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Research Article

Prediction of Air Quality Based on Hybrid Grey Double Exponential Smoothing Model

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To predict the concentration of air pollutants accurately, conformable fractional accumulation grey double exponential smoothing and adjacent accumulation grey double exponential smoothing are proposed, respectively. A hybrid model with conformable fractional accumulation and adjacent accumulation is proposed to further improve the prediction accuracy. The hybrid model is applied to the prediction of AQI, $PM_{2.5}$, and PM_{10}. Compared with the traditional double exponential smoothing model, the new model has a higher prediction accuracy.

1. Introduction

Air quality pollution can induce a variety of respiratory diseases and seriously endanger human health. Therefore, environmental pollution is highly valued. A series of policies to deal with environmental issues have also been proposed, such as the "Air Pollution Prevention and Control Action Plan" and "Notice of Winning the Three-Year Plan of Action to Defend the Blue Sky." With the vigorous control of the country, the air pollution problem has gradually improved. In order to have a better air level, it is necessary to continue to study air quality issues. Due to the promulgation of the new air quality standards, there are less available data on air quality under the new standards. Grey system theory provides an opportunity for accurate prediction of air quality due to its characteristics of "less data" and "poor information." Since Professor Deng published the first grey system paper "The control problems of grey systems" in "Systems & Control Letters" in 1982, this paper symbolizes the advent of the discipline of grey systems theory [1]. Since the emergence of the grey system theory, it has been widely used in various fields, such as construction demand [2], evaluating the interaction between cryptocurrencies [3], traffic-related emissions [4], energy consumption [5], environmental protection investment [6], medicine and health [7], and missile development cost [8]. With the development of grey system theory, different scholars have improved and expanded the grey system theory in terms of cumulative generation methods, parameters, and background values. For example, a grey multivariable convolution model with new information priority accumulation is proposed by Wu and Zhang [9]. Conformable fractional accumulation is proposed by Ma et al. [10]. Nonhomogenous discrete grey model with fractional-order accumulation is proposed by Wu et al. [11]. The error feedback is used to optimize the fractional accumulation grey model to predict per capita power generation by Yang and Xue [12]. Original condition, power exponent, and background value are optimized by Lu et al. The optimized nonlinear grey Bernoulli model is applied for traffic flow prediction [13]. Luo et al. used particle swarm optimization to optimize parameters and proposed a discrete grey polynomial model DGPM (1, 1, N) with new information priority accumulation, which effectively improved the prediction accuracy [14]. Xiong et al. proposed a new GM (1, 1) model to predict China's energy consumption and production based on the optimized original condition [15]. In order to enhance the forecasting accuracy of the traditional grey model, the grey model with the fractional Hausdorff derivative is proposed by Chen et al. [16]. All the above scholars have improved and optimized the grey model, but they are all a single model, and there is no hybrid prediction model.

The hybrid prediction method was first proposed by Bates and Granger [17]. Due to its applicability and high accuracy, it has received much attention. In predictive competitions, M-series competitions are very authoritative. The $M4$ competition is based on the previous three sessions of $M1$, $M2$, and $M3$. An important conclusion drawn from the $M4$ competition is that hybrid prediction can improve the calculation accuracy [18]. A hybrid approach for obtaining optimal cutting conditions during turning of commercially pure titanium (CP-TI) grade 2 was proposed by Khan and Maity [19]. This approach can generate adequate dimensional accuracy. The purpose of a hybrid model is to make up for the shortcomings of individual errors. For example, Zhu et al. believed that the existing prediction methods could not fully reflect the information of the pollution index sequence, so they proposed two hybrid models and conducted an empirical study taking Xingtai City as an example [20]. Gulia et al. combined the deterministic-based models with suitable statistical distribution models to predict the entire range of pollutant concentration distribution [21]. A hybrid model was established by Lai et al. from the perspective of air quality pollution sources. As emissions from pollution sources decrease, $PM_{2.5}$ would also decrease [22]. Smyl proposed the combination of exponential smoothing and neural networks to predict time series [23]. Sharma et al. combined online sequential extreme learning machines with empirical pattern decomposition algorithms to predict hourly air quality [24]. Wu and Lin considered air pollution factors, such as $PM_{2.5}$, PM_{10}, SO_2, CO, NO_2, and O_3. Wavelet decomposition, variational mode decomposition improved by sample entropy, and long-term memory neural network were used to predict air quality index [25]. The above scholars used various hybrid models to evaluate and predict air quality but did not involve the field of grey system. This paper introduces the grey accumulation operator into the statistical model. Then, the grey double exponential smooth hybrid forecasting model is proposed. The conformable fractional accumulation proposed by Ma and the adjacent accumulation will be introduced to grey double exponential smoothing, respectively, in this paper. Finally, the two grey double exponential smoothing prediction results are hybridized for prediction using the coefficient of variability. The validity of the model was verified by applying the hybrid model for the air quality prediction in Chongqing and Foshan.

This article is divided into four parts: Section 2 introduces the conformable fractional accumulation grey double exponential smoothing (CFES), adjacent accumulation grey double exponential smoothing (AAES), and hybrid model. Section 3 uses Nanchong as an example to predict AQI through CFES and AAES, respectively. Then, CFES and AAES are compared with the traditional grey double exponential smoothing (ES). In order to obtain better prediction results, the grey double exponential smoothing model of the two accumulation methods is used to make a hybrid prediction with the variation coefficient. Finally, this paper compares the hybrid model with ES. Section 4 is a summary of the full paper.

2. Grey Double Exponential Smoothing Model

CFES and AAES differ from ES in that grey exponential smoothing contains accumulation operators. The following model will explain in detail.

Assume that $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n-1), x^{(0)}(n)\}$ is a nonnegative time series. $x^{(r)}(k) = \sum_{i=1}^{k} (x^{(0)}(i)/i^{[r]-r})$ $(i = 1, 2, \ldots, k)$ is the conformable fractional accumulation generating operator of order r, $r \in (0, 1]$. In special cases, r takes 1 as the traditional grey first-order accumulation, and the accumulation operator is $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$ [26]. So $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), x^{(r)}(3), \ldots, x^{(r)}(n-1), x^{(r)}(n)\}$ can be deduced that the r-order accumulation sequence of $X^{(0)}$ is

$$\left[x^{(r)}(1), x^{(r)}(2), x^{(r)}(3), \cdots, x^{(r)}(k-1), x^{(r)}(k)\right] = \left[x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(k-1), x^{(0)}(k)\right] \cdot \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & \frac{1}{2^{[r]-r}} & \frac{1}{2^{[r]-r}} & \cdots & \frac{1}{2^{[r]-r}} & \frac{1}{2^{[r]-r}} \\ 0 & 0 & \frac{1}{3^{[r]-r}} & \cdots & \frac{1}{3^{[r]-r}} & \frac{1}{3^{[r]-r}} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{(k-1)^{[r]-r}} & \frac{1}{(k-1)^{[r]-r}} \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{(k)^{[r]-r}} \end{bmatrix}. \tag{1}$$

Equations (2)(5) are derived from [27, 28].

