

Research Article

Analysis of Complex Dynamics in Different Bargaining Systems

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This paper focuses on the bargaining behavior of supply chain members and studies the stability of the bargaining system. There are two forms of bargaining in the process of negotiation. One is separate bargaining, and the other is that the automobile manufacturers form an alliance and bargain with the supplier collectively. We explore the influence of bargaining power and adjustment speed on the stability of the dynamic system and find that both of the factors need to be small to maintain the stability of the supply chain. After comparing the two forms of bargaining in terms of profits and stable regions, we find that the collective bargaining is a pattern with the existence of risk and benefit simultaneously. In order to control chaos in collective bargaining to lower the risk, we adopt the delay feedback control method. With the introduction of the control factor, the system tends to be stable finally.

1. Introduction

The bargaining behavior is common in commercial operation and plays a critical role in the whole supply chain performance. Because of the fierce market competition, many manufacturers cannot raise their prices at will but rely more on bargaining to reduce costs to strive for greater profit space. The concept of common parts makes joint procurement possible, and many manufacturers become beneficiaries of collective bargaining. Automobile manufacturers are also one of them. Take the automobile industry as an example; after years of rapid development, the automobile industry has experienced a slowdown recently, which stimulates the fierce competition of the component market. In order to cut costs, most automobile manufacturers choose to purchase parts from external enterprises, while focusing on their core business. In the automobile manufacturing industry, the purchase cost accounts for a high proportion in the whole product cost, while the purchase cost of automobile parts is a large part of the purchase cost. Since that, the bargaining between automobile manufacturers and the supplier is of great significance. Formerly, manufacturers used to purchase components separately. With the prevalence of the supply chain and the win-win concept, many of

them prefer to form an alliance to bargain with the supplier for the sake of a stronger bargaining power. We used the generalized Nash bargaining framework to model the bargaining process and compared separate bargaining and collective bargaining on their performance of maintaining stability in the dynamic system.

Applying nonlinear dynamics theory to an economic system can provide a better understanding of its complex practice, and many scholars had made their attempt in different fields. Fibich and Gavish [1] applied a dynamical system as a new method to analyze the asymmetric first-price auction. The result proved that in the case of two different players, a unique equilibrium strategy exists. Chen et al. [2] analyzed a finite-level dynamic pricing model in which the demand for each period depends on both the current and past price. Chen and Gallego [3] analyzed the influence of dynamic pricing on public welfare and consumer surplus and proposed a dynamic pricing formulation to maximize welfare. Different behaviors of a dynamic system have corresponding explanations in economics. The equilibrium point in a dynamic system represents a stable state in the supply chain, and chaos represents disorder and risk. When variable spillover occurs, it means that in reality, some enterprises go bankrupt and have to withdraw from

the market. In this paper, some methods of nonlinear theory were used to study the dynamic characteristics of separate bargaining and collective bargaining. We analyzed the influence of bargaining power and adjustment speed on the dynamic system and presented some economic explanations for the results.

The inherent randomness of chaos makes the trajectory of a dynamic system difficult to predict. When chaos is beneficial to the system, conditions should be created to guide the system into a specific chaotic orbit. However, when chaos is harmful to the system, it should be controlled. And in most cases, chaos is not desirable and should be controlled via different methods according to its characteristics. Many methods have been proposed by scholars, such as the method of parameter adjustment and the adaptive chaos control method. Later in the paper, we will adopt the delay feedback control method to control the chaos occurred in the bargaining process because of its good tracking ability and stability.

The paper is organized as follows. In Section 2, the related literature is reviewed. We describe the model and compare the two forms of bargaining in Section 3. The delay feedback control method is used to control the chaos occurred in the system in Section 4. And finally, Section 5 draws conclusions.

2. Literature Review

2.1. Bargaining Behavior. One stream of literature that is related to our work is the one which studied bargaining behavior. Feng et al. [4] analyzed a bargaining game and sorted buyers to high type and low type to illustrate the significance of role forecasting accuracy on the supply chain. Aydin and Heese [5] used a bargaining framework to model the assortment selection process. The result showed that the improvements of products from a manufacturer benefit the other parts of the supply chain, even its competitors. Karagözoğlu and Riedl [6] studied the influence of performance information on bargaining and found that the result of bargaining is mainly equal share without performance information. A significant anchoring effect was found by Leider and Lovejoy [7] in their study of sequential bargaining in a two-tier supply chain with competition. Lee [8] introduced a noncooperative multilateral bargaining model for network restricted environment to characterize a significant condition for the efficient equilibrium. Davis and Hyndman [9] studied the bargaining behavior in a supply chain consisting of a retailer and a supplier. They found that when the quantity of orders is included, the efficiency of the supply chain is significantly improved. Haruvy et al. [10] studied the contract performance of bargaining behavior under the allowance of concessions made by the manufacturer. And, the result demonstrated that the contracts are efficient in this instance. Each paper focuses on different points of bargaining, but the ideas on the modeling of bargaining process are interlinked, which is worth learning from. There are also some scholars focusing on the analysis of different forms of bargaining. Guo and Iyer [11] compared two forms of bargaining in the supply chain containing one manufacturer and two retailers. They proved that

when the profitability is similar, simultaneous bargaining is optimal; otherwise, sequential bargaining is better. Hsu et al. [12] analyzed the two forms of leader-based collective bargaining: one is equal price LCB and the other is fixed price LCB. The result showed that the latter had better performance. Melkonyan et al. [13] applied virtual bargaining in competitive interactions to illustrate that it caused collusion in Bertrand instead of Cournot. Most of the above studies focused on a single period to make optimal decision. However, in real life, the manufacturer cannot fully know the information of the opponent, and his own decision-making in each period is not completely independent. Considering the complexity of the economy system in practical, we extend it to a dynamical system and present a multiperiod decision-making process for component bargaining.

2.2. Dynamical System. In recent years, a dynamical system has been widely used. Many researchers integrate nonlinear dynamics theory and complex system theory into the study of an economic system, which greatly enriches the study of long-term game complexity of an economic system. Zhang et al. [14] formulated a finite-level stochastic programming problem as a dynamic chance-constrained program and demonstrated the efficiency of the presented model. Besbes and Zeevi [15] considered a pricing problem with the demand curve unknown and illustrated the sufficiency of the linear model for dynamic pricing with demand learning. Coucheney et al. [16] obtained a new kind of continuous time learning dynamics, which is composed of a class replicator drift with penalty term adjustment in n -person games. Guo et al. [17] applied typical dynamic equilibrium algorithms such as the simplex gravity flow dynamics and the projected dynamical system to study the dynamic traffic equilibrium problem. Kim et al. [18] studied dynamic scheduling in a service system of multilevel. Ajorlou et al. [19] analyzed the optimal dynamical pricing problem in which the information of products can only be spread via words of mouth. By proving that the price fluctuation of nondurable products disappears after a limited time, the critical role of the product type in zero price sales optimization is further revealed. Ma and Xie [20] studied the effects of adjustment speed and loss sensitivity on supply chain stability and found that the supply chain would be stable when the retailer is not sensitive to the loss or adjusts the decision carefully. Camerer et al. [21] studied dynamic unstructured bargaining, which included deadlines and unilateral information about amounts. They used machine learning to prove that the characteristics of bargaining process recorded in the early stage of the game improve the prediction of disagreement. Jin [22] proposed a stable dynamical system for drivers to choose the departure time under a bottleneck. Li et al. [23] considered the issue of fair concern in pricing and discussed the stability of the Nash equilibrium point in the price game. Although the nonlinear dynamics theory has been used in some research studies on the price and quantity game, the market characteristics in different fields are different. Considering the difficulty and cost the manufacturers faced to obtain information about

their competitors, we adopt limited rational hypothesis and set up models under two different bargaining forms, which can further enrich the application of the theory in practice. We analyze the performance of two dynamic bargaining systems in terms of profit and stability and illustrate the different strength of bargaining power in different models. For the chaos appeared in the dynamic system, we not only analyze its influencing factors, but also present a method to control it.

Chaos control is significant since in many cases it may cause fluctuation in the supply chain and is not conducive to decision-making. Many methods have been proposed by scholars and widely used in chaos control. OGY is the earliest proposed method by Ott et al. [24] for chaos control. Askar [25] adopted the feedback control method to return the dynamic Stackelberg game to the stable region. Elsadany and Awad [26] used the feedback control method to control the disordered behaviors of the Bertrand competition market. Li et al. [27] realized effective chaotic control by the application of parameter adaptation method. Besides the above methods, the delay feedback control (DFC) is another method that is effective for chaos control and has been widely used in many other fields. Since DFC has good tracking ability and does not change the structure of the controlled system, we adopted it to stabilize the system in bargaining process.

