Discrete Dynamics in Nature and Society, 2002 VOL. 7 (4), pp. 261–268

Taylor & Francis
Taylor & Francis Group

Growth, Technology, and Environmental Change—Nonlinearity and Non-constant Returns

WEI-BIN ZHANG

Department of Management, Ritsumeikam Asia Pacific University, 1-1 Jumonjibaru, Beppu-Shi, Oita-Ken 874-8577, Japan

(Revised 14 April 2001)

This paper proposes a growth model with endogenous technology and environmental change. The economy consists of two sectors, production and environmental. The production sector produces goods with knowledge, labor, and capital as inputs under perfect competitive conditions. Knowledge accumulates through learning by doing. The environment is affected by production, consumption, the environmental sector's production efficiency, and the nature's purification. The simple model shows that it is difficult to explicitly judge the impact of factors such as environmental policy, knowledge accumulation efficiency and preference change on the environment.

Keywords: Nonlinearity; Non-constant returns; Environment; Growth

INTRODUCTION

Whether or not continued global economic growth is comparable with global environmental changes has become one of the major challenges in the future. It may be argued that in order to examine environmental issues, it is necessary to construct the models with genuine dynamic interactions between distribution of production factors, production, consumption, technology, and environment. The purpose of this study is to construct a macroeconomic model to analyze interdependence among economic development, environmental changes, and knowledge accumulation.

Both production and consumption may pollute the environment. Growth often implies worsened environmental conditions. But growth also implies a higher material standard of living, which will, through the demand for a better environment induces changes in the structure of the economy to improve the environment. As society accumulates more capital and makes progresses in technology, more resources may be used to protect, if not improve, the environment. It is well observed that a country in the beginning of its economic development will be experiencing a worsening of the environment, while a country in which growth has taken place over a longer period of time will be adjusting its patterns of growth in such a way that the environment in fact improves. There are dynamic tradeoffs among economic growth, consumption, pollution, and human efforts of protecting environment. Tradeoffs between consumption and pollution have been extensively analyzed since the seminal papers of Plouder (1972) and Forster (1973). There is a large amount of literature available on issues of interdependence between economic growth and environment (Smith, 1972; Fisher and Peterson, 1976; Maget, 1978; Kanemoto, 1980; Tietenberg, 1988; Krutilla, 1991; Falk and Mendelsohn, 1993; Barrett, 1991). It has become clear that it is not easy to analytically examine the economic growth with endogenous pollutant accumulation and environment policy. On the basis of these efforts, the purpose of this study is to show how the environment may interact with technological change and economic growth within a perfect competitive economy under the government's intervention in environment protection.

Economic growth and improved living standards are dependent on people's ability to create and utilize knowledge. On the other hand, knowledge accumulation is sustainable only with some economic bases. There is interdependence between knowledge creation and utilization and economic growth. The idea of endogenous knowledge growth is not new and it has been incorporated into economic analysis for a long time. It may be said that the modeling of interaction between economic growth and knowledge accumulation was initiated with Arrow's paper on learning by doing (Arrow, 1962) and Uzawa's paper on education and growth (Uzawa, 1965). There have been an increasing number of publications on relations between knowledge accumulation and economic development in the recent theoretical economic literature

ISSN 1026-0226 print/ISSN 1607-887X online

(e.g. Sato, 1996; Romer, 1986; Lucas, 1988; Matsuyama, 1999; Zhang, 1999, 2000). These approaches have provided insights into the complexity of modern economic development. But only a few models explicitly take account of environmental issues in growth models with endogenous capital and knowledge.

This study proposes a dynamic model to examine the issue of interdependence among economic growth, technological change, pollution, and government environmental policy. The government's environmental policy is to maximize the consumer's utility by allocating labor and capital resources for environment protection. Capital accumulation is endogenously determined. Pollutant accumulation speed is dependent on the production level, the level of consumption, natural purification power, and human efforts of purifying environment. The remainder of this paper is organized as follows. The second section defines the growth model with endogenous technological change and capital and pollutant accumulation. The third section analyzes the properties of the dynamic system. The fourth, fifth, and sixth sections, respectively, examine the effects of changes in the environmental policy, knowledge accumulation efficiency, and preference structure on the economic structure and environment. The seventh section concludes the study.

THE MODEL

We consider an economic system, which consists of two sectors, production and environmental. The production sector is similar to the standard one-sector neoclassical growth model (e.g. Zhang, 1999). Only one commodity is produced in the system. The commodity is assumed to be composed of homogeneous quality, and to be produced by employing three factors of production, namely knowledge, labor, and capital. At this initial stage, we neglect dynamics of population, assuming that the population is constant. The population is employed by the two sectors. The labor distribution is determined by the market mechanism. The environmental sector employs labor and capital to purify environment. The government finances the environment sector through taxing the production sector.

