GLOBAL ATTRACTIVITY OF POSITIVE PERIODIC SOLUTIONS FOR AN IMPULSIVE DELAY PERIODIC “FOOD LIMITED” POPULATION MODEL

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We will consider the following nonlinear impulsive delay differential equation $N'(t) = r(t)N(t)((K(t) - N(t - mw))/(K(t) + \lambda(t)N(t - mw)))$, a.e. $t > 0$, $t \neq t_k$, $N(t_k^+) = (1 + b_k)N(t_k)$, $K = 1, 2, \dots$, where m is a positive integer, $r(t)$, $K(t)$, $\lambda(t)$ are positive periodic functions of periodic ω . In the nondelay case ($m = 0$), we show that the above equation has a unique positive periodic solution $N^*(t)$ which is globally asymptotically stable. In the delay case, we present sufficient conditions for the global attractivity of $N^*(t)$. Our results imply that under the appropriate periodic impulsive perturbations, the impulsive delay equation preserves the original periodic property of the nonimpulsive delay equation. In particular, our work extends and improves some known results.

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1. Introduction

The theory of impulsive differential equations has attracted the interest of many researchers in the past twenty years [1, 2, 9–14, 16] since they provide a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Such processes are often investigated in various fields of science and technology such as physics, population dynamics, ecology, biological systems, optimal control, and so forth. For details, see [1, 9] and references therein. Recently, the corresponding theory for impulsive functional differential equations has been studied by several authors [2, 5, 10–12, 15].

The nonlinear delay differential equation

$$N'(t) = rN(t)\left(\frac{K - N(t - \tau)}{K + \lambda N(t - \tau)}\right), \quad (1.1)$$

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where m is a positive integer and that

$$r(t), K(t), \lambda(t) \text{ are positive periodic functions of period } \omega \quad (1.2)$$

has been proposed by Smith (see [4]) for a “food-limited” population model. He demonstrated that the average growth rate is linear. It is not realistic (for the *Daphnia* populations). For more details of (1.1), one can refer to [3, 4, 7].

Recently, taking into account the effects of a periodically varying environment, Huo and Li [6] considered the following delay “food-limited” population model:

$$N'(t) = r(t)N(t) \left(\frac{K(t) - N(t - m\omega)}{K(t) + \lambda(t)N(t - m\omega)} \right) \quad (1.3)$$

and obtained sufficient conditions for existence and global attractivity of positive periodic of (1.3).

With the ideal of impulsive perturbation, we will study the existence of positive periodic solution of the following impulsive delay periodic “food-limited” population model:

$$\begin{aligned} N'(t) &= r(t)N(t) \left(\frac{K(t) - N(t - m\omega)}{K(t) + \lambda(t)N(t - m\omega)} \right), \quad \text{a.e. } t > 0, t \neq t_k, \\ N(t_k^+) &= (1 + b_k)N(t_k), \quad k = 1, 2, \dots, \end{aligned} \quad (1.4)$$

where m is a positive integer, $r(t), K(t), \lambda(t)$ are positive periodic functions of period $\omega > 0$.

Next, we will consider (1.4). In the nondelay case ($m = 0$), we will show that (1.4) has a unique positive periodic solution $N^*(t)$, which is global asymptotically stable. In the delay case, we will establish sufficient conditions for the global attractivity of $N^*(t)$. Our results imply that under the appropriate periodic impulsive perturbations, the impulsive delay equation (1.4) preserves the original periodicity of the nonimpulsive delay differential equation. In particular, our work extends the results of Huo and Li [6] for the nonimpulsive delay population model.

For the system (1.4), we make the following assumptions:

- (A₁) $0 < t_1 < t_2 < \dots$ are fixed impulsive points with $t_k \rightarrow +\infty$ as $k \rightarrow \infty$;
- (A₂) $\{b_k\}$ is a real sequence and $b_k > -1, k = 1, 2, \dots$;
- (A₃) $r(t), K(t), \lambda(t)$ and $\prod_{0 < t_k < t} (1 + b_k)$ are periodic functions with period $\omega > 0, m \geq 0$ is an integer.