3. Model Description

We consider a supply chain that consisted of two manufacturers (marked as $i = 1$ and 2) and one supplier. The two manufacturers purchase the common component from the supplier to produce similar products which induces the competition between them. We assume the manufacturer i 's inverse demand function as follows: $p_i = a - q_i - bq_j$, $i, j = 1, 2, i \neq j$, in which p_i is the market price for the products made by the manufacturer i , q_i and q_j represent the quantity of the two products, a is the market capacity, and b is the competition intensity. Similar settings have been used in the supply chain literature (e.g., [12, 28]). Since the price of the manufacturer i is influenced more by his/her own quantity than their competitors [29], b should be in the range of $0 < b < 1$. Here, we normalize a to one and assume the suppliers cost c as zero to simplify calculation. The manufacturers can purchase the component separately or they can form a bargaining alliance and designate the leader to collectively negotiate with the supplier. We used the GNB framework to model the process of the bargaining, which is commonly used in the supply chain in terms of bargaining between buyers and sellers. We assume the bargaining power of the manufacturer i towards the supplier is k_i and the supplier's bargaining power towards the manufacturer i is $1 - k_i$, correspondingly.

3.1. Separate Bargaining. In the case of separate bargaining, the two manufacturers bargain with the supplier separately, and the decision process is as follows: firstly, the manufacturer i decides the procurement quantity q_i and then bargains with the supplier for the wholesale price w . The profit functions of the manufacturer i and the supplier can be described as follows:

$$\begin{aligned}\pi_i(q_i, q_j, w_i) &= (1 - q_i - bq_j - w_i)q_i, \\ \pi_s(q_i, q_j, w_i, w_j) &= w_i q_i + w_j q_j.\end{aligned}\quad (1)$$

We use the GNB framework to solve this case, and this framework has been widely used in the economic system [30]. We assume that two negotiations are carried on at the same time, and no information is shared between manufacturers, so q_j indicates the value estimated by the manufacturer i . Since the supplier has the responsibility to keep business confidential, this assumption is common in practice. Meanwhile, in an order period, renegotiation is not allowed, which in other words means that once the negotiation is failed, the manufacturer cannot launch another negotiation and both sides of the bargaining will gain no profit. Based on the GNB framework, we can formulate the bargaining problem as follows:

$$\max_{w_i} \pi_i(q_i, q_j, w_i)^{k_i} [\pi_s(q_i, q_j, w_i, w_j) - w_j q_j]^{1-k_i}. \quad (2)$$

Lemma 1. *The wholesale price is $w_i(q_i, q_j) = (1 - k_i)(1 - q_i - bq_j)$, and the manufacturer's profit is $\pi_i(q_i, q_j, w_i(q_i, q_j)) = k_i(1 - q_i - bq_j)q_i$.*

As shown in the above formula, the wholesale price is based on manufacturers' procurement quantity and the bargaining power of both. When the bargaining power of the manufacturer is extremely high, the wholesale price is close to zero; in other words, the manufacturer will gain the profit of the whole supply chain as more as possible. This can also be demonstrated in the second formula of profit. On the contrary, when the bargaining power of the manufacturer is rather small, the profit space will be tremendously compressed.

Static analysis is commonly used in the existing literature to analyze the bargaining problem in the supply chain. However, under the complicated environment, bargaining between both sides is a long-term and complex process, which cannot be solved by only one game. Next, we will employ the method of dynamic game to conduct multi-period adjustment to achieve the equilibrium solution and analyze the system stability.

Because of the lack of market information, the manufacturers are not always rational. For a better practical fit, we assume that the manufacturer adopts the adjustment rule called the gradient adjustment mechanism where they decide their procurement quantity based on the counterpart and margin profit in the last period. The application of this mechanism can also be found in other literatures [31, 32]. The mechanism can be formulated as follows:

$$\begin{cases} q_1(t+1) = q_1(t) + \alpha q_1(t) \frac{\partial \pi_1}{\partial q_1(t)}, \\ q_2(t+1) = q_2(t) + \beta q_2(t) \frac{\partial \pi_2}{\partial q_2(t)}. \end{cases} \quad (3)$$

Under this rule, the manufacturers will increase their procurement quantity in period $t + 1$ if the margin profit in period t is positive and will reduce the order quantity on the contrary. The parameters $\alpha > 0$ and $\beta > 0$ are with respect to the decision adjustment speed. The larger the parameters are, the greater change of procurement quantity will be in the next order period.

With the gradient adjustment mechanism, the manufacturers will stop adjusting their procurement quantity when $q_i(t + 1) = q_i(t)$. By solving the equation $q_i(t + 1) = q_i(t)$, we can achieve four equilibrium solutions as follows: $E_0(0, 0)$, $E_1(0, 1/2)$, $E_2(1/2, 0)$, and $E^*(1/(b + 2), t/(b + 2))$, among which E_0 , E_1 , and E_2 are the boundary equilibrium points and E^* is the only Nash equilibrium point.

Equilibrium points' stability is decided by the characteristic roots of the Jacobi matrix. When the absolute values of the characteristic roots are less than one, the equilibrium point tends to be stable, or not, otherwise. And, the Jacobi matrix can be shown as follows:

$$J = \begin{bmatrix} \frac{\partial q_1(t+1)}{\partial q_1(t)} & \frac{\partial q_1(t+1)}{\partial q_2(t)} \\ \frac{\partial q_2(t+1)}{\partial q_1(t)} & \frac{\partial q_2(t+1)}{\partial q_2(t)} \end{bmatrix} = \begin{bmatrix} j_{11} & -bk_1\alpha q_1 \\ -bk_2\beta q_2 & j_{22} \end{bmatrix}, \quad (4)$$

where $j_{11} = 1 + \alpha k_1(1 - 4q_1 - bq_2)$ and $j_{22} = 1 + \beta k_2(1 - 4q_2 - bq_1)$.

Proposition 1. *Equilibrium solutions $E_0(0, 0)$, $E_1(0, 1/2)$, and $E_2(1/2, 0)$ are not stable.*

More detailed proof can be found in Appendix. Lacking of stability implies that the solution cannot return to a fixed position in a certain period, and in this bargaining problem in the supply chain, it means the quantity will not turn to a fixed value after iterations during multiple periods. At these three boundary equilibrium points, at least one of the manufacturers decides not to place order, which signifies abandoning the next bargaining cycle. This will do harm to his/her interests or even force him/her to withdraw from the market in the long term. Therefore, the supply chain is not sustainable and unpredictable, which brings more difficulty to decision-making.

In terms of the Nash equilibrium point, since the characteristic roots of its Jacobi matrix are difficult to compute, we can use the Jury criterion [33] to judge it. And, the two-dimensional Jury criterion can be shown as follows:

$$\begin{cases} 1 + \text{Tr} + \text{Det} > 0, \\ 1 - \text{Tr} + \text{Det} > 0, \\ 1 - \text{Det} > 0, \end{cases} \quad (5)$$

where Tr and Det are the trace and the determinant of the Jacobi matrix, respectively.

Proposition 2. *The conditions for the stability of the supply chain can be described as follows:*

$$\begin{cases} 0 < \alpha k_1 < 2 + b, \\ 0 < \beta k_2 < \frac{4(2 - \alpha k_1 + b)}{4 - \alpha k_1(2 - b)}. \end{cases} \quad (6)$$

Proposition 2 demonstrates that the bargaining power and adjustment speed should be small at the same time to ensure the stability of the system. It is said that when α and β are large, the manufacturers are quite sensitive to the profit they have had in the last period, and a small fluctuation will induce a substantial change in their decision. A strong bargaining power implies a high expectation of profit it can share in a supply chain, following with a high possibility of the failure of negotiation between the supplier and manufacturers, which is not advantageous to the stability of the supply chain.

To better explain the influence of parameters in Proposition 2, we adopt numerical analysis, where we set $b = 0.1$. The 2D bifurcation diagram is shown in Figure 1, using the dual-layer iterative algorithm, which can also be found in [32, 34]. The stable region is marked in blue. When the bargaining power and adjustment of both manufacturers are small, the bargaining system maintains stability. However, the increase of either factor will cause damage to stability, which brings more loss risk and decision-making difficulty to members in the system. Figure 2 focuses on the adjustment speed and depicts the parameter basin with respect to α and β . The system changes from the stable region (marked in green), through the cycle-2 (marked in blue) and the cycle-4 (marked in purple), to the chaotic region (marked in light gray). When the value of adjustment speed is quite big, the system may even enter the divergence region (marked in dark gray) finally.

