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Research Article

Observer-Based Leader-Following Consensus of General Linear Multiagent Systems Based on Novel Event Trigger Mechanism with Input Time Delay under Directed Graphs

Hong Zhang, Changshun Chen, and Feng Wei

School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan 430074, China

Correspondence should be addressed to Hong Zhang; sunracer@hust.edu.cn

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This paper considers the tracking and containment consensus for the general linear systems with input time delays under directed communication networks. The distributed observer-based algorithm on the basis of event-triggering mechanism will be designed by using only neighboring agents information. In this way, we can save network resource effectively. The event-based protocol with input time delays will be proposed for the leader-follower systems. Appropriate feedback gain matrices and trigger parameters can be designed by using Lyapunov stability theory. Based on the designed control algorithm, if the feedback gain matrices and the event trigger are designed appropriately, the leader-follower general linear system can eventually reach tracking and containment consensus. Then, two simulation results are provided to demonstrate the practicability of the theoretical analysis.

1. Introduction

In recent years, the multiagent systems have been paid considerable attentions in various scientific communities, such as spacecraft formation flying, complex network control, and collaborative monitoring [1, 2]. Consensus is one of the important issues that is worth studying in the field of multiagent systems' cooperative control. For the consensus issues of multiagent systems, the main work is to design a suitable information exchange method so that all agents can agree on the state of interest. The research studies on multiagent systems' consensus issues mainly focus on communication topology, agent dynamic systems, and control method, which emphasize the specific analysis of the whole system. And some related results have been obtained. Consensus protocols are proposed in [3–6] for a set of single, double integrators, general linear, and Lipschitz nonlinear systems with undirected and directed network topologies.

The earlier related research studies mainly focus on the leaderless consensus issues, which means the final state of the agents will be not set in advance. However, there might be one or even more leaders in the agent network to realize that the agents reach the state we expected in some practical applications. The leader-follower tracking control issue is an extremely important part of the multiagent systems' consensus issues, which needs the states of all followers follow that of the leader. And in this way, consensus can be achieved more quickly and efficiently. By only using the neighboring information, the tracking control algorithm for the leader-follower single-integrator system with a single leader is designed in [7, 8]. Considering the actual situation, consensus-tracking control strategies are put forward in [9–12] for the leader-follower systems with time delays, noises, external disturbances, and so forth. Under the designed algorithms, control goals can be reached; that is to say, the leader-follower systems can achieve consensus. Considering the situation of multiple leaders, as a special branch of multiagent systems' control methods, containment has been widely used. The main goal of containment control is to ensure that the states of all followers can eventually converge to the convex hull spanned by that of the leaders. Up to now, there are many related outstanding achievements. The distributed control strategy is investigated to solve the containment consensus issues for not only discrete-time but also continuous-time general linear systems in [13]. The leader-follower systems can achieve containment consensus by this means. In [14], the authors study the distributed containment control issues and design the corresponding control algorithm for the general linear system which can make the leader-follower system reach containment consensus. They designed distributed output-feedback controllers based on the relative information of neighboring agents for the leader-follower systems. The authors in [15] proposed a stochastic sampling control algorithm to solve the issue of formation-containment for the leader-follower linear system. Under this control algorithm, the update frequency of the controller and energy can be reduced. The containment control protocol for heterogeneous linear systems on the basis of neighboring agents' relative information is designed in [16], which will guarantee that the novel system can reach containment consensus. The authors in [17] further extended the containment control method of general linear systems to directed random networks. The fully distributed control algorithm by using the formation-containment control method for heterogeneous linear systems is proposed in [18]. The formation-containment consensus issue is studied by an observer with directed graph. However, these mentioned references do not focus on the research on the update frequency of the controller.

Furthermore, the updates frequency of the controllers can be significantly reduced by using the event-based control method, which may save communication resources of the communication networks. The main idea of the event-triggered control method is that it depends on the predefined trigger condition to update the controller. In order to solve the problem of event-triggered consensus, we need to design the distributed event-based control protocols, which include the event-based control strategies and the triggering functions. In the past decade, the event-based consensus problem has been studied a lot. In [19], a new distributed sampled-data control protocol is designed for fractional-order multiagent systems under directed graph. Event-triggered and self-triggered control protocols are designed for single-integrator agents systems with undirected communication topologies in [20–22]. The event-triggered problems with general linear systems are studied in [23–26]. In [27], the work is extended to the leader-follower systems with event-triggered consensus issues. The distributed control algorithm by using the event-triggered method is designed to let the leader-follower system reach consensus. An adaptive distributed observer-based control strategy by using the event-triggered method is designed in [28] to achieve the predicted control target. The authors in [29] proposed distributed adaptive event-triggered algorithms for both the leaderless and leader-follower linear systems on the basis of the local sampled state information. In [30], the authors designed an event-triggered consensus algorithm for the second-order hybrid systems with continuous-time and discrete-time individuals. In practice, there are many constraints that affect the analysis of the multiagent systems' consistency issues, such as time delays, noises, and external disturbance. Furthermore, time delays are considered into the multiagent systems' consensus issues. In [31], the event-based control law of the leader-follower linear system with input time delay is designed. By algebraic Riccati equation-based method and low-gain output-feedback mechanism, the observer-based protocol is designed to solve the issue of edge-consensus in [32]. The distributed observer-based tracking and containment control algorithms by using the event-triggered control method are, respectively, proposed for general linear systems with time delays in [33, 34]. In [35], a novel distributed tracking control algorithm with input time delays is designed for leader-follower linear systems. A new event trigger is designed to let the multiagent networks reach tracking consensus as well as save network energy. As far as we know, the containment consensus issues with input time delays by using the distributed event-triggered control approach have not been adequately studied, which inspires the current work.

In this paper, we will consider the tracking and containment consensus issues for the leader-follower general linear systems with time delays under directed graph. And the observer-based event-triggered controller will be designed to satisfy control requirements. The main contributions of this work can be summarized as follows. First of all, two fully distributed observer-based control protocols by using the event-triggered method are proposed for both tracking control and containment control problems of the leader-follower linear systems with time delays. Then, appropriate feedback gain matrix and trigger parameters can be obtained by Lyapunov stability theory and matrix analysis. At last, under the proposed protocol, it is shown that the leader-follower linear systems with time delays can eventually achieve tracking and containment consensus.

The remainder of this paper is organized as follows. In Section 2, we give some basic notations and graph theory to be used as well as state the problem to be solved. By using some assumptions and lemmas, the main results are given in Section 3, which contains tracking control and containment control individuals. Section 4 shows the simulation examples to confirm our theoretical analysis. At last, conclusions are drawn in Section 5.

Notations: $R^{n\times n}$ represents the set of $n\times n$ order real matrices. A^T means the transposed matrix of the real matrix A. I_N denotes the identity matrix of order N. $A\otimes B$ denotes the Kronecker product of matrices A and B. λ_m means the maximum eigenvalue of a real matrix. $\text{conv}\{x_1, x_2, \ldots, x_N\} = \left\{\sum_{i=1}^N \alpha_i x_i | \alpha_i \in R, \alpha_i \geq 0, \sum_{i=1}^N \alpha_i = 1\right\}$ represents the convex hull of the set $x_1, x_2, \ldots, x_N$. $\text{dis}\{x, U\} = \inf_{y\in U}\|x-y\|$ represents the Euclidean distance between the point $x \in R$ and the set $U \subseteq R$.

2. Preliminaries and Problem Formulation

2.1. Graph Theory. In this paper, the communication of N agents with each other can be regarded as a network topology, which is represented as directed graph G. The directed graph G can be denoted by $G = (\nu, \varepsilon)$, where $\nu = 1, 2, \ldots, N$ represents the set of agents and $\varepsilon \in \nu \times \nu$ represents the set of edges. The weighted adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ is described as $a_{ii} = 0, a_{ij} = 1$ if $(j, i) \in \varepsilon$, and $a_{ij} = 0$ while others. The Laplacian matrix of

$GL = [L_{ij}] \in R^{n\times n}$ is described as $L_{ii} = \sum_{j\neq i} a_{ij}$ and $L_{ij} = -a_{ij}$ if $i \neq j$.