Throughout this paper we always assume that a product equals unit if the number of factors is zero. Let

$$\bar{f} = \frac{1}{m\omega} \int_0^{m\omega} f(t)dt, \quad f_m = \min_{t \in [0, \omega]} f(t), \quad f_M = \max_{t \in [0, \omega]} f(t), \quad (1.5)$$

where f is a periodic continuous positive function with period ω . We will consider (1.4) with the initial condition

$$N(t) = \varphi(t), \quad \text{for } t \in [-m\omega, 0], \varphi(t) \in \mathbb{C}([-m\omega, 0], [0, \infty)), \quad \varphi(0) > 0. \quad (1.6)$$

Definition 1.1. A function $N \in ([-m\omega, +\infty), (0, +\infty))$ is said to be a solution of (1.4) on $[-m\omega, +\infty)$ if

- (i) $N(t)$ is absolutely continuous on each interval $(0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \dots$;
- (ii) for any t_k , $k = 1, 2, \dots$, $N(t_k^+)$ and $N(t_k^-)$ exist and $N(t_k^-) = N(t_k)$;
- (iii) $N(t)$ satisfies the former equation of (1.4) for a.e. in $[0, +\infty) \setminus \{t_k\}$ and satisfies the latter equation for every $t = t_k$, $k = 1, 2, \dots$

Under the hypotheses (A_1) – (A_3) , we consider the nonimpulsive delay differential equation

$$z'(t) = r(t)z(t) \left(\frac{\rho(t) - z(t - m\omega)}{\rho(t) + \lambda(t)z(t - m\omega)} \right), \quad (1.7)$$

with initial condition

$$z(t) = \varphi(t), \quad \text{for } t \in [-m\omega, 0], \quad \varphi(t) \in \mathbb{C}([-m\omega, 0], [0, \infty)), \quad \varphi(0) > 0, \quad (1.8)$$

where

$$\rho(t) = \frac{K(t)}{\prod_{0 \leq t_k < t - m\omega} (1 + b_k)}, \quad \text{for } t > 0. \quad (1.9)$$

By a solution of (1.7) and (1.8) we mean an absolutely continuous function $z(t)$ defined on $[-m\omega, +\infty)$ satisfying (1.7) a.e. for $t \geq 0$ and $z(t) = \varphi(t)$ on $[-m\omega, 0]$.

The following lemma will be used in the proofs of our results. The proof is similar to that of [16, Theorem 1]. We will omit it here.

LEMMA 1.2. Assume that (A_1) – (A_3) hold. Then

- (i) if $z(t)$ is a solution of (1.7) on $[-m\omega, +\infty)$, then $N(t) = \prod_{0 < t_k < t} (1 + b_k)z(t)$ is a solution of (1.4) on $[-m\omega, +\infty)$;
- (ii) if $N(t)$ is a solution of (1.4) on $[-m\omega, +\infty)$, then $z(t) = \prod_{0 < t_k < t} (1 + b_k)^{-1}N(t)$ is a solution of (1.7) on $[-m\omega, +\infty)$.

It is easy to know that the solutions of (1.7) are defined on $[-m\omega, +\infty]$ and are positive on $[0, \infty)$.

2. Results in the nondelay case

In this section, we will study the periodic and asymptotic behavior of all solutions of (1.7) without delay, that is,

$$z'(t) = r(t)z(t) \left(\frac{\rho(t) - z(t)}{\rho(t) + \lambda(t)z(t)} \right). \quad (2.1)$$

We will prove that there exists a unique positive periodic solution z^* which is global asymptotically stable.

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THEOREM 2.1. Assume that (A_1) – (A_3) hold. Then

- (a) there exists a unique ω -periodic positive solution $\bar{z}(t)$ of (2.1),
- (b) for every other positive solution $z(t)$ of (2.1),

$$\lim_{t \rightarrow \infty} [z(t) - \bar{z}(t)] = 0. \quad (2.2)$$

Proof. To prove (a), we define function

$$f(z) = \frac{r\rho - rz}{\rho + \lambda z}, \quad z \in (0, \infty), \quad (2.3)$$

where r, ρ , and λ are positive constants. Clearly, $f(0) = r$, $f(\infty) = -r/\lambda$, and

$$f'(z) = -\frac{r\rho(1+\lambda)}{(\rho + \lambda z)^2} < 0, \quad \forall z > 0. \quad (2.4)$$

Thus, $f(z) = 0$ has a unique positive root $z_0 \in (0, \infty)$. Furthermore, $f(z) > 0$, $z \in [0, z_0]$ and $f(z) < 0$, $z \in [z_0, \infty)$.