It is assumed that the labor and capital markets are perfectly competitive and the labor and capital are always fully employed. We introduce:

N—the fixed labor force;
$K(t)$ and $F(t)$—the total capital and the output at time t;
$N_i(t)$ and $K_i(t)$—the labor force and capital stocks employed by the production sector;
$N_e(t)$ and $K_e(t)$—the labor force and capital stocks employed by the environmental sector;
$C(t)$—the consumption level of goods;
$E(t)$—the level of pollutant stocks;
$r(t)$ and $w(t)$—the rate of interest and the wage rate, respectively; and
τ—the fixed tax rate, $0 < \tau < 1$.

There are three factor inputs, knowledge, capital and labor, in economic production. We assume that the environmental quality may affect productivity of production units such as hotels, restaurants, and hospitals and deteriorate machines. We specify production function as follows

$$F(t) = Z^m K_i^\alpha N_i^\beta \exp(-h_p E),$$
$$\alpha + \beta = 1, \quad \alpha, \beta > 0, \quad m, h_p \geq 0 \tag{2.1}$$

where $Z(t)$ is the level of knowledge at time t and m is the knowledge utilization efficiency parameter of the production sector. We introduce knowledge stock $Z(t)$ of the system. In this study, the concept of knowledge refers to disembodied knowledge. Knowledge means ideas and theories, which exist, for instance in books and journals. They are free for anyone to utilize. Knowledge has the characteristics of public good in the sense that utilization of knowledge by any economic sector will not affect that by any other sectors. New theories in mathematics, theoretical physics, economics, philosophy and the like are accessible to the public, almost as soon as they are discovered. Knowledge is not a direct input to production, but may affect human capital (which is an input to production). We assume that knowledge may indirectly affect economic production in the way that human capital accumulation is affected by knowledge and human capital is a direct input to production. The term, $\exp(-h_p E)$, in $F(t)$ means that productivity is negatively related to the pollution level. In this study, we neglect possible impact of the environment on productivity, i.e. $h_p = 0$. It can be seen that this omission will not significantly affect our analytical results.

We select the commodity to serve as numeraire. The marginal conditions are given by

$$r = \frac{(1-\tau)\alpha F}{K_i}, \quad w = \frac{(1-\tau)\beta F}{N_i}. \tag{2.2}$$

The income Y from the interest and wage payments at time t is given by

$$Y = rK + wN. \tag{2.3}$$

We now describe the dynamics of the stock $E(t)$ of pollutants. We assume that pollutants are created through two sources, production and consumption. Pollutants may be reduced by two ways. The nature may *treat* certain pollutants in a similar way to that of waste treatment plants. Some of the pollutants may naturally disappear without any human efforts. Pollutants may be treated by using capital and labor. We specify the dynamics of the stock of pollutants as follows

$$\frac{dE}{dt} = q_f F + q_c C - Q_e - q_0 E \tag{2.4}$$

in which q_f, q_c, and q_0 are positive parameters and

$$Q_e(t) = f(E)Z^n K_e^u N_e^v \tag{2.5}$$

where u and v are positive parameters, n is the knowledge utilization efficiency parameter of the environmental sector, and $f(E)$ (≥ 0) is a function of E. The term $q_f F$ means that pollutants that are produced during production processes are linearly positively proportional to the output level (Gruver, 1976; Fisher and Peterson, 1976; Stephens, 1976). The term $q_c C$ means that in consuming one unit of the good, the quantity $q_c C$ is left as waste. The parameter q_c depends on the technology and environmental sense of consumers. The parameter q_0 is called the rate of natural purification. The term $q_0 E$ measures the rate that the nature purifies environment. The term $Z^n K_e^u N_e^v$ in Q_e means that the purification rate of environment is positively related to knowledge utilization efficiency, capital and labor inputs (Mäler, 1974). The function $f(E)$ in $Q_e(t) = f(E)Z^n K_e^u N_e^v$ implies that the purification efficiency is dependent on the scale of pollutants at time t. It is not easy to generally specify how the purification efficiency is related to the scale of pollutants. For simplicity, we specify f as follows $f(E) = q_e E^v$ where $q_e > 0$ and $v > 0$ are parameters. The function has the following properties

$$f(0) = 0, \quad \lim_{E\to\infty} f(E) = \infty, \quad \frac{\mathrm{d}f}{\mathrm{d}E} > 0, \quad \frac{\mathrm{d}^2 f}{\mathrm{d}^2 E} < 0.$$

Obviously, when E is very large, the specified functional form is problematic. At this initial stage of investigation, we accept the above-specified form. In order to describe the behavior of households, we define a variable

$$E^* = E_0 - E \tag{2.6}$$

where E_0 is called the threshold of pollution level. For instance, consumption of nuclear-generated electricity brings about the creation of radionuclides that cause death or severe mutation, when threshold concentrations are exceeded. Electricity production using coal creates atmospheric CO_2 concentrations which, at sufficiently high levels, may cause dramatic changes. We assume that the critical level is known. This assumption may be relaxed (Cropper, 1976; Smith, 1972; Clarke and Reed, 1994).

