Hindawi
Complexity
Volume 2021, Article ID 9939559, 6 pages
https://doi.org/10.1155/2021/9939559

Research Article

On Fault-Tolerant Resolving Sets of Some Families of Ladder Networks

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Received 1 April 2021; Accepted 5 May 2021; Published 17 May 2021

Academic Editor: Muhammad Javaid

In computer networks, vertices represent hosts or servers, and edges represent as the connecting medium between them. In localization, some special vertices (resolving sets) are selected to locate the position of all vertices in a computer network. If an arbitrary vertex stopped working and selected vertices still remain the resolving set, then the chosen set is called as the fault-tolerant resolving set. The least number of vertices in such resolving sets is called the fault-tolerant metric dimension of the network. Because of the variety of applications of the metric dimension in different areas of sciences, many generalizations were proposed, and fault tolerant is one of them. In this paper, we computed the fault-tolerant metric dimension of triangular snake, ladder, Mobius ladder, and hexagonal ladder networks. It is important to observe that, in all these classes of networks, the fault-tolerant metric dimension and metric dimension differ by 1.

1. Introduction

Let $G = (V(G), E(G))$ be a simple connected graph, where $V(G)$ and $E(G)$ are the set of vertices and edges, respectively. The distance $d(u_1, u_2)$ between a pair of vertices $u_1, u_2 \in V(G)$ is the length of the shortest path joining them. A vertex u resolves or distinguishes a pair u_1, u_2 if $d(u_1, u) \neq d(u_2, u)$. The representation of an arbitrary vertex u corresponding to an ordered nonempty subset of vertices $Q = \{q_1, q_2, \ldots, q_h\} \subset V(G)$ is the h-component vector $r(u|Q) = (d(u, q_1), d(u, q_2), \ldots, d(u, q_h))$. A subset $Q \subset V(G)$ is named the resolving set if any pair $u_1, u_2 \in V(G)$ possesses the condition of unique representation, that is, $r(u_1|Q) \neq r(u_2|Q)$. The least possible cardinality of Q is named the metric dimension of the graph G and symbolized as $\dim(G)$. A resolving set containing a least possible number of elements is named as basis. A basis set Q_f of G is called the fault-tolerant basis set if, for every $u \in Q_f$, the subset (Q_f/u) again resolves the vertices of G. The least number of vertices in Q_f is termed as the fault-tolerant metric dimension of G and is denoted by $\dim_f(G)$. Chaudhry et al. [1] proved an important relationship between the fault-tolerant metric dimension and the metric dimension of a graph G.

$$\dim_f(G) \geq \dim(G) + 1. \tag{1}$$

First time, the idea of metric dimension was studied by Slater [2] in 1975 and later by Harary and Melter [3] in 1976. Chartrand et al. [4] considered metric bases as sensors. If any of the sensors did not operate correctly, then we do not have sufficient knowledge to deal with the intruders. Hernando et al. [5] presented the idea of fault-tolerant metric dimension to overcome this kind of problems. Fault-tolerant basis set delivers accurate information despite being one of the sensors is not working.

Because of different applications, the study of fault-tolerant resolving sets of different networks is as sundry as the study of the metric dimension is.

It was proved that computing the metric dimension is NP-complete [6]. However, the idea of uniquely identifying every vertex in a graph based on distance is very useful. The application of this concept in chemical structures was presented by Chartrand el al. [4]. In robot navigation, the work by Khuller et al. [7] motivated the researchers for the theoretical investigation of the metric dimension. The resolving set has found a number of applications. For more details, see [8–10]. Generally, computing the fault-tolerant metric dimension is also an NP-complete problem. Computing the fault-tolerant metric dimension is considered as one of the interesting but difficult problems in combinatorics. So far, only few structures have been investigated. In the initial paper, Hernando et al. [5] computed the fault-tolerant basis sets of tree T. For a graph G, they derived a connection between the fault-tolerant metric dimension and the metric dimension, that is, $\dim_f(G) \leq \dim(G)(1 + 2 \times 5^{\dim(G)-1})$. Javaid et al. [11] studied this parameter of cycle graph C_n and discussed some bounds on the partition dimension. For more results related to the fault-tolerant metric dimension, the interested readers can see [12–23].