Now, we define two functions

$$f_1(z) = \frac{r_m \rho_m - r_M z}{\rho_M + \lambda_M z}, \quad f_2(z) = \frac{r_M \rho_M - r_m z}{\rho_m + \lambda_m z}. \quad (2.5)$$

Then from the above discussion, for $z > 0$, $f_1(z)$ and $f_2(z)$ have zeros z_1 and z_2 , respectively, that is, $f_1(z_1) = 0$ and $f_2(z_2) = 0$. Further, since

$$\frac{r_m \rho_m - r_M z_1}{\rho_M + \lambda_M z_1} < \frac{r_M \rho_M - r_m z_2}{\rho_m + \lambda_m z_2} = 0, \quad (2.6)$$

it is clear that $z_2 > z_1$. Now, suppose $z(t) = z(t, 0, z_0)$ with $z_0 > 0$ is the unique solution of (2.1) through $(0, z_0)$. We claim that $z_0 \in [z_1, z_2]$ implies that $z(t) \in [z_1, z_2]$ for all $t \geq 0$. Otherwise, let $t^* = \inf\{t > 0 \mid z(t) > z_2\}$. Then, there exists $\bar{t} \geq t^*$ such that $z(\bar{t}) > z_2$ and $z'(\bar{t}) \geq 0$. However, then from (2.1) and the fact that $z(\bar{t}) > z_2$, we conclude that for $\bar{t} \geq t^*$

$$\begin{aligned} z'(\bar{t}) &= r(\bar{t})z(\bar{t}) \left[\frac{\rho(\bar{t}) - z(\bar{t})}{\rho(\bar{t}) + \lambda(\bar{t})z(\bar{t})} \right] \\ &< z(\bar{t}) \left[\frac{r_M \rho_M - r_m z_2}{\rho_m + \lambda_m z_2} \right] = 0, \end{aligned} \quad (2.7)$$

which is a contradiction. By a similar argument, we can show that $z(t) > z_1$ for all $t \geq 0$. Hence, in particular, $z_\omega = z(\omega, 0, z_0) \in [z_1, z_2]$.

We define a mapping $F : [z_1, z_2] \rightarrow [z_1, z_2]$ as follows: for each $z_0 \in [z_1, z_2]$, $F(z_0) = z_\omega$. Since the solution $z(t, 0, z_0)$ depends continuously on the initial value z_0 , it follows that F is continuous and maps the interval $[z_1, z_2]$ into itself. By Brouwer fixed theorem, F has a fixed point $\bar{z}_0$. In view of the periodicity of $r(t)$, $\rho(t)$, and $\lambda(t)$, it follows that the unique solution $\bar{z}(t) = z(t, 0, \bar{z}_0)$ of (2.1) through the initial point $(0, z_0)$ is a positive periodic solution of period ω . The proof of (a) is complete.

Now we will prove (b). Assume that $z(t) > \bar{z}(t)$ for t sufficiently large (the proof when $z(t) < \bar{z}(t)$ is similar and will be omitted).

Set

$$z(t) = \bar{z}(t)e^{w(t)}. \quad (2.8)$$

Then, $w(t) > 0$ for t sufficiently large, and satisfies

$$w'(t) + \frac{r(t)\rho(t) - r(t)\bar{z}(t)}{\rho(t) + \lambda(t)\bar{z}(t)} - \frac{r(t)(\rho(t) - \bar{z}(t)e^{w(t)})}{\rho(t) + \lambda(t)\bar{z}(t)e^{w(t)}} = 0, \quad (2.9)$$

or

$$w'(t) + \frac{r(t)\rho(t)\bar{z}(t)(e^{w(t)} - 1)(1 + \lambda(t))}{(\rho(t) + \lambda(t)\bar{z}(t))(\rho(t) + \lambda(t)\bar{z}(t)e^{w(t)})} = 0. \quad (2.10)$$

However, since $e^{w(t)} - 1 > 0$ for t sufficiently large, it follows that

$$w'(t) = -\frac{r(t)\rho(t)\bar{z}(t)(e^{w(t)} - 1)(1 + \lambda(t))}{(\rho(t) + \lambda(t)\bar{z}(t))(\rho(t) + \lambda(t)\bar{z}(t)e^{w(t)})} < 0. \quad (2.11)$$

Thus, $w(t)$ is decreasing, and therefore $\lim_{t \rightarrow \infty} w(t) = \alpha \in [0, \infty)$. Now we will prove that $\alpha = 0$. If $\alpha > 0$, then there exists $\epsilon > 0$ and $T_\epsilon > 0$ such that $t \geq T_\epsilon$, $0 < \alpha - \epsilon < w(t) < \alpha + \epsilon$. However, from (2.11), we have

$$w'(t) \leq -\frac{r_m \rho_m \bar{z}_m (e^{\alpha - \epsilon} - 1)(1 + \lambda_m)}{(\rho_M + \lambda_M \bar{z}_M)(\rho_M + \lambda_M \bar{z}_M e^{\alpha + \epsilon})}, \quad t \geq T_\epsilon. \quad (2.12)$$

