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Research Article

A New Stability Criterion for Systems with Distributed Time-Varying Delays via Mixed Inequalities Method

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This paper is concerned with the delay-dependent stability of systems with distributed time-varying delays. The novelty relies on the use of some new inequalities which are less conservative than some existing inequalities. A less conservative stability criterion is obtained by constructing some new augmented Lyapunov–Krasovskii functionals, which are given in terms of linear matrix inequalities. The effectiveness of the presented criterion is demonstrated by two numerical examples.

1. Introduction

Consider the systems with distributed time-varying delays:

$$\dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\int_{t-h(t)}^{t} x(s)\mathrm{d}s, \tag{1}$$

$$\begin{aligned} x(t) &= \phi(t), \\ t &\in [-h, 0], \end{aligned} \tag{2}$$

where $x(t) \in R^n$ is the system state, $A, B, C \in R^{n\times n}$ are constant matrices, and $h(t)$ is the time-varying delay satisfying

$$0 \le h(t) \le h, \quad -u \le \dot{h}(t) \le u < 1. \tag{3}$$

Since time delays occur in many dynamic systems, stability analysis of the time delay system [1–5] has become a hot topic in the past few decades. Due to the representation of linear systems with time-varying delays, the delay-dependent stability analysis via the LKF method has attracted much attention. The conservatism of the LKF method comes from two aspects: the construction of the LKF and the bound on its derivative. Selecting the LKF is crucial to derive less conservative criteria. An augmented LKF [6] is proposed to reduce the conservatism in the early literature. Recently, a new augmented LKF [7] is introduced by employing the information of a second-order Bessel–Legendre inequality. It is necessary to take the derivative of the LKF to derive a stability criterion. The difficulty lies in the bounds of the integrals that arise in the derivative of the LKF. There are two main methods for dealing with such integrals: the free-weighting matrix method [8] and the integral inequality method. The integral inequality method includes various integral inequalities, such as Jensen inequality [9–11], Wirtinger-based inequality [12–15], free matrix-based inequality [16, 17], auxiliary function-based inequality [18], relaxed integral inequality [19], and Bessel–Legendre inequality [20]. Very recently, the improved inequality-based functions approach [21] is proposed to derive less conservative results for systems with time-varying delays. However, when estimating $\dot{V}(x_t)$, $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s$ is only estimated as $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \ge (1/b-a)\Omega_1^T R\Omega_1 + (3/b-a)\Omega_2^T R\Omega_2 + (5/b-a)\Omega_3^T R\Omega_3$. Ω_1, Ω_2, and Ω_3 are the same as in Lemma 2. Then, a new integral inequality was proposed in [22] to further reduce the conservatism. But the integral inequality can only deal with the constant time delay. On the other hand, stability analysis for systems with distributed delays is of both practical and theoretical importance. Then, it is desirable to extend the system model to include distributed delays. In recent years, the stability analysis of systems with distributed delays has been received considerable attention [23–27]. But only the authors in [25, 26] consider the systems with distributed time-varying delays.

This paper is concerned with the delay-dependent stability of systems with distributed time-varying delays. Based on some new inequalities and some new augmented LKFs, a less conservative stability criterion is obtained in terms of LMIs. Our paper has two characteristics: (1) the integral $\int_a^b \dot{x}^T(s)R\dot{x}(s)ds$ is estimated as $\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \geq (1/b-a)\Omega_1^T R\Omega_1 + (3/b-a)\Omega_2^T R\Omega_2 + (5/b-a)\Omega_3^T R\Omega_3 + (7/b-a)\Omega_4^T R\Omega_4$, which includes those in [9, 13, 20] as special cases. $\Omega_i, i = 1,2,3,4$, is the same as in Lemma 2. (2) An augmented LKF which contains more information about $h(t)$ is proposed to reduce the conservatism. The effectiveness of the presented criterion is demonstrated by two numerical examples.