2.2. Problem Formulation. Consider a leader-follower general linear system with a single leader and N followers. For a leader-follower system, the follower can receive information and the leader cannot receive information. The state update of the agents depends on the communication network. The communication network among agents can be regarded as a directed graph $G = (\nu, \varepsilon)$. Each node represents an agent, and each edge represents the communication between neighboring agents. The existence of an edge between the two agents indicates that there is information exchange between them.

With loss of generality, we suppose that 0 represents the leader and others represent the followers. The expression of the followers' dynamics can be described as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \\ y_i(t) &= Cx_i(t), \quad t \geq 0, i \in \nu, \end{aligned} \tag{1}$$

where $x_i(t) \in R^n, u_i(t) \in R^r$, and $y_i(t) \in R^m$, respectively, represent the state, control input, and output of the followers. The expression of the leader' dynamic can be described as follows:

$$\dot{x}_0(t) = Ax_0(t), \quad t \geq 0, \tag{2}$$

where $x_0(t) \in R^n$ means the state of the leader.

Definition 1. If the states of each followers satisfy

$$\lim_{x\longrightarrow\infty} \left\| x_i(t) - x_0(t) \right\| = 0, \quad i = 1, 2, \ldots, N, \tag{3}$$

then we think the leader-follower multiagent systems reach tracking consensus.

Definition 2. If the states of each followers satisfy

$$\lim_{t\longrightarrow\infty} \operatorname{dis}\left\{x_i(t), \operatorname{conv}\left\{x_j(t) | j \in R\right\}\right\} = 0, \quad i \in F, \tag{4}$$

in other words, the multiagent system control goal has been achieved, then we think the leader-follower systems reach containment consensus.

In practice, the full state information of the agent maybe unavailable due to the physical limitation, and we can consider the following state observer to solve this problem:

$$\begin{aligned} \dot{\overline{x}}_i(t) &= A\overline{x}_i(t) + Bu_i(t) + F\left(\overline{y}_i(t) - y_i(t)\right), \\ \overline{y}_i(t) &= C\overline{x}_i(t), \quad t \geq 0, i \in \nu, \end{aligned} \tag{5}$$

where $\overline{x}_i(t) \in R^n$ and $\overline{y}_i(t) \in R^m$ are the state and output of the observers and $F \in R^{n\times m}$ is the feedback gain matrix which is derived in the following section. If the leader-follower multiagent system wants to reach consensus, some assumptions are indispensable.

Assumption 1. For the agent system we used, the system matrix pairs need to satisfy that (A, C) is observable and (A, B) is controllable.

Assumption 2. There is at least one leader with a directional path to each follower in the multiagent system network.

Generally speaking, the communication among agents is through the network, which will cause transmission delays and should not be ignored. In addition, the update of relative state is continuous, but it is unnecessary in actual situation; in order to save network energy, we can update the control input by some events, that is, event-triggered control method. Therefore, we can consider the distributed tracking control algorithm with input time delays as follows:

$$u_i(t) = -Kq_i\left(t_k^i - \tau\right), \quad t \in \left[t_k^i, t_{k+1}^i\right], \tag{6}$$

where $\tau \geq 0$ is the input time delay of the network, $q_i(t) = \sum_{j=1}^{N} a_{ij}(\overline{x}_i(t) - \overline{x}_j(t)) + h(\overline{x}_i(t) - x_0(t))$, h represents the adjacency matrix between the leader and the followers, and $K \in R^{r\times n}$ is the feedback gain matrix to be determined. Let us combine (1) and (5), and the expression of control system can be written as follows:

$$\dot{\overline{x}}_i(t) = A\overline{x}_i(t) + Bu_i(t) + \mathrm{FC}\left(\overline{x}_i(t) - x_i(t)\right), \tag{7}$$

for $t \geq 0, i \in \nu$. Further, combining with (6), we can obtain

$$\dot{\overline{x}}_i(t) = A\overline{x}_i(t) - \mathrm{BK}q_i\left(t_k^i - \tau\right) + \mathrm{FC}\left(\overline{x}_i(t) - x_i(t)\right), \tag{8}$$

for $t \geq 0, i \in \nu$. Then, let $z_i(t) = \overline{x}_i(t) - x_i(t)$ be the state error between the observer and the agent. Let $\varsigma_i(t) = \overline{x}_i(t) - x_0(t)$ be the state error between the leader and follower. Let $e_i(t) = q_i(t_k^i) - q_i(t)$ be the state error between the triggering time and the real time, then the following expression can be obtained:

$$\begin{aligned} \dot{\varsigma}_i(t) &= A\varsigma_i(t) - \mathrm{BK}e_i(t-\tau) - \mathrm{BK}q_i(t-\tau) + \mathrm{FC}z_i(t) \\ &= A\varsigma_i(t) - \mathrm{BK}e_i(t-\tau) - \mathrm{BK}\sum_{j=1}^{N} a_{ij}\left(\overline{x}_i(t-\tau) - \overline{x}_j(t-\tau)\right) - \mathrm{BK}h\varsigma_i(t-\tau) + \mathrm{FC}z_i(t) \\ &= A\varsigma_i(t) - \mathrm{BK}e_i(t-\tau) - \mathrm{BK}\sum_{j=1}^{N} a_{ij}\left(\varsigma_i(t-\tau) - \varsigma_j(t-\tau)\right) - \mathrm{BK}h\varsigma_i(t-\tau) + \mathrm{FC}z_i(t). \end{aligned} \tag{9}$$

Let $\eta = L + h$, $\varsigma(t) = [\varsigma_1(t), \varsigma_2(t), \ldots, \varsigma_N(t)]^T$, and $e(t) = [e_1(t), e_2(t), \ldots, e_N(t)]^T$, then we can use the Kronecker product of matrix to obtain the following dynamic:

$$\begin{aligned}\dot{\varsigma}(t) &= (I_N \otimes A)\varsigma(t) - (I_N \otimes \mathrm{BK})e(t-\tau) - (\eta \otimes \mathrm{BK})\varsigma(t-\tau) + (I_N \otimes \mathrm{FC})z(t),\\ \dot{z}(t) &= [I_N \otimes (A + \mathrm{FC})]z(t).\end{aligned} \tag{10}$$

Lemma 1. *From the previous assumption, we can obtain $\eta = L + h$ is a positive definite matrix.*

Lemma 2. *For given orthogonal matrix U and $\eta > 0$, we can obtain that $U^T \eta U = \theta = diag(\nu_1, \nu_2, \ldots, \nu_N)$.*

Lemma 3. *For given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, where S_{11}, S_{12}, and S_{22} are compatible dimension matrices, the following descriptions are equivalent [35]:*

(1) $S < 0$

(2) $S_{22} < 0, S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$

Lemma 4. *From the previous assumption, let $(\alpha_1^T, \ldots, \alpha_N^T)^T = (-L_1^{-1}L_2 \otimes I_M)(\beta_1^T, \ldots, \beta_M^T)^T$, where $\alpha_i \in R^n$ and $\beta_i \in R^n$. Then, we can obtain $\alpha_i \in conv\{\beta_1, \ldots, \beta_M\}$ [34].*

3. Main Result

3.1. Tracking Control. In this section, we will discuss the distributed tracking consensus issue for the leader-follower linear system with input time delay by using the event-triggered method. And sufficient event-triggering function and distributed control algorithm are given to achieve our control goals; that is to say, the tracking consensus can be reached for the leader-follower general linear system under the proposed protocol.