CFES based on the $X^{(r)}$ sequence is

$$S^{(1)}(k) = \alpha x^{(r)}(k) + (1-\alpha)S^{(1)}(k-1), \tag{2}$$

$$S^{(2)}(k) = \alpha S^{(1)}(k) + (1-\alpha)S^{(2)}(k-1). \tag{3}$$

$S^{(1)}(k)$ is the grey single exponential smoothing value of the k period, and $S^{(2)}(k)$ is the grey double exponential smoothing value of the k period.

$$a_k = 2S^{(1)}(k) - S^{(2)}(k), \tag{4}$$

$$b_k = \frac{\alpha}{1-\alpha}\left[S^{(1)}(k) - S^{(2)}(k)\right]. \tag{5}$$

The fitting formula is

$$\hat{x}^{(r)}(k) = a_k + b_k. \tag{6}$$

The prediction formula is

$$\hat{x}^{(r)}(k) = a_k + mb_k, \tag{7}$$

where m is the out-of-sample size.

Finally, we transform the $\hat{x}^{(r)}(k)$ back to the $\hat{x}^{(0)}(k)$:

$$\hat{x}^{(0)}(k) = k^{(1-r)}\left(\hat{x}^{(r)}(k) - \hat{x}^{(r)}(k-1)\right). \tag{8}$$

The original sequence of $X^{(r)}$ is $\hat{x}^{(0)}(i) = k^{(1-r)}(\hat{x}^{(r)}(i) - \hat{x}^{(r)}(i-1))$ $(i = 2, 3, \ldots, k)$.

Furthermore, the original sequence can be also expressed in the form of a matrix as follows:

$$\begin{aligned}
&\left[\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \hat{x}^{(0)}(3), \ldots, \hat{x}^{(0)}(k-1), \hat{x}^{(0)}(k)\right] \\
&= \left[\hat{x}^{(r)}(1), \hat{x}^{(r)}(2), \hat{x}^{(r)}(3), \ldots, \hat{x}^{(r)}(k-1), \hat{x}^{(r)}(k)\right]\begin{bmatrix} 1 & -2^{(1-r)} & \cdots & 0 & 0 \\ 0 & 2^{(1-r)} & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -(k-1)^{(1-r)} & 0 \\ 0 & 0 & \cdots & (k-1)^{(1-r)} & -k^{(1-r)} \\ 0 & 0 & \cdots & 0 & k^{(1-r)} \end{bmatrix} \\
&= \left[a_1 + b_1, a_2 + b_2, \ldots, a_k + b_k\right]\begin{bmatrix} 1 & -2^{(1-r)} & \cdots & 0 & 0 \\ 0 & 2^{(1-r)} & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -(k-1)^{(1-r)} & 0 \\ 0 & 0 & \cdots & (k-1)^{(1-r)} & -k^{(1-r)} \\ 0 & 0 & \cdots & 0 & k^{(1-r)} \end{bmatrix}.
\end{aligned} \tag{9}$$

The prediction reduction formula is

$$\begin{aligned}
&\left[\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(k), \ldots \hat{x}^{(0)}(k+m)\right] \\
&= \left[a_1 + b_1, a_2 + b_2, \ldots, a_k + b_k, a_k + 2b_k, \ldots, a_k + mb_k\right]\begin{bmatrix} 1 & -2^{(1-r)} & \cdots & 0 \\ 0 & 2^{(1-r)} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & -(k+m)^{(1-r)} \\ 0 & 0 & \cdots & (k+m)^{(1-r)} \end{bmatrix}.
\end{aligned} \tag{10}$$

The above is the calculation method of the CFES. The difference between the AAES and the CFES is the accumulated method.

Assume that $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n-1), x^{(0)}(n)\}$ is a nonnegative time series:

$$\begin{gathered}
x^{(1)}(1) = x^{(0)}(1), \\
x^{(1)}(i) = \lambda x^{(0)}(i-1) + x^{(0)}(i), \\
(i = 2, 3, \ldots, n),
\end{gathered} \tag{11}$$

where $\lambda \in (-1, 1)$ is a parameter accumulated nearby, and its purpose is to adjust the weight between old and new information. After accumulation, we can obtain $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n-1), x^{(1)}(n)\}$, and the accumulation sequence of $X^{(0)}$ can be expressed as follows:

$$
\begin{aligned}
&\left[x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n-1), x^{(1)}(n)\right] \\
&= \left[x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n-1), x^{(0)}(n)\right]
\begin{bmatrix}
1 & \lambda & 0 & \cdots & 0 & 0 \\
0 & 1 & \lambda & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \lambda & 0 \\
0 & 0 & 0 & \cdots & 1 & \lambda \\
0 & 0 & 0 & \cdots & 0 & 1
\end{bmatrix}.
\end{aligned} \tag{12}
$$

Then, we use $X^{(1)}$ for traditional double exponential smoothing. Finally, equation (13) is used to transform the prediction value to original sequence:

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \lambda \widehat{x}^{(0)}(k-1). \tag{13}$$

So the original sequence of $X^{(0)}$ can be also expressed as follows:

$$
\begin{aligned}
&\left[\widehat{x}^{(0)}(1), \widehat{x}^{(0)}(2), \widehat{x}^{(0)}(3), \ldots, \widehat{x}^{(0)}(k)\right] \\
&= \left[\widehat{x}^{(r)}(1), \widehat{x}^{(r)}(2), \widehat{x}^{(r)}(3), \ldots, \widehat{x}^{(r)}(k-1), \widehat{x}^{(r)}(k)\right]
\begin{bmatrix}
1 & -\lambda & \lambda^2 & \ldots & (-1)^{(n-1)}\lambda^{(n-1)} \\
0 & 1 & -\lambda & \ldots & (-1)^{(n-1)}\lambda^{(n-2)} \\
0 & 0 & 1 & \ldots & (-1)^{(n-1)}\lambda^{(n-3)} \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \ldots & \lambda^2 \\
0 & 0 & 0 & \ldots & -\lambda \\
0 & 0 & 0 & \ldots & 1
\end{bmatrix} \\
&= \left[a_1 + b_1, a_2 + b_2, \cdots, a_n + b_n\right]
\begin{bmatrix}
1 & -\lambda & \lambda^2 & \ldots & (-1)^{(n-1)}\lambda^{(n-1)} \\
0 & 1 & -\lambda & \ldots & (-1)^{(n-1)}\lambda^{(n-2)} \\
0 & 0 & 1 & \ldots & (-1)^{(n-1)}\lambda^{(n-3)} \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \ldots & \lambda^2 \\
0 & 0 & 0 & \ldots & -\lambda \\
0 & 0 & 0 & \ldots & 1
\end{bmatrix}.
\end{aligned} \tag{14}
$$

The prediction values can be shown in the following formula.