But, now an integration of (2.12) from T_ϵ to ∞ immediately gives a contradiction. Hence, $\alpha = 0$ and therefore $w(t)$ tends to zero as $t \rightarrow \infty$. Thus, we have

$$\lim_{t \rightarrow \infty} [z(t) - \bar{z}(t)] = \lim_{t \rightarrow \infty} \bar{z}(t)(e^{w(t)} - 1) = 0. \quad (2.13)$$

The proof of (b) is completed. $\square$

Remark 2.2. From the proof of Theorem 2.1, it follows that the unique ω -periodic positive solution $\bar{z}(t)$ of (2.1) satisfies $z_1 \leq \bar{z}(t) \leq z_2$. Thus, an interval for the location of $\bar{z}(t)$ is readily available.

3. Results in the delay case

In this section, we will consider the periodic delay differential equation (1.7). It is easy to see that the unique periodic positive solution $\bar{z}(t)$ of (2.1) is also a periodic positive solution of (1.7). Conversely, if (1.7) and (1.8) have an ω -periodic solution $\bar{z}(t)$, then such a solution is also a periodic solution of (2.1). Hence (1.7) has a unique ω -periodic solution $\bar{z}(t)$.

In the following we first prove that every positive solution of (1.7) which does not oscillate about $\bar{z}(t)$ converges to $\bar{z}(t)$. Finally, we will establish sufficient conditions for $\bar{z}(t)$ to be a global attractor of all other positive solutions of (1.7).

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THEOREM 3.1. *Assume that (A_1) – (A_3) hold. Let $z(t)$ be a positive solution of (1.7) which does not oscillate about $\bar{z}(t)$. Then*

$$\lim_{t \rightarrow \infty} [z(t) - \bar{z}(t)] = 0. \quad (3.1)$$

Proof. The proof is similar to that of Theorem 2.1 and it will be omitted. $\square$

To show that $\bar{z}(t)$ is a global attractor of (1.7) we also need the following lemma.

LEMMA 3.2. *Assume that (A_1) – (A_3) hold, and let $z(t)$ be a positive solution of (1.7) which oscillates about $\bar{z}(t)$. Then, there exists a T such that for all $t \geq T$,*

$$Z_1 = z_1 \exp \left(\frac{r(t)\rho(t) - r(t)z_2}{\rho(t) + \lambda(t)z_2} \right) m\omega \leq z(t) \leq z_2 \exp m\omega \overline{r(t)} = Z_2. \quad (3.2)$$

Proof. First we will show the following inequality:

$$z(t) \leq z_2 \exp m\omega \overline{r(t)} = Z_2. \quad (3.3)$$

Let $m\omega \leq t_1 < t_2 < \dots < t_l < \dots$ be a sequence of zeros $z(t) - \bar{z}(t)$ with $\lim_{l \rightarrow \infty} t_l = \infty$. Our strategy is to show that the upper bound holds in each interval (t_l, t_{l+1}) . For this, let $\zeta_l \in (t_l, t_{l+1})$ be a point where $z(t)$ attains its maximum in (t_l, t_{l+1}) . Then, it suffices to show that

$$z(\zeta_l) \leq z_2 \exp m\omega \overline{r(t)} = Z_2. \quad (3.4)$$

We can assume that there exists a ζ_l where $z(\zeta_l) > z_2$, otherwise there is nothing to prove. Since $z'(\zeta_l) = 0$, it follows that

$$0 = z'(\zeta_l) < z(\zeta_l) \left(\frac{r_M \rho_M - r_m z(\zeta_l - m\omega)}{\rho_m + \lambda_m z(\zeta_l - m\omega)} \right), \quad (3.5)$$

and hence

$$\frac{r_M \rho_M - r_m z(\zeta_l - m\omega)}{\rho_m + \lambda_m z(\zeta_l - m\omega)} > 0. \quad (3.6)$$