At first, we put forward the following event-triggered function for each follower:

$$t_{k+1}^i = \inf\{t\colon t > t_k^i, f_i(t) < 0\}, \tag{11}$$

where

$$f_i(t) = a\|e_i(t)\|^2 - b\|q_i(t)\|^2 - c\|q_i(t_k^i - \tau)\|^2, \tag{12}$$

a, b, and c are derived in the following section and t_κ^i denotes the k_{th} triggering time of the k_{th} follower, which is defined by $f_i(t) = 0$.

Theorem 1. *Consider the leader-follower linear system (1), the observer (5), the control algorithm (6), and the triggering conditions (11). The leader-follower linear system can reach tracking consensus by distributed event-triggered control protocol if there exist $P > 0$, $Q > 0$, $D > 0$, $R > 0$, $H > 0$, $W > 0$, F, and K satisfying:*

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & 0 & M_{25} \\ * & * & M_{33} & 0 & M_{35} \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned}
M_{11} &= \theta \otimes (\mathrm{PA} + A^T P) - \frac{1}{\tau}(\theta \otimes W) + (\theta \otimes R) + b(\theta^2 \otimes H),\\
M_{12} &= -2\theta \otimes \mathrm{PBK},\\
M_{13} &= -2\theta^2 \otimes \mathrm{PBK} + \frac{1}{\tau}(\theta \otimes W),\\
M_{14} &= 2\theta \otimes \mathrm{PFC},\\
M_{15} &= U^T \otimes A,\\
M_{22} &= -I_N \otimes H,\\
M_{25} &= -U^T \eta \otimes (\mathrm{BK})^T,\\
M_{33} &= -\theta \otimes R + b(\theta^2 \otimes D) - \frac{1}{\tau}(\theta \otimes W),\\
M_{35} &= -U^T \otimes (\mathrm{BK})^T,\\
M_{44} &= 2I_N \otimes (\mathrm{QA} + \mathrm{QFC}),\\
M_{45} &= -U^T \otimes (\mathrm{FC})^T,\\
M_{55} &= -\frac{1}{\tau}(\eta^{-1} \otimes W^{-1}).
\end{aligned} \tag{14}$$

That is, all followers can finally track the leader as well as we can avoid the Zeno behavior through the tracking control protocol we designed.

Proof. Assuming that all proposed assumptions are satisfied, then consider the following Lyapunov–Krasovskii functional $V(t) = V_1(t) + V_2(t) + V_3(t)$ with

$$
\begin{aligned}
V_1(t) &= \varsigma^T(t)(\eta \otimes P)\varsigma(t) + z^T(t)(I_N \otimes Q)z(t),\\
V_2(t) &= \int_{t-\tau}^{t} \left(e^T(s)(I_N \otimes H)e(s) + \varsigma^T(s)(\eta \otimes R)\varsigma(s)\right)\mathrm{d}s,\\
V_3(t) &= \int_0^{\tau}\int_{t+\theta}^{t} \dot{\varsigma}^T(s)(\eta \otimes W)\dot{\varsigma}(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned}
\tag{15}
$$

where $P > 0, Q > 0, H > 0, R > 0$, and $W > 0$, so we can get that $V(t)$ is positive definite. Then, the time derivation of $V(t)$ can be given as follows:

$$
\begin{aligned}
\dot{V}_1(t) &= 2\varsigma^T(t)(\eta \otimes P)\dot{\varsigma}(t) + 2z^T(t)(I_N \otimes Q)\dot{z}(t),\\
\dot{V}_2(t) &= e^T(t)(I_N \otimes H)e(t) - e^T(t-\tau)(I_N \otimes H)e(t-\tau) + \varsigma^T(t)(\eta \otimes R)\varsigma(t) - \varsigma^T(t-\tau)(\eta \otimes R)\varsigma(t-\tau),\\
\dot{V}_3(t) &= \tau\dot{\varsigma}^T(t)(\eta \otimes W)\dot{\varsigma}(t) - \int_{t-\tau}^{t} \dot{\varsigma}^T(s)(\eta \otimes W)\dot{\varsigma}(s)\mathrm{d}s.
\end{aligned}
\tag{16}
$$

Let $\delta(t) = (U^T \otimes I_N)\varsigma(t)$, $\widehat{e}(t) = (U^T \otimes I_N)e(t)$, and $\widehat{z}(t) = (U^T \otimes I_N)z(t)$, we can get

$$
\begin{aligned}
\dot{V}_1(t) &= 2\delta^T(t)\left[(\theta \otimes \mathrm{PA})\delta(t) - (\theta \otimes \mathrm{PBK})\widehat{e}(t-\tau) - \left(\theta^2 \otimes \mathrm{PBK}\right)\delta(t-\tau) + (\theta \otimes \mathrm{PFC})\widehat{z}(t)\right]\\
&\quad + 2\widehat{z}^T(t)\left[I_N \otimes (\mathrm{QA} + \mathrm{QFC})\right]\widehat{z}(t),\\
\dot{V}_2(t) &= \widehat{e}^T(t)(I_N \otimes H)\widehat{e}(t) - \widehat{e}^T(t-\tau)(I_N \otimes H)\widehat{e}(t-\tau)\\
&\quad + \delta^T(t)(\theta \otimes R)\delta(t) - \delta^T(t-\tau)(\theta \otimes R)\delta(t-\tau),\\
\dot{V}_3(t) &= \tau\dot{\varsigma}^T(t)(\eta \otimes W)\dot{\varsigma}(t) - \int_{t-\tau}^{t} \dot{\delta}^T(s)(\eta \otimes W)\dot{\delta}(s)\mathrm{d}s\\
&\le \tau\dot{\varsigma}^T(t)(\eta \otimes W)\dot{\varsigma}(t) - \frac{1}{\tau}(\delta(t) - \delta(t-\tau))^T(\theta \otimes W)(\delta(t) - \delta(t-\tau))\\
&= \tau\left[\delta^T(t)\left(\theta \otimes A^T\mathrm{WA}\right)\delta(t) - \delta^T(t)\left(\theta \otimes A^T\mathrm{WBK}\right)\widehat{e}(t-\tau)\right.\\
&\quad - \delta^T(t)\left(\theta^2 \otimes A^T\mathrm{WBK}\right)\delta(t) + \delta^T(t)\left(\theta \otimes A^T\mathrm{WFC}\right)\widehat{z}(t)\\
&\quad - \widehat{e}^T(t-\tau)\left(\theta \otimes (\mathrm{BK})^T\mathrm{WA}\right)\delta(t) + \widehat{e}^T(t-\tau)\left(\theta \otimes (\mathrm{BK})^T\mathrm{WBK}\right)\widehat{e}(t-\tau)\\
&\quad + \widehat{e}^T(t-\tau)\left(\theta^2 \otimes (\mathrm{BK})^T\mathrm{WBK}\right)\delta(t-\tau) - \widehat{e}^T(t-\tau)\left(\theta \otimes (\mathrm{BK})^T\mathrm{WFC}\right)\widehat{z}(t)\\
&\quad - \delta^T(t-\tau)\left(\theta^2 \otimes (\mathrm{BK})^T\mathrm{WA}\right)\delta(t) + \delta^T(t-\tau)\left(\theta^2 \otimes (\mathrm{BK})^T\mathrm{WBK}\right)\widehat{e}(t-\tau)\\
&\quad + \delta^T(t-\tau)\left(\theta^3 \otimes (\mathrm{BK})^T\mathrm{WBK}\right)\delta(t-\tau) - \delta^T(t-\tau)\left(\theta^2 \otimes (\mathrm{BK})^T\mathrm{WFC}\right)\widehat{z}(t)\\
&\quad + \widehat{z}^T(t)\left(\theta \otimes (\mathrm{FC})^T\mathrm{WA}\right)\delta(t) - \widehat{z}^T(t)\left(\theta \otimes (\mathrm{FC})^T\mathrm{WBK}\right)\widehat{e}(t-\tau)\\
&\quad \left. - \widehat{z}^T(t)\left(\theta^2 \otimes (\mathrm{FC})^T\mathrm{WBK}\right)\delta(t-\tau) + \widehat{z}^T(t)\left(\theta \otimes (\mathrm{FC})^T\mathrm{WFC}\right)\widehat{z}(t)\right]\\
&\quad - \frac{1}{\tau}(\delta(t) - \delta(t-\tau))^T(\theta \otimes W)(\delta(t) - \delta(t-\tau)).
\end{aligned}
\tag{17}
$$

