Letter to the Editor

Comment on “Modified Weights-of-Evidence Modeling with Example of Missing Geochemical Data”

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Even when restricted to a particular method, the undertaking by Zhang and Agterberg [1] seems surprising to review a topic as complex as estimating probabilities [2–5] without a single mathematical definition or formula. Thus, it becomes almost inevitable that this review provokes comments for reasons of mathematical clarification, in particular by this author because he has fragmentarily been referred to at several instances.

This comment intends to clarify the required mathematical modeling assumptions and the mutual relationship of mathematical methods, namely, weights-of-evidence, modified weights-of-evidence, and logistic regression of prospectivity modeling, and in particular, their possibilities to handle missing data as addressed by Zhang and Agterberg [1]. Their communication [1] solely argues with practical applications. However, what do practical applications of mathematical methods convey about the methods and their relationships? The mathematics of methods, e.g., properties of estimators, cannot be compared by the way of practical applications, i.e., by estimates. Case studies may be useful to compare the performance of methods in terms of cpu time and storage consumption.

Joint conditional independence of predictor variables, given the target variable, is a property of a set of random variables, and it cannot be controlled by any method applied to the variables. If it is satisfied, contrasts of weights-of-evidence and logistic regression parameters with indicator variables are the same; in particular, they are independent of one another. If it is not satisfied, the logistic regression parameters are different from the contrasts of weights-of-evidence, while the latter is still independent and the former is dependent to counterbalance the violation of conditional independence. Including interaction terms of those random variables which cause the violation of conditional independence in the logistic regression model exactly compensates for the lack of conditional independence. Weights-of-evidence cannot reasonably include interaction terms. Thus, in case of lacking conditional independence, there is no need to turn weights-of-evidence into modified weights-of-evidence [6] by successively applying first weights-of-evidence to the initial indicator variables even though the application is not mathematically authorized, and secondly, logistic regression to the corrupted weights to yield corrected weights.

Moreover, this comment elaborates on the case of missing data; it reveals the details of the strategy numerically realized by weights-of-evidence, and it shows that logistic regression can easily be adjusted. Assuming that logistic regression and modified weights-of-evidence are “equivalent” as Zhang et al. claim [1], then why not use logistic regression in the first place? Since logistic regression does not only apply to random indicator variables as weights-of-evidence but also to continuous random variables, an indicator transformation according to a user defined threshold becomes obsolete.
2. A False Model Cannot Decrease Bias

How can a method [7] "significantly reduce the bias" (p. 2) and "diminish lack of CI (conditional independence) bias in WofE (Weights-of-Evidence) results" (p. 2) that was not just illustrated by way of a counter-example but explicitly proven to be mathematically false [8] by grossly mistaking that the logit of a sum is the sum of logits ([8]; p. 406)?


How can “. . . better results generally (can) be obtained by other methods such as boosting [22–24] . . .” (p. 2) as put forward by Cheng [9]; in [24] by Zhang and Agterberg, even though it was explicitly shown and concluded [10] that “Boost weights-of-evidence does not generally reduce not to mention significantly reduce the effect of lacking joint conditional independence. Its application yields corrupted predicted conditional probabilities and corrupted spatial patterns of prospectivity as weights-of-evidence does. In particular, its results depend on the sequential processing order of predictors.” [10]; (p. 404). “. . . criteria for the user decision how to choose the sequence are not provided.” [10]; (p. 404). This publication [10] is not referred to nor included in the authors’ references.


Joint conditional independence of predictor variables, given the target variable, is required to simplify multifoldly conditional probabilities involved in Bayes theorem for several variables to simply conditional probabilities. It is a required modeling assumption of weights-of-evidence to ensure its features and proper estimates of probabilities. If this assumption is relaxed to “features and proper estimates of probabilities. If this as-modeling assumption of weights-of-evidence to ensure its variables to simply conditional probabilities. It is a required additional probabilities involved in Bayes theorem for several the target variable, is required to simplify multifoldly conditional independence mathematically unauthorized and erroneous weights-of-evidence must not be applied, and (ii) in case of joint conditional independence of all \( B_\ell \), given \( T \), as required by weights-of-evidence logistic regression parameters \( \beta_\ell \) like contrasts \( C_\ell \) can be interpreted in terms of conditional odds ratios:

\[
\exp(\beta_\ell) = \exp(C_\ell) = \frac{O(T = 1|B_\ell = 1)}{O(T = 1|B_\ell = 0)} \quad \ell = 1, \ldots, m.
\]

(1)

The odds ratio is a measure of association; it approximates how much more likely (or unlikely) it is for the target to be present \( (T = 1) \) among those with \( B_\ell = 1 \) than among those with \( B_\ell = 0 \).

Moreover, since “the WofE contrast \( [C_\ell] \) measures the strength of spatial correlation between a point pattern and an indicator pattern,” (p. 2) so do the logistic regression parameters \( \beta_\ell \) as they are identical for jointly conditionally independent indicator predictor variables discussed as follows.

Logistic regression applies to continuous random predictor variables and does not require an indicator transformation by thresholding. Logistic regression is the canonical generalization of weights-of-evidence, or the other way round, weights-of-evidence is the special case of logistic regression if the predictor variables are dichotomous and jointly conditionally independent, given the dichotomous target variable.

5. How Missing Data Are Accounted for?

The “category” of missing data is technically often referred to as not available (NA) whether the data are categorical or not. The authors’ review is restricted to dichotomous predictor variables indicating the presence or absence of a given phenomenon. The “powerful tool to deal with missing data” (p. 1) comes down to “Pixel or unit area input for MWofE (Modified Weights-of-Evidence) consists of positive and negative weights for the presence and absence of a pattern plus zeros for missing data” (p. 1). Why is “logistic regression as such (is) not capable of handling missing data” (p. 1)?