In Figures 3 and 4, the bifurcation and corresponding LLE diagram show that a manufacturer with a stronger bargaining power will probably induce the system to fall into an unstable region. The formation of chaotic attractors demonstrates the track of the system. We take k_1 for example in Figure 5 and k_2 is similar to it. With the increase of k_1 ($k_1 = 4.465, 6.025, 6.155$, and 6.545), the track tends to be disordered. Strong bargaining power increases the contradiction of negotiation, and sometimes the manufacturers need to sacrifice certain current profit for the sake of long-term benefit.

3.2. Collective Bargaining. Besides separate bargaining, manufacturers can also form an alliance to negotiate with the supplier collectively. Leader-based collective bargaining is one of the most popular forms of collective bargaining in practice. In this case, manufacturers decide their procurement quantity q_i, q_j at first, and then they form an alliance and designate manufacturer i (here, we assume $k_i > k_j$) to negotiate with the supplier with the total quantity $q_i + q_j$ for the wholesale price w . Since the quantity under collective bargaining ($q_i + q_j$) is larger than that under separate one (q_i), we defined k as the new bargaining power and $k > k_i$. No renegotiation is permitted. If succeed, both manufacturers will get the component with the wholesale price; otherwise, none of the three parts in this supply chain will gain profits.

The profit functions of manufacturers and the supplier can be described as follows:

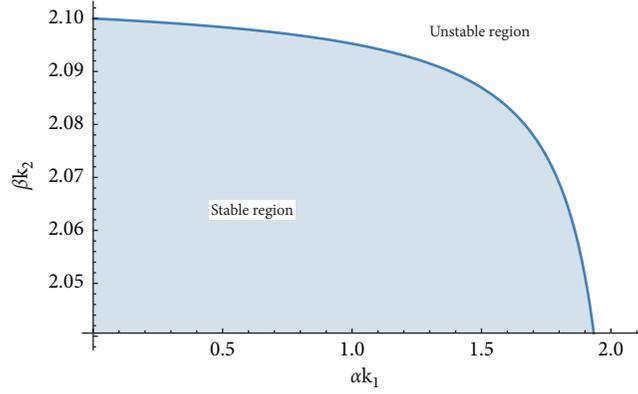
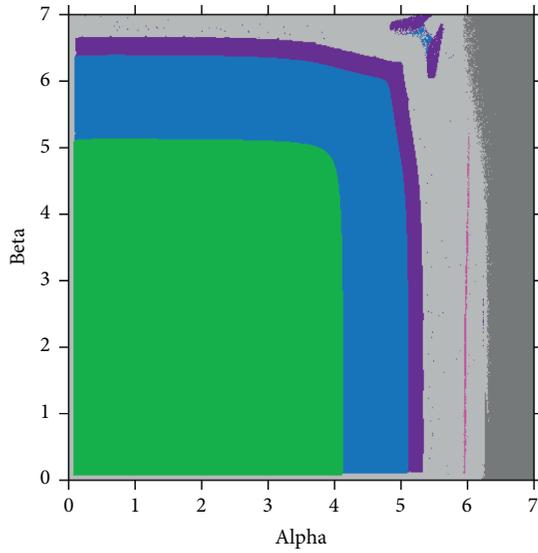


FIGURE 1: The stable region of the separate bargaining system.

FIGURE 2: The parameter basin with respect to α and β .

$$\begin{aligned}\pi_i(q_i, q_j, w_i) &= (1 - q_i - bq_j - w)q_i, \\ \pi_s(q_i, q_j, w_i, w_j) &= w(q_i + q_j).\end{aligned}\quad (7)$$

Based on the GNB framework, the bargaining problem between the leader manufacturer and the supplier can be formulated as

$$\max_w \pi_i(q_i, q_j, w)^k [w(q_i + q_j)]^{1-k}. \quad (8)$$

Lemma 2. The outcomes of the wholesale price is

$$w(q_i, q_j) = (1 - k)(1 - q_i - bq_j), \quad (9)$$

and the manufacturers' profit is

$$\begin{aligned}\pi_1(q_i, q_j) &= k(1 - q_1 - bq_2)q_1, \\ \pi_2(q_i, q_j) &= [k - q_2 - bq_1 + (1 - k)(q_1 + bq_2)]q_2.\end{aligned}\quad (10)$$

Both functions of the wholesale price and the profit have the same form as the counterpart under separate bargaining. But, considering the enhanced bargaining power, it is actually different. Compared with the separate bargaining case, with the stronger bargaining power, the wholesale price is more close to zero and the manufacturer will gain more share of the whole profit in the supply chain. This provides manufacturers with a stand point to form an alliance willingly.

Likewise, under the gradient adjustment mechanism, the system can be formulated as

$$\begin{cases} q_1(t+1) = q_1(t) + \alpha q_1(t)k(1 - 2q_1(t) - bq_2(t)), \\ q_2(t+1) = q_2(t) + \beta q_2(t)(k - 2q_2(t) - bq_1(t) + (1 - k)(q_1(t) + 2bq_2(t))). \end{cases}\quad (11)$$

By solving the equation $q_1(t+1) = q_1(t)$, we can achieve four equilibrium solutions as

$$\begin{aligned}E_0 &(0, 0), \\ E_1 &\left(0, \frac{k}{2 - 2b(1 - k)}\right), \\ E_2 &\left(\frac{1}{2}, 0\right), \\ E^* &\left(\frac{2 - 2b + bk}{4 - b^2 - 3b + 3bk}, \frac{1 - b + k}{4 - b^2 - 3b + 3bk}\right),\end{aligned}\quad (12)$$

in which E_0 , E_1 , and E_2 are the boundary equilibrium points and E^* is the only Nash equilibrium point.

Proposition 3. Equilibrium solutions E_0 , E_1 , and E_2 are not stable.

At least one of the quantity decisions is set to zero in these three boundary equilibrium points, which is the lack of economic significance and is not sustainable. It means that the manufacturer chooses to abandon the next bargaining period, which may cause damage to him/her and may even

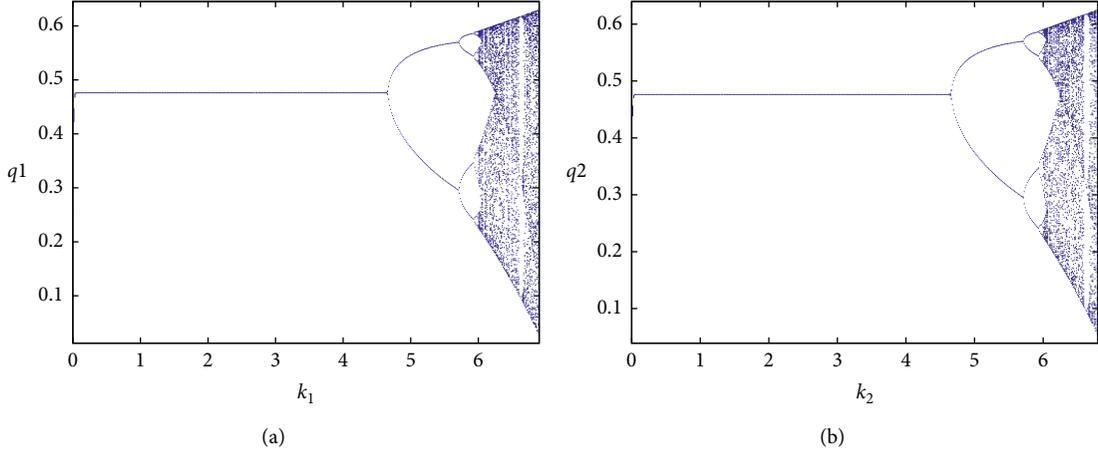


FIGURE 3: The bifurcation diagram of the model with respect to k_1 and k_2 .