Then, according to the proposed event-triggering function (12), we can easily get

$$\begin{aligned}\hat{e}^T(t)(I_N\otimes H)\hat{e}(t)&\le\lambda_m(H)\sum_{i=1}^{N}\|e_i(t)\|^2\le b\sum_{i=1}^{N}\|q_i(t)\|^2+c\sum_{i=1}^{N}\|q_i(t-\tau)\|^2\\&=b\varsigma^T(t)\left(\eta^2\otimes I_N\right)\varsigma(t)+c\varsigma^T(t-\tau)\left(\eta^2\otimes I_N\right)\varsigma(t-\tau)\\&\le k_1\delta^T(t)\left(\theta^2\otimes H\right)\delta(t)+k_2\delta^T(t-\tau)\left(\theta^2\otimes D\right)\delta(t-\tau),\end{aligned}\tag{18}$$

where $a=\lambda_m(H)$, $b=k_1\lambda_m(H)$, and $c=k_2\lambda_m(D)$. Then, the following expression can be summarized:

$$\dot{V}(t)\le\varepsilon^T(t)\widehat{M}\varepsilon(t),\tag{19}$$

where

$$\begin{aligned}&\varepsilon(t)=\left[\delta^T(t),\hat{e}^T(t-\tau),\delta^T(t-\tau),\hat{z}(t)\right],\\&\widehat{M}=\begin{bmatrix}\widehat{M}_{11}&\widehat{M}_{12}&\widehat{M}_{13}&\widehat{M}_{14}\\ *&\widehat{M}_{22}&\widehat{M}_{23}&\widehat{M}_{24}\\ *&*&\widehat{M}_{33}&\widehat{M}_{34}\\ *&*&*&\widehat{M}_{44}\end{bmatrix},\\&\widehat{M}_{11}=\theta\otimes\left(\mathrm{PA}+A^TP\right)+\tau\left(\theta\otimes A^T\mathrm{WA}\right)-\frac{1}{\tau}(\theta\otimes W)+(\theta\otimes R)+k_1\left(\theta^2\otimes H\right),\\&\widehat{M}_{12}=-2\theta^2\otimes\mathrm{PBK}-\tau\left(\theta\otimes A^T\mathrm{WBK}\right),\\&\widehat{M}_{13}=-2\theta^2\otimes\mathrm{PBK}-\tau\left(\theta^2\otimes A^T\mathrm{WBK}\right)+\frac{1}{\tau}(\theta\otimes W),\\&\widehat{M}_{14}=2\theta\otimes\mathrm{PFC}+\tau\left(\theta\otimes A^T\mathrm{WFC}\right),\\&\widehat{M}_{22}=-(I_N\otimes H)+\tau\left(\theta\otimes(\mathrm{BK})^T\mathrm{WBK}\right),\\&\widehat{M}_{23}=\tau\left(\theta^2\otimes(\mathrm{BK})^T\mathrm{WBK}\right),\\&\widehat{M}_{24}=-\tau\left(\theta\otimes(\mathrm{BK})^T\mathrm{WFC}\right),\\&\widehat{M}_{33}=-\theta\otimes R+\tau\left(\theta^3\otimes(\mathrm{BK})^T\mathrm{WBK}\right)+k_2\left(\theta^2\otimes D\right)-\frac{1}{\tau}(\theta\otimes W),\\&\widehat{M}_{34}=-\tau\left(\theta^2\otimes(\mathrm{BK})^T\mathrm{WFC}\right),\\&\widehat{M}_{44}=\tau\left(\theta\otimes(\mathrm{FC})^T\mathrm{WFC}\right).\end{aligned}\tag{20}$$

By (19), we can get that only if $\widehat{M}<0$ and $\varepsilon(t)\neq 0$, $\dot{V}(t)<0$ is established. In other words, the control error system (10) can be proven to be progressively stable. Then, according to Lemma 3, $\widehat{M}<0$ can be equivalent to

$$M=\begin{bmatrix}M_{11}&M_{12}&M_{13}&M_{14}&M_{15}\\ *&M_{22}&0&0&M_{25}\\ *&*&M_{33}&0&M_{35}\\ *&*&*&M_{44}&M_{45}\\ *&*&*&*&M_{55}\end{bmatrix}<0,\tag{21}$$

where

$$M_{11} = \theta\otimes\left(\mathrm{PA} + A^T P\right) - \frac{1}{\tau}(\theta\otimes W) + (\theta\otimes R) + b\left(\theta^2\otimes H\right),$$
$$M_{12} = -2\theta\otimes\mathrm{PBK},$$
$$M_{13} = -2\theta^2\otimes\mathrm{PBK} + \frac{1}{\tau}(\theta\otimes W),$$
$$M_{14} = 2\theta\otimes\mathrm{PFC},$$
$$M_{15} = U^T\otimes A,$$
$$M_{22} = -I_N\otimes H,$$
$$M_{25} = -U^T\eta\otimes(\mathrm{BK})^T,$$
$$M_{33} = -\theta\otimes R + b\left(\theta^2\otimes D\right) - \frac{1}{\tau}(\theta\otimes W),$$
$$M_{35} = -U^T\otimes(\mathrm{BK})^T,$$
$$M_{44} = 2I_N\otimes(\mathrm{QA} + \mathrm{QFC}),$$
$$M_{45} = -U^T\otimes(\mathrm{FC})^T,$$
$$M_{55} = -\frac{1}{\tau}\left(\eta^{-1}\otimes W^{-1}\right). \tag{22}$$

So, we can obtain that $\lim_{t\longrightarrow\infty}\varsigma(t) = 0$ by using Lyapunov stability theory; that is to say, all followers of the multiagent system can finally track the leader. Zeno behavior is an important problem for the event-triggered control method. We should ensure strictly positive triggering interval to avoid it. The proof is as follows. The time derivative of $\|e_i(t)\|$ for $t\in(t_k^i, t_{k+1}^i)$ can be obtained as follows:

$$\begin{aligned}\frac{d\|e_i(t)\|}{dt} &\le \|\dot{e}_i(t)\| = \|-\dot{q}_i(t)\|\\ &= \left\|-\sum_{j=1}^{N} a_{ij}\left(\dot{\bar{x}}_i(t) - \dot{\bar{x}}_j(t)\right) + h\left(\dot{\bar{x}}_i(t) - \dot{x}_0(t)\right)\right\|\\ &= \left\|-Aq_i(t) - \left(\sum_{j=1}^{N} a_{ij}B\left(u_i(t) - u_j(t)\right) + \mathrm{FC}\left(z_i(t) - z_j(t)\right) + hBu_i(t) + hFCz_i(t)\right\|\\ &\le \|A\|\|e_i(t)\| + \omega,\end{aligned} \tag{23}$$

where $\omega = \max_{t\in[t_k^i, t_{k+1}^i))}\|Aq_i(t_k^i) + \sum_{j=1}^{N} a_{ij}B(u_i(t) - u_j(t)) + \mathrm{FC}(z_i(t) - z_j(t)) + hBu_i(t) + hFCz_i(t)\|$; then, consider a nonnegative function that satisfies the following conditions:

$$\begin{aligned}\dot{\varphi} &= \|A\|\varphi + \omega,\\ \varphi(0) &= \left\|e_i\left(t_k^i\right)\right\| = 0,\end{aligned} \tag{24}$$

and hence we can obtain $\|e_i(t\| \le \varphi(t - t_k^i)$, where $\varphi(t)$ is the analytical solution to (24), which is given by $\varphi(t) = (\omega/\|A\|)(e^{\|A\|t} - 1)$. In order to ensure the event trigger function $f_i(t)\le 0$, we give the following sufficient condition:

$$a\|e_i(t)\|^2 \le c\left\|q_i\left(t_k^i - \tau\right)\right\|^2, \tag{25}$$

so we can easily get $\|e_i(t)\| \le \sqrt{(c/a)}\|q_i(t_k^i - \tau)\|$; let $\rho_k^i = \sqrt{(c/a)}\|q_i(t_k^i - \tau)\|$. It follows

$$\left\|e_i\left(t_{k+1}^i\right)\right\| = \rho_k^i \le \left(\frac{\omega}{\|A\|}\right)\left(e^{\|A\|\left(t_{k+1}^i - t_k^i\right)} - 1\right). \tag{26}$$

