cstc2019jcyj-msxmX0767).

References

[1] J. L. Deng, "Control problems of grey systems," *Systems & Control Letters*, vol. 1, no. 5, pp. 288–294, 1982.

[2] B. Zeng, H. Duan, and Y. Zhou, "A new multivariable grey prediction model with structure compatibility," *Applied Mathematical Modelling*, vol. 75, pp. 385–397, 2019.

[3] S. Ding, Y. G. Dang, N. Xu, J. J. Wang, and Z. D. Xu, "A novel grey model based on the trends of driving factors and its application," *The Journal of Grey System*, vol. 30, no. 3, pp. 105–126, 2018.

[4] S. Liu, Y. Yang, N. Xie, and J. Forrest, "New progress of grey system theory in the new millennium," *Grey Systems: Theory and Application*, vol. 6, no. 1, pp. 2–31, 2016.

[5] B. Zeng, H. Duan, Y. Bai, and W. Meng, "Forecasting the output of shale gas in China using an unbiased grey model and weakening buffer operator," *Energy*, vol. 151, pp. 238–249, 2018.

[6] X. Ma, X. Mei, W. Wu, X. Wu, and B. Zeng, "A novel fractional time delayed grey model with grey wolf optimizer and its applications in forecasting the natural gas and coal consumption in Chongqing, China," *Energy*, vol. 178, pp. 487–507, 2019.

[7] W. Meng, D. L. Yang, and H. Huang, "Prediction of China's sulfur dioxide emissions by discrete grey model with fractional order generation operators," *Complexity*, vol. 2018, Article ID 8610679, 13 pages, 2018.

[8] H. M. Duan and X. P. Xiao, "A multimode dynamic short-term traffic flow grey prediction model of high dimension tensors," *Complexity*, vol. 2019, Article ID 9162163, 18 pages, 2019.

[9] B. Zeng, Y. T. Tan, H. Xu, Q. Jing, L. Y. Wang, and X. Y. Zhou, "Forecasting the electricity consumption of commercial sector in Hong Kong using a novel grey dynamic prediction model," *The Journal of Grey System*, vol. 30, pp. 157–172, 2018.

[10] Y. Bai, Z. Chen, J. Xie, and C. Li, "Daily reservoir inflow forecasting using multiscale deep feature learning with hybrid models," *Journal of Hydrology*, vol. 532, pp. 193–206, 2016.

[11] L. Wu, S. Liu, Y. Yang, L. Ma, and H. Liu, "Multi-variable weakening buffer operator and its application," *Information Sciences*, vol. 339, pp. 98–107, 2016.

[12] M. Zhou, B. Zeng, and W. H. Zhou, "A hybrid grey prediction model for small oscillation sequence based on information decomposition," *Complexity*, vol. 2020, Article ID 5071267, 13 pages, 2020.

[13] J. J. Wang, Y. G. Dang, J. Ye, N. Xu, and J. Wang, "An improved grey prediction model based on matrix representations of the optimized initial value," *The Journal of Grey System*, vol. 30, no. 3, pp. 143–156, 2018.

[14] K. Li, L. Liu, J. Zhai, T. M. Khoshgoftaar, and T. Li, "The improved grey model based on particle swarm optimization algorithm for time series prediction," *Engineering Applications of Artificial Intelligence*, vol. 55, pp. 285–291, 2016.

[15] B. Zeng and C. Li, "Improved multi-variable grey forecasting model with a dynamic background-value coefficient and its application," *Computers & Industrial Engineering*, vol. 118, pp. 278–290, 2018.

[16] L. L. Pei, W. M. Chen, J. H. Bai, and Z. X. Wang, "The improved GM(1, *N*) models with optimal background values: a case study of Chinese High-tech Industry," *The Journal of Grey System*, vol. 27, no. 3, pp. 223–233, 2015.

[17] X. Ma, W. Wu, B. Zeng, Y. Wang, and X. Wu, "The conformable fractional grey system model," *ISA Transactions*, 2019, In press.

[18] S. Mao, M. Gao, X. Xiao, and M. Zhu, "A novel fractional grey system model and its application," *Applied Mathematical Modelling*, vol. 40, no. 7-8, pp. 5063–5076, 2016.

[19] B. Zeng and S. Liu, "A self-adaptive intelligence gray prediction model with the optimal fractional order accumulating operator and its application," *Mathematical Methods in the Applied Sciences*, vol. 40, no. 18, pp. 7843–7857, 2017.