When

$$\begin{aligned}
k = 1,\quad & \hat{x}^{(0)}(1) = \hat{x}^{(1)}(1),\\
k = 2,\quad & \hat{x}^{(0)}(2) = \hat{x}^{(1)}(2) - \lambda\hat{x}^{(0)}(1)\\
& \qquad = \hat{x}^{(1)}(2) - \lambda\hat{x}^{(1)}(1),\\
k = 3,\quad & \hat{x}^{(0)}(3) = \hat{x}^{(1)}(3) - \lambda\hat{x}^{(0)}(2)\\
& \qquad = \hat{x}^{(1)}(3) - \lambda\left[\hat{x}^{(1)}(2) - \lambda\hat{x}^{(1)}(1)\right]\\
& \qquad = \hat{x}^{(1)}(3) - \lambda\hat{x}^{(1)}(2) + \lambda^2\hat{x}^{(1)}(1).
\end{aligned} \tag{15}$$

Without loss of generality,

$$\begin{aligned}
k = 4,\quad & \hat{x}^{(0)}(4) = \hat{x}^{(1)}(4) - \lambda\hat{x}^{(0)}(2)\\
& = \hat{x}^{(1)}(4) - \lambda\left[\hat{x}^{(1)}(3) - \lambda\hat{x}^{(1)}(2) + \lambda^2\hat{x}^{(1)}(1)\right]\\
& = \hat{x}^{(1)}(4) - \lambda\hat{x}^{(1)}(3) + -\lambda^2\hat{x}^{(1)}(2) - \lambda^3\hat{x}^{(1)}(1).
\end{aligned} \tag{16}$$

If n is an even number, then

$$\begin{aligned}
\hat{x}^{(0)}(n-1) &= \hat{x}^{(1)}(n-1) - \lambda\hat{x}^{(0)}(n-2)\\
&= \hat{x}^{(1)}(n-1) - \lambda\hat{x}^{(0)}(n-2) + \cdots\\
&\quad + \lambda^{(n-3)}\hat{x}^{(1)}(2) - \lambda^{(n-2)}\hat{x}^{(1)}(1),\\
\hat{x}^{(0)}(n) &= \hat{x}^{(1)}(n) - \lambda\hat{x}^{(1)}(n-1) + \cdots\\
&\quad + \lambda^{(n-2)}\hat{x}^{(1)}(2) - \lambda^{(n-1)}\hat{x}^{(1)}(1).
\end{aligned} \tag{17}$$

If n is an odd number, then

$$\begin{aligned}
\hat{x}^{(0)}(n-1) &= \hat{x}^{(1)}(n-1) - \lambda\hat{x}^{(1)}(n-2)\\
&= \hat{x}^{(1)}(n-1) - \lambda\hat{x}^{(1)}(n-2) + \cdots\\
&\quad + \lambda^{(n-3)}\hat{x}^{(1)}(2) - \lambda^{(n-2)}\hat{x}^{(1)}(1),\\
\hat{x}^{(0)}(n) &= \hat{x}^{(1)}(n) - \lambda\hat{x}^{(1)}(n-1)\\
&\quad + \cdots - \lambda^{(n-2)}\hat{x}^{(1)}(2) - \lambda^{(n-1)}\hat{x}^{(1)}(1).
\end{aligned} \tag{18}$$

When the prediction of a single model is completed, CFES and AAES are hybridized with the variation coefficient to determine the weight. The hybrid formula is shown in equations (19)(24). Suppose there are m single prediction models, and each original sequence has n data; then, the matrix formed by the sequence values $x_{ij}^{(0)}$ of a single prediction model is $X_{ij}^{(0)}$:

$$\mathrm{X}_{ij}^{(0)} = \begin{bmatrix} x_{11}^{(0)} & \cdots & x_{1n}^{(0)} \\ \vdots & \ddots & \vdots \\ x_{m1}^{(0)} & \cdots & x_{mn}^{(0)} \end{bmatrix}, \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \tag{19}$$

The mean of the individual sequence prediction is

$$\overline{x}_i^{(0)} = \frac{1}{n}\sum_{i=1}^{n} x_{ij}^{(0)}. \tag{20}$$

The standard deviation of the single prediction series is

$$S_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(x_{ij}^{(0)} - \overline{x}_i^{(0)}\right)^2}. \tag{21}$$

The variation coefficient of the single method is

$$V_i = \frac{S_i}{\overline{x}_i^{(0)}}, \quad i = 1, 2, \ldots, m. \tag{22}$$

We can obtain the weight of the hybrid model is

$$w_i = \frac{V_i}{\sum_{i=1}^{m} V_i}. \tag{23}$$

We can hybridize the individual forecasting sequences from the obtained weights:

$$Y = \sum_{i=1}^{m} w_i y_j. \tag{24}$$

In this paper, CFES and AAES are used to compare with ES, respectively. Then, the two models are hybridized with the variation coefficient to compare with ES again to prove that the cumulative grey double exponential smoothing has a better prediction effect. This article uses MAPE as the prediction criterion, as shown in following equation:

$$\mathrm{MAPE} = \frac{1}{n}\sum_{k=1}^{n}\frac{\left|x^{(0)}(k) - \hat{x}^{(0)}(k)\right|}{x^{(0)}(k)}. \tag{25}$$

3. Application of Grey Double Exponential Smoothing in Air Quality Prediction

This part will use data to verify the validity of CFES, AAES, and hybrid model, respectively. The data in this article come from https://www.aqistudy.cn/historydata/.