Then, we can obtain

$$\tau_k^i = t_{k+1}^i - t_k^i \ge \frac{1}{\|A\|}\ln\left(\frac{\|A\|\rho_k^i}{\omega} + 1\right). \tag{27}$$

We can know that the trigger interval always exists and is strictly positive at any time. Only when $t \longrightarrow \infty$, we can obtain $t_k^i \longrightarrow \infty$ and $k \longrightarrow \infty$. Therefore, all the agents will not exist Zeno behavior at any finite time. So, the proof is thus completed. □

3.2. Containment Control. In this section, we will discuss the distributed containment consensus problem for the leader-follower linear system with input time delay by using the event-triggered method. And sufficient event-triggering function and distributed control algorithm are given to achieve our control goals; that is to say, the containment consensus can be reached for the general linear system under the proposed protocol. The general linear system contains M leaders and N followers. Without loss of generality, we assume that the followers are labeled as $F = \{1, 2, \ldots, N\}$ and the leaders are labeled as $R = \{N+1, N+2, \ldots, N+M\}$, and the communication among agents can also be defined as $G = (\nu, \varepsilon)$. Under the previous assumptions, we can obtain that the Laplacian matrix of G can be written as $L = \begin{bmatrix} L_1 & L_2 \\ 0 & 0 \end{bmatrix}$, where L_1 is also a positive definite matrix. The dynamics of the followers can be expressed as (1), and the expression of the leaders' dynamics can be described as follows:

$$\dot{x}_i(t) = Ax_i(t), \quad t \ge 0, i \in R, \tag{28}$$

where $x_i(t) \in R^n$ means the state of the leaders. For convenience, let ${}^*f = ({}^*1^T, \ldots, {}^*N^T)^T$ represents the information of the followers and ${}^*r = ({}^*N+1^T, \ldots, {}^*N+M^T)^T$ represents the information of the leaders. We can consider the distributed event-triggered containment control protocol with input time delays as follows:

$$u_i(t) = -Kq_i\left(t_k^i - \tau\right), \quad t \in \left[t_k^i, t_{k+1}^i\right], \tag{29}$$

where $q_i(t) = \sum_{j\in F} a_{ij}(\overline{x}_i(t) - \overline{x}_j(t)) + \sum_{j\in R} a_{ij}(\overline{x}_i(t) - x_j(t))$. Then, the expression of control system can be written as follows:

$$\begin{aligned} \dot{x}_r(t) &= (I_M \otimes A)x_r(t), \\ \dot{\overline{x}}_f(t) &= (I_N \otimes A)\overline{x}_f(t) - (I_N \otimes \mathrm{BK})e_f(t-\tau) - (L_1 \otimes \mathrm{BK})\overline{x}_f(t-\tau) - (L_2 \otimes \mathrm{BK})x_r(t-\tau) + (I_N \otimes \mathrm{FC})z_f(t), \\ \dot{z}_f(t) &= [I_N \otimes (A + \mathrm{FC})]z_f(t). \end{aligned} \tag{30}$$

Let $\varsigma_f(t) = \overline{x}_f(t) + (L_1^{-1}L_2 \otimes I_M)x_r(t)$. Then, we have the following expression:

$$\begin{aligned} \dot{\varsigma}_f(t) &= (I_N \otimes A)\varsigma_f(t) - (I_N \otimes \mathrm{BK})e_f(t-\tau) - (L_1 \otimes \mathrm{BK})\varsigma_f(t) + (I_N \otimes \mathrm{FC})z_f(t), \\ \dot{z}_f(t) &= [I_N \otimes (A + \mathrm{FC})]z_f(t). \end{aligned} \tag{31}$$

Theorem 2. *Consider the leader-follower linear system (1), the observer (5), the control algorithm (29), and the triggering conditions (11). The leader-follower linear system can reach containment consensus by distributed event-triggered control protocol if there exists $P > 0$, $Q > 0$, $D > 0$, $R > 0$, $H > 0$, and $W > 0$ and F and K satisfied:*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & 0 & 0 & \Phi_{25} \\ * & * & \Phi_{33} & 0 & \Phi_{35} \\ * & * & * & \Phi_{44} & \Phi_{45} \\ * & * & * & * & \Phi_{55} \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned}
\Phi_{11} &= \theta\otimes\left(\mathrm{PA}+A^{T}P\right)-\frac{1}{\tau}(\theta\otimes W)+(\theta\otimes R)+b\left(\theta^{2}\otimes H\right),\\
\Phi_{12} &= -2\theta\otimes \mathrm{PBK},\\
\Phi_{13} &= -2\theta^{2}\otimes \mathrm{PBK}+\frac{1}{\tau}(\theta\otimes W),\\
\Phi_{14} &= 2\theta\otimes \mathrm{PFC},\\
\Phi_{15} &= U^{T}\otimes A,\\
\Phi_{22} &= -I_{N}\otimes H,\\
\Phi_{25} &= -U^{T}L_{1}\otimes(\mathrm{BK})^{T},\\
\Phi_{33} &= -\theta\otimes R+b\left(\theta^{2}\otimes D\right)-\frac{1}{\tau}(\theta\otimes W),\\
\Phi_{35} &= -U^{T}\otimes(\mathrm{BK})^{T},\\
\Phi_{44} &= 2I_{N}\otimes(\mathrm{QA}+\mathrm{QFC}),\\
\Phi_{45} &= -U^{T}\otimes(\mathrm{FC})^{T},\\
\Phi_{55} &= -\frac{1}{\tau}\left(L_{1}^{-1}\otimes W^{-1}\right).
\end{aligned}\tag{33}$$

Then, we can get that the leader-follower linear system can eventually achieve containment consensus.

Proof. Assuming that all proposed assumptions are satisfied, then consider the following Lyapunov–Krasovskii functional$V(t)=V_{1}(t)+V_{2}(t)+V_{3}(t)$ with

$$\begin{aligned}
V_{1}(t) &= \varsigma_{f}^{T}(t)\left(L_{1}\otimes P\right)\varsigma_{f}(t)+z_{f}^{T}(t)\left(I_{N}\otimes Q\right)z_{f}(t),\\
V_{2}(t) &= \int_{t-\tau}^{t}\left(e_{f}^{T}(s)\left(I_{N}\otimes H\right)e_{f}(s)+\varsigma_{f}^{T}(s)\left(L_{1}\otimes R\right)\varsigma_{f}(s)\right)\mathrm{d}s,\\
V_{3}(t) &= \int_{0}^{\tau}\int_{t+\theta}^{t}\dot{\varsigma}_{f}^{T}(s)\left(L_{1}\otimes W\right)\dot{\varsigma}_{f}(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned}\tag{34}$$

where $P>0, Q>0, H>0, R>0$, and $W>0$, so we can get that $V(t)$ is positive definite. Then, the time derivation of $V(t)$ can be given as follows:

$$\begin{aligned}
\dot{V}_{1}(t) &= 2\varsigma_{f}^{T}(t)\left(L_{1}\otimes P\right)\dot{\varsigma}_{f}(t)+2z_{f}^{T}(t)\left(I_{N}\otimes Q\right)\dot{z}_{f}(t),\\
\dot{V}_{2}(t) &= e_{f}^{T}(t)\left(I_{N}\otimes H\right)e_{f}(t)-e_{f}^{T}(t-\tau)\left(I_{N}\otimes H\right)e_{f}(t-\tau)\\
&\quad+\varsigma_{f}^{T}(t)\left(L_{1}\otimes R\right)\varsigma_{f}(t)-\varsigma_{f}^{T}(t-\tau)\left(L_{1}\otimes R\right)\varsigma_{f}(t-\tau),\\
\dot{V}_{3}(t) &= \tau\dot{\varsigma}_{f}^{T}(t)\left(L_{1}\otimes W\right)\dot{\varsigma}_{f}(t)-\int_{t-\tau}^{t}\dot{\varsigma}_{f}^{T}(s)\left(L_{1}\otimes W\right)\dot{\varsigma}_{f}(s)\mathrm{d}s.
\end{aligned}\tag{35}$$