The core objective of this paper is to find some classes of graphs satisfying the equation $\dim(G) = \dim_f(G) + 1$. It has been observed that if G is ladder $(L_m, m \geq 3)$, Möbius ladder $(\mathrm{ML}_m, m \geq 4)$, and hexagonal Möbius ladder graph $(\mathrm{HML}_m, m \geq 2)$, then the metric dimension and fault-tolerant metric dimension differ by 1. Also, for triangular snake graph TS_n, the above equality holds if and only if $n = 5, 7$. In order to prove these results, we need some results from the literature.

Theorem 1 (see [4]). *ie metric dimension of a graph is 1 iff $G \cong P_n$.*

Theorem 2 (see [24]). *Let ML_m be a Möbius ladder graph. Then,*

$$\dim(\mathrm{ML}_m) = \begin{cases} 3 & \text{if } m \geq 4 \text{ even}, \\ 4 & \text{if } m \geq 5 \text{ odd}. \end{cases} \tag{2}$$

Theorem 3 (see [25]). *Let HML_m be a hexagonal Möbius ladder graph. Then,*

$$\dim(\mathrm{HML}_m) = \begin{cases} 2 & \text{if } m = 2, \\ 3 & \text{if } m \geq 3. \end{cases} \tag{3}$$

Theorem 4 (see [5]). *Let L_n be a ladder graph with $n \geq 3$. Then,*

$$\dim(L_n) = 2. \tag{4}$$

2. Fault-Tolerant Metric Dimension of the Triangular Snake Graph

This section deals with the metric dimension and fault-tolerant metric dimension of the triangular snake graph. Let P_n be a path with vertex set $V(P_n) = \{v_1, v_2, \ldots, v_n\}$, where $n = 2s + 1$ is an odd integer. A triangular snake graph TS_n is constructed from P_n by connecting v_{2i-1} with v_{2i+1} for $1 \leq i \leq s$. Note that s represents the cardinality of triangles in TS_n. The triangular snake graph is depicted in Figure 1. In the following Lemma 1, we calculate the metric dimension $\dim(\mathrm{TS}_n)$ of triangular snake graph TS_n.

Lemma 1. *Let TS_n be a triangular snake graph with $s \geq 2$. Then,*

$$\dim(\mathrm{TS}_n) = 2. \tag{5}$$

Proof. From Theorem 1, it follows that $\dim(\mathrm{TS}_n) \geq 2$. To prove that $\dim(\mathrm{TS}_n) = 2$, we prove that $\dim(\mathrm{TS}_n) \leq 2$. Let $Q = \{v_1, v_n\}$ be a resolving set. Then, the representations of vertices v_t regarding the resolving set Q are as follows:

$$r(v_t|Q) = \begin{cases} \left(\frac{t-1}{2}, \frac{2s-t-1}{2}\right) & \text{if } t = 1, 3, 5, \ldots, 2s-1, \\ \left(\frac{t}{2}, \frac{2s-t+2}{2}\right) & \text{if } t = 2, 4, 6, \ldots, 2s-2, \\ \left(\frac{t}{2}, 1\right) & \text{if } t = 2s, \\ \left(\frac{t}{2}, 0\right) & \text{if } t = 2s+1. \end{cases} \tag{6}$$

As all the vertices have unique representations regarding the resolving set Q, hence, $\dim(\mathrm{TS}_n) \leq 2$. □

Theorem 5. *Let TS_n be a triangular snake graph; then,*

$$\dim_f(\mathrm{TS}_n) = \begin{cases} 3 & \text{if } s = 2, 3, \\ 4 & \text{if } s \geq 4. \end{cases} \tag{7}$$

Proof. First, we show that $\dim_f(\mathrm{TS}_n) = 3$ when $s = 2, 3$. To prove this, it is enough to show that $\dim_f(\mathrm{TS}_n) \leq 3$. The inequality $\dim_f(\mathrm{TS}_n) \geq 3$ follows from Lemma 1 and equation (1). If we take $Q_f = \{v_1, v_4, v_{2s+1}\}$, then it satisfies the condition of the fault-tolerant resolving set. The following is the representation of any vertex v_t of the graph TS_n with respect to Q_f:

$$r(v_t|Q_f) = \left(\lfloor \frac{t}{2} \rfloor, 2\left\lceil \left|\frac{4-t}{2}\right| \right\rceil + z, \left\lceil \frac{2s+1-t}{2} \right\rceil\right), \tag{8}$$

where $z = 1$ when $t = 3, 5$ and zero, otherwise.

