Hindawi
Complexity
Volume 2023, Article ID 6628901, 10 pages
https://doi.org/10.1155/2023/6628901

Research Article

A New Robust Adaptive Control Method for Complex Nontriangular Nonlinear Systems

Lingrong Xue [1] and Zhen-Guo Liu [2]

[1] *School of Information, Shanxi University of Finance and Economics, Taiyuan 030006, China*
[2] *School of Automation and Software Engineering, Shanxi University, Taiyuan 030006, China*

Correspondence should be addressed to Zhen-Guo Liu; lzg819@163.com

Received 22 March 2023; Revised 21 May 2023; Accepted 12 June 2023; Published 28 June 2023

Academic Editor: Qingdu Li

Copyright © 2023 Lingrong Xue and Zhen-Guo Liu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The existing research studies on adaptive control frequently introduce many parameter estimations and lead to a complicated controller. This paper investigates the robust regulation issue for high-order system and plants to raise a new approach for adaptive control. Specifically, the considered system has odd system power, nontriangular form, and external disturbance. By introducing the transformations of a parameter estimation, the studied system is transformed into a new dynamic system. By employing fuzzy systems and some inequality skills, the appropriate bounds of nonlinear terms are established. Based on the adaptive method and homogeneous control, a recursive control design algorithm is provided to construct a new adaptive controller, which dominates those uncertain bounds and guarantees that the closed-loop system is semiglobally uniformly ultimately bounded (SUUB). The constructed controller employs only one adaptive law and has a much simpler form. Simulation examples verify the validness of the presented method.

1. Introduction

In engineering, plenty of practical plants [1, 2] suffer from various disturbances, which produce bad influences to system responses and stability. For such kind of systems, robust control approaches played an important role in stabilizing the system [3]. On the other hand, multiple uncertainties exist in nonlinear systems and often add the obstacles to design the controllers [4, 5]. To overcome the difficulty, it is necessary to employ an effect tool, i.e., the adaptive technique, which mainly utilizes the idea of certainty equivalence [6]. In addition, some nonlinear systems may have nontriangular form [7, 8], which makes the traditional methods such as the backstepping method and the sliding mode approach difficult to apply. Since different nonlinear systems own their special structures, it is unable to present a universal control strategy to regulate all systems [9]. Up to now, there still exist many challenging control issues for nonlinear systems.

Controls of nonlinear systems have been a hot topic in recent years [10–12]. Scholars mainly employed the idea of state feedback or output feedback during the design procedure. Specifically, in control design steps, state feedback control utilizes all the system state signals, while output feedback control uses partial measurable signals and the signals obtained from the constructed state observer. For systems with uncertainties, many excellent strategies were reported over the past years. For instance, to achieve the asymptotic tracking control, the authors in [13] presented an adaptive σ-modification control method for systems that contained uncertain control directions and the authors in [14] raised a novel adaptive recursive RISE control for system with mismatched uncertainties. To obtain the asymptotic prescribed tracking performance, the authors in [15] proposed a RBFNN-based adaptive control algorithm for complicated systems with heavy modeling uncertainties. To study the finite-time control problem, the authors in [16] provided a fuzzy based event-triggered controller for uncertain systems, and the authors in [17] studied quantized systems and proposed an finite-time controller using the adaptive strategy. When the system involves unmeasurable states, the authors in [18] raised an adaptive neural output feedback scheme, and the authors in [19] gave a new extended-state-observer based adaptive control strategy. The authors in [20] further studied stochastic systems using the dynamic gain and the state observer. For systems with external disturbances, some robust control approaches were employed, see [21] and references therein.

When the system powers are odd numbers, the related control problems are more difficult. One reason lies in that many existing approaches are not applicable since the system is not feedback linearization and the upper bounds of nonlinear terms are difficult to identify. To tackle the control problems of systems with odd system powers, the authors in [22] proposed the adding a power integrator (API) scheme. Later, the domination idea and homogeneous properties were applied to solve the regulation of homogeneous systems [23]. Recently, these methods have been extended to solve many nonlinear systems. Particularly, by employing API scheme and dynamic gain strategy, the authors in [24] designed a stable controller for time-delay systems. Utilizing homogeneous domination (HD) approach, the authors in [25] studied nonlinear systems and constructed a homogeneous controller to make the system globally stable. For some uncertain systems, the authors in [26, 27] considered systems with different structures and designed the associated controller. Actually, control issues of nontriangular systems are more challenging. So far, intelligent control strategies such as fuzzy control, neural network control, and evolutionary computation have been applied to regulate different systems. However, the obtained controller tends to be very complicated. Naturally, a problem is: *Is it possible to present an adaptive control method which leads to a simpler controller for nontriangular systems with odd system power?*

This work tries to give a solution to the problem. The main contributions are as follows:

(i) In this work, a new adaptive control strategy is proposed. During the control design steps, the adaptive-law-based transformation is introduced and a new dynamic system is obtained. By choosing an appropriate Lyapunov function using a recursive design method, a control input is designed without using the information of the unknown functions. By using homogeneous domination method and fuzzy systems, the unknown functions are dominated via the designed adaptive law at last steps. In this way, this algorithm yields a new adaptive controller by allowing the control input and the adaptive law to be designed separately.

(ii) The studied system is more general. It can be seen from three aspects. The studied system has high-order power, which makes many methods for low-order system difficult to work. Since the system is in nontriangular structure, it is not easy to utilize the existing methods for triangular systems to study the control problem [23, 25]. Besides, the existing of unknown functions and disturbance further add new obstacles to the control design.

2. Problem Formulation

Consider nontriangular nonlinear system

$$\begin{cases} \dot{x}_1 = c_1(t)x_2^p + f_1(\theta, x_1, \ldots, x_n) + w(t), \\ \dot{x}_2 = c_2(t)x_3^p + f_2(\theta, x_1, \ldots, x_n) + w(t), \\ \vdots \\ \dot{x}_n = c_n(t)u^p + f_n(\theta, x_1, \ldots, x_n) + w(t), \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^\top \in \mathscr{R}$ is the state vector, $u \in \mathscr{R}$ denotes the system input, θ is the unknown parameter vector, $w(t)$ is the disturbance, $f_i(\cdot)$ is the continuous function, and p is the odd system power. For $1 \le i \le n$, $c_i(t)$ satisfies $c_0 \le c_i(t) \le c_m$, where $c_0 > 0$ and $c_m > 0$ are known constants. $|w(t)| \le w_0$, where $w_0 > 0$ is a constant.