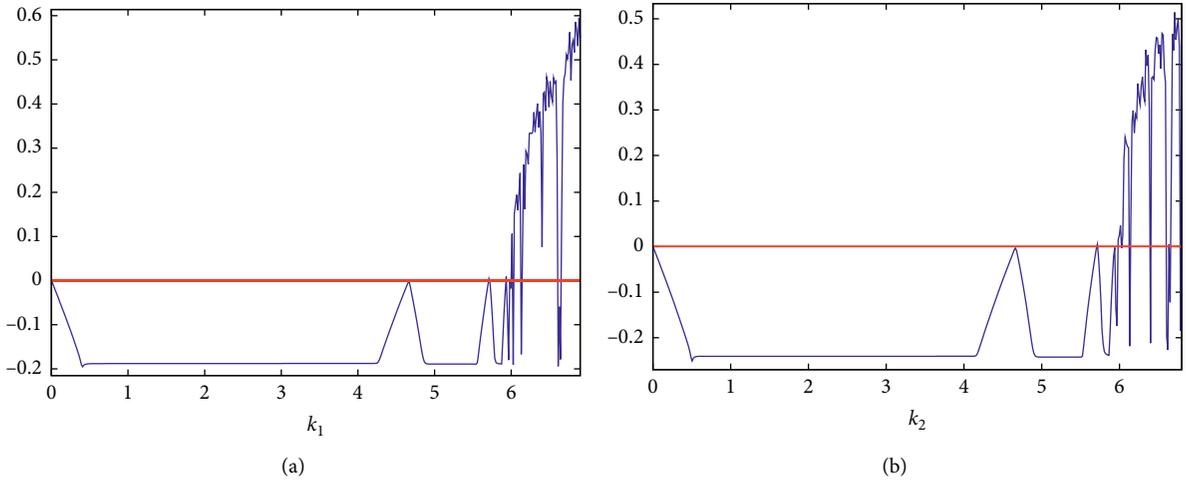


FIGURE 4: LLE diagram of the model with respect to k_1 and k_2 .

be forced out of the market in the long run. Therefore, boundary equilibrium points cannot maintain stability and is not sustainable in reality. Since the absolute values of the characteristic roots are less than one, it means that the three boundary equilibrium points are lacking stability and will approach to chaos after a few iterations. Under these circumstances, the supply chain is in a shambles, and the

economic activities in the supply chain are unpredictable, which add more difficulty for decision-making.

Proposition 4. *In terms of the Nash equilibrium point, the conditions under the Jury criterion for the stability of the supply chain can be described as follows:*

$$\left\{ \begin{array}{l} 0 < \alpha < \frac{4(b(3+b-3k)-4)(4-3b-b^2+3bk+((b-1)b-1)k-(b-1)^2)\beta}{((k-2)b+2)k(4b(3+b-3k)-16+((b-1)^2(4+b)+4(1+b-b^2)k-bk^2)\beta)}, \\ 0 < \beta < \frac{b(3+b-3k)-4}{((b-1)b-1)k-(b-1)^2}. \end{array} \right. \quad (13)$$

Proposition 4 presents that the adjustment speed should be controlled below the threshold so that the stability can be guaranteed. And, the effect of bargaining power is significant

but complex. For the sake of the better explanation of Proposition 4, we picture the stable region of the collective bargaining system in Figure 6.

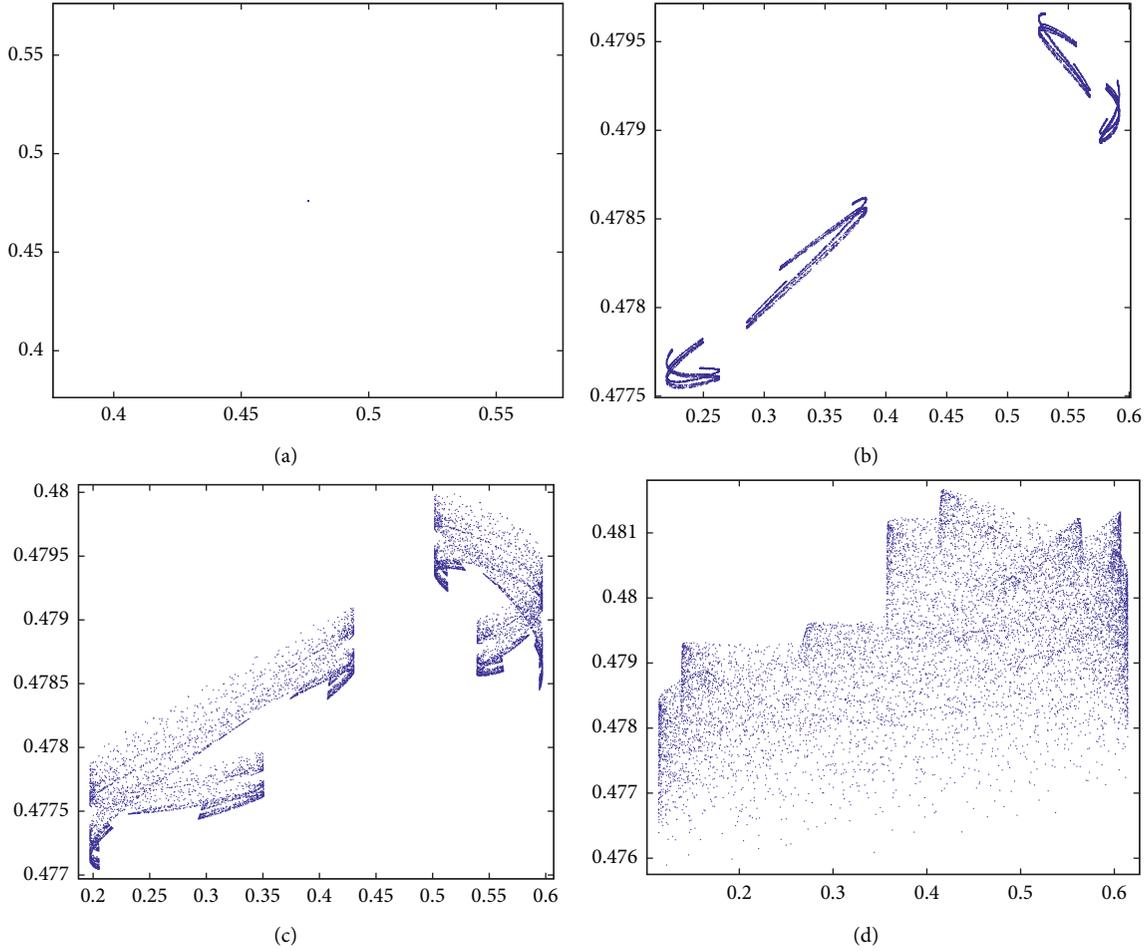


FIGURE 5: The stable region of both bargaining systems.

Just as stated in Proposition 4, the system will stay stable when the bargaining power and adjustment of both manufacturers are small simultaneously. Figures 7 and 8 depict the bifurcation of the model in terms of the adjustment speed. And, Figure 9 shows the track of the system from stable to chaos with the increase of α ($\alpha = 2.594, 3.398, 3.465$, and 3.599). We take α , for example, and β is similar to it. When one of the manufacturers increases the speed of decision adjustment, it means that he/she may make a big change while making the next decision. This change causes the fluctuation of not only ones own decision but also the decision of the others. Due to the sensibility of the dynamical system, the fluctuation will be magnified and lead the system to chaos.

3.3. Performance Comparison. In practice, risk and profit are two key factors emphasized by enterprises. Considering that, we will present the comparisons on these two factors between separate bargaining and collective bargaining in the following.

Proposition 5. *Total profit of the alliance under collective bargaining is greater than that under separate bargaining at the Nash equilibrium point, if $b < b^*$, while chaos will cause damage to profits of both forms.*

According to Lemma 1 and the Nash equilibrium point, the sum of the two manufacturers' profits in the separate bargaining system is $(\pi_1 + \pi_2)^S = (k_1 + k_2)/(2 + b)^2$. Similarly, the gross profit of the alliance formed by these manufacturers in the collective bargaining system is $(\pi_1 + \pi_2)^C = ((1 - b)^3 + (1 - b)^2(6 + b)k + (1 - b)(1 + 6b)k^2 + (b + b^2)k^3)/(4 - b^2 - 3b - 3bk)^2$. Since $k > k_1 > k_2$, the total profit $(\pi_1 + \pi_2)^C$ is greater than $(\pi_1 + \pi_2)^S$ when b satisfies $b < b^*$. More detailed proof can be found in the online appendix. When b is large, the competition intensity between manufacturers is great and the quantity the other manufacturer purchased will lay more influence on one's own sale price. Since the influence is negative, it will diminish the sale price and shrink the profit. Meanwhile, considering the profit expression from above, we can see that profit is closely related to bargaining power. Since bargaining power affects the profit distribution between the supplier and the manufacturer, when the supplier bargains with the powerful buyer, his/her share will be compressed. The enhanced bargaining power has won manufacturers' lower wholesale prices, which means more profit margins. Therefore, the manufacturer will obtain more when his bargaining power is close to one. Especially, when his bargaining power is equal to one, the manufacturer will obtain the whole surplus of the negotiation.