We assume that the disutility that the society experiences from pollution is a continuous function of the environmental pollution stock. It is assumed that the utility level $U(t)$ that a typical household obtains is dependent on the consumption level $C(t)$ of commodity, the environmental condition $E^*(t)$ and the net savings $S(t)$. The utility function is specified as follows

$$U(t) = E^{*\zeta} C^{\xi} S^{\lambda}, \quad \zeta, \xi, \lambda > 0 \tag{2.7}$$

in which ζ, ξ, and λ, respectively, are the propensities to enjoy environment, to consume goods, and to save.

Consumers get income Y from the interest and the wage payments. They can also sell their properties, which are equal to K, to purchase consumption goods and make investment. The *total available budget* for savings and consumption is thus equal to

$$Y^* = Y + K.$$

We assume that the consumers pay the depreciation of capital goods, which they own. The total amount is equal to $\delta_k K$ where δ_k is the depreciation rate of physical capital. At each point of time, the consumers would distribute among savings (S), consumption of goods (C), and payment for depreciation ($\delta_k K$) where δ_k is the fixed depreciation rate of capital. The budget constraint is thus given by

$$C + \delta_k K + S = Y^* = Y + K. \tag{2.8}$$

The households determine C and S with the level of E^* as given. Maximizing U subject to Eq. (2.8) yields

$$C = \xi\rho Y + (1 - \delta_k)\xi\rho K, S = \lambda\rho Y + (1 - \delta_k)\lambda\rho K \tag{2.9}$$

where $\rho \equiv 1/(\xi + \lambda)$.

It is assumed that the savings is equal to investment. The change in the households' wealth is equal to the net savings minus the wealth sold at time t, i.e.

$$\frac{\mathrm{d}K}{\mathrm{d}t} = S - K.$$

Substituting S in Eq. (2.9) into the above equation yields

$$\frac{\mathrm{d}K}{\mathrm{d}t} = \lambda\rho Y - (\xi + \delta_k\lambda)\rho K. \tag{2.10}$$

We now determine how the government determines the number of labor force and the level of capital employed for purifying pollution. The government budget is given by

$$rK_e + wN_e = \tau F. \tag{2.11}$$

We assume that the government will employ the labor force and capital stocks for purifying the environment in such a way that the purification rate achieves its maximum under the given budget constraint. The government's optimal problem is given by

$$\text{Max } Q_e = f(E)Z^n K_e^u N_e^v \quad \text{s.t.}: rK_e + wN_e = \tau F.$$

The optimal solution is given by

$$rK_e = \tau u v_0 F, \quad wN_e = \tau v v_0 F \tag{2.12}$$

where $v_0 \equiv 1/(u+v)$. The product of the production sector is equal to the consumption and the net savings, i.e.

$$C + S - K + \delta_k K = F. \tag{2.13}$$

We assume that the labor and capital are fully employed

$$K_i + K_e = K, \quad N_i + N_e = N. \tag{2.14}$$

There are different ways of creating new knowledge. In the economic literature, processes of knowledge creation through learning by doing and pure and applied research are well modeled. In this study, for simplicity, we assume that knowledge accumulation is through learning by doing. We may introduce research and development activities in the way as in Zhang (1999). We propose the following possible dynamics of knowledge

$$\frac{dZ}{dt} = \frac{\tau_i F}{Z^{\varepsilon}} - \delta_z Z \tag{2.15}$$

in which τ_i, ε, and δ_z are parameters. We require τ_i, and δ_z to be non-negative. We interpret $\tau_i F/Z^{\varepsilon}$ as the contribution to knowledge accumulation through the production sector's learning by doing. In order to explain Eq. (2.15), we consider a case in which knowledge is a function of the total production output during a certain historical period

$$Z(t) = a_1 \left\{ \int_0^t F(\theta) d\theta \right\}^{a_2} + a_3$$

in which a_1, a_2 and a_3 are positive parameters. The above mentioned equation implies that the knowledge accumulation through learning by doing exhibits decreasing (increasing) returns to scale in the case of $a_2 < (>) 1$. We interpret a_1 and a_3 as the measurements of the efficiency of learning by doing by the production sector. Taking the derivatives of the equation yields

$$\frac{dZ}{dt} = \frac{\tau_i F}{Z^{\varepsilon}}$$

in which $\tau_i \equiv a_1 a_2$ and $\varepsilon \equiv 1 - a_2$. Adding the depreciation part to the above equation yields Eq. (2.15).