Remark 1. It shows that system (1) has odd system power and is in a nontriangular form. It cannot be regulated with the conventional methods like the backstepping control and the sliding control for triangular systems. The existing methods for odd system power such as the API scheme and the HD approach are also inapplicable since it is difficult to deal with those nontriangular uncertain nonlinear terms. In this work, by introducing new transformations, we will transform the system into a new one. Then, we propose a fuzzy approximation based HD method for control design.

The following Lemmas are needed later.

Lemma 2 (See [25]). *The inequality* $|\kappa G^\delta H^\lambda| \le c|G|^{\delta+\lambda} + \lambda/(\delta+\lambda)(\delta/c(\delta+\lambda))^{(\delta/\lambda)}|\kappa|^{\delta+\lambda/\lambda}|H|^{\delta+\lambda}$ *holds, where* $\kappa, G, H \in \mathscr{R}$ *are any constants,* $\delta, \lambda, c \in \mathscr{R}$ *are positive constants.*

Lemma 3 (See [27]). *For any constant* $\delta_0 > 0$ *and a continuous function* $\pi_i(\theta, X)$ *defined on the compact set* Λ*, a fuzzy system* $\Psi_i^\top L_i(X)$ *exists and guarantees* $\sup_{X\in\Lambda}|\pi_i(\theta, X) - \Psi_i^\top L_i(X)| \le \delta_0$.

3. Main Results

Introduce the transformations

$$\eta_i = \frac{x_i}{H^{r_i}(\widehat{\vartheta}(t))}, \nu(t) = \frac{u(t)}{H^{r_{n+1}}(\widehat{\vartheta}(t))}, i = 1, \ldots, n, \tag{2}$$

where $r_1 = r_0 \ge 1$, $r_{i+1}p = r_i + r_0$, $\widehat{\vartheta}(t)$ is the estimation of parameter ϑ to be defined later and satisfies $\widehat{\vartheta}(t) \ge 1$, $H(\widehat{\vartheta})$ is selected such that $H(\widehat{\vartheta}) \ge 1$, $dH(\widehat{\vartheta})/d\widehat{\vartheta} > 0$. Then, we transform system (1) into

$$\begin{cases} \dot{\eta}_i = H^{r_0}(\widehat{\vartheta})c_i(t)\eta_{i+1}^p - \dfrac{r_i}{H(\widehat{\vartheta})}\dfrac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}\eta_i + F_i(\cdot) + W_i(\cdot), i = 1, \ldots, n-1, \\ \dot{\eta}_n = H^{r_0}(\widehat{\vartheta})c_n(t)\nu^p - \dfrac{r_n}{H(\widehat{\vartheta})}\dfrac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}\eta_n + F_n(\cdot) + W_n(\cdot), \end{cases} \tag{3}$$

where $F_j(\cdot) = H^{-r_j}(\widehat{\vartheta})f_j$, $W_j(\cdot) = H^{-r_j}(\widehat{\vartheta})w(t)$, $1 \le j \le n$.

Remark 4. In (3), the adaptive law should be designed such that $\widehat{\vartheta} \ge 0$, $H(\widehat{\vartheta}) \ge 1$. In this way, it can be utilized to dominate some nonlinear terms raised during the control design. For details, see the following.

Next, we give the following results.

Theorem 5. *System (1) has the adaptive controller*

$$\begin{cases} u = -H^{r_{n+1}}(\widehat{\vartheta})\beta_n S_n, \\ \dot{\widehat{\vartheta}} = \dfrac{\lambda}{H^q(\widehat{\vartheta})} \displaystyle\sum_{j=1}^{n} S_j^{2p}, \widehat{\vartheta}(0) \ge 1, \end{cases} \tag{4}$$

where r_n, q, λ and β_j are constants to be specified, $\widehat{\vartheta}$ is estimation of an unknown parameter ϑ defined later, $S_1 = \eta_1$, $S_j = \eta_j - \eta_j^*$, $\eta_j^* = -\beta_{j-1}S_{j-1}$, $j = 2, \ldots, n$. It ensures that all system solutions are semiglobally uniformly ultimately bounded (SUUB).

Proof. We first provide the control design procedure. Later, we perform the stability analysis. □

Part 6. Control design procedure.

Initial Step 7. Choosing the function $V_1 = 1/(p+1)S_1^{p+1}$, we get

$$\begin{aligned} \dot{V}_1 &= S_1^p\left(H^{r_0}(\widehat{\vartheta})c_1(t)\eta_2^p - \frac{r_1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}\eta_1 + F_1(\cdot) + W_1(\cdot)\right) \\ &= H^{r_0}(\widehat{\vartheta})c_1(t)S_1^p\left(\eta_2^p - \eta_2^{*p}\right) + H^{r_0}(\widehat{\vartheta})c_1(t)S_1^p\eta_2^{*p} - \frac{r_1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_1^{p+1} + S_1^p(F_1 + W_1). \end{aligned} \tag{5}$$

By Lemma 3, there exists a fuzzy system $\Psi_1^{\top}L_1$ such that $f_1 = \Psi_1^{\top}L_1 + \delta_1, |\delta_1| \le \delta_0$. Then, we obtain

$$\begin{aligned} S_1^p F_1 &= H^{-r_1}(\widehat{\vartheta})S_1^p\left(\Psi_1^{\top}L_1 + \delta_1\right) \\ &\le H^{-r_1}(\widehat{\vartheta})|S_1|^p\left(\left|\Psi_1^{\top}L_1\right| + |\delta_0|\right) \\ &\le H^{-r_1}(\widehat{\vartheta})\left(a_1\left\|\Psi_1\right\|^2 S_1^{2p} + \frac{1}{4a_1}\left\|L_1\right\|^2 + a_1S_1^{2p} + \frac{1}{4a_1}\delta_0^2\right), \end{aligned} \tag{6}$$

$$S_1^p W_1 = H^{-r_1}(\widehat{\vartheta})S_1^p w_1 \le H^{-r_1}(\widehat{\vartheta})\left(a_1S_1^{2p} + \frac{1}{4a_1}w_0^2\right). \tag{7}$$

Choosing $\eta_2^* = -(na_1/c_0)^{1/p}S_1 =: -\beta_1 S_1$ and substituting (6) and (7) into (5), we have

$$\begin{aligned} \dot{V}_1 \le & -n\rho_0 a_1 H^{r_0}(\widehat{\vartheta})S_1^{2p} - \frac{h_1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_1^{p+1} \\ & + H^{r_0}(\widehat{\vartheta})c_1(t)S_1^p\left(\eta_2^p - \eta_2^{*p}\right) \\ & + H^{-r_1}(\widehat{\vartheta})\left(2a_1 + a_1\left\|\Psi_1\right\|^2\right)S_1^{2p} + \frac{D_1}{H^{r_1}(\widehat{\vartheta})a_1}, \end{aligned} \tag{8}$$

where $\rho_0 = 1$, $h_1 = r_1$, $D_1 = 1/4\left(\|L_1\|^2 + \delta_0^2 + w_0^2\right)$.