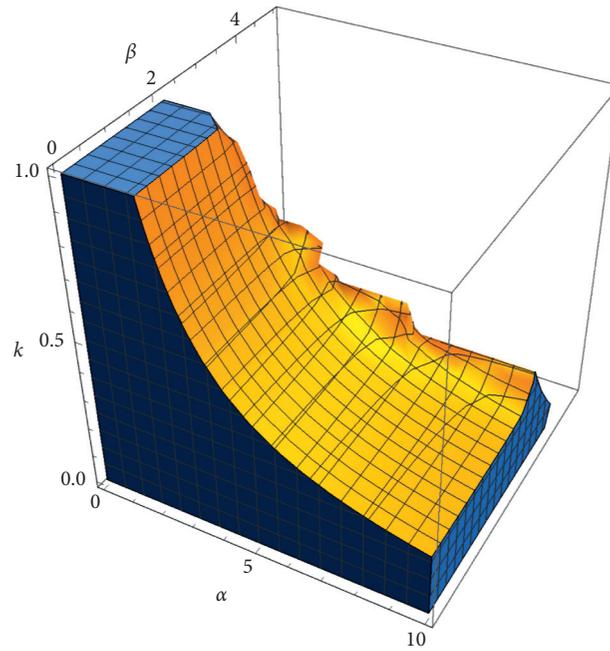


FIGURE 6: The stable region of the collective bargaining system.

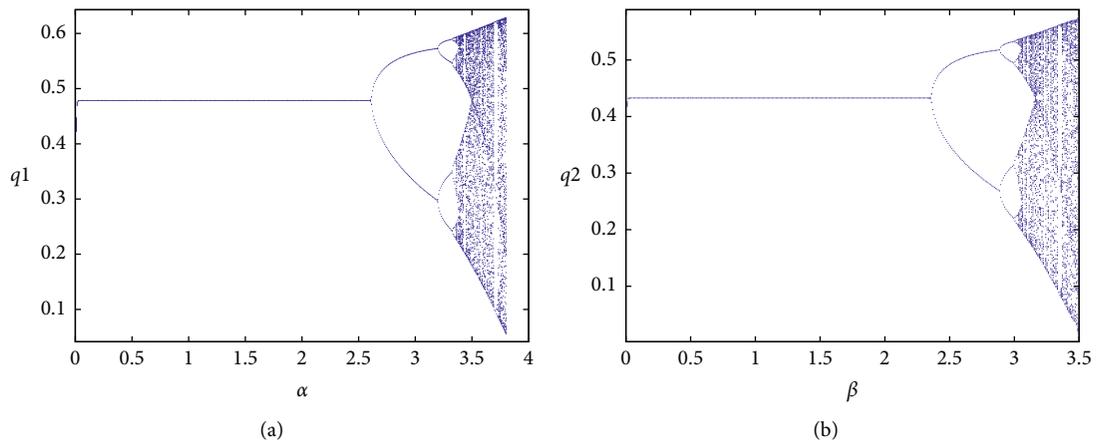


FIGURE 7: The bifurcation diagram of the model with respect to α and β .

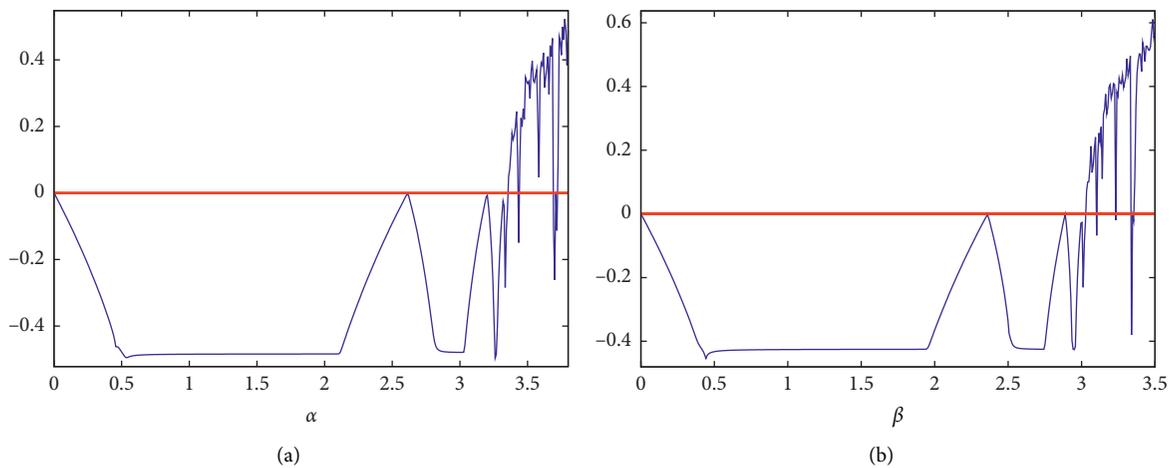


FIGURE 8: LLE diagram of the model with respect to α and β .

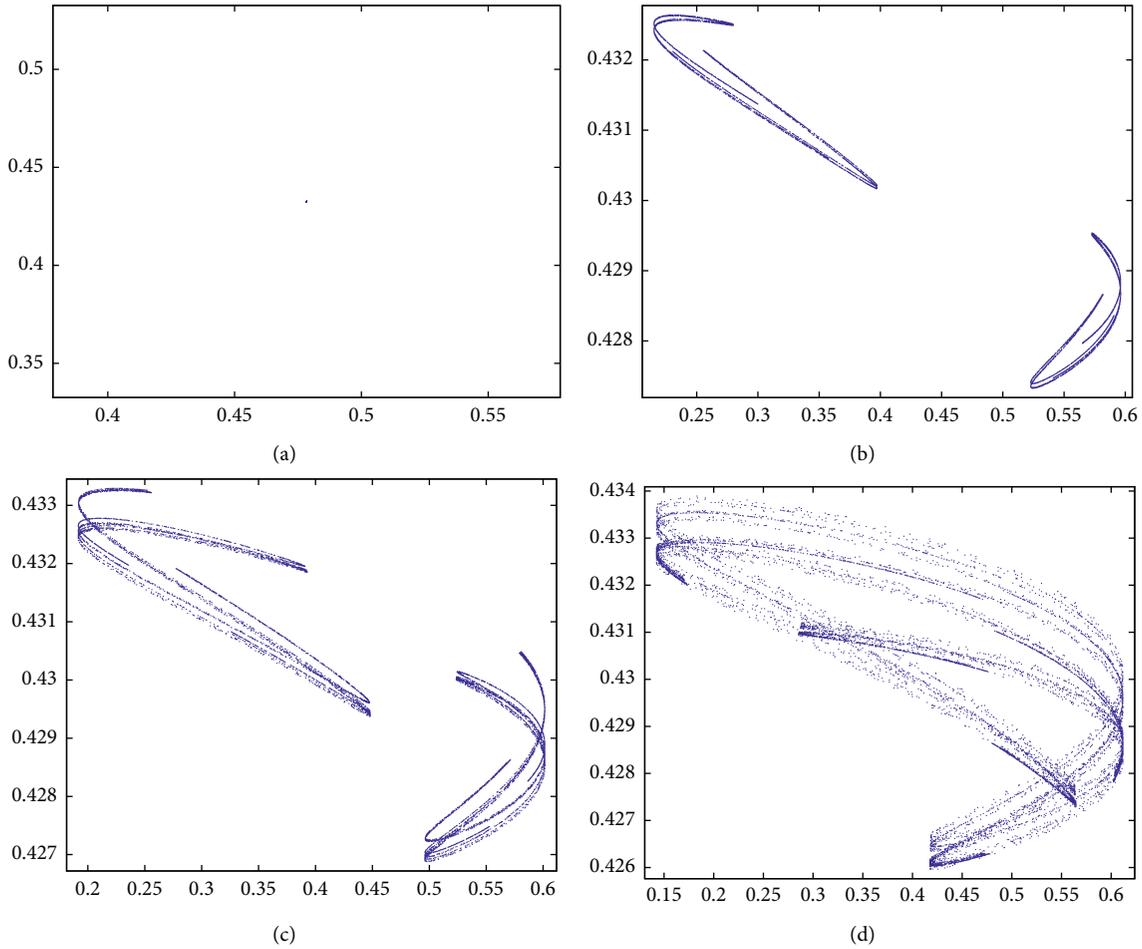


FIGURE 9: The formation of chaotic attractors.

Figure 10 shows the sum of both manufacturers' profits under separate (marked in blue) and collective (marked in red) bargaining. For ease of observation, the profits in the chaotic region are represented by a mean value [35, 36]. With the increase in adjustment speed, the system enters into chaos, and the profits of both forms cut down and eventually become irregular. Chaos will cause the market to fluctuate continuously, and manufacturers will be more sensitive to their current profit to adjust production more frequently and substantially, which adds more difficulty to decision-making. Influenced by both one's own and the opponent's decision, the market price fluctuates greatly, so do the profits of manufacturers.

Proposition 6. *Stability of the supply chain under separate bargaining is better than that under collective bargaining.*

To focus on the common factor, i.e., adjustment speed, we fixed bargaining power as $k = 0.8$, $k_1 = 0.5$, and $k_2 = 0.4$. Based on the above analysis, we can obtain the stable region (marked in green) of both separate bargaining 11(a) and collective bargaining 11(b) in Figure 11. Same as before, the chaotic region and the divergence region are marked in light gray and dark gray, respectively. Obviously, the stable regions of the collective bargaining system shrink a lot compared with the separate bargaining system.