Thus, if $z(t)$ attains its maximum at ζ_l , then it follows (cf. see the proof of Theorem 2.1) that $z(\zeta_l - m\omega) < z_2$. Since $z(\zeta_l) > z_2$ and $z(\zeta_l - m\omega) < z_2$, we can let $\bar{\zeta}_l$ be the first zero of $z(t) - z_2$ in $(\zeta_l - m\omega, \zeta_l)$, that is, $z(\bar{\zeta}_l) = z_2$. Integrating (1.7) from $\bar{\zeta}_l$ to ζ_l , we get

$$\begin{aligned} \ln \frac{z(\zeta_l)}{z(\bar{\zeta}_l)} &= \int_{\bar{\zeta}_l}^{\zeta_l} \left(\frac{r(t)\rho(t) - r(t)z(t - m\omega)}{\rho(t) + \lambda(t)z(t - m\omega)} \right) dt \\ &< \int_{\bar{\zeta}_l}^{\zeta_l} \frac{\rho(t)r(t)}{\rho(t)} dt < \overline{r(t)} m\omega, \end{aligned} \quad (3.7)$$

which immediately gives (3.4).

Now, we will show the following inequality for $t \geq T_1 + m\omega$:

$$Z_1 = z_1 \exp \left(\int_0^{m\omega} \left(\frac{r(t)\rho(t) - r(t)z_2}{\rho(t) + \lambda(t)z_2} \right) dt \right) \leq z(t). \quad (3.8)$$

For this, following as above let $\mu_l \in (t_l, t_{l+1})$ be a point where $z(t)$ attains its minimum in (t_l, t_{l+1}) . Then, it suffices to show that

$$Z_1 = z_1 \exp \left(\int_0^{m\omega} \left(\frac{r(t)\rho(t) - r(t)z_2}{\rho(t) + \lambda(t)z_2} \right) dt \right) \leq z(\mu_l). \quad (3.9)$$

Since $Z_2 > z_2 > z_1$, it follows that

$$\frac{r_m \rho_m - r_M z_2}{\rho_M + \lambda_M z_2} < 0. \quad (3.10)$$

Thus, $Z_1 < z_1$. Now, assume that there exists a $\mu_l \geq T_1 + m\omega$ where $z(\mu_l) < z_1$, otherwise there is nothing to prove. Since $z'(\mu_l) = 0$, we have

$$0 = z'(\mu_l) > z(\mu_l) \left(\frac{r_m \rho_m - r_M z(\mu_l - m\omega)}{\rho_M + \lambda_M z(\mu_l - m\omega)} \right), \quad (3.11)$$

and hence

$$\frac{r_m \rho_m - r_M z(\mu_l - m\omega)}{\rho_M + \lambda_M z(\mu_l - m\omega)} < 0. \quad (3.12)$$

Thus, it is necessary that $z(\mu_l - m\omega) > z_1$. Hence, there exists a $\bar{\mu}_l \in (\mu_l - m\omega, \mu_l)$ where $z(\bar{\mu}_l) = z_1$. Integrating (1.7) from $\bar{\mu}_l$ to μ_l , and using $z(t) \leq z_2$ and (3.10), we get

$$\begin{aligned} \ln \frac{z(\mu_l)}{z(\bar{\mu}_l)} &= \int_{\bar{\mu}_l}^{\mu_l} \left(\frac{r(t)\rho(t) - r(t)z(t - m\omega)}{\rho(t) + \lambda(t)z(t - m\omega)} \right) dt \\ &> \int_{\bar{\mu}_l}^{\mu_l} \left(\frac{r(t)\rho(t) - r(t)z_2}{\rho(t) + \lambda(t)z_2} \right) dt \\ &> m\omega \overline{\left(\frac{r(t)\rho(t) - r(t)z_2}{\rho(t) + \lambda(t)z_2} \right)}, \end{aligned} \quad (3.13)$$

which immediately leads to (3.9). $\square$

The following result provides sufficient conditions for the global attractivity of $\bar{z}(t)$.

THEOREM 3.3. *Assume that (A_1) – (A_3) hold, and*

$$\lim_{t \rightarrow \infty} \int_{t-m\omega}^t \left((1 + \lambda(s)) \frac{K(s)r(s)Y_2(K(s) + \lambda(s)Y_1)}{(K(s) + \lambda(s)Y_1)K(s)^2} \right) ds < \frac{\pi}{2}, \quad (3.14)$$

where Z_1 and Z_2 are as in (3.2). Then,

$$\lim_{t \rightarrow \infty} [z(t) - \bar{z}(t)] = 0. \quad (3.15)$$

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Proof. In the nondelay case we have established (3.15) in Theorem 2.1(b), and for the positive solutions of (1.7) which are nonoscillatory about $\bar{z}(t)$ we have shown (3.15) in Theorem 3.1. Thus, it remains to prove (3.15) for the positive solutions of (1.7) which oscillate about $\bar{z}(t)$. If $z(t)$ is an arbitrary positive solution of (1.7), then from the transformation (2.8), (1.7) reduces to

$$w'(t) + (\lambda(t) + 1) \frac{K(t)r(t)\bar{z}(t)(e^{w(t-m\omega)} - 1)}{(K(t) + \lambda(t)\bar{z}(t-m\omega))(K(t) + \lambda(t)\bar{z}(t-m\omega)e^{w(t-m\omega)})} = 0. \quad (3.16)$$