Step k $(k \ge 2)$. Suppose that for step $k-1$ there exists a continuous function V_{k-1} and transformations $S_j = \eta_j - \eta_j^*$, $\eta_j^* = -\beta_{j-1}S_{j-1}$, $j = 2, \ldots, k$ which satisfy

$$\begin{aligned} \dot{V}_{k-1} \le & -(n-k+2)\rho_0 \cdots \rho_{k-2}\sum_{j=1}^{k-1} H^{r_0}(\widehat{\vartheta})a_jS_j^{2p} - \sum_{j=1}^{k-1}\frac{h_{k-1}}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_j^{p+1} \\ & + c_{k-1}H^{r_0}(\widehat{\vartheta})S_{k-1}^p\left(\eta_k^p - \eta_k^{*p}\right) + \sum_{i=1}^{k-2}\rho_i \cdots \rho_{k-2}\left(\sum_{j=1}^{i} H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p}\right. \\ & \left. + \sum_{j=1}^{i} H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j\right) + \sum_{j=1}^{k-1} H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p} + \sum_{j=1}^{k-1} H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j. \end{aligned} \tag{9}$$

In this step, choose $V_k = \rho_{k-1}V_{k-1} + 1/(p+1)S_k^{p+1}$, which leads to

$$\begin{aligned}\dot{V}_k &= \rho_{k-1}\dot{V}_{k-1} + S_k^p\left(\dot{\eta}_k - \dot{\eta}_k^*\right)\\ &\le -(n-k+2)\rho_0\cdots\rho_{k-1}\sum_{j=1}^{k-1}H^{r_0}(\widehat{\vartheta})a_jS_j^{2p} - \sum_{j=1}^{k-1}\frac{\rho_{k-1}h_{k-1}}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_j^{p+1}\\ &\quad + \rho_{k-1}c_{k-1}H^{r_0}(\widehat{\vartheta})S_{k-1}^p\left(\eta_k^p - \eta_k^{*p}\right) + \sum_{i=1}^{k-1}\rho_i\cdots\rho_{k-1}\left(\sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p}\right.\\ &\quad \left. + \sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j\right) + S_k^p\left(H^{r_0}(\widehat{\vartheta})c_k(t)\eta_{k+1}^p - \frac{r_k}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}\eta_k + F_k + W_k - \sum_{j=1}^{k-1}\frac{\partial\eta_k^*}{\partial\eta_j}\dot{\eta}_j\right).\end{aligned} \tag{10}$$

By Lemma 2, we obtain

$$\rho_{k-1}c_{k-1}H^{r_0}(\widehat{\vartheta})S_{k-1}^p\left(\eta_k^p - \eta_k^{*p}\right) \le H^{r_0}(\widehat{\vartheta})\phi_{k1}S_k^{2p} + H^{r_0}(\widehat{\vartheta})\frac{\rho_0\cdots\rho_{k-1}}{3}S_{k-1}^{2p}, \tag{11}$$

where $\phi_{k1} > 0$ is a constant. Noting that $H(\widehat{\vartheta}) \ge 1$, we obtain

$$-S_k^p\frac{r_k}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}\eta_k \le -r_kb_0\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_k^{p+1} + B_{k1}\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_{k-1}^{p+1}, \tag{12}$$

where $0 < b_0 < 1$ and $B_{k1} > 0$ are constants. By Lemma 3, there is a fuzzy system $\Psi_k^\top L_k$ such that $f_k = \Psi_k^\top L_k + \delta_k$, $|\delta_k| \le \delta_0$. By Lemma 2, we get $S_k^p\Psi_k^\top L_k \le a_k\|\Psi_k\|^2S_k^{2p} + \|L_k\|^2/(4a_k)$, $S_k^p\delta_k \le a_kS_k^{2p} + \delta_k^2/(4a_k)$, and $S_k^pw \le a_kS_k^{2p} + w_0^2/(4a_k)$, which and (3) yield that

$$S_k^p\left(F_k + W_k\right) \le H^{-r_k}(\widehat{\vartheta})\left(2a_k + a_k\left\|\Psi_k\right\|^2\right)S_k^{2p} + H^{-r_k}(\widehat{\vartheta})\frac{D_k}{4a_k}, \tag{13}$$

where a_k is a positive constant, $D_k = \|L_k\|^2 + \delta_0^2 + w_0^2$. Similarly, using Lemma 3, it is deduced that

$$\begin{aligned}-S_k^p\sum_{j=1}^{k-1}\frac{\partial\eta_k^*}{\partial\eta_j}\dot{\eta}_j &\le H^{r_0}(\widehat{\vartheta})\phi_{k2}S_k^{2p} + H^{r_0}(\widehat{\vartheta})\frac{2\rho_0\cdots\rho_{k-1}}{3}\sum_{j=1}^{k-1}S_j^{2p} + \frac{r_kb_0}{2}\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_k^{p+1}\\ &\quad + B_{k2}\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}\sum_{j=1}^{k-1}S_j^{p+1} + \sum_{j=1}^{k-1}H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p} + \sum_{j=1}^{k-1}H^{-r_j}(\widehat{\vartheta})\frac{D_j}{4a_j},\end{aligned} \tag{14}$$

where ϕ_{k2} and B_{k2} are positive constants, $D_j = 1/4(\|L_j\|^2 + \delta_0^2 + w_0^2)$. Considering that $\eta_{k+1}^p = \eta_{k+1}^p - \eta_{k+1}^{*p} + \eta_{k+1}^{*p}$ and choosing $\eta_{k+1}^* = -(((n-k+1)a_k + \phi_{k1} + \phi_{k2})/c_0)^{1/p}S_k$, it follows that

$$\begin{aligned}\dot{V}_k \le & -(n-k+1)\rho_0\cdots\rho_{k-1}\sum_{j=1}^{k}H^{r_0}(\widehat{\vartheta})a_jS_j^{2p} - h_k\sum_{j=1}^{k}\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_j^{p+1} \\ & + c_kH^{r_0}(\widehat{\vartheta})S_k^p\left(\eta_{k+1}^p - \eta_{k+1}^{*p}\right) + \sum_{i=1}^{k-1}\rho_i\cdots\rho_{k-1}\left(\sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p}\right. \\ & \left. + \sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j\right) + \sum_{j=1}^{k}H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p} + \sum_{j=1}^{k}H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j,\end{aligned} \quad (15)$$

where $h_k = \min\{\rho_{k-1}h_{k-1} - B_{k1} - B_{k2}, r_kb_0/2\}$. This completes the recursive design.