Now, we prove that $\dim_f(\mathrm{TS}_n) \leq 4$ when $s \geq 4$. For this, we construct a fault-tolerant resolving set. Take

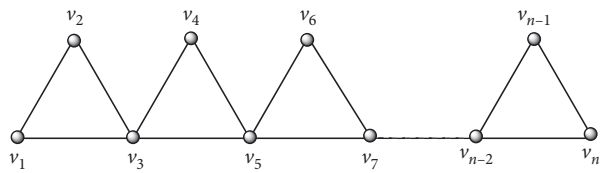

Figure 1: Triangular snake graph TS_n.

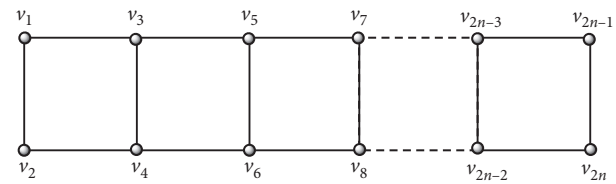

Figure 2: Ladder graph L_m.

$Q_f = \{v_1, v_2, v_{2s-2}, v_{2s}\}$; the following is the representation of any vertex of graph with Q_f:

$$r(v_t|Q_f) = \left(\lceil\frac{t-1}{2}\rceil, \lceil\left|\frac{t-2}{2}\right|\rceil + z, \lceil\left|\frac{2s-2-t}{2}\right|\rceil + z, \lceil\left|\frac{2s-t}{2}\right|\rceil + z\right), \tag{9}$$

where $z = 1$ for $t =$ even and zero, otherwise. Hence, it follows from the above discussion that $\dim_f(TS_n) \leq 4$ since every vertex of TS_n has a unique representation regarding resolving set Q_f.

We prove the reverse inequality $\dim_f(TS_n) \geq 4$ by the contradiction method. Suppose $\dim_f(TS_n) < 4$, it follows from Lemma 1 that $\dim_f(TS_n) = 3$. Let us consider some resolving set Q_f with cardinality 3. □

Case 1. Let $Q_f = \{v_{1+l}, v_{2+l}, v_{3+l}\}$, where $l \equiv 0 \pmod 3$. If we choose $Q_f = \{v_1, v_2, v_3\}$, then $r(v_4|Q_f) = r(v_5|Q_f)$. For any other choice of Q_f, we get $r(v_1|Q_f) = r(v_2|Q_f)$.

Case 2. Let $Q_f = \{v_i, v_{1+l}, v_{2+l}\}$ or $Q_f = \{v_i, v_{1+l}, v_{3+l}\}$, where $1 \leq i \leq 2$ and $l \geq 3$. In other words, we are taking the first vertex from the first triangle and the remaining vertices from any arbitrary triangle. Then, it is easy to observe that there exists an integer $6 \leq j \leq n-1$ such that $r(v_j|Q_f) = r(v_{j+1}|Q_f)$.

Case 3. Let $Q_f = \{v_i, v_j, v_k\}$, where $1 \leq i, j, k \leq n$. Then, $r(v_1|Q_f) = r(v_2|Q_f)$.

Hence, it follows from the above discussion that $\dim_f(TS_n) \neq 3$ when $s \geq 4$. This implies that $\dim_f(TS_n) \geq 4$ when $s \geq 4$. This completes the proof.

3. Fault-Tolerant Metric Dimension of Ladder Graphs

A ladder graph is obtained by the Cartesian product of path P_n with path P_2 and is denoted by L_m. Observe that $m = 2n$. The ladder graph is shown in Figure 2. In the following theorem, we compute the fault-tolerant metric dimension of the ladder graph.

Theorem 6. *Let L_m be a ladder graph with $m \geq 3$. Then,*

$$\dim_f(L_m) = 3. \tag{10}$$

Proof. The inequality $\dim_f(L_m) \geq 3$ follows from Theorem 4 and equation (1). To prove $\dim_f(L_m) \leq 3$, we construct a fault-tolerant resolving set. Let $Q_f = \{v_1, v_3, v_{2n-1}\}$; then, the representation of any vertex v_t of the graph with Q_f can be computed as

$$r(v_t|Q_f) = \left(\lceil\frac{t-1}{2}\rceil, \lceil\left|\frac{t-3}{2}\right|\rceil + w, \lceil\left|\frac{2n-1-t}{2}\right|\rceil + z\right), \tag{11}$$

where $w = 1$ for $t = 2$ and $z = 1$ for $t =$ even and zero, otherwise. Observe that all the vertices of L_m have a unique representation regarding fault-tolerant set Q_f. Hence, $\dim_f(L_m) = 3$.