Assuming \( m \) jointly conditionally independent indicator predictor variables \( B_1, \ldots, B_m \), given the dichotomous target variable \( T \) the weights-of-evidence are individually determined by counting events, e.g., the ordinary weights,

\[
W_\ell(i) = \ln \frac{P(B_\ell = i|T = 1)}{P(B_\ell = i|T = 0)} \quad i = 0, 1,
\]

(2)

of \( B_\ell \) are estimated, provided that \( B_\ell \) is not missing for any unit area as follows:
assuming joint conditional independence of all of the corresponding logistic regression model \([11, 12]\) as we will see in digital map image.

Provided that where 

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and contrasts \(\hat{C}_\ell = \hat{W}_\ell(1) - \hat{W}_\ell(0)\) are identical with the parameters of the corresponding logistic regression model \([11, 12]\) as we assumed joint conditional independence of all \(B_\ell\) given \(T\).

Otherwise, a linear relationship between weights-of-evidence and logistic regression parameters does not exist \([11, 12]\). It should be noted that the total number of events \(B_\ell = 1\) or \(B_\ell = 0\) is \(n\), which is the total number of pixels of the digital map image.

To keep the notation light, just the indicator variable \(B_{\ell_0}\) is assumed to be missing for some \(0 < n_{\text{NA}} < n\) pixels. Then, in equations (3) and (4), \(n\) has to be replaced by \(n' = n - n_{\text{NA}}\), thus leading generally to a biased estimate \(\hat{W}_{\ell_0}'\) of \(W_{\ell_0}\) only. The corresponding practical "modified" weights-of-evidence model is as follows:

(i) For pixels where \(B_{\ell_0}\) is missing,

\[
\hat{\logit}(T = 1 | B_1, \ldots, B_m) = \hat{\logit}(T = 1) + \sum_{\ell: B_\ell = 0} \hat{W}_\ell(1) + \sum_{\ell: B_\ell = 0} \hat{W}_\ell(0) + \sum_{\ell: B_\ell = \text{NA}} \hat{W}_\ell(\text{NA})
\]

(ii) For pixels where \(B_{\ell_0}\) is not missing,

\[
\hat{\logit}(T = 1 | B_1, \ldots, B_{\ell_0} = \text{NA}, \ldots, B_m) = \hat{\logit}(T = 1) + \sum_{\ell: B_\ell = 0 \neq \ell_0} \hat{W}_\ell(1) + \sum_{\ell: B_\ell = 0 \neq \ell_0} \hat{W}_\ell(0) + \sum_{\ell: B_\ell = \text{NA}} \hat{W}_\ell(\text{NA}) = 0
\]

where \(\hat{W}(0) = \sum_{\ell \neq \ell_0} \hat{W}_\ell(0)\).
\[
\logit(T = 1 | B_1, \ldots, B_{\ell} \neq \text{NA}, \ldots, B_m) = \logit(T = 1) + \sum_{\ell: B_\ell = 0} W_\ell(1) + \sum_{\ell: B_\ell = 0} \bar{W}_\ell(0) - W'_\ell(1) \tag{7}
\]

with \( \bar{W}(0) = \sum_{\ell: \neq B_\ell} \bar{W}_\ell(0) + \bar{W}'_\ell(0) \) and \( \bar{C}_\ell = W'_\ell(1) - \bar{W}'_\ell(0) \).

Thus, for pixels where \( B_\ell \) is missing, it is just omitted by setting its weights to 0; for pixels where \( B_\ell \) is not missing, its weights \( W'_\ell \) and its contrast \( C'_\ell \) are biased as their support does not cover the entire map image. Due to the assumption of joint conditional independence of all \( B_\ell, \ell = 1, \ldots, m \), given \( T \) their weights and contrasts are independent whatever their common support in terms of total numbers of pixels. Thus, the contrasts \( C_\ell, \ell \neq B_\ell, \) remain unaffected. Nevertheless, equation (7) represents a mixture of two different models while equation (6) represents a restricted model.

A similar procedure applies to ordinary logistic regression, i.e., the two logistic regression models are as follows:

(i)

\[
\logit_0(T = 1|B_1, \ldots, B_{\ell} = \text{NA}, \ldots, B_m) = \tilde{\beta}_0 + \sum_{\ell: \neq B_\ell} \hat{\beta}_\ell B_\ell \tag{8}
\]

supported by all pixels of the digital map image, and

(ii)

\[
\logit_1(T = 1|B_1, \ldots, B_{\ell} \neq \text{NA}, \ldots, B_m) = \tilde{\beta}_0 + \sum_{\ell: B_\ell = 1} \hat{\beta}_\ell B_\ell \tag{9}
\]

supported by those pixels for which \( B_\ell \) is not NA.

In contrast to modified weights-of-evidence, all parameters of the latter model, equation (9), are affected by the fact that \( B_\ell \) is missing for some pixels. Since logistic regression does not require a mathematical modeling assumption as restrictive as joint conditional independence of all \( B_\ell \), given \( T \), its parameters are not independent which allows to some extent to account for any kind of interdependencies. Including interaction terms turns logistic regression nonlinear and allows to compensate any lack of joint conditional independence exactly, i.e., logistic regression for dichotomous variables including proper interaction terms recovers the true conditional probability exactly; thus, it is optimum [11].

Depending on what caused realizations of \( B_\ell \) to be missing at some locations, an alternative option to resolve that problem might be spatial interpolation as the data are georeferenced.

**Conflicts of Interest**

The author declares that there are no conflicts of interest regarding the publication of this paper.

**References**