We have thus defined the model. The model has 14 endogenous variables, Z, K_i, K_n, N_i, N_n, E^*, K, C, S, E, r, w, Y, F, and U. It is easy to check that the system has the same number of independent equations. We now examine the behavior of the system.

PROPERTIES OF THE DYNAMIC SYSTEM

First, we show that the dynamics can be represented by a three-dimensional differential equations system. Then, we provide conditions for existence of equilibria and for stability.

By Eq. (2.3) and $N_i + N_n = N$, we have

$$Y = rK + wN_i + wN_n. \tag{3.1}$$

Substituting Eq. (2.2) and wN_n in Eq. (2.12) into Eq. (3.1) yields

$$Y = \frac{(1-\tau)KF}{K_i} + \beta F(1-\tau) + vv_0\tau F. \tag{3.2}$$

By Eqs. (2.8) and (2.13), $Y = F$. Substituting this equation into Eq. (3.2) yields

$$K_i = \frac{(1-\tau)K}{1 - \beta(1-\tau) - vv_0\tau}. \tag{3.3}$$

By Eq. (3.3) and $K_i + K_e = K$, we solve

$$K_i = \alpha_i K, \quad K_e = \alpha_e K \tag{3.4}$$

where

$$\alpha_i \equiv \frac{\alpha(1-\tau)}{1 - \beta(1-\tau) - vv_0\tau},$$

$$\alpha_e = \frac{uv_0\tau}{1 - \beta(1-\tau) - vv_0\tau}.$$

By Eq. (3.4), we conclude that for a given tax rate, the capital inputs of the two sectors are linearly and positively proportional to the total capital stocks at any point of time.

By Eq. (2.3) and $K_i + K_e = K$, we get

$$Y = rK_i + rK_e + wN. \tag{3.5}$$

Substituting Eq. (2.2) and rK_e in Eq. (2.12) into Eq. (3.5) yields

$$Y = \alpha F(1-\tau) + uv_0\tau F + \frac{\beta NF(1-\tau)}{N_i}. \tag{3.6}$$

By $Y = F$, Eq. (3.6) and $N_i + N_n = N$, we solve

$$N_i = \beta_i N, \quad N_e = \beta_e N \tag{3.7}$$

where

$$\beta_i \equiv \frac{\beta(1-\tau)}{1 - \alpha(1-\tau) - uv_0\tau},$$

$$\beta_e = \frac{vv_0\tau}{1 - \alpha(1-\tau) - uv_0\tau}.$$

Then, for a given tax rate, the labor distribution of the two sectors are linearly and positively in proportion to the total population at any point of time.

Summarizing the discussion, we get the following lemma.

LEMMA 3.1 For any given positive levels of $Z(t)$, $K(t)$, and $E(t)$ at any given point of time, all the variables in the system can be expressed as functions of $Z(t)$, $K(t)$, and

$E(t)$ by the following procedure: K_i and K_e by Eq. (3.4) $\rightarrow$ N_i and N_e by Eq. (3.7) $\rightarrow$ F by Eq. (2.1) $\rightarrow$ r and w by Eq. (2.2) $\rightarrow$ Y by Eq. (2.3) $\rightarrow$ $Q_e(t) = q_e E^v Z^n K_e^u N_e^v \rightarrow C$ and S by Eq. (2.9) $\rightarrow$ E^* by Eq. (2.6) $\rightarrow$ U by Eq. (2.7).

By the above procedure, $Y = F$, Eqs. (2.4), (2.10), and (2.15), we represent the dynamics of the economic system in terms of the following three differential equations:

$$\frac{dK}{dt} = \lambda\rho\alpha_i^\alpha\beta_i^\beta Z^m K^\alpha - (\xi + \delta_k\lambda)\rho K,$$

$$\frac{dZ}{dt} = \frac{\alpha_i^\alpha\beta_i^\beta\tau_i Z^m K^\alpha}{Z^\varepsilon} - \delta_z Z,$$

$$\frac{dE}{dt} = \lambda_i Z^m K^\alpha + (1-\delta_k)q_c\xi\rho K - \lambda_e Z^n E^v K^u - q_0 E \tag{3.8}$$

where

$$\lambda_i \equiv (q_f + \xi\rho q_c)\alpha_i^\alpha\beta_i^\beta, \quad \lambda_e \equiv q_e\alpha_e^\alpha\beta_e^\beta.$$