Let $\delta_{f}(t)=(U^{T}\otimes I_{N})\varsigma_{f}(t)$, $\widehat{e}_{f}(t)=(U^{T}\otimes I_{N})e_{f}(t)$, and $\widehat{z}_{f}(t)=(U^{T}\otimes I_{N})z_{f}(t)$, we can get

$$\begin{aligned}
\dot{V}_{1}(t) &= 2\delta_{f}^{T}(t)\left[(\theta\otimes \mathrm{PA})\delta_{f}(t)-(\theta\otimes \mathrm{PBK})\widehat{e}_{f}(t-\tau)-\left(\theta^{2}\otimes \mathrm{PBK}\right)\delta_{f}(t-\tau)+(\theta\otimes \mathrm{PFC})\widehat{z}_{f}(t)\right]\\
&\quad+2\widehat{z}_{f}^{T}(t)\left[I_{N}\otimes(\mathrm{QA}+\mathrm{QFC})\right]\widehat{z}_{f}(t),\\
\dot{V}_{2}(t) &= \widehat{e}_{f}^{T}(t)\left(I_{N}\otimes H\right)\widehat{e}_{f}(t)-\widehat{e}_{f}^{T}(t-\tau)\left(I_{N}\otimes H\right)\widehat{e}_{f}(t-\tau)\\
&\quad+\delta_{f}^{T}(t)(\theta\otimes R)\delta_{f}(t)-\delta_{f}^{T}(t-\tau)(\theta\otimes R)\delta_{f}(t-\tau),\\
\dot{V}_{3}(t) &= \tau\dot{\varsigma}_{f}^{T}(t)\left(L_{1}\otimes W\right)\dot{\varsigma}_{f}(t)-\int_{t-\tau}^{t}\dot{\delta}_{f}^{T}(s)\left(L_{1}\otimes W\right)\dot{\delta}_{f}(s)\mathrm{d}s\\
&\le \tau\dot{\varsigma}_{f}^{T}(t)\left(L_{1}\otimes W\right)\dot{\varsigma}_{f}(t)-\frac{1}{\tau}\left(\delta_{f}(t)-\delta_{f}(t-\tau)\right)^{T}(\theta\otimes W)\left(\delta_{f}(t)-\delta_{f}(t-\tau)\right)\\
&= \tau\Big[\delta_{f}^{T}(t)\left(\theta\otimes A^{T}\mathrm{WA}\right)\delta_{f}(t)-\delta_{f}^{T}(t)\left(\theta\otimes A^{T}\mathrm{WBK}\right)\widehat{e}_{f}(t-\tau)\\
&\quad-\delta_{f}^{T}(t)\left(\theta^{2}\otimes A^{T}\mathrm{WBK}\right)\delta_{f}(t)+\delta_{f}^{T}(t)\left(\theta\otimes A^{T}\mathrm{WFC}\right)\widehat{z}_{f}(t)\\
&\quad-\widehat{e}_{f}^{T}(t-\tau)\left(\theta\otimes(\mathrm{BK})^{T}\mathrm{WA}\right)\delta_{f}(t)+\widehat{e}_{f}^{T}(t-\tau)\left(\theta\otimes(\mathrm{BK})^{T}\mathrm{WBK}\right)\widehat{e}_{f}(t-\tau)\\
&\quad+\widehat{e}_{f}^{T}(t-\tau)\left(\theta^{2}\otimes(\mathrm{BK})^{T}\mathrm{WBK}\right)\delta_{f}(t-\tau)-\widehat{e}_{f}^{T}(t-\tau)\left(\theta\otimes(\mathrm{BK})^{T}\mathrm{WFC}\right)\widehat{z}_{f}(t)\\
&\quad-\delta_{f}^{T}(t-\tau)\left(\theta^{2}\otimes(\mathrm{BK})^{T}\mathrm{WA}\right)\delta_{f}(t)+\delta_{f}^{T}(t-\tau)\left(\theta^{2}\otimes(\mathrm{BK})^{T}\mathrm{WBK}\right)\widehat{e}_{f}(t-\tau)\\
&\quad+\delta_{f}^{T}(t-\tau)\left(\theta^{3}\otimes(\mathrm{BK})^{T}\mathrm{WBK}\right)\delta_{f}(t-\tau)-\delta_{f}^{T}(t-\tau)\left(\theta^{2}\otimes(\mathrm{BK})^{T}\mathrm{WFC}\right)\widehat{z}_{f}(t)\\
&\quad+\widehat{z}_{f}^{T}(t)\left(\theta\otimes(\mathrm{FC})^{T}\mathrm{WA}\right)\delta_{f}(t)-\widehat{z}_{f}^{T}(t)\left(\theta\otimes(\mathrm{FC})^{T}\mathrm{WBK}\right)\widehat{e}_{f}(t-\tau)\\
&\quad-\widehat{z}_{f}^{T}(t)\left(\theta^{2}\otimes(\mathrm{FC})^{T}\mathrm{WBK}\right)\delta_{f}(t-\tau)+\widehat{z}_{f}^{T}(t)\left(\theta\otimes(\mathrm{FC})^{T}\mathrm{WFC}\right)\widehat{z}_{f}(t)\Big]\\
&\quad-\frac{1}{\tau}\left(\delta_{f}(t)-\delta_{f}(t-\tau)\right)^{T}(\theta\otimes W)\left(\delta_{f}(t)-\delta_{f}(t-\tau)\right).
\end{aligned}\tag{36}$$

Then, according to the proposed event-triggering function (12), we can easily obtain

$$\begin{aligned}&\hat{e}^T(t)(I_N\otimes H)\hat{e}(t)\le\lambda_m(H)\sum_{i=1}^{N}\|e_i(t)\|^2\\&\le b\sum_{i=1}^{N}\|q_i(t)\|^2+c\sum_{i=1}^{N}\|q_i(t-\tau)\|^2\\&=b\varsigma_f^T(t)\left(L_1^2\otimes I_N\right)\varsigma_f(t)+c\varsigma_f^T(t-\tau)\left(L_1^2\otimes I_N\right)\varsigma_f(t-\tau)\\&\le k_1\delta_f^T(t)\left(\theta^2\otimes H\right)\delta_f(t)+k_2\delta_f^T(t-\tau)\left(\theta^2\otimes D\right)\delta_f(t-\tau).\end{aligned}\tag{37}$$