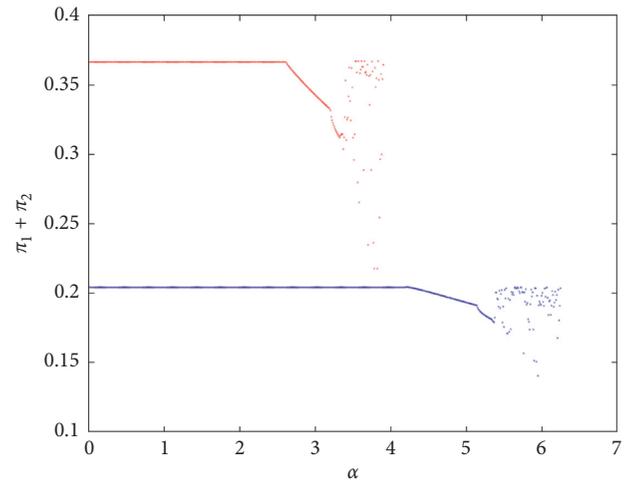


FIGURE 10: The total profit of manufacturers.

In collective bargaining, manufacturers form the alliance designate manufacturer 1 as a leader to negotiate with the supplier. Since the leader pools the quantity of both manufacturers, a slight fluctuation will bring great changes to the system. Therefore, the adjustment speed should be controlled to a smaller extent. Compared with separate

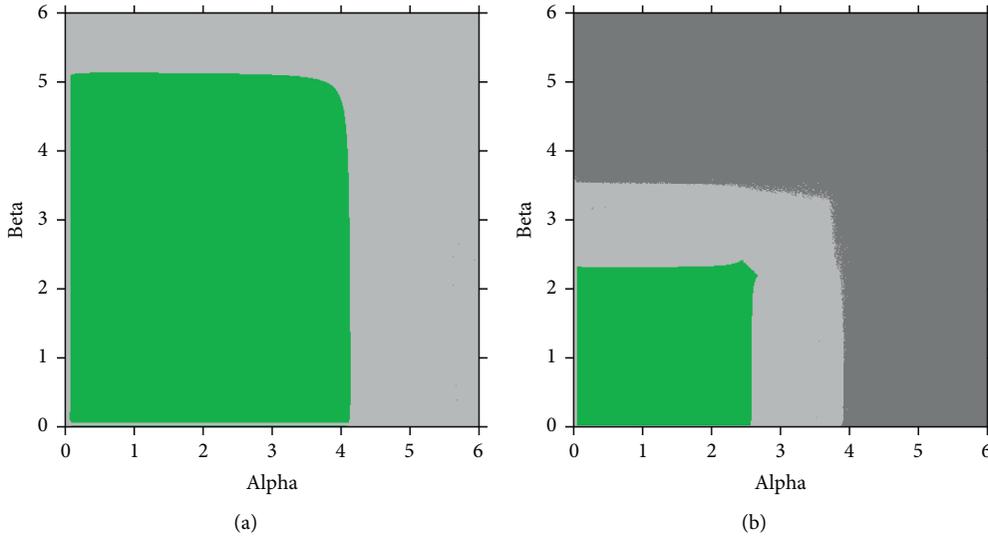


FIGURE 11: The stable region of both bargaining systems.

bargaining, the adjustment speed of the follower not only influences his/her own decision but also has indirect impact on the alliance. With the double influence, the collective bargaining system will be easier to be chaotic if the adjustment speed increased. Meanwhile, according to Proposition 2, the range of the stable region will contract with the enlargement of k . Since $k > k_i$, manufacturer i possesses a stronger bargaining power under collective bargaining and the enhanced bargaining power will also account for the reduction of the stable region.

We depict the bifurcation diagrams of the profit under separate bargaining in Figure 12 and collective bargaining in Figure 13. Profits of manufacturer 1 are marked in blue and pink with respect to α and β . The counterparts of manufacturer 2 are marked in green and red, respectively. Comparing Figure 12 with 13, we also find that though the increase in both adjustment speeds will cause the system to enter into chaos, similarly, the influence of them will be different between separate and collective bargaining systems. For separate bargaining, the bifurcation and chaos appear earlier with the increase in α than β , but in the case of collective bargaining, the situation is reversed.

The transformation of relationship focus between manufacturers, from competition to cooperation, can explain the change of adjustment speed status. In separate bargaining, competition is more emphasized, and the position of the manufacturer determines the influence of his/her adjustment speed. Since we assumed that manufacturer 1 is more powerful than manufacturer 2, his/her adjustment speed has more influence on the system. When two manufacturers form an alliance in collective bargaining, competition is weakened and cooperation plays a more critical role in the bargaining system. Since manufacturer 1 is the representative of the alliance to negotiate with the supplier, the adjustment speed of manufacturer 2 not only influences his/her own decision directly but also has an impact on the alliance. Because the influence of β is double, it has more influence in collective bargaining.

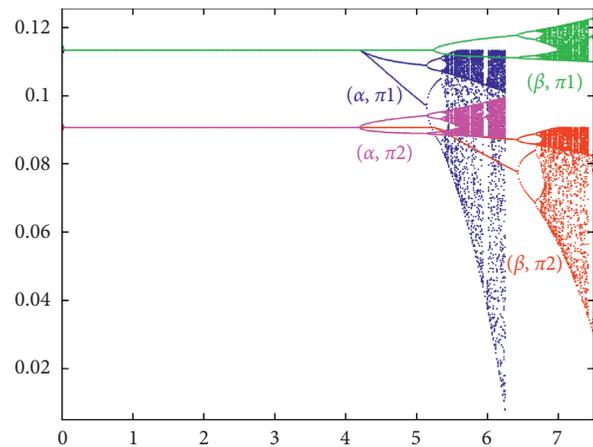


FIGURE 12: The bifurcation diagram of the profit under separate bargaining.

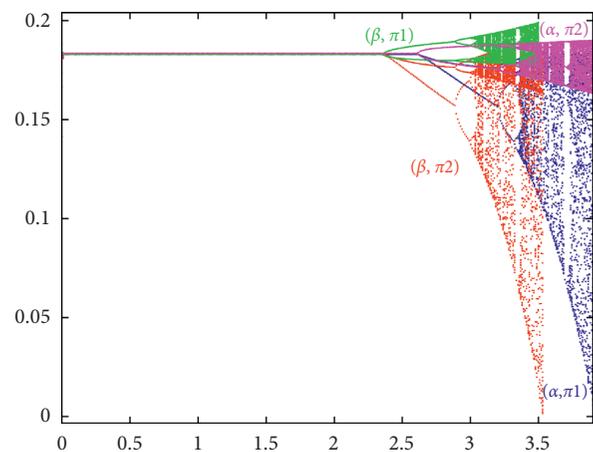


FIGURE 13: The bifurcation diagram of the profit under collective bargaining.

Propositions 5 and 6 imply that the collective bargaining brings more profit and risk at the same time. In order to develop the strength and avoid the weakness, the controlling of the supply chain state is significant. In the following, we will present the chaos controlling method and use it to maintain the stability of the supply chain.

4. Chaos Control

Chaos is inherently random, nonlinear, and sensitive to an initial value. Sometimes, chaos benefits firms [37]. However, in the supply chain and other economic systems, chaos is harmful in many cases. It makes manufacturers difficult to make accurate decisions and causes profit fluctuation. And, for the enterprise, there is even a risk of being forced out of the market in the long run. According to Proposition 5, chaos will cause damage to both the separate bargaining system and the collective bargaining system, which makes profits fall and show irregular fluctuations. In order to maintain profit advantage of collective bargaining, it is very

important to take effective methods according to the characteristics of the system to control the chaos.