3.1. Application of CFES in Air Quality Prediction.

The whale algorithm is proposed by Mirjalili and Lewis [29]. The algorithm mimics the social behavior of humpback whales, and it is inspired by the bubble-net hunting strategy. The whale algorithm is used to search for the smallest fitting error in this paper. The original sequence is $X^{(0)} = \{51, 78, 90, 83, 72, 80, 84, 94, 133, 109\}$. The conformable fractional order is $r = 0.5563$, and the smoothing coefficient $\alpha = 0.5032$. Then, $X^{(0.5563)} = \{51, 108.35, 163.63, 208.50, 243.75, 279.88, 315.31, 352.67, 402.85, 442.09\}$. For convenience of calculation, the first value of the original sequence is used as the original value of the single exponential smoothing value and the double exponential smoothing value, respectively. Equations (2) and (3) are used to calculate $S^{(1)}(k)$ and $S^{(2)}(k)$. The results are shown as follows:

$$\begin{aligned}
S^{(1)}(k) = &\{51, 79.86, 122.02, 165.54, 204.90, 242.64, 279.21,\\
& 316.18, 359.79, 401.21\},\\
S^{(2)}(k) = &\{51, 65.52, 93.95, 129.98, 167.68, 205.40, 242.55,\\
& 279.60, 319.96, 360.85\}.
\end{aligned} \tag{26}$$

We can obtain a_k and b_k by equations (4) and (5):

$$\begin{aligned} a_k &= \{51, 94.20, 150.08, 201.10, 242.12, 279.87, \\ &\quad 315.87, 352.76, 399.63, 441.57\}, \\ b_k &= \{0, 14.52, 28.43, 36.03, 37.70, 37.72, 37.14, \\ &\quad 37.06, 40.36, 40.89\}. \end{aligned} \tag{27}$$

So $a_n = 441.57$ and $b_n = 40.89x^{(0.5563)}(k) = \{51, 108.72, 178.51, 237.12, 279.82, 317.59, 353.01, 389.81, 439.99, 482.46, 523.35, 564.24, 605.13\}$ The data of December 1–10, 2019 were used to predict the AQI concentration value of December 11–13. The final prediction is

$$\begin{aligned} x^{(0)}(k) = \{&51, 78.51, 113.62, 108.42, 87.20, 83.62, \\ &84.00, 92.58, 133.00, 117.97, 118.48, 123.14, 127.60\}. \end{aligned} \tag{28}$$

The prediction results between ES and CFES are shown in Table 1. In Table 1, in order to facilitate the comparison of the prediction accuracy of the two models, the smoothing coefficient between ES and CFES takes the same value $\alpha = 0.5032$. As can be seen in Table 1, the prediction error of ES is 18.82%, and the prediction error of CFES is 8.08%. It can be seen that CFES makes the prediction error significantly lower.

3.2. Application of AAES in Air Quality Prediction. The original sequence is $X^{(0)} = \{51, 78, 90, 83, 72, 80, 84, 94, 133, 109\}$. The whale algorithm is used to search for the smallest fitting error. The adjacent accumulation is $\lambda = -0.3863$, and the smoothing coefficient $\alpha = 0.4995$. The sequence $X^{(1)} = \{51, 58.30, 59.87, 48.23, 39.94, 52.19, 53.10, 61.55, 96.69, 57.62\}$ is obtained by adjacent accumulation. We can get $S^{(1)}(k)$ and $S^{(2)}(k)$ by equations (2) and (3):

$$\begin{aligned} S^{(1)} &= \{51, 54.65, 57.25, 52.75, 46.35, 49.26, 51.18, 56.36, 76.50, 67.07\}, \\ S^{(2)} &= \{51, 52.82, 55.04, 53.89, 50.12, 49.70, 50.44, 53.39, 64.94, 66.00\}, \end{aligned} \tag{29}$$

where a_k and b_k can be obtained by equations (4) and (5):

$$\begin{aligned} a_k &= \{51, 56.47, 59.47, 51.60, 42.57, 48.83, 51.92, 59.32, 88.07, 68.14\}, \\ b_k &= \{0, 1.82, 2.21, -1.14, -3.77, -0.43, 0.74, 2.96, 11.54, 1.07\}. \end{aligned} \tag{30}$$

So $a_n = 68.14$ and $b_n = 1.07$.

$$\begin{aligned} x^{(1)}(k) = \{&51, 58.29, 61.69, 50.46, 38.81, 48.40, 52.66, \\ &62.28, 99.61, 69.21, 70.27, 71.34, 72.41\}. \end{aligned} \tag{31}$$

The final prediction is

$$\begin{aligned} \hat{x}^{(0)}(k) = \{&51, 77.99, 91.82, 85.93, 72.00, 76.22, 82.10, \\ &94.00, 135.92, 121.71, 117.29, 116.65, 117.47\}. \end{aligned} \tag{32}$$

Table 1: Comparison of CFES and ES prediction results.

Date	Actual value	ES	CFES
11-Dec	102	128.95	118.48
12-Dec	118	135.39	123.14
13-Dec	123	141.83	127.60
MAPE		18.82%	8.08%

As can be seen in Figure 1, the fitted value and predicted value of the AAES are closer to the actual value. The fitting error and forecasting error of AAES are 2.64% and 6.88%, respectively. The fitting error and forecasting error of ES are 4.94% and 18.93%. It can be seen that AAES has a higher prediction accuracy than ES.

Forecasting AQI in Nanchong demonstrated that the grey double exponential smoothing with conformable fractional accumulation and adjacent accumulation can improve the prediction effect. It is proved that the grey double exponential smoothing with accumulation mode is meaningful. In order to obtain a more accurate prediction effect, a hybrid prediction model is proposed.

3.3. Application of Hybrid Model in Air Quality Prediction

Case 1. Forecasting AQI, $PM_{2.5}$, and PM_{10} in Chongqing.

Compared with ES, the grey exponential smoothing with the accumulation operator has improved the prediction accuracy to a certain extent, but individual prediction values sometimes have large errors. The conformable fractional accumulation is the principle of old information first, and adjacent accumulation is the principle of new information first. In order to make the model more applicable, a hybrid model is used to balance new information with old information to make up for the shortcomings of a single model.

Table 2 shows the original data of Chongqing AQI, $PM_{2.5}$, and PM_{10} in December 2019. Chongqing is foggy and is known as “fog Chongqing.” The annual average foggy day in Chongqing is 104 days. The average annual foggy day in London, England, which is known as the world ’s foggy city, is only 94 days. Therefore, it is necessary to study Chongqing air quality.

This section uses AQI as an example to elaborate the model. In the ES model, the fitting error is minimal when $\alpha = 0.4$.