In the last step, we choose $V_n = \rho_{n-1}V_{n-1} + 1/p + 1\,S_n^{p+1} + \rho/2\lambda\widetilde{\vartheta}^2$, where $\lambda > 0$ is a constant, $\rho = \min_{1\le i\le n}\{\rho_0\cdots\rho_{n-1}a_i\}$, $\widetilde{\vartheta} = \widehat{\vartheta} - \vartheta$. Using the similar procedure, we obtain the control input

$$v = -\left(\frac{a_n + \phi_{n1} + \phi_{n2}}{c_0}\right)^{\frac{1}{p}}S_n =: -\beta_nS_n, \quad (16)$$

which gives that

$$\begin{aligned}\dot{V}_n \le & -\rho_0\cdots\rho_{n-1}\sum_{j=1}^{n}H^{r_0}(\widehat{\vartheta})a_jS_j^{2p} - h_n\sum_{j=1}^{n-1}\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_j^{p+1} \\ & + \sum_{i=1}^{n-1}\rho_i\cdots\rho_{n-1}\left(\sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p} + \sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j\right) \\ & + \sum_{j=1}^{n}H^{-r_j}(\widehat{\vartheta})\left(2a_j + a_j\left\|\Psi_j\right\|^2\right)S_j^{2p} + \sum_{j=1}^{n}H^{-r_j}(\widehat{\vartheta})a_j^{-1}D_j + \frac{\rho}{\lambda}\widetilde{\vartheta}\dot{\widehat{\vartheta}}.\end{aligned} \quad (17)$$

Noting that

$$\begin{aligned}&\sum_{i=1}^{n-1}\rho_i\cdots\rho_{n-1}\sum_{j=1}^{i}H^{-r_j}(\widehat{\vartheta})2a_jS_j^{2p} \\ &\le H^{-q}(\widehat{\vartheta})\sum_{i=1}^{n-1}\left(\rho_i\cdots\rho_{n-1}\left(2a_1S_1^{2p} + \cdots + 2a_iS_i^{2p}\right)\right) \\ &= H^{-q}(\widehat{\vartheta})\rho_1\cdots\rho_{n-1}2a_1S_1^{2p} \\ &\quad + H^{-q}(\widehat{\vartheta})\rho_2\cdots\rho_{n-1}\left(2a_1S_1^{2p} + 2a_2S_2^{2p}\right) \\ &\quad + H^{-q}(\widehat{\vartheta})\rho_3\cdots\rho_{n-1}\left(2a_1S_1^{2p} + 2a_2S_2^{2p} + 2a_3S_3^{2p}\right) \\ &\quad + \cdots + H^{-q}(\widehat{\vartheta})\rho_{n-1}\left(2a_1S_1^{2p} + \cdots + 2a_{n-1}S_{n-1}^{2p}\right) \\ &= H^{-q}(\widehat{\vartheta})\sum_{i=1}^{n-1}\left(\sum_{j=i}^{n-1}\rho_j\cdots\rho_{n-1}\right)\cdot 2a_iS_i^{2p},\end{aligned} \quad (18)$$

where $q = \min_{1\le j\le n}\{r_j\}$. Defining $A = \max_{1\le i\le n-1}\{\sum_{j=i}^{n-1}\rho_j\cdots\rho_{n-1}\cdot 2a_i + 2a_i, 2a_n\}$, $D = \max_{1\le i\le n-1}\{\sum_{j=i}^{n-1}\rho_j\cdots\rho_{n-1}a_j^{-1}D_j + a_j^{-1}D_j, a_n^{-1}D_n\}$, we have

$$\begin{aligned}\dot{V}_n \le & -\rho\sum_{j=1}^{n}H^{r_0}(\widehat{\vartheta})S_j^{2p} - h_n\sum_{j=1}^{n-1}\frac{1}{H(\widehat{\vartheta})}\frac{dH(\widehat{\vartheta})}{d\widehat{\vartheta}}\dot{\widehat{\vartheta}}S_j^{p+1} \\ & + \frac{A\left(1 + \max_{1\le j\le n}\left\|\Psi_j\right\|^2\right)}{H^q(\widehat{\vartheta})}\sum_{j=1}^{n}S_j^{2p} + \frac{D}{H^q(\widehat{\vartheta})} + \frac{\rho}{\lambda}\widetilde{\vartheta}\dot{\widehat{\vartheta}}.\end{aligned} \quad (19)$$

Defining $\vartheta = A(1 + \max_{1\le j\le n}\|\Psi_j\|^2)/\rho$ and noting that $-h_n\sum_{j=1}^{n-1}1/H(\widehat{\vartheta})dH(\widehat{\vartheta})/d\widehat{\vartheta}\dot{\widehat{\vartheta}}S_j^{p+1} \le 0$, we have

$$\begin{aligned}\dot{V}_n \le & -\rho(1-\epsilon)\sum_{j=1}^{n}H^{r_0}(\widehat{\vartheta})S_j^{2p} \\ & -\rho\left(\epsilon H^{r_0}(\widehat{\vartheta}) - \frac{\widehat{\vartheta}}{H^q(\widehat{\vartheta})}\right)\sum_{j=1}^{n}S_j^{2p} \\ & + \frac{\rho}{\lambda}\widetilde{\vartheta}\left(\dot{\widehat{\vartheta}} - \frac{\lambda}{H^q(\widehat{\vartheta})}\sum_{j=1}^{n}S_j^{2p}\right) + \frac{D}{H^q(\widehat{\vartheta})},\end{aligned} \quad (20)$$

where $0 < \epsilon < 1$ is a constant. Choosing $H(\widehat{\vartheta}) = (\widehat{\vartheta}/\epsilon)^{1/r_0+q}$ and

$$\dot{\widehat{\vartheta}} = \frac{\lambda}{H^q(\widehat{\vartheta})} \sum_{j=1}^{n} S_j^{2p}, \widehat{\vartheta}(0) \geq 1, \tag{21}$$

we obtain

$$\dot{V}_n \leq -\overline{\rho} \sum_{j=1}^{n} S_j^{2p} + \overline{D}, \tag{22}$$

where $\overline{\rho} \leq \rho(1-\epsilon)$ and $\overline{D} = D$ are constants.