The Möbius ladder was first introduced by Richard Guy and Frank Harary [26]. It is a three-regular graph created by a cycle C_n by joining vertices u, v of the cycle iff $d(u, v) = \operatorname{diam}(C_n)$. Möbius ladder has an even number of vertices. It is denoted by M_n, where $n = 2m$. The vertices which satisfy this condition are called antipodal. The graph of the Möbius ladder is shown in Figure 3. Möbius ladders have several uses in different fields of sciences such as stereochemistry, computer networks, and electrotechnology. In the next result, we find the fault-tolerant metric dimension of the Möbius ladder. □

Theorem 7. *Let M_n be a Möbius ladder graph. Then,*

$$\dim_f(M_n) = \begin{cases} 4 & \text{if } m \geq 4 \text{ and } m = \text{even}, \\ 5 & \text{if } m \geq 5 \text{ and } m = \text{odd}. \end{cases} \tag{12}$$

Proof. Let $m \geq 4$ be an even integer. The inequality $\dim_f(M_n) \geq 4$ follows from Theorem 2 and equation (1). To prove $\dim_f(M_n) \leq 4$, we construct a set satisfying the definition of fault-tolerant metric dimension. Let $Q_f = \{v_1, v_2, v_m, v_{m+1}\}$; then, the distances of any vertex v_t of the graph corresponding to Q_f can be computed as follows.

Distance of vertex v_t from vertex v_1 is

$$d(v_t, v_1) = \begin{cases} t-1 & \text{if } t = 1, 2, \ldots, \lfloor\frac{m+2}{2}\rfloor, \\ m-t+2 & \text{if } t = \lfloor\frac{m+2}{2}\rfloor + 1, \ldots, m+1, \\ t-m & \text{if } t = m+2, \ldots, \lceil\frac{m+2}{2}\rceil, \\ 2m-t+1 & \text{if } t = \lceil\frac{m+2}{2}\rceil + 1, \ldots, 2m. \end{cases} \tag{13}$$

Distance of vertex v_t from vertex v_2 is

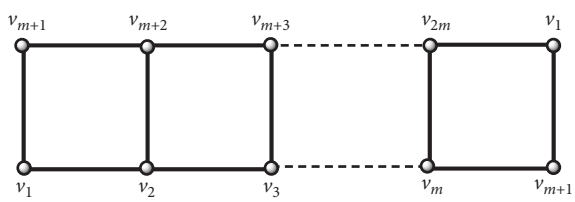

Figure 3: Möbius ladder graph M_n.

$$d(v_t, v_2) = \begin{cases} |t-2| & \text{if } t = 1, 2, \ldots, \lceil \frac{m+4}{2} \rceil, \\ m - t + 3 & \text{if } t = \lceil \frac{m+2}{2} \rceil + 1, \ldots, m+1, \\ t - m - 1 & \text{if } t = m+2, \ldots, \lfloor \frac{3m}{2} \rfloor + 1, \\ 2m - t + 2 & \text{if } t = \lfloor \frac{3m}{2} \rfloor + 2, \ldots, 2m. \end{cases} \tag{14}$$

Distance of vertex v_t from vertex v_m is

$$d(v_t, v_m) = \begin{cases} |m-t| & \text{if } t = \lfloor \frac{m}{2} \rfloor, \lfloor \frac{m}{2} \rfloor + 1, \ldots, m+1, \\ t+1 & \text{if } t = 1, 2, \ldots, \lfloor \frac{m}{2} \rfloor - 1, \\ t - m & \text{if } t = m+2, \ldots, \lceil \frac{3m}{2} \rceil, \\ 2m - t + 1 & \text{if } t = \lceil \frac{3m}{2} \rceil + 1, \ldots, 2m. \end{cases} \tag{15}$$

Distance of vertex v_t from vertex v_{m+1} is

$$d(v_t, v_{m+1}) = \begin{cases} t & \text{if } t = 1, 2, \ldots, \lfloor \frac{m+1}{2} \rfloor, \\ m - t + 1 & \text{if } t = \lfloor \frac{m+1}{2} \rfloor + 1, \ldots, m+1, \\ t - m - 1 & \text{if } t = m+2, \ldots, \lceil \frac{3m}{2} \rceil + 1, \\ 2m - t + 2 & \text{if } t = \lceil \frac{3m}{2} \rceil + 2, \ldots, 2m. \end{cases} \tag{16}$$

Since all the vertices of M_n have unique representations regarding the resolving set Q_f, hence, $\dim_f(M_n) = 4$.