It is direct to check that the dynamic system has a unique equilibrium, given by

$$Z = \left\{\left(\frac{\xi}{\lambda} + \delta_k\right)^{\alpha/\beta}\frac{\delta_z}{\tau_i\alpha_i^{\alpha/\beta}\beta_i}\right\}^{1/x},$$

$$K = \left(\frac{\lambda\alpha_i^\alpha\beta_i^\beta}{\xi + \delta_k\lambda}\right)^{1/\beta} Z^{m/\beta},$$

$$\lambda_i Z^m K^\alpha + (1-\delta_k)q_c\xi\rho K = \lambda_e Z^n E^v K^u + q_0 E \tag{3.9}$$

in which

$$x \equiv \frac{\beta}{m} - \varepsilon - 1.$$

We require $x \neq 0$. We thus have a unique equilibrium. By the first equation, we explicitly solve Z. So the second one gives the value of K. In the last equation in Eq. (3.9), the right-hand side is constant (because we have solved K and Z). It is direct to check that the last equation (with E as a single variable) has a unique solution for $0 < E < +\infty$.

The three eigenvalues, ϕ_j, $j = 1, 2, 3$, are given by

$$\phi_{1,2} = -\frac{A}{2} \pm \left\{\frac{A^2}{4} + \rho\delta_z\beta x(\xi + \lambda\delta_k)\right\}^{1/2},$$

$$\phi_3 = -\nu\lambda_e Z^n E^{v-1} K^u - q_0 \tag{3.10}$$

where

$$A \equiv \rho\beta(\xi + \lambda\delta_k) + \frac{\alpha\delta_z\alpha_i^\alpha\beta_i^\beta}{\beta} - \delta_z x.$$

If $x < 0$, then $\mathrm{Re}\{\phi_j\} < 0$, $j = 1, 2$. In this case, the unique equilibrium is stable. If $x > 0$, $\mathrm{Re}\{\phi_1\} < 0$. In this case, the unique equilibrium is unstable.

PROPOSITION 3.1 In the case of $x < (>) 0$, the dynamic system has a unique stable (unstable) equilibrium.

The stability of the system is determined by the parameter $x \equiv m/\beta - \varepsilon - 1$. As m is the production sector's knowledge utilization efficiency parameter and ε is the return to scale effects of knowledge in knowledge accumulation, we may interpret x as the measurement of return to scale effects of knowledge in the whole system. We may thus make the following interpretation of the parameter x. We say that the knowledge utilization and creation of the production sector exhibits increasing (decreasing) return to scale effects in the dynamic system when $x > (<) 0$. The above proposition simply says that if the knowledge utilization and creation of the production sector exhibits increasing (decreasing) return to scale effects, then the dynamic system is unstable (stable). This conclusion is intuitively acceptable.

In the remainder of this study, we examine the impact of changes in some parameters on the long-run equilibrium.

THE TAX POLICY AND THE LONG-RUN EQUILIBRIUM

This section examines the impact of changes in the tax rate, τ, on the system. Taking derivatives of Eq. (3.9) with respect to τ yields

$$\frac{1}{Z}\frac{dZ}{d\tau} = \frac{1}{(1+\varepsilon)K}\frac{dK}{d\tau} = \frac{\alpha^*}{\beta x},$$

$$(v\lambda_e Z^n E^{v-1} K^u + q_0)\frac{dE}{d\tau} = \left\{(\alpha\varepsilon + \alpha + m - \varepsilon u + u + n)\lambda_i Z^{m-1} K^\alpha + (1 + \varepsilon - \varepsilon u - u - n)(1-\delta_k)\frac{q_c\xi\rho K}{Z} + (\varepsilon u + u + n\frac{q_0 E}{Z}\right\}\frac{dZ}{d\tau} - \alpha^*\lambda_i Z^m K^\alpha - \left[\frac{\alpha u}{\{1-(1-\tau)\beta - vv_0\tau\}\tau} - \frac{\beta v}{\{1-(1-\tau)\alpha - uv_0\tau\}\tau}\right]\lambda_e Z^n E^v K^u \tag{4.1}$$

in which

$$\alpha^* \equiv \left\{\frac{\alpha u}{1-\beta(1-\tau)-vv_0\tau}+\frac{\beta v}{1-\alpha(1-\tau)-uv_0\tau}\right\} \times \frac{v_0}{1-\tau}.$$