As one of the methods of chaos control, the delay feedback control method does not change the structure of the controlled system and has good tracking ability and stability. Its main idea is to feedback partial information of the output signal of the system, instead of the external input, to the control system with delay time. The control system can be described as follows:

$$x_i(t+1) = f(x_i(t), u_i(t)), \quad i = 1, 2, \quad (14)$$

where $x(t)$ is the state variable and $u(t)$ is the control signal. The specific form of the control signal is as follows:

$$u_i(t) = \begin{cases} \delta(x_1(t+1-\tau) - x_1(t+1)), & i = 1, \\ \lambda(x_2(t+1-\tau) - x_2(t+1)), & i = 2, \end{cases} \quad (15)$$

where δ and λ are the controlling factors and $\tau(t > \tau)$ is the length of lag time. Here, we assume τ as one period. The control system can be formulated as

$$\begin{cases} q_1(t+1) = q_1(t) + \frac{\alpha q_1(t)k(1-2q_1-bq_2)}{1+\delta}, \\ q_2(t+1) = q_2(t) + \frac{\beta q_2(t)(k-2q_2-bq_1+(1-k)(q_1+2bq_2))}{1+\lambda}, \end{cases} \quad (16)$$

and the Jacobi matrix of the control system is

$$J = \begin{bmatrix} 1 + \frac{\alpha k(1-4q_1-bq_2)}{1+\delta} & \frac{-\alpha k b q_1}{1+\delta} \\ \frac{\beta q_2(-b+1-k)}{1+\lambda} & 1 + \frac{\beta(k-4q_2-bq_1+(1-k)(q_1+4bq_2))}{1+\lambda} \end{bmatrix}. \quad (17)$$

At the Nash equilibrium point $E^*((2-2b+bk)/(4-b^2-3b+3bk), q(1-b+k)/(4-b^2-3b+3bk))$, the above Jacobi matrix can be formulated as follows:

$$J = \begin{bmatrix} 1 + \frac{\alpha k(4+2b(k-2))}{(1+\delta)(b^2-4-3b(k-1))} & \frac{-\alpha k b(2-2b+bk)}{(1+\delta)(4-b^2-3b+3bk)} \\ \frac{\beta(-b+1-k)(1-b+k)}{(1+\lambda)(4-b^2-3b+3bk)} & 1 + \beta \frac{2(1-b^2(k-1)+k+b(k^2-2))}{(1+\lambda)(b^2-4-3b(k-1))} \end{bmatrix}. \quad (18)$$

From the former numerical analysis, we know that the system is chaotic when $\alpha = 3.5$, $\beta = 3.3$, and $k = 0.8$. But now, the matrix of the control system has the form as

$$J = \begin{bmatrix} 1 - \frac{2.68}{1+\delta} & \frac{-0.13}{1+\delta} \\ \frac{0.14}{1+\lambda} & 1 - \frac{2.80}{1+\lambda} \end{bmatrix}. \quad (19)$$

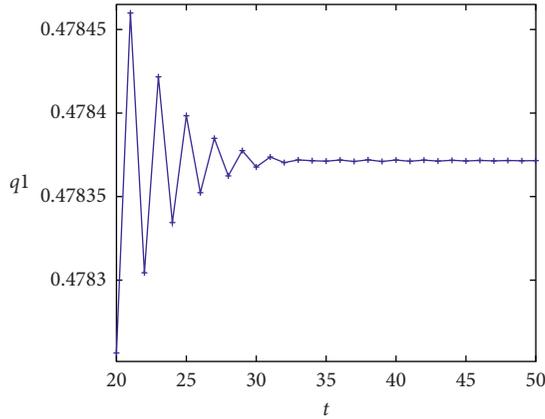


FIGURE 14: The quantity fluctuations of manufacturer 1.

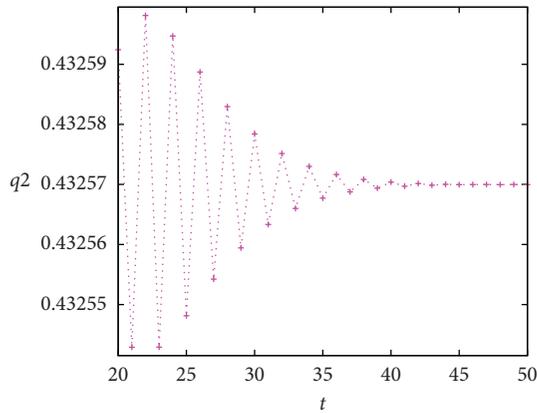


FIGURE 15: The quantity fluctuations of manufacturer 2.

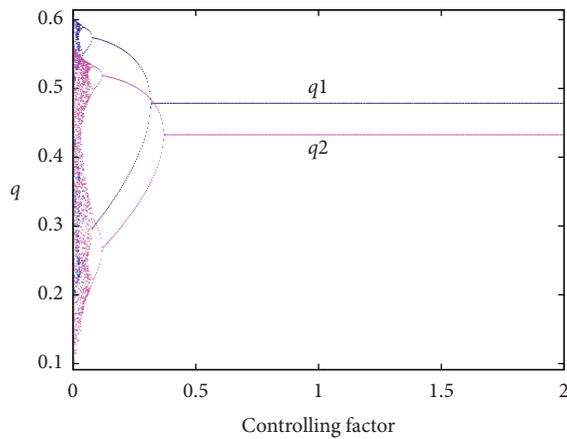


FIGURE 16: The bifurcation diagram of the controlling factors.

The stability is achieved when the Jury criterion is satisfied. Therefore, the control system is stable around the Nash equilibrium point when

$$\begin{cases} 0.14 + \delta(\lambda - 0.40) - 0.34\lambda > 0, \\ 2.80\delta - 2.04 + 2.68\lambda > 0. \end{cases} \quad (20)$$

As shown in Figures 14 and 15, when $\delta = 0.5$ and $\lambda = 0.6$, the quantity tends to the fixed Nash equilibrium value (0.478, 0.433) after several iterations. Obviously, it can also be seen from Figure 16 that the system has changed from chaos to stability with the enhancement of controlling factors δ and λ . The delayed feedback control method is effective for chaos control. Under these circumstances, manufacturers can adjust their decision by not only taking the profit of the last period as the benchmark, but also taking the profit of the previous periods as the reference to improve the stability and effectiveness of decision-making. With the application of controlling factors, manufacturers can guide the system back to stability by increasing the value of controlling factors. Therefore, the manufacturers can make more efficient decision and lower the risk that chaos brought at the same time, which are advantageous for the whole supply chain.

5. Conclusion

This paper analyzed the system of two forms of bargaining: the separate one and the collective one. The boundary equilibrium points of both are unstable. In fact, neither of the manufacturers wants to keep his quantity at zero because that would cost him market share. Therefore, the motivation to change will damage the stability of the bargaining system. The Nash equilibrium point is sensitive to the value of adjustment speed and bargaining power of manufacturers. The result demonstrated that the bargaining power and adjustment speed should be small at the same time to ensure the stability of the system. The adjustment speed reflects the sensitivity of manufacturers to the profits of the last period. When the adjustment speed is large, it will magnify the fluctuation in the system and cause chaos. Meanwhile, the bargaining power should be small to reduce the probability of negotiation failure. Comparing these two forms of bargaining, we found that the collective bargaining brings about more profits and risks at the same time. However, chaos will cause damage to the gross profit of manufacturers, which reduces the profit advantage of collective bargaining. By introducing the controlling factors, the delay feedback method can control the chaos effectively. It means that manufacturers can make their decisions not only based on the information from the last period but also from previous periods to improve decision effectiveness. This result can help manufacturers make decisions accurately and is beneficial for maintaining the stability of the economic system.

Appendix

Proofs for Main Results

Proof of Lemma 1. w can be solved by deriving formula (2), and then π can be described with w as $\pi_i(q_i, q_j, w_i(q_i, q_j)) = k_i(1 - q_i - bq_j)q_i$.

Proof of Proposition 1. The four equilibrium points is the solution of $q_i(t+1) = q_i(t)$ can be specified as follows:

$$\begin{cases} q_1(t) = q_1(t) + \alpha q_1(t)k_1(1 - 2q_1(t) - bq_2(t)), \\ q_2(t) = q_2(t) + \beta q_2(t)k_2(1 - 2q_2(t) - bq_1(t)). \end{cases} \quad (\text{A.1})$$

And, can take $E_0(0, 0)$, $E_1(0, 1/2)$, $E_2(1/2, 0)$, and $E^*(1/(b+2), 1/(b+2))$ into the Jacobi matrix.

The Jacobi matrix of $E_0(0, 0)$ is

$$J(E_0) = \begin{bmatrix} 1 + \alpha k_1 & 0 \\ 0 & 1 + \beta k_2 \end{bmatrix}. \quad (\text{A.2})$$

The characteristic roots of $J(E_0)$ is $\lambda_1 = 1 + \alpha k_1$ and $\lambda_2 = 1 + \beta k_2$. Since $\alpha, \beta > 0$ and $k_1, k_2 \in (0, 1)$, the absolute value of both characteristic roots is more than one, which means that E_0 is unstable. The Jacobi matrix of $E_1(0, 1/2)$ is

$$J(E_1) = \begin{bmatrix} 1 + \frac{1}{2}\alpha k_1(2-b) & 0 \\ -\frac{1}{2}\beta k_2 & 1 - \beta k_2 \end{bmatrix}. \quad (\text{A.3})$$

One of the characteristic roots is $\lambda = 1 + (1/2)\alpha k_1(2-b)$, which is more than one.