Throughout this paper, the set S^n denotes the set of symmetric matrices and the set S_+^n denotes the set of symmetric positive definite matrices. For any square matrix P, we define $\mathrm{Sym}(P) = P + P^T$.

2. Main Results

In this section, the following lemmas are introduced to derive the main results.

Lemma 1 (see [20]). *For any matrices $\Theta \in S_+^n$, $M_1, M_2 \in R^{m\times n}$, $\Upsilon \in R^{2n\times m}$, and $\forall \alpha \in (0,1)$, the inequality*

$$-\Upsilon^T \begin{bmatrix} \frac{1}{\alpha}\Theta & 0 \\ 0 & \frac{1}{1-\alpha}\Theta \end{bmatrix} \Upsilon \leq -\Upsilon^T\Sigma(\alpha)\Upsilon - \mathrm{Sym}\left(\Upsilon^T \begin{bmatrix} (1-\alpha)M_1^T \\ \alpha M_2^T \end{bmatrix}\right) + \alpha M_1\Theta^{-1}M_1^T + (1-\alpha)M_2\Theta^{-1}M_2^T, \tag{4}$$

holds, where

$$\Sigma(\alpha) = \begin{bmatrix} (2-\alpha)\Theta & 0 \\ 0 & (1+\alpha)\Theta \end{bmatrix}. \tag{5}$$

Lemma 2 (see [22]). *For a matrix $R \in S_+^n$ and any continuously differentiable function x: $[a,b] \longrightarrow R^n$, the inequality*

$$\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \geq \frac{1}{b-a}\Omega_1^T R\Omega_1 + \frac{3}{b-a}\Omega_2^T R\Omega_2 + \frac{5}{b-a}\Omega_3^T R\Omega_3 + \frac{7}{b-a}\Omega_4^T R\Omega_4, \tag{6}$$

holds, where

$$\begin{aligned} \Omega_1 &= x(b) - x(a), \\ \Omega_2 &= x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds, \\ \Omega_3 &= x(b) - x(a) + \frac{6}{b-a}\int_a^b x(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_u^b x(s)ds\,du, \\ \Omega_4 &= x(b) + x(a) - \frac{12}{b-a}\int_a^b x(s)ds + \frac{60}{(b-a)^2}\int_a^b\int_u^b x(s)ds\,du \\ &\quad - \frac{120}{(b-a)^3}\int_a^b\int_u^b\int_v^b x(s)ds\,dv\,du. \end{aligned} \tag{7}$$

Lemma 3 (see [28]). *Suppose that $\Omega, \Omega_{ij}\,(i,j = 1,2)$ are the constant matrices of appropriate dimensions, $\alpha \in [0,1]$, $\beta \in [-u,u]$, $0 \leq u < 1$, then*

$$\Omega + \alpha\Omega_{11} + (1-\alpha)\Omega_{12} + \beta\Omega_{21} + (1-\beta)\Omega_{22} < 0, \tag{8}$$

holds if and only if the following inequalities hold:

$$\begin{aligned} &\Omega + \Omega_{11} - u\Omega_{21} + (1+u)\Omega_{22} < 0, \\ &\Omega + \Omega_{12} - u\Omega_{21} + (1+u)\Omega_{22} < 0, \\ &\Omega + \Omega_{11} + u\Omega_{21} + (1-u)\Omega_{22} < 0, \\ &\Omega + \Omega_{12} + u\Omega_{21} + (1-u)\Omega_{22} < 0. \end{aligned} \tag{9}$$

Based on Lemmas 1–3, a novel stability criterion is derived for system (1) with distributed time-varying delays.