[20] B. Zeng and C. Li, "Forecasting the natural gas demand in China using a self-adapting intelligent grey model," *Energy*, vol. 112, pp. 810–825, 2016.

[21] X. Ma, M. Xie, W. Wu, B. Zeng, Y. Wang, and X. Wu, "The novel fractional discrete multivariate grey system model and its applications," *Applied Mathematical Modelling*, vol. 70, pp. 402–424, 2019.

[22] S. H. Mao, X. P. Xiao, M. Y. Gao, X. P. Wang, and Q. He, "Nonlinear fractional order grey model of urban traffic flow short term prediction," *The Journal of Grey System*, vol. 30, no. 4, pp. 1–17, 2018.

[23] J. Ye, Y. Dang, S. Ding, and Y. Yang, "A novel energy consumption forecasting model combining an optimized DGM (1,1) model with interval grey numbers," *Journal of Cleaner Production*, vol. 229, pp. 256–267, 2019.

[24] B. Zeng, G. Chen, and S.-f. Liu, "A novel interval grey prediction model considering uncertain information," *Journal of the Franklin Institute*, vol. 350, no. 10, pp. 3400–3416, 2013.

[25] B. Zeng, M. Y. Tong, and X. Ma, "A new-structure grey Verhulst model: development and performance comparison," *Applied Mathematical Modelling*, vol. 81, pp. 522–537, 2020.

[26] H. Hao, Q. Zhang, Z. Wang, and J. Zhang, "Forecasting the number of end-of-life vehicles using a hybrid model based on grey model and artificial neural network," *Journal of Cleaner Production*, vol. 202, pp. 684–696, 2018.

[27] X. Ma and Z. B. Liu, "The GMC(1, *N*) model with optimized parameters and its application," *The Journal of Grey System*, vol. 29, pp. 1–17, 2017.

[28] Z. M. Yaseen, M. Fu, C. Wang, W. H. M. W. Mohtar, R. C. Deo, and A. El-Shafie, "Application of the hybrid artificial neural network coupled with rolling mechanism and grey model algorithms for streamflow forecasting over multiple time horizons," *Water Resources Management*, vol. 32, no. 5, pp. 1883–1899, 2018.
[29] S. Xu, B. Zou, S. Shafi, and T. Sternberg, "A hybrid Grey-Markov/LUR model for PM_{10} concentration prediction under future urban scenarios," *Atmospheric Environment*, vol. 187, pp. 401–409, 2018.
[30] Z. Yin, X. Luo, S. Fang, and X. Guo, "Intelligent forecasts and evaluation of financial risk assets based on Grey Markov chain model," *Journal of Intelligent & Fuzzy Systems*, vol. 35, no. 3, pp. 2679–2684, 2018.
[31] Y.-H. Wu and H. Shen, "Grey-related least squares support vector machine optimization model and its application in predicting natural gas consumption demand," *Journal of Computational and Applied Mathematics*, vol. 338, pp. 212–220, 2018.
[32] Q. Song and A.-m. Wang, "Simulation and prediction of alkalinity in sintering process based on grey least squares support vector machine," *Journal of Iron and Steel Research International*, vol. 16, no. 5, pp. 1–6, 2009.
[33] X. Ma, Y.-s. Hu, and Z.-b. Liu, "A novel kernel regularized nonhomogeneous grey model and its applications," *Communications in Nonlinear Science and Numerical Simulation*, vol. 48, pp. 51–62, 2017.
[34] X. Ma, "A brief introduction to the grey machine learning," *The Journal of Grey System*, vol. 31, pp. 1–12, 2019.
[35] S. F. Liu, Y. J. Yang, and J. Forrest, *Grey Data Analysis Methods, Models and Applications*, Springer-Verlag, Berlin, Germany, 2016.
[36] S. Mao, M. Zhu, X. Yan et al., "Modeling mechanism of a novel fractional grey model based on matrix analysis," *Journal of Systems Engineering and Electronics*, vol. 27, no. 5, pp. 1040–1053, 2016.
[37] J. L. Deng, *Grey Theory Basis*, Huazhong University of Science and Technology Press of China, Wuhan, China, 2002, in Chinese.
[38] S. F. Liu and Y. Lin, *An Introduction to Grey Systems*, IIGS Academic Publisher, Slippery Rock, PA, USA, 1998.