We can obtain

$$\begin{aligned} S^{(1)} &= \{46, 54.40, 62.64, 75.18, 78.71, 77.23, 77.14, \\ &\quad 89.08, 105.05, 113.83\}, \\ S^{(2)} &= \{46, 49.36, 54.67, 62.88, 69.21, 72.42, \\ &\quad 74.30, 80.22, 90.15, 99.62\}, \end{aligned} \tag{33}$$

by

$$\begin{aligned} S^{(1)}(k) &= 0.4 \times X^{(0)}(k) + 0.6 \times S^{(1)}(k-1), \\ S^{(2)}(k) &= 0.4 \times S^{(1)}(k) + 0.6 \times S^{(2)}(k-1). \end{aligned} \tag{34}$$

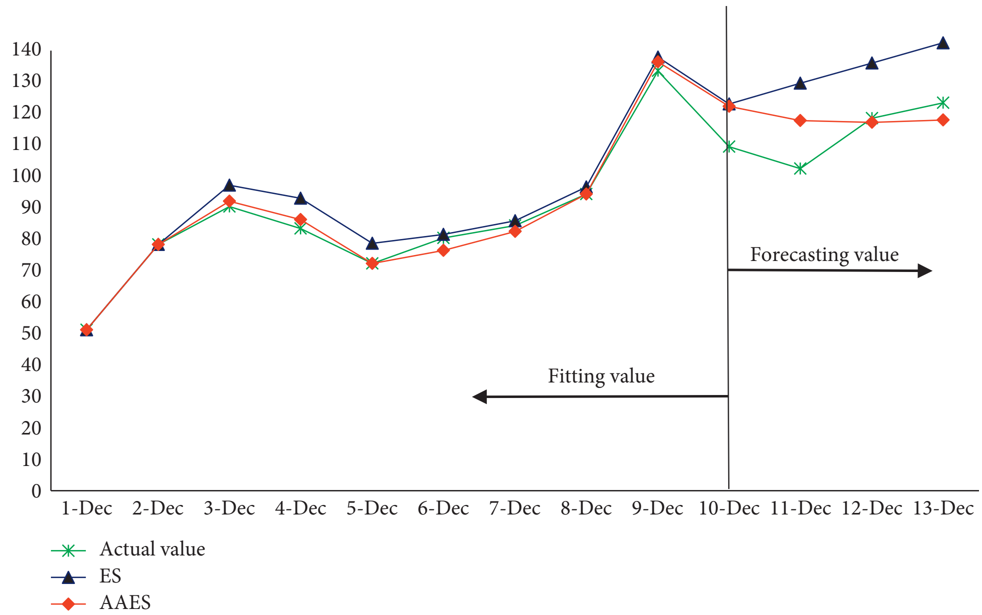

FIGURE 1: Comparison of AAES and ES forecasting results.

TABLE 2: The original data of Chongqing AQI, $PM_{2.5}$, and PM_{10} in December 2019.

Date	AQI	$PM_{2.5}$	PM_{10}
1-Dec	46	32	46
2-Dec	67	46	75
3-Dec	75	55	86
4-Dec	94	70	105
5-Dec	84	62	88
6-Dec	75	55	84
7-Dec	77	56	84
8-Dec	107	80	121
9-Dec	129	98	138
10-Dec	127	96	137
11-Dec	135	103	142
12-Dec	143	109	146
13-Dec	145	111	156

We can obtain

$$\begin{aligned} a_k &= \{46, 59.44, 70.61, 87.49, 88.21, 82.04, \\ &\quad 79.97, 97.95, 119.95, 128.04\}, \\ b_k &= \{0, 3.36, 5.31, 8.20, 6.33, 3.21, 1.89, 5.91, 9.93, 9.47\}, \end{aligned} \tag{35}$$

by

$$\begin{aligned} a_k &= 2S^{(1)}(k) - S^{(2)}(k), \\ b_k &= 0.67 \times \left(S^{(1)} - S^{(2)}\right). \end{aligned} \tag{36}$$

So $a_n = 128.04$ and $b_n = 9.47$.

The forecasting result of ES can be obtained as follows:

$$\begin{aligned} \hat{x}^{(0)}(k) = \{&46, 62.80, 75.92, 95.70, 94.54, 85.24, 81.85, \\ &103.86, 129.88, 137.51, 146.98, 156.45, 165.93\}. \end{aligned} \tag{37}$$

When the conformable fractional grey double exponential smoothing takes $r = 0.5385$ and $\alpha = 0.5$,

$$\begin{aligned} \hat{x}^{(0.5385)}(k) = \{&46, 67, 95.20, 115.41, 103.66, 84.44, 77.00, \\ &103.36, 131.79, 135.14, 124.60, 129.70, 134.58\}. \end{aligned} \tag{38}$$

When the adjacent accumulation grey double exponential smoothing takes $\lambda = -0.4540$ and $\alpha = 0.5122$,

$$\begin{aligned} \hat{x}^{(1)}(k) = \{&46, 67, 74.99, 94.00, 87.20, 75.04, 73.96, \\ &105.14, 135.54, 138.98, 144.96, 152.08, 159.72\}, \end{aligned} \tag{39}$$

where $\hat{x}^{(0.5385)}(k)$ obtained by CFES and $\hat{x}^{(1)}(k)$ obtained by AAES are combined using the variation coefficient. A matrix $X_{ij}^{(0)}$ composed of the sequence values $x_{ij}^{(0)}$ of a single prediction model obtained by two accumulation methods is

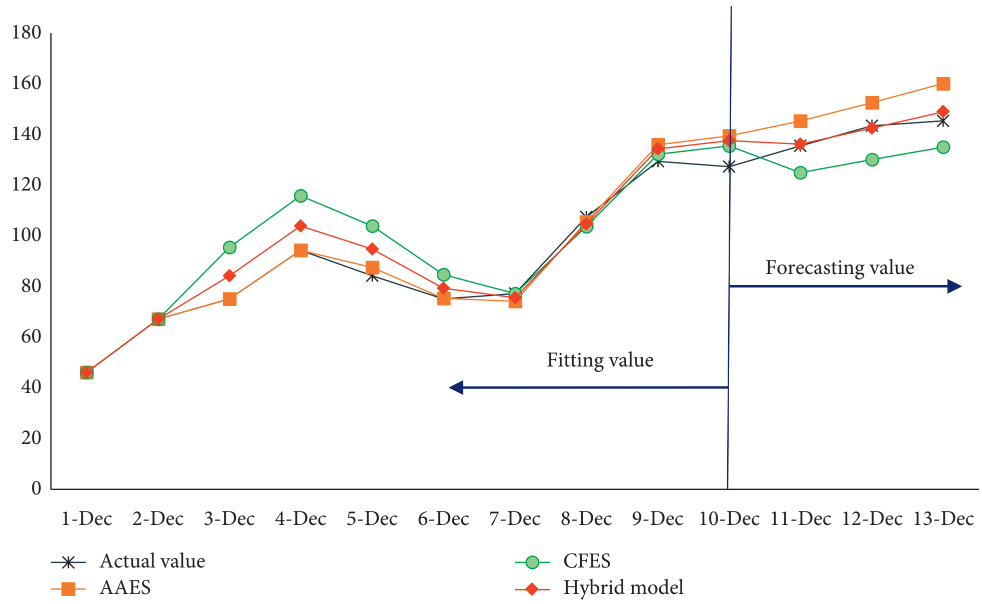

Figure 2: Prediction results of AQI concentration in four models.