Theorem 1. *For given scalars $h > 0, u > 0$, if there exist matrices $P, Q_1, Q_2, \in S_+^{2n}$, $Q_3 \in S_+^n$, $M_1, M_2 \in R^{10n\times 4n}$, such that the LMI*

$$\Phi(\alpha,\beta) = \begin{bmatrix} \phi(\alpha,\beta) - \Upsilon^T\Sigma(\alpha)\Upsilon - \mathrm{Sym}\left(\Upsilon^T\begin{bmatrix} (1-\alpha)M_1^T \\ \alpha M_2^T \end{bmatrix}\right) & * \\ \alpha M_1^T + (1-\alpha)M_2^T & -\Theta \end{bmatrix} < 0, \tag{10}$$

holds for $\alpha = \{0,1\}$, $\dot{h}(t) = \beta = \{-u,u\}$, i.e.,

$$\Phi(0,-u) < 0, \tag{11}$$

$$\Phi(0,u) < 0, \tag{12}$$

$$\Phi(1,-u) < 0, \tag{13}$$

$$\Phi(1,u) < 0, \tag{14}$$

then, system (1) is asymptotically stable, where

$$
\begin{aligned}
\phi(\alpha,\beta) &= Sym\left(\Pi_1^T P \Pi_2\right) + \Pi_3^T Q_1 \Pi_3 \\
&\quad - (1-\beta)\Pi_4^T Q_1 \Pi_4 + \mathrm{Sym}\left(\Pi_5^T Q_2 \Pi_6\right) + h^2 \varepsilon_0^T Q_3 \varepsilon_0, \\
\Pi_1 &= \left[\, \varepsilon_1^T \quad \alpha h \varepsilon_5^T + (1-\alpha) h \varepsilon_6^T \,\right]^T, \\
\Pi_2 &= \left[\, A\varepsilon_1^T + B\varepsilon_2^T + C\alpha h \varepsilon_5^T \quad \varepsilon_1^T - \varepsilon_3^T \,\right]^T, \\
\Pi_3 &= \left[\, \varepsilon_1^T \quad A\varepsilon_1^T + B\varepsilon_2^T + C\alpha h \varepsilon_5^T \,\right]^T, \\
\Pi_4 &= \left[\, \varepsilon_2^T \quad \varepsilon_4^T \,\right]^T, \\
\Pi_5 &= \left[\, \varepsilon_1^T \quad \varepsilon_2^T \,\right]^T, \\
\Pi_6 &= \left[\, A\varepsilon_1^T + B\varepsilon_2^T + C\alpha h \varepsilon_5^T \quad (1-\beta)\varepsilon_4^T \,\right]^T, \\
\Pi_7 &= \varepsilon_1 - \varepsilon_2, \\
\Pi_8 &= \varepsilon_1 + \varepsilon_2 - 2\varepsilon_5, \\
\Pi_9 &= \varepsilon_1 - \varepsilon_2 + 6\varepsilon_5 - 12\varepsilon_7, \\
\Pi_{10} &= \varepsilon_1 + \varepsilon_2 - 12\varepsilon_5 + 60\varepsilon_7 - 120\varepsilon_9, \\
\Pi_{11} &= \varepsilon_2 - \varepsilon_3, \\
\Pi_{12} &= \varepsilon_2 + \varepsilon_3 - 2\varepsilon_6, \\
\Pi_{13} &= \varepsilon_2 - \varepsilon_3 + 6\varepsilon_6 - 12\varepsilon_8, \\
\Pi_{14} &= \varepsilon_2 + \varepsilon_3 - 12\varepsilon_6 + 60\varepsilon_8 - 120\varepsilon_{10}, \\
\varepsilon_0 &= A\varepsilon_1 + B\varepsilon_2 + C\alpha h \varepsilon_5, \\
\Upsilon &= \left[\, \Pi_7^T \quad \Pi_8^T \quad \Pi_9^T \quad \Pi_{10}^T \quad \Pi_{11}^T \quad \Pi_{12}^T \quad \Pi_{13}^T \quad \Pi_{14}^T \,\right]^T, \\
\Theta &= \mathrm{diag}\left(Q_3, 3Q_3, 5Q_3, 7Q_3\right),
\end{aligned}
\tag{15}
$$

and $\varepsilon_i \in \mathscr{R}^{n\times 10n}$ *is defined as* $\varepsilon_i = \left[\, 0_{n\times(i-1)n} \quad I_n \quad 0_{n\times(10-i)n} \,\right]$ *for* $i = 1, 2, \ldots, 10$.