Part 8. Stability analysis. Define the set $\Lambda = \{S \mid \sum_{j=1}^{n} S_j^{2p} \leq \overline{D}/\overline{\rho}\}$, where $S = [S_1, \ldots, S_n]^\top$. From (22), it follows that S will stay in Λ for all $S \in \Lambda$. For S which satisfies $\sum_{j=1}^{n} S_j^{2p} > \overline{D}/\rho$, S will converge into the set Λ. Hence, the vector S is bounded. Actually, $\widehat{\vartheta}(t)$ is also bounded on $[0, +\infty)$. We prove it by contradiction. Suppose that there is a finite time T which satisfies $\lim_{t \longrightarrow T} \widehat{\vartheta}(t) = +\infty$. Then, integrate both sides of (21) and noting that $H^q(\widehat{\vartheta}) \geq 1$, we obtain

$$\begin{aligned} \widehat{\vartheta}(T) - \widehat{\vartheta}(0) &= \int_0^T \dot{\widehat{\vartheta}}(l) dl = \int_0^T \frac{\lambda}{H^q(\widehat{\vartheta}(l))} \sum_{j=1}^{n} S_j^{2p}(l) \mathrm{d}l \\ &\leq \lambda \int_0^T \sum_{j=1}^{n} S_j^{2p}(l) \mathrm{d}l < \lambda_0, \end{aligned} \tag{23}$$

where $\lambda_0 > 0$ is a constant. Hence, $\widehat{\vartheta}(T) \leq \widehat{\vartheta}(0) + \lambda_0$, which is in contradiction with $\lim_{t \longrightarrow T} \widehat{\vartheta}(t) = +\infty$. Therefore, $\widehat{\vartheta}(t)$ is bounded. Utilizing $S_1 = \eta_1, S_j = \eta_j - \eta_j^* = \eta_j - \beta_{j-1} S_{j-1}, j = 2, \ldots, n$, it yields that $\eta_i, 1 \leq i \leq n$ are bounded. By the definition of $H^q(\widehat{\vartheta})$ and the boundedness of $\widehat{\vartheta}$, we know that $x_i, 1 \leq i \leq n$ are bounded. So, for any initial condition, all the system signals are bounded. Thus, the system is SUUB.

Remark 6. This paper presents an adaptive control strategy, which guarantees a SUUB stability result. The advantages of the result are as follows: (1) the result is applicable to systems that require less stronger conditions. In many existing results, the nonlinear conditions should satisfy some growing conditions, such as linear growing condition in [28], high-order growing condition [26], and homogeneous growing condition [23]. These conditions are somewhat stronger. Also, the systems should satisfy the triangular form. In this work, the nonlinear functions can be in a more general form which covers all those conditions. Besides, the considered system has a nontriangular structure. (2) The result is achieved via a much simpler adaptive controller. Different from the methods [2, 6, 24], this paper employs one adaptive law and does not introduce many complicated nonlinear terms in the virtual controllers to dominate the uncertainties in each steps. Instead, we utilize the adaptive law to dominate them in the last design steps. Hence, the presented adaptive controller has a much simpler form. (3) The result can be extended to asymptotic stability under some nonlinear growing conditions. For instance, suppose that the disturbance $w = 0$ and the functions satisfy $f_i \leq \overline{\theta} \sum_{i=1}^{n} |x_i|^p$. Following the similar deductions, we can obtain $S_k^p F_k \leq H^{r_0 - a}(\widehat{\vartheta}) S_k^{2p} + H^{r_0 - a}(\widehat{\vartheta}) \overline{\theta} C_k \sum_{i=1}^{k} |S_i|^{2p}$, where $a > 0, C_k > 0$ are constants. Following the similar design steps, we can design the adaptive controller such that the derivative of the Lyapunov function satisfies $\dot{V}_n \leq -\overline{\rho} \sum_{j=1}^{n} S_j^{2p}$. By the Lyapunov stability theorem, we can show that the system is globally asymptotically stability.

4. Simulation Example

Example 1. We study the single-link robot system (see [29]):

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 - Bx_2 - g\sin(x_1) + \mu(x_1), \\ \dot{x}_3 = u - \dfrac{K}{M} x_2 - \dfrac{L}{M} x_3, \end{cases} \tag{24}$$

where B, K, L are all unknown parameters, $\mu(x_1) = \sin(x_1)$. With the help of the method in Theorem 5, we obtain the adaptive controller

$$\begin{cases} u = -H^{r_4}(\widehat{\vartheta}) \beta_3 S_3, \\ \dot{\widehat{\vartheta}} = \dfrac{\lambda}{H^q(\widehat{\vartheta})} \sum_{i=1}^{3} S_i^2, \widehat{\vartheta}(0) = 1, \end{cases} \tag{25}$$

where $S_1 = x_1/H^{r_1}(\widehat{\vartheta})$, $S_2 = x_2/H^{r_2}(\widehat{\vartheta}) + \beta_1 S_1$, and $S_3 = x_3/H^{r_3}(\widehat{\vartheta}) + S_2$. In the simulation, $H(\widehat{\vartheta}) = (\widehat{\vartheta}/\epsilon)^{1/r_0 + q}$, the system parameters are selected as $M = 1$, $L = 5$, $B = 1$, $g = 10$, and $K = 10$. The design parameters are $q = 1$, $\epsilon = 2/3$, $r_0 = r_1 = 1$, $r_2 = 2$, $r_3 = 3$, $r_4 = 4$, $\beta_1 = 1.5, \beta_2 = 2$, $\beta_3 = 7$, and $\lambda_2 = 10$. The initial conditions are given as $\xi_1(0) = 0.5$, $\xi_2(0) = 0$, and $\xi_3(0) = 0$. To make a comparison, the simulation is also performed using the fuzzy method in [30] under the same condition. The trajectories of system signals are provided in Figures 1–4. As can be seen, with the method of this paper, the states x_1, x_2, x_3 have smaller amplitudes. In detail, under the presented method, the amplitudes of x_1, x_2, x_3 are about 0.08, 1.1, and 2.5. Under the method in [30], the amplitudes of x_1, x_2, x_3 are about 0.2, 3, and 18. The states x_1, x_2, x_3 can converge to the neighborhood of the original faster. Specifically, under the presented method, the convergence time of the states x_1, x_2, x_3 are about 2 s, 2 s, and 2 s. Under the method in [30], the convergence times of the states x_1, x_2, x_3 are about 3 s, 3 s, and 3 s. Besides, the amplitude of the control input u in this paper is smaller. More exactly, under the presented method, the amplitude of u is about 20. Under the method in [30], the amplitude of u is about 500. Thus, the presented method is effective and has better system response.