We see that the return to scale parameter, x, plays an important role in determining the impact of changes in the environment policy on the equilibrium levels of capital and knowledge. The sign of $\mathrm{d}Z/\mathrm{d}\tau$ and $\mathrm{d}K/\mathrm{d}\tau$ is the same as that of x. Here, we require $\varepsilon+1>0$, which simply implies that in the case of $\varepsilon<0$, the increasing return to scale in knowledge accumulation is not too strong. In the case of $x<(>)0$, a decrease in the tax rate increases (reduces) the levels, Z and K, of knowledge and capital stocks. The tax policy has the opposite effects on knowledge and capital accumulation, when knowledge exhibits increasing or decreasing return to scale effects in the dynamic system. It is very difficult to judge the impact of changes in the tax rate on the pollution level. In the case of $\varepsilon+1-u\varepsilon-u-n>0$, and $x<0$, we have: $\mathrm{d}E/\mathrm{d}\tau<0$. But it is difficult to judge the other cases.

By Eqs. (3.4) and (3.7), the impact on capital and labor distribution are given by

$$\frac{1}{K_\mathrm{i}}\frac{\mathrm{d}K_\mathrm{i}}{\mathrm{d}\tau}=\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau}-\frac{uv_0}{\{1-vv_0\tau-\beta(1-\tau)\}(1-\tau)},$$

$$\begin{aligned}\frac{1}{K_\mathrm{e}}\frac{\mathrm{d}K_\mathrm{e}}{\mathrm{d}\tau}&=\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau}\\&\quad+\frac{\alpha}{\{1-vv_0\tau-\beta(1-\tau)\}\tau},\frac{1}{N_\mathrm{i}}\frac{\mathrm{d}N_\mathrm{i}}{\mathrm{d}\tau}\\&=-\frac{vv_0}{\{1-vv_0\tau-\beta(1-\tau)\}(1-\tau)}\\&>0,\frac{\mathrm{d}N_\mathrm{e}}{\mathrm{d}\tau}=-\frac{\mathrm{d}N_\mathrm{i}}{\mathrm{d}\tau}<0.\end{aligned}\tag{4.2}$$

As τ is decreased in the case of $\mathrm{d}E/\mathrm{d}\tau<0$, the capital stock, K_i, employed by the production sector is increased, the capital stock, K_e, employed by the environmental sector may be either increased or decreased. As τ is decreased, in the case of $\mathrm{d}E/\mathrm{d}\tau>0$, K_i may be either increased or decreased, K_e is decreased. As τ is decreased, more (less) labor force is employed by the production (environmental) sector.

By $F=Y=(\xi/\lambda+\delta_0)K$ and $C=\xi K/\lambda$, we have

$$\frac{1}{F}\frac{\mathrm{d}F}{\mathrm{d}\tau}=\frac{1}{Y}\frac{\mathrm{d}Y}{\mathrm{d}\tau}=\frac{1}{C}\frac{\mathrm{d}C}{\mathrm{d}\tau}=\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau}.\tag{4.3}$$

The change rates of the output level, the net income, and the consumption level have the same sign as that of $\mathrm{d}K/\mathrm{d}\tau$.

By Eq. (2.2), we get

$$\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}=\frac{\beta-vv_0}{1-vv_0-\beta(1-\tau)},$$

$$\frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}\tau}=-\frac{\alpha-uv_0}{1-vv_0-\alpha(1-\tau)}+\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau}.\tag{4.4}$$

We see that the rate of interest and wage rate may be either increased or decreased.

KNOWLEDGE ACCUMULATION EFFICIENCY

We now examine the effects of changes in knowledge accumulation efficiency τ_i on the system. By Eq. (3.9), we have

$$\begin{aligned}\frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}\tau_\mathrm{i}}&=\frac{\beta}{mK}\frac{\mathrm{d}K}{\mathrm{d}\tau_\mathrm{i}}=-\frac{1}{\tau_\mathrm{i}x},\quad (v\lambda_\mathrm{e}K^uE^{v-1}+q_0)K\frac{\mathrm{d}E}{\mathrm{d}\tau_\mathrm{i}}\\&=\left\{\left(1-u-\frac{\beta n}{m}\right)\lambda_\mathrm{i}Z^mK^\alpha\right.\\&\quad+\left(1-u-\frac{\beta n}{m}\right)(1-\delta_\mathrm{k})q_\mathrm{c}\xi\rho K\\&\quad+\left(u+\frac{\beta n}{m}\right)q_0E\frac{\mathrm{d}K}{\mathrm{d}\tau_\mathrm{i}}.\end{aligned}\tag{5.1}$$