Proof. Introduce an LKF candidate as

$$
V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t), \tag{16}
$$

where

$$
\begin{aligned}
V_1(x_t) &= \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}, \\
V_2(x_t) &= \int_{t-h(t)}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T Q_1 \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} \mathrm{d}s, \\
V_3(x_t) &= \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T Q_2 \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}, \\
V_4(x_t) &= h \int_{t-h}^{t} \int_{u}^{t} \dot{x}^T(s) Q_3 \dot{x}(s) \mathrm{d}s\, \mathrm{d}u.
\end{aligned}
\tag{17}
$$

Calculate the derivative of $V(x_t)$ along the solution of system (1) as follows:

$$
\begin{aligned}
\dot{V}_1(x_t) &= 2\begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)\mathrm{d}s \end{bmatrix}^T P \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-h) \end{bmatrix} \\
&= 2\begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)\mathrm{d}s \end{bmatrix}^T P \begin{bmatrix} Ax(t) + Bx(t-h(t)) + C\int_{t-h(t)}^{t} x(s)\mathrm{d}s \\ x(t) - x(t-h) \end{bmatrix},
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\dot{V}_2(x_t) &= \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T Q_1 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - (1-\dot{h}(t)) \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}^T Q_1 \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix} \\
&= \begin{bmatrix} x(t) \\ Ax(t) + Bx(t-h(t)) + C\int_{t-h(t)}^{t} x(s)\mathrm{d}s \end{bmatrix}^T Q_1 \begin{bmatrix} x(t) \\ Ax(t) + Bx(t-h(t)) + C\int_{t-h(t)}^{t} x(s)\mathrm{d}s \end{bmatrix} \\
&\quad - (1-\dot{h}(t)) \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}^T Q_1 \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix},
\end{aligned}
\tag{19}
$$

$$
\dot{V}_3(x_t) = 2\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T Q_2 \begin{bmatrix} \dot{x}(t) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix}, \tag{20}
$$

$$= 2\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T Q_2 \begin{bmatrix} Ax(t) + Bx(t-h(t)) + C\int_{t-h(t)}^{t} x(s)\mathrm{d}s \\ (1-\beta)\dot{x}(t-h(t)) \end{bmatrix}, \tag{21}$$

$$\dot{V}_4(x_t) = h^2\dot{x}^T(t)Q_3\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s. \tag{22}$$

According to (18)–(22), we can obtain

$$\begin{aligned}\dot{V}(x_t) = \zeta^T(t)\big\{&\mathrm{sym}\big(\Pi_1^T P\Pi_2\big) + \Pi_3^T Q_1\Pi_3 - (1-\beta)\Pi_4^T Q_1\Pi_4 \\ &+ \mathrm{Sym}\big(\Pi_5^T Q_2\Pi_6\big) + h^2\varepsilon_0^T Q_3\varepsilon_0\big\}\zeta(t) \\ &- h\int_{t-h}^{t}\dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s,\end{aligned} \tag{23}$$