Table 3: The forecasting results of $PM_{2.5}$ concentration.

Date Parameter	CFES $\alpha = 0.5$, $r = 0.5216$	AAES $\alpha = 0.5041$, $\lambda = -0.4905$	Hybrid model $w_c = 0.45, w_A = 0.55$	ES $\alpha = 0.4$
1-Dec	32	32	32	32
2-Dec	46.00	45.99	45.99	43.20
3-Dec	68.96	54.58	61.03	54.88
4-Dec	85.78	70.00	77.07	71.10
5-Dec	77.01	64.78	70.27	70.37
6-Dec	62.26	55.27	58.40	63.16
7-Dec	56.00	53.64	54.70	60.06
8-Dec	77.06	77.93	77.54	70.50
9-Dec	100.27	102.52	101.51	98.57
10-Dec	102.71	105.38	104.18	104.47
MAPE	9.83%	2.69%	5.81%	5.60%
11-Dec	93.85	109.91	102.71	112.01
12-Dec	97.84	115.27	107.45	119.56
13-Dec	101.66	121.03	112.34	127.10
MAPE	9.18%	7.16%	0.97%	10.98%

$$X_{ij}^{(0)} = \begin{bmatrix} 46 & 67 & 74.99 & 94.00 & 87.20 & 75.04 & 73.96 & 105.14 & 135.54 & 138.98 & 144.96 & 152.08 & 159.72 \\ 46 & 67 & 95.20 & 115.41 & 103.66 & 84.44 & 77.00 & 103.36 & 131.79 & 135.14 & 124.60 & 129.70 & 134.58 \end{bmatrix}. \tag{40}$$

We can get $\overline{x}_{ij}^{(0)} = \begin{bmatrix} 104.20 \\ 103.68 \end{bmatrix}$, $S_i = \begin{bmatrix} 32.80 \\ 26.15 \end{bmatrix}$, $V_i = \begin{bmatrix} 0.31 \\ 0.25 \end{bmatrix}$, and $w_i = \begin{bmatrix} 0.56 \\ 0.44 \end{bmatrix}$ by equations (11)(14).

So the final fitting values and forecasting values can be obtained by w_i:

$$Y = \{46 \quad 67 \quad 83.98 \quad 103.53 \quad 94.52 \quad 79.22 \quad 75.31 \quad 104.35 \quad 133.87 \quad 137.27 \quad 135.90 \quad 142.12 \quad 148.83\}. \tag{41}$$

As can be seen from Figure 2, the fitting error of CFES is large and the deviation from the true value is higher, but the fitting of the AAES model to the actual value is higher. After combining the two models of CFES and AAES, the curve fitting effect of the hybrid model is significantly better than the CFES model. Judging from the prediction results, the

Table 4: The forecasting results of PM_{10} concentration.

Date	CFES	AAES	Hybrid model	ES
Parameter	$\alpha = 0.5000$, $r = 0.5488$	$\alpha = 0.4588$, $\lambda = -0.5153$	$w_c = 0.46$, $w_A = 0.54$	$\alpha = 0.5$
1-Dec	46	46	46	46
2-Dec	75.00	74.56	74.76	75.00
3-Dec	108.51	87.18	96.90	93.25
4-Dec	129.48	105.00	116.15	115.00
5-Dec	109.94	93.05	100.74	100.94
6-Dec	92.84	84.00	88.03	90.19
7-Dec	84.00	80.93	82.33	85.95
8-Dec	116.88	114.41	115.53	121.41
9-Dec	142.03	141.09	141.52	147.17
10-Dec	145.09	147.28	146.28	150.32
MAPE	9.72%	2.65%	5.87%	5.90%
11-Dec	135.48	154.40	145.78	161.09
12-Dec	140.90	162.00	152.39	171.87
13-Dec	146.08	169.84	159.02	182.64
MAPE	4.82%	9.52%	2.99%	16.08%

Table 5: The forecasting results of AQI concentration.

Date	Actual value	CFES	AAES	Hybrid model
Parameter	—	$\alpha = 0.5174$, $r = 0.4383$	$\alpha = 0.4605$, $\lambda = 0.1814$	$w_c = 0.3943$, $w_A = 0.6057$
1-Dec	63	63	63	63
2-Dec	44	45.53	44.60	44.97
3-Dec	56	70.75	53.66	60.40
4-Dec	58	74.14	58.00	64.36
5-Dec	54	64.60	55.69	59.21
6-Dec	58	61.53	58.33	59.59
7-Dec	57	57.05	58.11	57.69
8-Dec	67	64.71	67.02	66.11
9-Dec	100	100.00	100.00	100.00
10-Dec	108	115.69	116.97	116.46
MAPE	—	9.40%	1.95%	4.38%
11-Dec	111	97.46	126.93	115.31
12-Dec	112	102.35	138.16	124.04
13-Dec	118	107.05	149.16	132.56
MAPE	—	10.03%	21.37%	8.99%

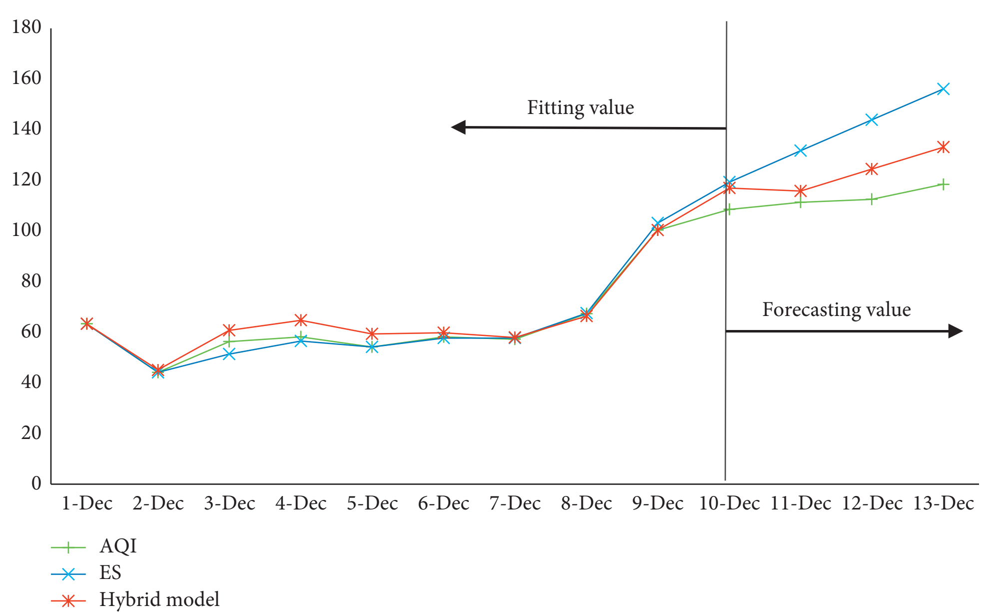

Figure 3: Compared forecasting results between hybrid model and ES.