Then, the following expression can be summarized:

$$\dot{V}(t)\le\varphi^T(t)\hat{\Phi}\varphi(t),\tag{38}$$

where

$$\begin{aligned}&\varphi(t)=\left[\delta_f^T(t),\hat{e}_f^T(t-\tau),\delta_f^T(t-\tau),\hat{z}_f(t)\right],\\&\hat{\Phi}=\begin{bmatrix}\hat{\Phi}_{11}&\hat{\Phi}_{12}&\hat{\Phi}_{13}&\hat{\Phi}_{14}\\ *&\hat{\Phi}_{22}&\hat{\Phi}_{23}&\hat{\Phi}_{24}\\ *&*&\hat{\Phi}_{33}&\hat{\Phi}_{34}\\ *&*&*&\hat{\Phi}_{44}\end{bmatrix},\\&\hat{\Phi}_{11}=\theta\otimes\left(\mathrm{PA}+A^TP\right)+\tau\left(\theta\otimes A^T\mathrm{WA}\right)-\frac{1}{\tau}(\theta\otimes W)+(\theta\otimes R)+k_1\left(\theta^2\otimes H\right),\\&\hat{\Phi}_{12}=-2\theta^2\otimes\mathrm{PBK}-\tau\left(\theta\otimes A^T\mathrm{WBK}\right),\\&\hat{\Phi}_{13}=-2\theta^2\otimes\mathrm{PBK}-\tau\left(\theta^2\otimes A^T\mathrm{WBK}\right)+\frac{1}{\tau}(\theta\otimes W),\\&\hat{\Phi}_{14}=2\theta\otimes\mathrm{PFC}+\tau\left(\theta\otimes A^T\mathrm{WFC}\right),\\&\hat{\Phi}_{22}=-(I_N\otimes H)+\tau\left(\theta\otimes(\mathrm{BK})^T\mathrm{WBK}\right),\\&\hat{\Phi}_{23}=\tau\left(\theta^2\otimes(\mathrm{BK})^T\mathrm{WBK}\right),\\&\hat{\Phi}_{24}=-\tau\left(\theta\otimes(\mathrm{BK})^T\mathrm{WFC}\right),\\&\hat{\Phi}_{33}=-\theta\otimes R+\tau\left(\theta^3\otimes(\mathrm{BK})^T\mathrm{WBK}\right)+k_2\left(\theta^2\otimes D\right)-\frac{1}{\tau}(\theta\otimes W),\\&\hat{\Phi}_{34}=-\tau\left(\theta^2\otimes(\mathrm{BK})^T\mathrm{WFC}\right),\\&\hat{\Phi}_{44}=\tau\left(\theta\otimes(\mathrm{FC})^T\mathrm{WFC}\right).\end{aligned}\tag{39}$$

By (38), we can get that only if $\hat{\Phi}<0$ and $\varphi(t)\neq0$, $\dot{V}(t)<0$ is established. In other words, the control error system (31) can be proven to be progressively stable. Then, according to Lemma 3, $\hat{\Phi}<0$ can be equivalent to

$$\Phi=\begin{bmatrix}\Phi_{11}&\Phi_{12}&\Phi_{13}&\Phi_{14}&\Phi_{15}\\ *&\Phi_{22}&0&0&\Phi_{25}\\ *&*&\Phi_{33}&0&\Phi_{35}\\ *&*&*&\Phi_{44}&\Phi_{45}\\ *&*&*&*&\Phi_{55}\end{bmatrix}<0,\tag{40}$$

where

$$\Phi_{11} = \theta \otimes \left(PA + A^T P\right) - \frac{1}{\tau}(\theta \otimes W) + (\theta \otimes R) + b\left(\theta^2 \otimes H\right),$$

$$\Phi_{12} = -2\theta \otimes PBK,$$

$$\Phi_{13} = -2\theta^2 \otimes PBK + \frac{1}{\tau}(\theta \otimes W),$$

$$\Phi_{14} = 2\theta \otimes PFC,$$

$$\Phi_{15} = U^T \otimes A,$$

$$\Phi_{22} = -I_N \otimes H,$$

$$\Phi_{25} = -U^T L_1 \otimes (BK)^T,$$

$$\Phi_{33} = -\theta \otimes R + b\left(\theta^2 \otimes D\right) - \frac{1}{\tau}(\theta \otimes W),$$

$$\Phi_{35} = -U^T \otimes (BK)^T,$$

$$\Phi_{44} = 2I_N \otimes (QA + QFC),$$

$$\Phi_{45} = -U^T \otimes (FC)^T,$$

$$\Phi_{55} = -\frac{1}{\tau}\left(L_1^{-1} \otimes W^{-1}\right). \quad (41)$$

Then, we have $\lim_{t\longrightarrow\infty}\varsigma_f(t) = 0$ by using Lyapunov stability theory, that is, $\lim_{t\longrightarrow\infty}\|\overline{x}_f(t) + (L_1^{-1}L_2 \otimes I_M)x_r(t)\| = 0$ and $\lim_{t\longrightarrow\infty}z_f(t) = 0$. Thus, we can get that $\lim_{t\longrightarrow\infty}\|x_f(t) + (L_1^{-1}L_2 \otimes I_M)x_r(t)\| = 0$. Then, according to Lemma 4, distributed control goals can be achieved; that is, the leader-follower system can finally reach the containment consensus. The proof of Zeno behavior is similar to that in Theorem 1. And the proof is thus completed. □

4. Simulation Example

First, we discuss the situation of the leader-follower linear system with a single leader, and the MAS with identical agent dynamics is as follows:

$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix},$$
$$A = \begin{bmatrix} 0 & 0.4 \\ 0.2 & -0.6 \end{bmatrix}, \quad (42)$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Then, we consider that the communication connection weights are $a_{12} = a_{13} = a_{24} = a_{34} = 1$, $h_1 = 1$, and others are zero. Let $x_0(t) = [-2 \ \ 3]^T$, $x_1(t) = [-10 \ \ 1]^T$, $x_2(t) = [8 \ \ -2]^T$, $x_4(t) = [-12 \ \ 4]^T$, and $x_4(t) = [13 \ \ 4]^T$. We should note that when the event trigger method is used to deal with the delay problem, it is necessary to ensure that the delay is bigger than the event trigger time interval; otherwise, the time delay problem will become meaningless; therefore, we will choose the largest possible time delay bigger than the trigger time interval. So, we choose the appropriate input time delay $\tau = 0.1s$. By solving (13) in Theorem 1, we can get $K = [1.6538 \ \ 1.7246]$ and $F = [1.3429 \ \ 0.8573]^T$. The results are shown in Figures 1–3.

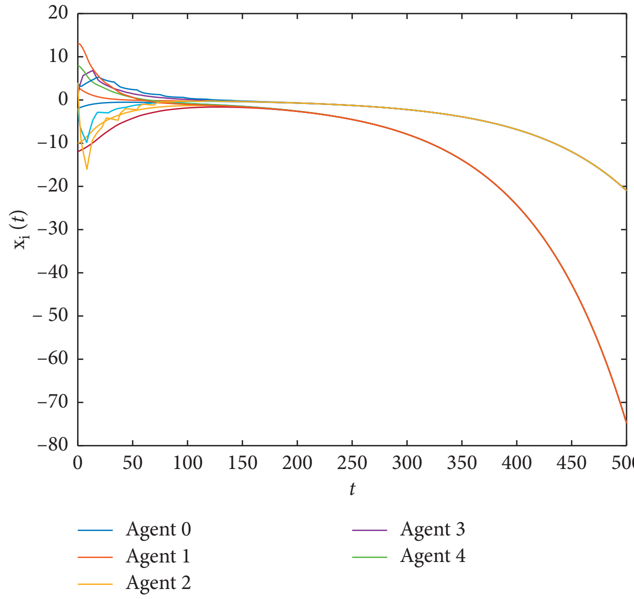

FIGURE 1: The states for tracking control.