E_2 can be proved similarly as E_1 .

Proof of Proposition 2. The Jacobi matrix of $E^*(1/(b+2), 1/(b+2))$ is

$$J(E^*) = \begin{bmatrix} \frac{2 - 2\alpha k_1 + b}{2 + b} & \frac{b\alpha k_1}{2 + b} \\ \frac{b\beta k_2}{2 + b} & \frac{2 - 2\beta k_2 + b}{2 + b} \end{bmatrix}. \quad (\text{A.4})$$

The trace and determinant of $J(E^*)$ is

$$\begin{aligned} \text{Tr}(J(E^*)) &= \frac{2[2 - \alpha k_1 - \beta k_2 + b]}{2 + b}, \\ \text{Det}(E^*) &= \frac{2 + b - 2(\alpha k_1 + \beta k_2) + \alpha k_1 \beta k_2(2 - b)}{2 + b}. \end{aligned} \quad (\text{A.5})$$

Solving three inequalities: $1 + \text{Tr} + \text{Det} > 0$, $1 - \text{Tr} + \text{Det} > 0$, and $1 - \text{Det} > 0$ at the same time and Proposition 2 can be achieved.

Proof of Lemma 2. The proof of Lemma 2 is similar to that of Lemma 1.

Proof of Proposition 3. In terms of collective bargaining, the gradient adjustment mechanism of manufacturer 1 is the same as that in separate bargaining, while the formula of manufacturer 2 is adjusted as

$$q_2(t+1) = q_2(t) + \alpha q_2(t)(k - 2q_2 - bq_1 + (1-k)(q_1 + 2bq_2)). \quad (\text{A.6})$$

When $q_i(t+1) = q_i(t)$ is satisfied, we can obtain four solutions: $(0, 0)$, $(0, k/(2 - 2b(1-k)))$, $(1/2, 0)$, and $((2 - 2b + bk)/(4 - b^2 - 3b + 3bk), (1 - b + k)/(4 - b^2 - 3b + 3bk))$, namely, E_0, E_1, E_2 , and E^* .

The Jacobi matrix of E_0 is

$$J(E_0) = \begin{bmatrix} 1 + \alpha k & 0 \\ 0 & 1 + \beta k \end{bmatrix}. \quad (\text{A.7})$$

The absolute values of both characteristic roots are more than 1, and that means E_0 is unstable.

The Jacobi matrix of E_1 is

$$J(E_1) = \begin{bmatrix} 1 + \frac{\alpha k(2 - 2b + bk)}{2 - 2b(1 - k)} & 0 \\ \beta(1 - b - k) \frac{k}{2 - 2b(1 - k)} & 1 - \beta k^2 \end{bmatrix}. \quad (\text{A.8})$$

One of the characteristic roots is $\lambda'_1 = 1 + (\alpha k(2 - 2b + bk)/2 - 2b(1 - k)) > 1$, so E_1 is unstable.

Similarly, the instability of E_2 can be proved.

Proof of Proposition 4. The Jacobi matrix of $E^*((2 - 2b + bk)/(4 - b^2 - 3b + 3bk), (1 - b + k)/(4 - b^2 - 3b + 3bk))$ is

$$J(E^*) = \begin{bmatrix} 1 + \frac{\alpha k(4b - 4 - 2bk)}{4 - b^2 - 3b + 3bk} & \frac{-b\alpha k(2 - 2b + bk)}{4 - b^2 - 3b + 3bk_1} \\ \frac{\beta(1 - b - k)(1 - b + k)}{4 - b^2 - 3b + 3bk} & 1 + \frac{\beta(-2k + 2b^2k - 2bk - 2 + 4b - 2b^2)}{4 - b^2 - 3b + 3bk} \end{bmatrix}. \quad (\text{A.9})$$

The trace and determinant of $J(E^*)$ is

$$\begin{aligned} \text{Tr}(J(E^*)) &= 2 + \frac{(4b\alpha - 4\alpha + 2b^2\beta - 2\beta - 2b\beta)k - 2b\alpha k^2 - 2\beta + 4b\beta - 2b^2\beta}{4 - b^2 - 3b + 3bk}, \\ \text{Det}(J(E^*)) &= \text{Tr}(J(E^*)) - 1 + \frac{\alpha\beta k(2 - 2b + bk)(4k - 4b^2k + 4bk + 4 - 7b - 6b^2 + b^3 - bk^2)}{(4 - b^2 - 3b + 3bk)^2}. \end{aligned} \quad (\text{A.10})$$

According to the Jury criterion, the following inequalities are solved:

$$\begin{cases} 2\text{Tr} + \frac{\alpha\beta k(2 - 2b + bk)(4k - 4b^2k + 4bk + 4 - 7b - 6b^2 + b^3 - bk^2)}{(4 - b^2 - 3b + 3bk)^2} > 0, \\ \frac{\alpha\beta k(2 - 2b + bk)(4k - 4b^2k + 4bk + 4 - 7b - 6b^2 + b^3 - bk^2)}{(4 - b^2 - 3b + 3bk)^2} > 0, \\ -\text{Tr} - \frac{\alpha\beta k(2 - 2b + bk)(4k - 4b^2k + 4bk + 4 - 7b - 6b^2 + b^3 - bk^2)}{(4 - b^2 - 3b + 3bk)^2} > 0. \end{cases} \quad (\text{A.11})$$

And, Proposition 4 can be obtained.

Proof of Proposition 5. The sum of two manufacturers' profits under separate bargaining is

$$\begin{aligned} (\pi_1 + \pi_2)^S &= k_1 \left(1 - \frac{1+b}{2+b}\right) \frac{1}{2+b} + k_2 \left(1 - \frac{1+b}{2+b}\right) \frac{1}{2+b} \\ &= \frac{k_1 + k_2}{(2+b)^2}. \end{aligned} \quad (\text{A.12})$$

The gross profit of the alliance of these two under collective bargaining is

$$\begin{aligned} (\pi_1 + \pi_2)^C &= k(1 - q_1 - bq_2)q_1 + [k - q_2 - bq_1 + (1 - k)(q_1 + bq_2)]q_2 \\ &= k(1 - q_1 - bq_2)(q_1 + q_2) + q_2(q_1 - q_2)(1 - b) \\ &= \frac{k(2 - 3b + bk + k)(2 - 2b + bk) + (1 - b + k)(1 - b)(1 - b + bk - k)}{(4 - b^2 - 3b - 3bk)^2} \\ &= \frac{(1 - b)^3 + (1 - b)^2(6 + b)k + (1 - b)(1 + 6b)k^2 + (b + b^2)k^3}{(4 - b^2 - 3b - 3bk)^2}. \end{aligned} \quad (\text{A.13})$$

The comparison between $(\pi_1 + \pi_2)^S$ and $(\pi_1 + \pi_2)^C$ is equivalent to that between $(k_1 + k_2)/(2 + b)^2$ and $[(1 - b)^3 + (1 - b)^2(6 + b)k + (1 - b)(1 + 6b)k^2 + (b + b^2)k^3]/(4 - b^2 - 3b - 3bk)^2$.

Since $((k_1 + k_2)/(2 + b)^2) < (2k/(2 + b)^2) < (2k/(4 - b^2 - 3b - 3bk))$, the comparison can be further transformed into the comparison between $2k$ and $(1 - b)^3 + (1 - b)^2(6 + b)k + (1 - b)(1 + 6b)k^2 + (b + b^2)k^3$.

Suppose $f(k) = 2k - (1 - b)^3 - (1 - b)^2(6 + b)k - (1 - b)(1 + 6b)k^2 - (b + b^2)k^3$. Then, the first derivative of $f(k)$ is $f'(k) = 2 - (1 - b)^2(6 + b) - 2k(1 - b)(1 + 6b) - 3k^2(b + b^2)$, and the second derivative of it is $f''(k) = -2(1 - b)(1 + 6b) - 6k(b + b^2)$. $f''(k) < 0$ is always satisfied in the range of $k \in (0, 1)$, which means that $f'(k)$ is monotonically decreasing. $f'(1) = 2 - (1 - b)^2(6 + b) - 2(1 - b)(1 + 6b) - 3(b + b^2)$ is always negative when

$b \in (0, 1)$. And, in this range, $f'(0) = 2 - (1 - b)^2(6 + b)$ continues to increase with the increase in b from negative to positive. There exists a threshold value b^* , smaller than at which $f'(0) \leq 0$. Since $f(0) < 0$, b^* ensures that $f(k) < 0$ is negative when $k \in (0, 1)$. From the above derivation, $(\pi_1 + \pi_2)^S < (\pi_1 + \pi_2)^C$ can be proved.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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