Let

$$G(t, u) = (\lambda(t) + 1) \frac{K(t)r(t)\bar{z}(t)(e^u - 1)}{(K(t) + \lambda(t)\bar{z}(t-m\omega))(K(t) + \lambda(t)\bar{z}(t-m\omega)e^u)}. \quad (3.17)$$

Then, we have

$$\frac{\partial G(t, u)}{\partial u} = (\lambda(t) + 1) \frac{K(t)r(t)\bar{z}(t)(e^u - 1)(K(t) + \lambda(t)\bar{z}(t-m\omega))e^u}{(K(t) + \lambda(t)\bar{z}(t-m\omega))(K(t) + \lambda(t)\bar{z}(t-m\omega)e^u)^2}. \quad (3.18)$$

Equation (3.16) is the same as

$$w'(t) + G(t, w(t-m\omega)) - G(t, 0) = 0. \quad (3.19)$$

Clearly, by the mean value theorem (3.19) can be written as

$$w'(t) + F(t)w(t-m\omega) = 0, \quad (3.20)$$

where

$$\begin{aligned} F(t) &= \left. \frac{\partial G(t, u)}{\partial u} \right|_{u=\xi(t)} \\ &= (\lambda(t) + 1) \frac{K(t)r(t)\bar{z}(t)(K(t) + \lambda(t)\bar{z}(t-m\omega))e^{\xi(t)}}{(K(t) + \lambda(t)\bar{z}(t-m\omega))(K(t) + \lambda(t)\bar{z}(t-m\omega)e^{\xi(t)})^2}, \end{aligned} \quad (3.21)$$

and $\xi(t)$ lies between 0 and $w(t-m\omega)$, that is, $\min\{0, w(t-m\omega)\} < \xi(t) < \max\{0, w(t-m\omega)\}$. Then by Lemma 3.2, we obtain

$$\begin{aligned} F(t) &\geq (\lambda(t) + 1) \frac{K(t)r(t)Z_1(K(t) + \lambda(t)Z_1)}{(K(t) + \lambda(t)Z_2)(K(t) + \lambda(t)(Z_2/Z_1)Z_2)}^2, \\ F(t) &\leq (\lambda(t) + 1) \frac{K(t)r(t)Z_2(K(t) + \lambda(t)Z_1)}{(K(t) + \lambda(t)Z_1)(K(t))^2}. \end{aligned} \quad (3.22)$$

Thus, in view of (3.14), we find

$$\lim_{t \rightarrow \infty} \int_{t-m\omega}^t F(s)ds \leq \lim_{t \rightarrow \infty} \int_{t-m\omega}^t \left((1 + \lambda(s)) \frac{K(s)r(s)Z_2(K(s) + \lambda(s)Z_1)}{(K(s) + \lambda(s)Z_1)K(s)^2} \right) ds < \frac{\pi}{2}. \quad (3.23)$$

But, now by a known result in [8], every solution of (3.20) satisfies $\lim_{t \rightarrow \infty} w(t) = 0$, and hence $\lim_{t \rightarrow \infty} [z(t) - \bar{z}(t)] = 0$. $\square$

Remark 3.4. Our results in this paper indicate that under the appropriate periodic impulsive perturbations, the impulsive delay model (1.4) preserves the original periodicity of the nonimpulsive delay model (1.7).

Remark 3.5. Next, we consider the following nonimpulsive delay periodic “food-limited” population model:

$$x'(t) + \frac{\alpha V(t)x(t)x^n(t - m\omega)}{\theta^n + x^n(t - m\omega)} = \lambda(t). \quad (3.24)$$

By employing Theorem 3.3, we obtained sufficient conditions of global attractivity of positive periodic solution of (3.24). The conditions which we have obtained are given in terms of the averages of the related parameters over an interval of the common period. So our results generalized the main results in [6] which are given in terms of supremum and infimum of the parameters.

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