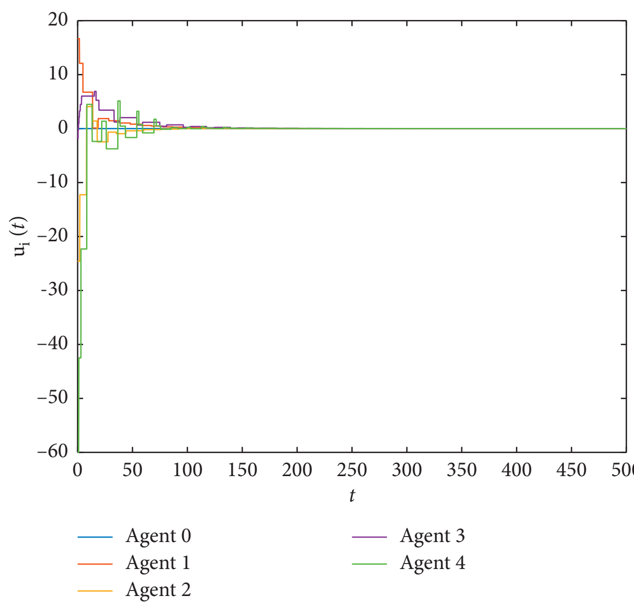

FIGURE 2: The control inputs for tracking control.

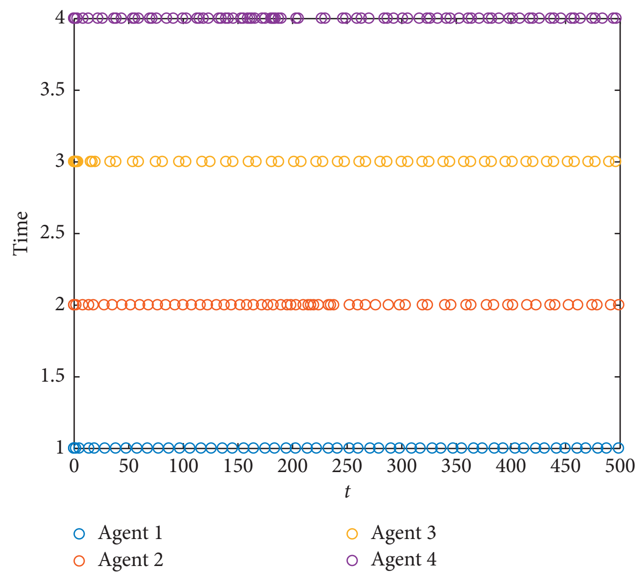

Figure 3: The event-triggered time for tracking control.

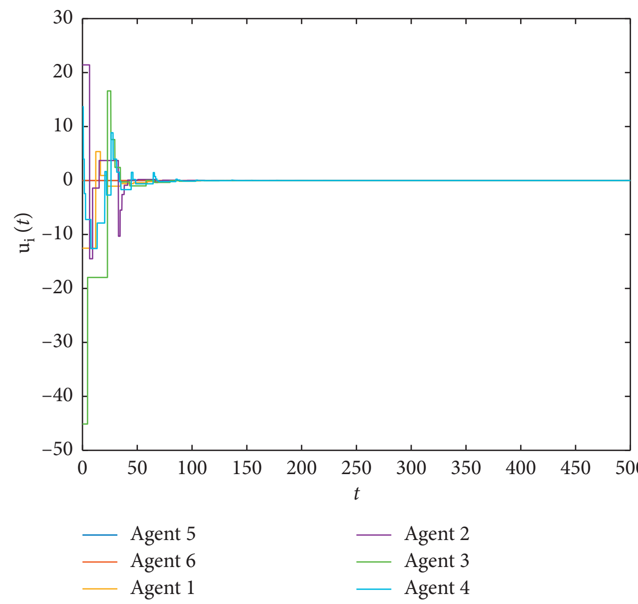

Figure 5: The control inputs for containment control.

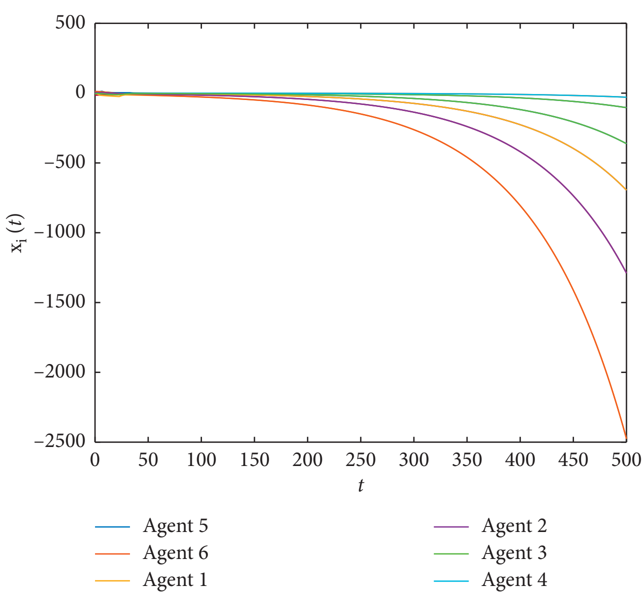

Figure 4: The states for containment control.

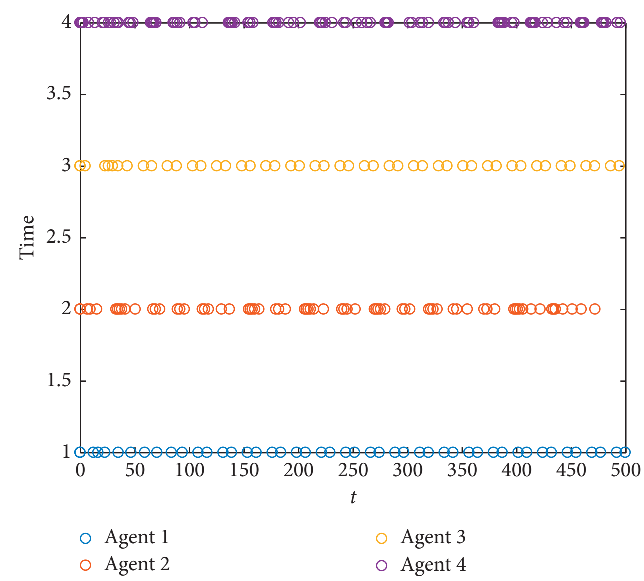

Figure 6: The event-triggered time for containment control.

Then, we discuss the situation of the leader-follower linear system with four followers and two leaders, the followers are marked as $\{1, 2, 3, 4\}$, and the leaders are marked as $\{5, 6\}$. The MAS with identical agent dynamics is as follows:

$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix},$$
$$A = \begin{bmatrix} 0 & 0.4 \\ 0.2 & -0.6 \end{bmatrix}, \quad (43)$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Let $x_1(t) = [9 \;\; -3]^T$, $x_2(t) = [-15 \;\; 5]^T$, $x_3(t) = [12 \;\; 3]^T$, $x_4(t) = [4 \;\; -4]^T$, $x_5(t) = [-1 \;\; 1]^T$, and $x_6(t) = [-11 \;\; 1]^T$. As with the above conditions, we choose the appropriate input time delay $\tau = 0.1s$. By solving (32) in Theorem 2, we can get $K = [1.8356 \;\; 1.4528]$ and $F = [1.3429 \;\; 1.2176]^T$. The results are shown in Figures 4–6.

The following information can be obtained through the above simulation results:

(1) For the leader-follower linear system with a single leader, by designing appropriate feedback gain matrix and event trigger parameters, the followers'

states can be same as the leader and the followers' control inputs tend to zero eventually

(2) For the leader-follower linear system with multiple leaders, by designing appropriate feedback gain matrix and event trigger parameters, the system can reach containment consensus and the followers' control inputs also tend to zero eventually

5. Conclusions

This paper considered the observer-based tracking and containment control on the basis of event-triggering mechanism for the leader-follower linear systems with input time delays. A fully distributed delay-dependent containment controller and event trigger have been designed for the general linear systems under directed graphs. Under the designed algorithms, the leader-follower linear system can achieve tracking consensus and containment. Two simulation results showed the effect of the designed control protocol. In the future, we will study the containment problems of the leader-follower systems with switching networks by using the event-triggered method.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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