Table 6: The forecasting results of $PM_{2.5}$ concentration.

Date	Actual value	CFES	AAES	Hybrid model
Parameter	—	$\alpha = 0.5029$, $r = 0.3760$	$\alpha = 0.4573$, $\lambda = 0.4571$	$w_c = 0.4057$, $w_A = 0.5943$
1-Dec	35	35	35	35
2-Dec	19	19.11	19.00	19.04
3-Dec	29	35.21	28.77	31.38
4-Dec	32	40.67	32.00	35.52
5-Dec	26	32.97	28.58	30.36
6-Dec	30	32.03	30.18	30.93
7-Dec	34	34.33	34.51	34.44
8-Dec	33	33.00	34.65	33.98
9-Dec	51	49.94	50.86	50.49
10-Dec	66	69.03	69.36	69.23
MAPE	—	9.03%	2.32%	4.95%
11-Dec	66	53.80	70.86	63.94
12-Dec	67	56.80	80.13	70.67
13-Dec	78	59.71	85.85	75.25
MAPE	—	19.05%	12.34%	4.04%

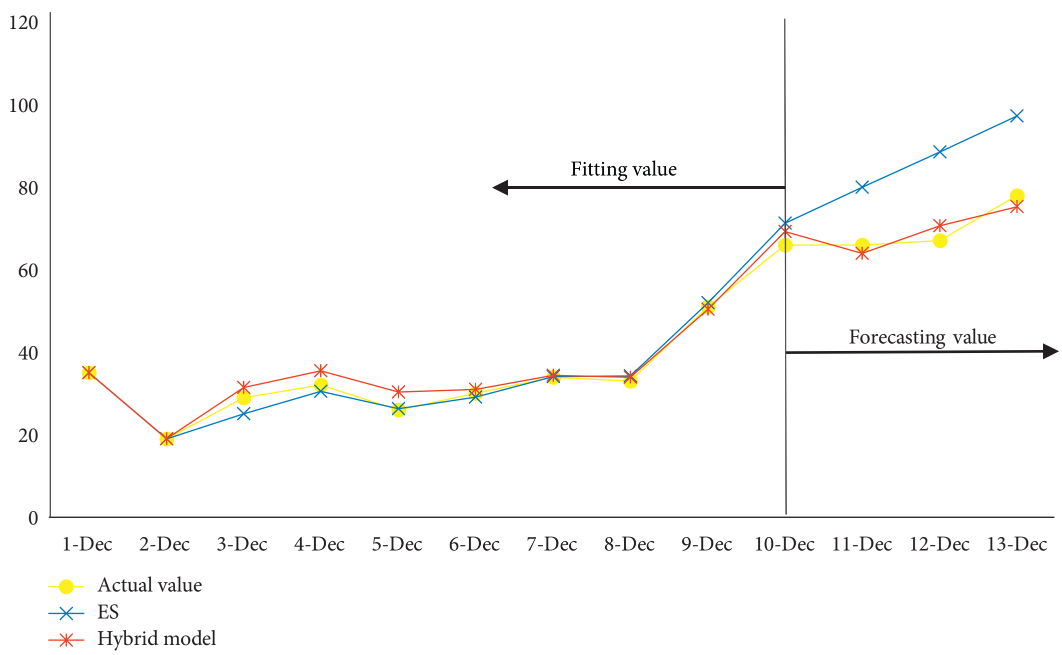

Figure 4: Compared forecasting results between hybrid model and ES in $PM_{2.5}$.

predicted value polyline of the combined model is closer to the actual value polyline, so it can explain the superiority of the hybrid model compared to the single model.

Table 3 is the $PM_{2.5}$ concentration values predicted by ES, CFES, AAES, and hybrid model. The smoothing coefficients and parameters of CFES and AAES are obtained by using the whale algorithm in Matlab R2014b. The prediction error obtained by the hybrid model in Table 3 is 0.97%, which is far lower than the traditional double exponential smoothing prediction error of 10.98%, so the hybrid model has better prediction performance.

Table 4 shows the predicted concentrations of PM_{10} under the four models. In $PM_{2.5}$ concentration prediction, the single prediction error of CFES is relatively large. In PM_{10} concentration prediction, the single prediction error of AAES is relatively large. The hybrid model makes up for the lack of single errors, integrates ES new information and old information first, and improves prediction accuracy. Comparing the hybrid model with ES, the prediction error of ES is significantly higher than the prediction error of the hybrid model, so the hybrid model has more advantages.

The above content uses Chongqing as an example to explain the grey double exponential smoothing. The following content further verifies the model with Foshan, and the calculation process is the same, so it will not be repeated.

Case 2. Forecasting AQI, $PM_{2.5}$, and PM_{10} in Foshan

Foshan is located in the central and southern part of Guangdong Province, China. It is an important part of the Pearl River Delta economic circle. While vigorously

Table 7: The forecasting results of PM_{10} concentration.