Example 2. Consider the system

$$\begin{cases} \dot{x}_1 = c_1 x_2^3 + \theta_1 x_1 x_2 + w(t), \\ \dot{x}_2 = c_2 u^3 + \theta_2 x_2^2, \end{cases} \tag{26}$$

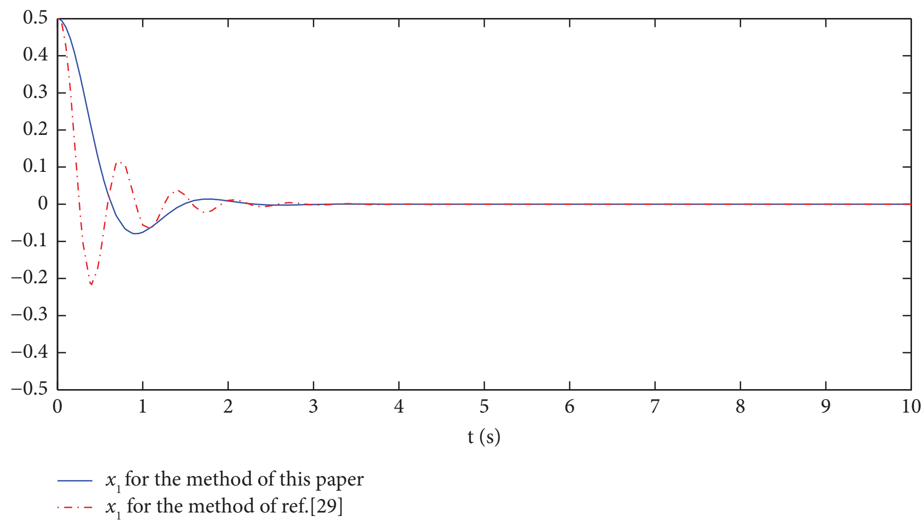

Figure 1: Trajectory of x_1 in system (24).

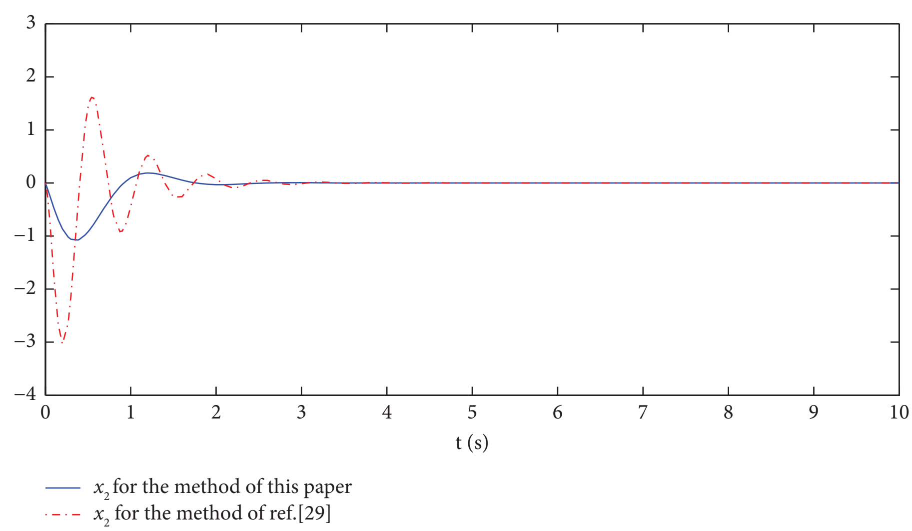

Figure 2: Trajectory of x_2 in system (24).

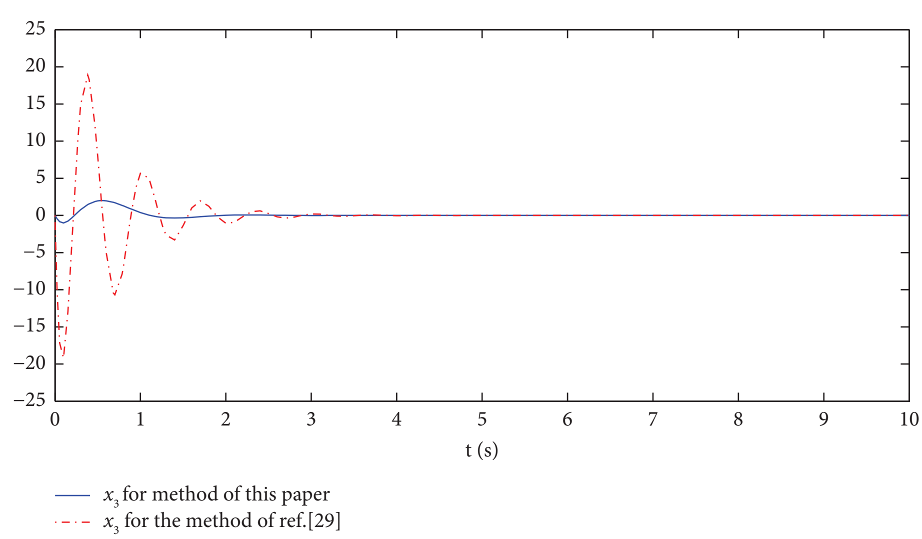

Figure 3: Trajectory of x_3 in system (24).

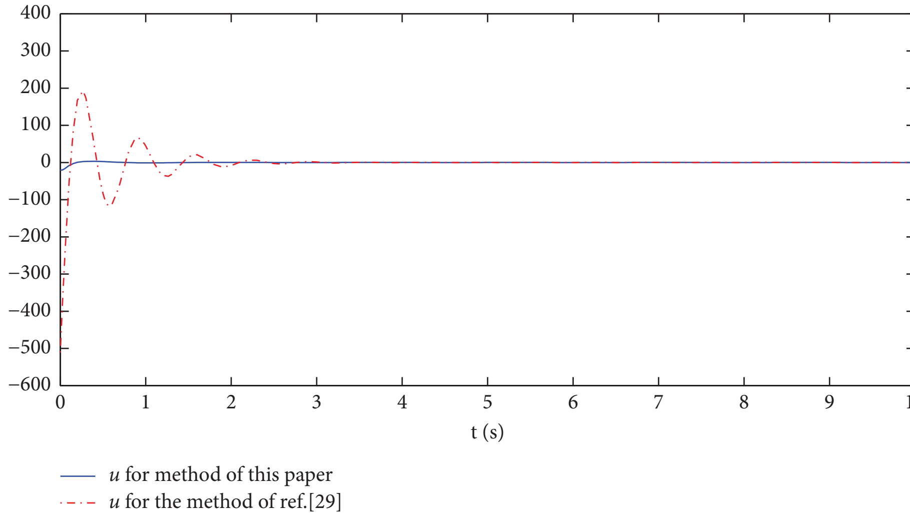

Figure 4: Trajectory of u given in (25).