In the case of $x<(>)0$, an increase in knowledge accumulation efficiency increases (reduces) the equilibrium levels of knowledge and capital stocks. In the case of $1>u+\beta n/m$, the sign of $\mathrm{d}E/\mathrm{d}\tau_\mathrm{i}$ is the same as that of $\mathrm{d}K/\mathrm{d}\tau_\mathrm{i}$

Taking the derivatives of Eqs. (3.4) and (3.7) with respect to τ_i yields

$$\frac{1}{K_\mathrm{i}}\frac{\mathrm{d}K_\mathrm{i}}{\mathrm{d}\tau_\mathrm{i}}=\frac{1}{K_\mathrm{e}}\frac{\mathrm{d}K_\mathrm{e}}{\mathrm{d}\tau_\mathrm{i}}=\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau_\mathrm{i}},\quad \frac{\mathrm{d}N_\mathrm{i}}{\mathrm{d}\tau_\mathrm{i}}=\frac{\mathrm{d}N_\mathrm{e}}{\mathrm{d}\tau_\mathrm{i}}=0.\tag{5.2}$$

The capital stocks employed by each sector is increased (reduced) in the case of $x<(>)0$; the labor distribution is not affected.

By $F=Y=(\xi/\lambda+\delta_0)K$ and $C=\xi K/\lambda$, we have

$$\frac{1}{F}\frac{\mathrm{d}F}{\mathrm{d}\tau_\mathrm{i}}=\frac{1}{Y}\frac{\mathrm{d}Y}{\mathrm{d}\tau_\mathrm{i}}=\frac{1}{C}\frac{\mathrm{d}C}{\mathrm{d}\tau_\mathrm{i}}=\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau_\mathrm{i}}.\tag{5.3}$$

The output level, the net income, and the consumption level are increased (reduced) in the case of $x<(>)0$. By Eq. (2.2), we get

$$\frac{\mathrm{d}r}{\mathrm{d}\tau_\mathrm{i}}=0,\quad \frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}\tau_\mathrm{i}}=\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\tau_\mathrm{i}}.\tag{5.4}$$

THE PROPENSITY TO SAVE

It is important to examine how a shift in the preference structure may affect the pollution issue. We now examine how changes in the propensity λ to hold wealth affect the system. It should be remarked that an increase in the propensity to save implies a decrease in the propensity ξ to consume goods. Taking derivatives of Eq. (3.9) with respect to λ yields

$$\frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}\lambda} = -\frac{\alpha\xi}{(\xi+\delta_k\lambda)\beta\lambda x},$$

$$\frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\lambda} = -\frac{\alpha m/\beta - x}{(\xi+\delta_k\lambda)\beta\lambda x}\xi,\ (v\lambda_e Z^n E^{v-1}K^u + q_0)\frac{\mathrm{d}E}{\mathrm{d}\lambda}$$

$$= \left\{\frac{\alpha\lambda_i Z^m}{K^\beta} + (1-\delta_k)q_c\xi\rho - u\lambda_e Z^n E^v K^{u-1}\right\}$$

$$\frac{\mathrm{d}K}{\mathrm{d}\lambda} + (m\lambda_i Z^{m-1}K^\alpha - n\lambda_e Z^{n-1}E^v K^u)\frac{\mathrm{d}Z}{\mathrm{d}\lambda}$$

$$-(\alpha_i^\alpha\beta_i^\beta Z^m K^\alpha - K + \delta_k K)\xi q_c\rho^2. \qquad (6.1)$$

An increase in the propensity to save increases (reduces) the level of knowledge in the case of $x < (>)0$. An increase in λ increases (reduces) the level of capital stocks in the case of $\alpha m/x\beta < (>) 1$. It is not easy to explicitly judge the sign of $\mathrm{d}E/\mathrm{d}\lambda$.

Taking derivatives of Eqs. (3.4) and (3.7) with respect to λ yields

$$\frac{1}{K_i}\frac{\mathrm{d}K_i}{\mathrm{d}\lambda} = \frac{1}{K_e}\frac{\mathrm{d}K_e}{\mathrm{d}\lambda} = \frac{1}{K}\frac{\mathrm{d}K}{\mathrm{d}\lambda},\quad \frac{\mathrm{d}N_i}{\mathrm{d}\lambda} = \frac{\mathrm{d}N_e}{\mathrm{d}\lambda} = 0. \qquad (6.2)$$

Taking the derivatives of $F = Y = (\xi/\lambda + \delta_0)K$ and $C = \xi K/\lambda$ with respect to λ, we have

$$\frac{1}{F}\frac{\mathrm{d}F}{\mathrm{d}\lambda} = \frac{1}{Y}\frac{\mathrm{d}Y}{\mathrm{d}\lambda} = \frac{(x-m/\beta)\alpha\xi}{(\xi+\delta_k\lambda)\beta\lambda x},\ \frac{1}{C}\frac{\mathrm{d}C}{\mathrm{d}\lambda}$$

$$= -\frac{1}{\lambda} + \frac{(x-\alpha m/\beta)\xi}{(\xi+\delta_k\lambda)\beta\lambda x}. \qquad (6.3)$$

By Eq. (2.2), we directly get the impact on r and w.