Now, let $m \geq 5$ be an odd integer. To prove that $\dim_f(M_n) = 5$, we construct a resolving set satisfying the condition of the fault-tolerant metric dimension. Let $Q_f = \{v_1, v_2, v_{(m+5/2)}, v_{m+2}, v_{(3m+5/2)}\}$; then, the distance of any vertices v_t from the vertex $v_{(m+5/2)}$, v_{m+2}, and $v_{(3m+5/2)}$ is as follows.

Distance of vertex v_t from vertex $v_{(m+5/2)}$ is

$$d(v_t, v_{(m+5/2)}) = \begin{cases} \frac{m-3}{2} + t & \text{if } t = 1, 2, \\ \left| 3 - t + \frac{m-1}{2} \right| & \text{if } t = 3, 4, \ldots, m+1, \\ t - \frac{m+5}{2} & \text{if } t = m+2, m+3, \\ \left| t - \frac{3m+5}{2} \right| + 1 & \text{if } t = m+4, \ldots, 2m. \end{cases} \tag{17}$$

Distance of vertex v_t from vertex v_{m+2} is

$$d(v_t, v_{m+2}) = \begin{cases} |t-2| + 1 & \text{if } t = 1, 2, \ldots, \frac{m+1}{2}, \\ m - t + 2 & \text{if } t = \frac{m+3}{2}, \ldots, m+1, \\ t - m - 2 & \text{if } t = m+2, m+3, \ldots, \frac{3(m+1)}{2}, \\ 3 - t + 2m & \text{if } t = \frac{3m+5}{2}, \ldots, 2m. \end{cases} \tag{18}$$

Distance of vertex v_t from vertex $v_{(3m+5/2)}$ is

$$d(v_t, v_{(3m+5/2)}) = \begin{cases} \frac{m-5}{2} + t & \text{if } t = 1, 23, \\ \left| \frac{m+5}{2} - t \right| + 1 & \text{if } t = 4, \ldots, m+1, \\ \left| t + \frac{3m+5}{2} \right| & \text{if } t = m+2, m+3, \ldots, 2m. \end{cases} \tag{19}$$

Since all the vertices of M_n have unique representations regarding decided resolving set Q_f, hence, $\dim_f(M_n) = 5$.

A hexagonal Möbius ladder $\mathrm{HML}(m, n)$ is constructed by subdividing each horizontal edge of the square lattice; thus, we obtained a lattice in which each cycle has order six; after creating this lattice, twist the recently attained lattice 180° and join the utmost left and right vertices as shown in Figure 4. Hexagonal Möbius ladder $\mathrm{HML}(m, n)$ contains m horizontal and n vertical cycles. Let $\mathrm{HML}_m = \mathrm{HML}(m, 1)$ be

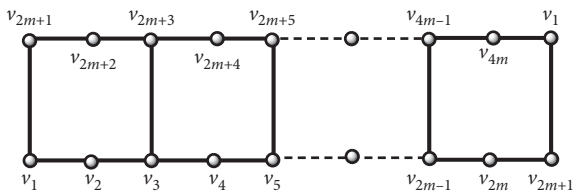

Figure 4: Hexagonal Möbius ladder graph HML_m.

a hexagonal Möbius ladder with m horizontal cycles and 1 vertical cycle. In the next theorem, we compute the fault-tolerant metric dimension of HML_m. □

Theorem 8. *Let HML_m be a hexagonal Möbius ladder graph. Then,*

$$\dim_f(\mathrm{HML}_m) = \begin{cases} 3 & \text{if } m = 2, \\ 4 & \text{if } m \geq 3. \end{cases} \tag{20}$$

Proof. Let $m = 2$; the inequality $\dim_f(\mathrm{HML}_m) \geq 3$ follows from Theorem 3 and equation (1). To prove $\dim_f(\mathrm{HML}_m) \leq 3$, we construct a set satisfying the definition of fault-tolerant metric dimension. Let $Q_f = \{v_1, v_4, v_6\}$; then, the distances of any vertex v_t of the graph corresponding to Q_f can be computed as follows.