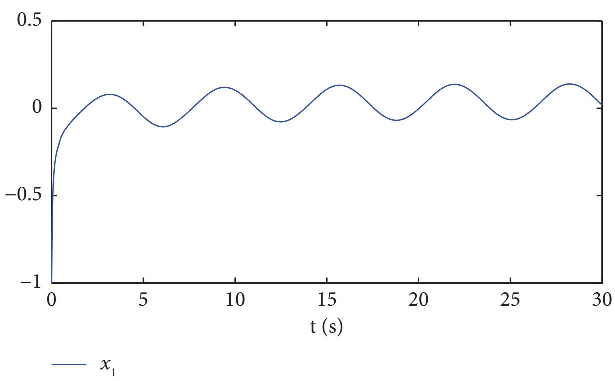

Figure 5: Trajectory of x_1 in system (26).

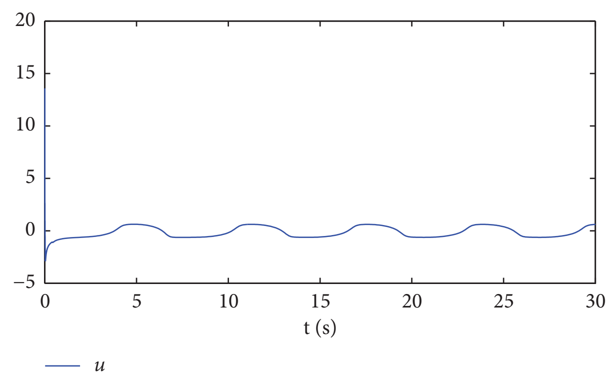

Figure 7: Trajectory of u provided in (27).

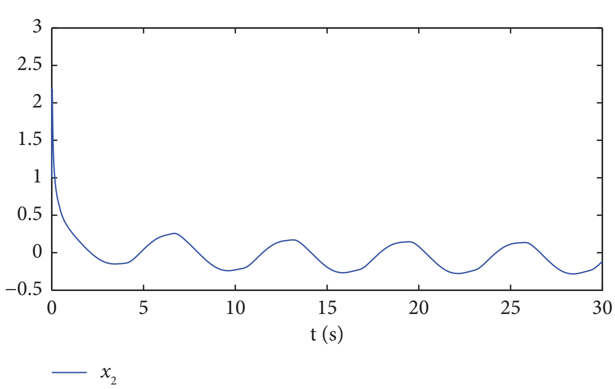

Figure 6: Trajectory of x_2 in system (26).

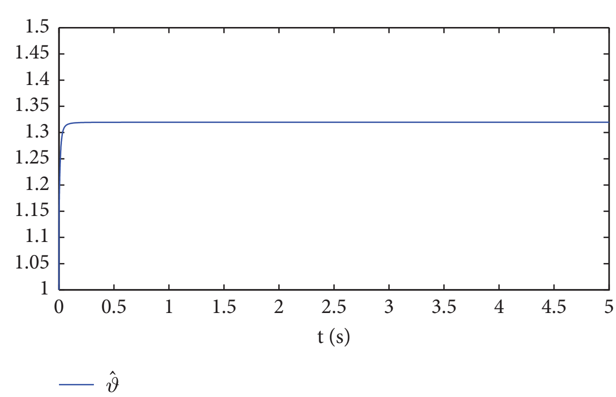

Figure 8: Trajectory of $\widehat{\vartheta}$ in (27).

where x_1, x_2 are the states, u is the input, θ_1, θ_2 are unknown functions, $w(t)$ is the disturbance. Let $H = (\widehat{\vartheta}/\epsilon)^{9/14}$ and choose $\epsilon = 1/2$, $r_0 = r_1 = 1$, $q = 5/9$, $r_2 = 2/3$, and $r_3 = 5/9$. Following the presented algorithm, we can construct the adaptive controller

$$\begin{cases} u = -H^{r_3}(\widehat{\vartheta})\beta_2 S_2, \\ \dot{\widehat{\vartheta}} = \dfrac{\lambda}{H^q(\widehat{\vartheta})}\left(S_1^{2p} + S_2^{2p}\right), \widehat{\vartheta}(0) = 1, \end{cases} \tag{27}$$

where $p = 3$, $S_1 = x_1/H^{r_1}(\widehat{\vartheta})$, and $S_2 = x_2/H^{r_2}(\widehat{\vartheta}) + \beta_1 S_1$.

In the simulation, we choose $c_1 = 1$, $c_2 = 1$, $\theta_1 = 1$, $\theta_2 = 1$, $w(t) = 0.1 \sin t$, $\beta_2 = 9$, $\beta_1 = 3$, and $\lambda = 500$. Figures 5–8 provide the trajectories of the system signals. As can be seen, the control effort is small. Besides, all system states are bounded. Hence, the provided strategy is valid.

5. Conclusion

In the research, the adaptive problem of the nontriangular system has been discussed. In the control design procedure, the function of parameter estimation is skillfully employed. By utilizing the idea of homogeneous control, fuzzy systems, and the adaptive scheme, a new robust adaptive controller has been constructed. Since homogeneous control is employed and only one parameter adaptive law is introduced, the obtained adaptive controller has a simpler form. Actually, future studies can focus on the following problems. For instance, if the plant contains time-delay, can we construct the similar stable controller? Can we extend the result to stochastic nonlinear system? If the system has unmodeled dynamics, can we construct the adaptive controller with similar form?

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

Anyone else who contributed to the manuscript but does not qualify for authorship has been acknowledged with their permission. There are no copyediting or translation services used for the preparation of the paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

This study was supported by the Fundamental Research Program of Shanxi Province (Grant nos. 202203021221004 and 202103021223308).

References

[1] X. Gao, T. Li, L. Yuan, and W. Bai, “Robust fuzzy adaptive output feedback optimal tracking control for dynamic positioning of marine vessels with unknown disturbances and uncertain dynamics,” *International Journal of Fuzzy Systems*, vol. 23, no. 7, pp. 2283–2296, 2021.