CONCLUDING REMARKS

This study proposed a dynamic model to examine the issues related to interdependence between economic growth, technological change, pollution and government environmental policy under perfectly competitive markets. In our model, knowledge accumulation is through learning by doing. Pollutant accumulation is dependent on the production, consumption, natural purification power, and human efforts to purify the environment. We showed that the dynamic system has a unique equilibrium. The unique equilibrium is either stable or unstable, depending on whether the system exhibits decreasing or increasing returns to scale. We also examined the effects of changes in some parameters on the long-run economic structure.

We may extend the model in different ways. For instance, we may extend the one-sector model to a model with multiple sectors. We may assume that the tax rate is an endogenous variable by specifying τ as a function of F and E at any point of time as $\tau = H(F, E, t)$ where t measures the impact of the households' preferences on the government's tax policy and other factors. It may be reasonable to require that for a given level of F, an increase in E tends to increase the tax rate, i.e. $H_E \geq 0$. But whether an increase in the output will increase the tax rate is difficult to predict. That is, H_F may be either positive or negative, depending on the social and environment consciousness of the society under consideration. We may also assume that the government's budget for environment protection is dependent on consumption level. In this case, the household budget constraint and the government budget constraint are changed.

References

Arrow, K.J. (1962) "The economic implications of learning by doing", *Review of Economic Studies* **29**, 155–173.

Barrett, S. (1991) "Economic growth and environmental preservation", *Journal of Environmental Economics and Management* **23**, 289–300.

Clarke, H.R. and Reed, W.J. (1994) "Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse", *Journal of Economic Dynamics and Control* **18**, 991–1010.

Cropper, M.L. (1976) "Regulating activities with catastrophic environmental effects", *Journal of Environmental Economics and Management* **3**, 1–15.

Falk, I. and Mendelsohn, R. (1993) "The economics of controlling stock pollutants: an efficient strategy for greenhouse cases", *Journal of Environmental Economics and Management* **25**, 76–88.

Fisher, A.C. and Peterson, F.A. (1976) "The environment in economics: a survey", *Journal of Economic Literature* **14**, 1–33.

Forster, B.A. (1973) "Optimal consumption planning in a polluted environment", *Economic Record* **49**, 534–545.

Gruver, G.W. (1976) "Optimal investment in pollution control capital in a neoclassical growth context", *Journal of Environmental Economics and Management* **3**, 165–177.

Kanemoto, Y. (1980) *Theories of Urban Externalities* (North-Holland, Amsterdam).

Lucas, R.E. (1988) "On the mechanics of economic development", *Journal of Monetary Economics* **22**, 3–42.

Mäler, K.G. (1974) *Environmental Economics—A Theoretical Inquiry* (Johns Hopkins University, Baltimore).

Matsuyama, K. (1999) "Growth through cycles", *Econometrica* **67**, 335–347.

Plouder, G.C. (1972) "A model of waste accumulation and disposal", *Canadian Journal of Economics* **5**, 119–125.

Romer, P.M. (1986) "Increasing returns and long-run growth", *Journal of Political Economy* **94**, 1002–1037.

Sato, R. (1996) *Growth Theory and Technical Change, the Selected Essays* (Edward Elgar, Cheltenham).

Smith, V.K. (1972) "Dynamics of waste accumulation disposal versus recycling", *Quarterly Journal of Economics* **86**, 601–616.

Stephens, J.K. (1976) "A relatively optimistic analysis of growth and pollution in a neoclassical framework", *Journal of Environmental Economics and Management* **3**, 85–96.

Tietenberg, T. (1988) *Environmental and Natural Resource Economics*, 2nd ed. (Scott Foresman, Glenview, IL).

Uzawa, H. (1965) "Optimal technical change in an aggregative model of economic growth", *International Economic Review* **6**, 18–31.

Zhang, W.B. (1999) *Capital and Knowledge—Dynamics of Economic Structures with Non-constant Returns* (Springer, Berlin).

Zhang, W.B. (2000) *A Theory of International Trade—Capital, Knowledge and Economic Structures* (Springer, Berlin).