where

$$\begin{aligned}\zeta(t) &= \big[(t)\,x^T(t)\ \ x^T(t-h(t))\ \ x^T(t-h)\ \ \dot{x}^T(t-h(t))\ \ \pi_1^T(t)\ \ \pi_2^T(t)\ \ \pi_3^T(t)\big]^T, \\ \pi_1(t) &= \left[\tfrac{1}{h(t)}\int_{t-h(t)}^{t} x^T(s)\mathrm{d}s\ \ \tfrac{1}{h-h(t)}\int_{t-h}^{t-h(t)} x^T(s)\mathrm{d}s\right]^T, \\ \pi_2(t) &= \left[\tfrac{1}{h(t)^2}\int_{t-h(t)}^{t}\int_{u}^{t} x^T(s)\mathrm{d}s\,\mathrm{d}u\ \ \tfrac{1}{(h-h(t))^2}\int_{t-h}^{t-h(t)}\int_{u}^{t-h(t)} x^T(s)\mathrm{d}s\right]^T, \\ \pi_3(t) &= \left[\tfrac{1}{h(t)^3}\int_{t-h(t)}^{t}\int_{u}^{t}\int_{v}^{t} x^T(s)\mathrm{d}s\,\mathrm{d}v\mathrm{d}u\ \ \tfrac{1}{(h-h(t))^3}\int_{t-h}^{t-h(t)}\int_{u}^{t-h(t)}\int_{v}^{t-h(t)} x^T(s)\mathrm{d}s\,\mathrm{d}v\mathrm{d}u\right]^T.\end{aligned} \tag{24}$$

Let $\alpha = (h(t)/h)$, then $1-\alpha = (h-h(t)/h)$, applying Lemma 2, and we have

$$\begin{aligned}&-h\int_{t-h}^{t}\dot{x}^T(s)Q_3\dot{x}(s) \\ &= -h\int_{t-h(t)}^{t}\dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s - h\int_{t-h}^{t-h(t)}\dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s \\ &\le -\frac{h}{h(t)}\zeta^T(t)\big(\Pi_7^T Q_3\Pi_7 + 3\Pi_8^T Q_3\Pi_8 + 5\Pi_9^T Q_3\Pi_9 + 7\Pi_{10}^T Q_3\Pi_{10}\big)\zeta(t) \\ &\quad - \frac{h}{h-h(t)}\zeta^T(t)\big(\Pi_{11}^T Q_3\Pi_{11} + 3\Pi_{12}^T Q_3\Pi_{12} + 5\Pi_{13}^T Q_3\Pi_{13} + 7\Pi_{14}^T Q_3\Pi_{14}\big)\zeta(t) \\ &= -\frac{1}{\alpha}\zeta^T(t)\big(\Pi_7^T Q_3\Pi_7 + 3\Pi_8^T Q_3\Pi_8 + 5\Pi_9^T Q_3\Pi_9 + 7\Pi_{10}^T Q_3\Pi_{10}\big)\zeta(t) \\ &\quad - \frac{1}{1-\alpha}\zeta^T(t)\big(\Pi_{11}^T Q_3\Pi_{11} + 3\Pi_{12}^T Q_3\Pi_{12} + 5\Pi_{13}^T Q_3\Pi_{13} + 7\Pi_{14}^T Q_3\Pi_{14}\big)\zeta(t) \\ &= -\zeta^T(t)\Upsilon^T\begin{bmatrix}\frac{1}{\alpha}\Theta & 0 \\ 0 & \frac{1}{1-\alpha}\Theta\end{bmatrix}\Upsilon\zeta(t).\end{aligned} \tag{25}$$

For any matrices $M_1, M_2 \in R^{10n\times 4n}$ and applying Lemma 1, we can obtain

$$\begin{aligned} &-\Upsilon^T \begin{bmatrix} \frac{1}{\alpha}\Theta & 0 \\ 0 & \frac{1}{1-\alpha}\Theta \end{bmatrix} \Upsilon \\ &\leq -\Upsilon^T \Sigma(\alpha)\Upsilon - \mathrm{Sym}\left(\Upsilon^T \begin{bmatrix} (1-\alpha)M_1^T \\ \alpha M_2^T \end{bmatrix}\right) \\ &\quad + \alpha M_1 \Theta^{-1} M_1^T + (1-\alpha) M_2 \Theta^{-1} M_2^T = \eta(\alpha). \end{aligned} \tag{26}$$

From (23)–(26), we get

$$\dot{V}(x_t) \leq \zeta(t)(\phi(\alpha,\beta) + \eta(\alpha))\zeta(t). \tag{27}$$