Date	Actual value	CFES	AAES	Hybrid model
Parameter	—	$\alpha = 0.5000$, $r = 0.5837$	$\alpha = 0.5037$, $\lambda = 0.2386$	$w_c = 0.3632$, $w_A = 0.6368$
1-Dec	75	75	75	75
2-Dec	44	44.00	43.90	43.94
3-Dec	61	74.02	57.77	63.67
4-Dec	66	83.19	66.00	72.24
5-Dec	58	72.08	60.22	64.52
6-Dec	65	71.21	65.22	67.40
7-Dec	63	66.22	64.40	65.06
8-Dec	60	59.99	60.81	60.51
9-Dec	97	94.36	97.00	96.04
10-Dec	120	126.12	129.08	128.01
MAPE	—	9.42%	2.08%	4.07%
11-Dec	122	95.41	138.30	122.73
12-Dec	126	98.93	152.98	133.35
13-Dec	154	102.28	166.35	143.08
MAPE	—	25.62%	14.26%	4.51%

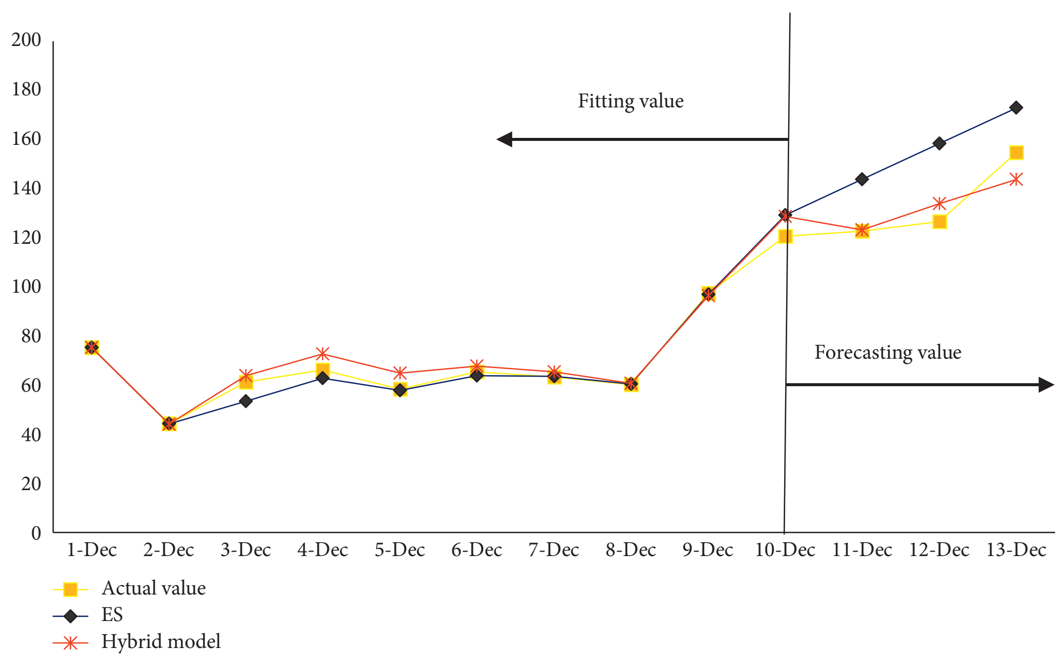

Figure 5: Compared forecasting results between hybrid model and ES in PM_{10}.

developing the economy, the country is also actively addressing air pollution. Compared with the air quality in December 2013, air quality has improved significantly in 2019. With the increasing pursuit in a better life, a high-quality atmospheric environment has received more attention. This section uses Foshan as an example to predict the three air quality indicators of AQI, $PM_{2.5}$, and PM_{10}.

Table 5 shows the prediction results of the AQI in Foshan City. The optimal parameters of CFES $\alpha = 0.5174$ and $r = 0.4383$ and the optimal parameters of AAES $\alpha = 0.4605$ and $\lambda = 0.1814$ are obtained from the whale algorithm search. The two sets of prediction results obtained based on the optimal parameters were used to make a fixed weight combination prediction using the variation coefficient. The CFES weight was 0.3943, and the AAES weight was 0.6057. It can be seen from Table 3 that the prediction error of the single model has exceeded 10% and is no longer suitable for single prediction, but the prediction error of CFES is only more than 0.03% of the MAPE standard, so a hybrid prediction can be considered. The hybrid fitting error is 4.38%, and the prediction error is 8.99%. Therefore, compared with the single model, the hybrid model can improve the prediction accuracy.

When using ES to predict AQI, the smoothing coefficient is determined to be 0.5 based on the minimum fitting error. The comparison between ES and the hybrid prediction results is shown in Figure 3. The fitting error of ES is 2.66%, the fitting error of the hybrid model is 4.38%, and the fitting error of the two models is far below 10%, while the prediction error of the two models is quite different, and the fitting error of ES is 25.92%, more than 10%, so it can be said that the combination model is meaningful.

It can be seen in Table 6 that the prediction error of CFES is 19.05%, the prediction error of AAES is 12.34%, the hybrid prediction error is 4.04%, and the prediction accuracy has been greatly improved. Compare the hybrid prediction result with the ES. As shown in Figure 4, the predicted polyline of the hybrid model is closer to the actual value of $PM_{2.5}$.

Table 7 is similar to Table 6, and Figure 5 is similar to Figure 4. The hybrid model is superior to the single model. In general, when the error of the single model is greater than or close to 10%, to obtain a higher fitting accuracy, a hybrid model can be used. In the above charts, the superiority of the hybrid model can be verified. The new information-first principle of adjacent accumulation and the old information-first principle of conformable fractional-order accumulation are fused in this paper to obtain a higher prediction accuracy. Therefore, the proposed hybrid model has certain practical significance.

4. Conclusion

The conformable fractional accumulation proposed by Ma is the old information first principle, and the adjacent accumulation is the new information-first principle. In this paper, the conformable fractional order and adjacent are added to the double exponential smoothing model, respectively. Taking the AQI of Nanchong in December 2019 as an example, two sets of grey exponential smoothing models are used to obtain two sets of sequences. In order to facilitate the comparison of model priorities, the conformable fractional smoothing coefficient is the same as the traditional double exponential smoothing coefficient, proving that CFES is better than ES. Under the condition that adjacent cumulative grey double exponential smoothing and traditional double exponential smoothing have the same smoothing coefficient, AAES is also better than ES. Therefore, it is proved that the exponential smoothing model with accumulation is better than the ES.

CFES and AAES models have their advantages and disadvantages in general. Generally speaking, the prediction accuracy of CFES is better than the fitting accuracy, while the fitting accuracy of AAES is better than its own prediction accuracy. Considering the two situations, in order to get more good results, this paper uses a combination of the variation coefficient of the CFES and AAES to obtain a fixed weighted combination of predicted values. From the case of this article, it can be seen that the hybrid prediction result is significantly better than the prediction result using the single grey double exponential smoothing model. Comparing the hybrid results with the prediction results of ES, it is also verified that the hybrid model is better than ES.

In general, from the perspective of predicting air quality, it is of practical significance to put two new grey accumulation methods of conformable fractional accumulation and adjacent accumulation into the exponential smoothing model and their hybrid model, respectively. In future work, we can combine these two new accumulation methods with Holt–Winters model for further research.

Data Availability

The data that support the findings of this study are openly available at https://www.aqistudy.cn/historydata/. The creator is Jie Wang, air data are available after December 2013, the title is air quality historical data query, and copyright is owned by aqistudy.cn.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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