Distance of vertex v_t from vertex v_1 is

$$d(v_t, v_1) = \begin{cases} 1 & \text{if } t = 2, 5, 8, \\ 2 & \text{if } t = 3, 4, 6, 7. \end{cases} \tag{21}$$

Distance of vertex v_t from vertex v_4 is

$$d(v_t, v_4) = \begin{cases} 1 & \text{if } t = 3, 5, \\ 2 & \text{if } t = 1, 2, 6, 7, \\ 3 & \text{if } t = 8. \end{cases} \tag{22}$$

Distance of vertex v_t from vertex v_6 is

$$d(v_t, v_6) = \begin{cases} 1 & \text{if } t = 5, 7, \\ 2 & \text{if } t = 1, 3, 4, 8, \\ 3 & \text{if } t = 2. \end{cases} \tag{23}$$

It is easy to see that every vertex of HML_m has its unique representation with the resolving set Q_f. Hence, $\dim_f(\mathrm{HML}_m) = 2$.

Let $m \geq 3$; to prove that $\dim_f(\mathrm{HML}_m) = 4$, it is enough to construct a fault-tolerant resolving set of four elements that satisfy the condition of fault-tolerant metric dimension. Let $Q_f = \{v_1, v_2, v_{2m}, v_{2m+2}\}$; then, the distance of any vertex v_t of HML_m with Q_f can be computed as follows.

Distance of vertex v_t from vertex v_1 is

$$d(v_t, v_1) = \begin{cases} t - 1 & \text{if } t = 1, 2, \ldots, m + 1, \\ 2m - t + 2 & \text{if } t = m + 2, \ldots, 2m + 1, \\ t - 2m & \text{if } t = 2m + 2, \ldots, 3m, \\ 4m - t + 1 & \text{if } t = 3m + 1, \ldots, 4m. \end{cases} \tag{24}$$

Distance of vertex v_t from vertex v_2 is

$$d(v_t, v_2) = \begin{cases} |t - 2| & \text{if } t = 1, 2, \ldots, m + 2, \\ 2m - t + 3 & \text{if } t = m + 3, \ldots, 2m + 1, \\ t - 2m - 1 & \text{if } t = 2m + 3, \ldots, 3m + 1, \\ t - 2m + 1 & \text{if } t = 2m + 2, \\ 4m - t + 2 & \text{if } t = 3m + 2, \ldots, 4m. \end{cases} \tag{25}$$

Distance of vertex v_t from vertex v_{2m} is

$$d(v_t, v_{2m}) = \begin{cases} t + 1 & \text{if } t = 1, 2, \ldots, m - 1, \\ |2m - t| & \text{if } t = m, \ldots, 2m + 1, \\ t - 2m & \text{if } t = 2m + 2, \ldots, 3m, \\ 4m - t + 1 & \text{if } t = 3m + 1, \ldots, 4m. \end{cases} \tag{26}$$

Distance of vertex v_t from vertex v_{2m+2} is

$$d(v_t, v_{2m+2}) = \begin{cases} |t - 2| + 1 & \text{if } t = 1, 3, 4, 5, \ldots, \lceil \frac{2m+1}{2} \rceil, \\ 3 & \text{if } t = 2, \\ 2m - t + 2 & \text{if } t = \lceil \frac{2m+1}{2} \rceil + 1, \ldots, 2m + 1, \\ t - 2m - 2 & \text{if } t = 2m + 2, 2m + 3, \ldots, \lceil \frac{6m+3}{2} \rceil, \\ 3 - t + 4m & \text{if } t = \lceil \frac{6m+3}{2} \rceil + 1, \ldots, 4m. \end{cases} \tag{27}$$

It is easy to see that all the vertices of HML_m have unique representations regarding resolving set Q_f. Hence, $\dim_f(\mathrm{HML}_m) = 3$. □

4. Conclusion

In this article, we study the fault-tolerant metric dimension of the triangular snake graph, ladder, Möbius ladder, and hexagonal Möbius ladder. It is known that, for cycle graph C_n, we have $\dim_f(C_n) = \dim(C_n) + 1$. Therefore, it is interesting to find some classes of graph in which this equality holds. Note that, in case of the triangular snake graph, ladder, Möbius ladder, and hexagonal Möbius ladder, we have $\dim_f = \dim + 1$.

Data Availability

No data were used to support this study.

Disclosure

This research was carried out as a part of the employment of the authors.

Conflicts of Interest

The authors declare no conflicts of interest.

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