By Lemma 2 [20], if the LMI (10) is true for $\alpha = \{0,1\}$, $\beta = \{-u, u\}$, then $\phi(\alpha,\beta) + \eta(\alpha) < 0$ holds for all $\alpha \in (0,1)$ and $\beta \in [-u, u]$. By Lemma 3, LMI (10) holds if and only if LMIs (11)–(14) hold. This completes the proof. □

Remark 1. The integral $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s$ in [9, 13, 20] is estimated as $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \geq (1/b-a)\Omega_1^T R\Omega_1$, $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \geq (1/b-a)\Omega_1^T R\Omega_1 + (3/b-a)\Omega_2^T R\Omega_2$, $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \geq (1/b-a)\Omega_1^T R\Omega_1 + (3/b-a)\Omega_2^T R\Omega_2 + (5/b-a)\Omega_3^T R\Omega_3$, respectively. In this paper, the integral $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s$ is estimated as $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \geq (1/b-a)\Omega_1^T R\Omega_1 + (3/b-a)\Omega_2^T R\Omega_2 + (5/b-a)\Omega_3^T R\Omega_3 + (7/b-a)\Omega_4^T R\Omega_4$, which includes those in [9, 13, 20] as special cases. So our method can yield less conservative results.

Remark 2. An augmented LKF which contains more information about time-varying delay $h(t)$ which was proposed to reduce the conservatism. $\dot{x}(t-h(t))$ is added as a state vector, which may yield less conservative criteria.

3. Numerical Examples

Two numerical examples are given to demonstrate advantages of the proposed criterion.

Example 1. Consider system (1) with

$$\begin{aligned} A &= \begin{bmatrix} 0.0 & 1.0 \\ -1.0 & -2.0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.0 & 0.0 \\ -1.0 & 1.0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{28}$$

For different u, Table 1 presents the allowable upper bound of $h(t)$, which guarantees the stability of system (1). Table 1 shows that our method produces the larger upper bound h than those in [7, 12, 13, 16, 17, 21]. In this sense, our stability criterion is less conservative than those in [7, 12, 13, 16, 17, 21].

Table 1: Upper bound of h for Example 1 with different u.

u	0.1	0.2	0.5	0.8
[13]	6.590	3.672	1.411	1.275
[12]	7.125	4.413	2.243	1.662
[16]	7.148	4.466	2.352	1.768
[17]	7.167	4.517	2.415	1.838
[7]	7.230	4.556	2.509	1.940
[21]	7.297	4.625	2.264	2.038
Theorem 1	10.095	6.808	3.676	2.615

Table 2: Upper bound of h for Example 2 with different u.

u	0.1	0.2	0.5	0.8
[13]	4.703	3.834	2.420	2.137
[11]	4.753	—	2.429	2.183
[16]	4.788	4.060	3.055	2.615
[15]	4.93	4.22	3.09	2.66
[7]	4.910	—	3.233	2.789
[21]	4.996	4.308	3.251	2.867
Theorem 1	5.650	4.913	3.793	3.251

Example 2. Consider system (1) with

$$\begin{aligned} A &= \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \\ B &= \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{29}$$

For different u, Table 2 presents the allowable upper bound of $h(t)$, which guarantees the stability of system (1). Table 2 shows that our method produces the larger upper bound h than those in [7, 11, 13, 15, 16, 21]. In this sense, our stability criterion is less conservative than those in [7, 11, 13, 15, 16, 21].

4. Conclusions

This paper focus on delay-dependent stability analysis for systems with distributed time-varying delays. The novelty relies on the use of some new inequalities which are less conservative than some existing inequalities. A less conservative stability criterion is obtained by constructing some new augmented LKFs. The effectiveness of the presented criterion is demonstrated by two numerical examples. In addition, the proposed method can be applied to stability analysis of other dynamic systems such as fuzzy systems with time-varying delay and neutral systems with time-varying delay.

Data Availability

No additional data are available for this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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