[2] C. Hua, P. X. Liu, and X. Guan, “Backstepping control for nonlinear systems with time delays and applications to chemical reactor systems,” *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3723–3732, 2009.

[3] R. Freeman and P. V. Kokotovic, *Robust Nonlinear Control Design: State-Space and Lyapunov Techniques*, Springer Science & Business Media, Berlin, Germany, 2008.

[4] G. Wang, C. Wang, and Y. Shen, “Distributed adaptive leader-following tracking control of networked Lagrangian systems with unknown control directions under undirected/directed graphs,” *International Journal of Control*, vol. 92, no. 12, pp. 2886–2898, 2019.

[5] G. Wang, C. Wang, and X. Cai, “Consensus control of output-constrained multiagent systems with unknown control directions under a directed graph,” *International Journal of Robust and Nonlinear Control*, vol. 30, no. 5, pp. 1802–1818, 2020.

[6] M. Krstic, P. V. Kokotovic, and I. Kanellakopoulos, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, Inc, Hoboken, NJ, USA, 1995.

[7] W. Zhang and W. Wei, “Disturbance-observer-based finite-time adaptive fuzzy control for non-triangular switched nonlinear systems with input saturation,” *Information Sciences*, vol. 561, pp. 152–167, 2021.

[8] Y. Li, L. Liu, and G. Feng, “Robust adaptive output feedback control to a class of non-triangular stochastic nonlinear systems,” *Automatica*, vol. 89, pp. 325–332, 2018.

[9] H. K. Khalil, *Nonlinear Systems*, Prentice Hall, Hoboken, NJ, USA, 2002.

[10] Y. He, X. H. Chang, and H. Wang, “Command-filtered adaptive fuzzy control for switched MIMO nonlinear systems with uknown dead zones and full state constraints,” *International Journal of Fuzzy Systems*, vol. 25, pp. 544–560, 2022.

[11] H. Su and W. Zhang, “Finite-time tracking control for a class of MIMO nonstrict-feedback nonlinear systems via adaptive fuzzy method,” *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 713–727, 2022.

[12] C. C. Cheng, T. Y. Lin, and Y. K. Li, “Design of adaptive sliding mode controllers for perturbed nonlinear systems with partial unmeasurable states and state constraints,” *Nonlinear Dynamics*, vol. 110, no. 2, pp. 1487–1499, 2022.

[13] G. Wang, Y. Ji, and C. Wang, “Asymptotic tracking control with preassigned transient performance for strict-feedback systems in the presence of unknown control directions,” *Journal of the Franklin Institute*, vol. 357, no. 1, pp. 206–228, 2020.

[14] W. Deng and J. Yao, “Asymptotic tracking control of mechanical servosystems with mismatched uncertainties,” *IEEE/ASME Transactions on Mechatronics*, vol. 26, no. 4, pp. 2204–2214, 2021.

[15] W. Deng, H. Zhou, J. Zhou, and J. Yao, “Neural network-based adaptive asymptotic prescribed performance tracking control of hydraulic manipulators,” *IEEE Transactions on Systems, Man, and Cybernetics. Systems*, vol. 53, no. 1, pp. 285–295, 2023.

[16] Z. Song, P. Li, Z. Sun, and Z. Wang, “Finite-time adaptive fuzzy event-triggered control of constrained nonlinear systems via bounded command filter,” *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 1, pp. 117–128, 2023.

[17] B. Meng, W. Liu, and X. Qi, “Disturbance and state observer-based adaptive finite-time control for quantized nonlinear systems with unknown control directions,” *Journal of the Franklin Institute*, vol. 359, no. 7, pp. 2906–2931, 2022.

[18] G. Zong, Y. Wang, H. R. Karimi, and K. Shi, “Observer-based adaptive neural tracking control for a class of nonlinear systems with prescribed performance and input dead-zone constraints,” *Neural Networks*, vol. 147, pp. 126–135, 2022.

[19] W. Deng and J. Yao, “Extended-state-observer-based adaptive control of electrohydraulic servomechanisms without velocity measurement,” *IEEE/ASME Transactions on Mechatronics*, vol. 25, no. 3, pp. 1151–1161, 2020.

[20] W. Li, X. Yao, and M. Krstic, "Adaptive-gain observer-based stabilization of stochastic strict-feedback systems with sensor uncertainty," *Automatica*, vol. 120, Article ID 109112, 2020.
[21] Y. X. Li, "Command filter adaptive asymptotic tracking of uncertain nonlinear systems with time-varying parameters and disturbances," *IEEE Transactions on Automatic Control*, vol. 67, no. 6, pp. 2973–2980, 2022.
[22] W. Lin and C. Qian, "Adaptive regulation of high-order lower-triangular systems: an adding a power integrator technique," *Systems & Control Letters*, vol. 39, no. 5, pp. 353–364, 2000.
[23] J. Polendo and C. Qian, "A generalized homogeneous domination approach for global stabilization of inherently nonlinear systems via output feedback," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 7, pp. 605–629, 2007.
[24] X. Zhang, W. Lin, and Y. Lin, "Nonsmooth feedback control of time-delay nonlinear systems: a dynamic gain based approach," *IEEE Transactions on Automatic Control*, vol. 62, no. 1, pp. 438–444, 2017.
[25] Z. G. Liu, Y. P. Tian, and Z. Y. Sun, "An adaptive homogeneous domination method to time-varying control of nonlinear systems," *International Journal of Robust and Nonlinear Control*, vol. 32, no. 1, pp. 527–540, 2022.
[26] F. Gao, X. Zhu, Y. Wu, J. Huang, and H. Li, "Reduced-order observer-based saturated finite-time stabilization of high-order feedforward nonlinear systems by output feedback," *ISA Transactions*, vol. 93, pp. 70–79, 2019.
[27] G. Shao, X. Zhao, R. Wang, and B. Yang, "Fuzzy tracking control for switched uncertain nonlinear systems with unstable inverse dynamics," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 1066–1072, 2018.
[28] M. S. Koo, H. L. Choi, and J. T. Lim, "Universal control of nonlinear systems with unknown nonlinearity and growth rate by adaptive output feedback," *Automatica*, vol. 47, no. 10, pp. 2211–2217, 2011.
[29] N. Duan, H. Min, and Z. Zhang, "Adaptive stabilization control for high-order nonlinear time-delay systems with its application," *Journal of the Franklin Institute*, vol. 354, no. 14, pp. 5825–5838, 2017.
[30] J. Liu, *Robot Control System Design and Matlab Simulation*, Press of Tsinghua University, Beijing, China, 2008.