Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2007, Article ID 71742, 15 pages
doi:10.1155/2007/71742

Research Article

Discrete Stochastic Dynamics of Income Inequality in Education: An Applied Stochastic Model and a Case Study

Ahmet Kara

Received 26 January 2007; Accepted 31 July 2007

This paper develops a model for the discrete stochastic dynamics of economic inequality-induced outcomes in education and demonstrates, in the context of the Turkish higher education sector, that higher incomes lead to higher performance levels; and hence income inequality matters. The paper formulates a stochastic subsidy policy that could help the sector to escape the low performance equilibria, and that could stabilize the sector at relatively higher performance levels.

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1. Introduction

In recent years, there has been a renewed interest in education, which has led to a number of interesting works, among which are (Dahlgaard et al. [1, 2], Kanji et al. [3, 4], Desimone [5], Pike [6], Fethke [7], Goffe and Sosin [8], Ellis [9], Häyrinen-Alestalo and Peltola [10], Johnes [11], Marginson [12] and Leonard [13]. These works represent only a subset of the service sector literature which includes a rich spectrum of works such as Parasuraman et al. [14–16], Oliver and Swan [17], Carman [18], Babakus and Boller [19], Cronin and Taylor [20, 21], Boulding et al. [22] and Rust and Zahorik [23], Teas [24], Yavas et al. [25], Bolton and Lemon [26], Dasu and Rao [27], Caruana et al. [28], Koerner [29], Lee et al. [30], Rust et al. [31], Kara and Kurtulmuş [32], Kara et al. [33], Kara et al. [34], Kopalle and Lehmann [35], Yavas and Shemwell [36], Sen [37], Sendi et al. [38], Costa and García [39], Kayis et al. [40], Stepanovich [41], Kara [42–44], and Shih and Fang [45].

These works explore various aspects of the service sector in general and education sector in particular. The range of the topics they cover is indeed impressive but not exhaustive. There are still a number of topics that remain underexplored. Among the topics

in question is the stochastic dynamics of inequality which we will examine in this paper. We will present a model of the stochastic dynamics of economic inequality-induced outcomes in education and demonstrate, in the context of the Turkish higher education sector, that higher incomes lead to higher performance levels; and hence income inequality matters. We will formulate a stochastic subsidy policy that could help the sector to escape the low performance equilibria, and that could stabilize the sector at relatively higher performance levels.

In Section 2 of the paper, we develop the model. (The author has previously done (or participated in) work on the dynamic analysis of sectors, such as Kara et al. [34], Kara and Kurtulmuş [32] and Kara [42, 43], none of which did deal with the dynamics of income inequality in education, which is the focus of this paper.) Section 3 presents the empirical results. The policy implications are articulated in Section 4. The concluding remarks follow in Section 5.

2. The model

Consider an education sector where suppliers (such as universities) provide a service, say x, to the customers. (Educational firms could provide multiple services, in which case x could be conceived as "a composite service" representing these services.) For the sake of simplicity, we will analyze the case of a representative supplier in the market. Let D_t denote a peculiarly defined concept, namely the *quantity demanded* for service x supplied by the firm, which indicates the degree to which customers are willing to buy the service at time t. D_t depends on the relative price of the service at time t (π_t^r), and customers' income at time t (M_{ts}), that is,

$$D_t = f\left(\pi_t^r, M_{ts}\right), \tag{2.1}$$

which is a peculiar demand function. Suppose that customers' income at time t (M_{ts}) has a nonstochastic component M_t, and a stochastic component, ε_t, which is a normally distributed white noise random variable, uncorrelated across time. It has zero mean and constant variance σ_ε^2. Suppose that,

$$\ln M_{ts} = \ln M_t + \varepsilon_t. \tag{2.2}$$

Let S_t denote a peculiarly defined concept, namely the *quantity supplied* for the service, which indicates the degree to which the supplier is willing to supply the service at time t. Suppose that S_t depends on the relative price of the service (π_t^r) as well as on the present and past performances (P_t, P_{t-1}), that is, the peculiar supply function is

$$S_t = f^s\left(\pi_t^r, P_t, P_{t-1}\right). \tag{2.3}$$

(The demand and supply equations could be obtained through utility maximization and profit maximization, resp.).

For analytical purposes, we will assume that the demand and supply functions have the following explicit forms:

$$\begin{aligned} \ln D_t &= \alpha_0 + \alpha_1 \ln M_{ts} + \alpha_2 \ln \pi_t^r + u_t, \\ \ln S_t &= \beta_0 + \beta_1 \ln P_t + \beta_2 \ln P_{t-1} + \beta_3 \ln \pi_t^r + v_t, \end{aligned} \tag{2.4}$$

where u_t and v_t are independent normally distributed white noise stochastic terms uncorrelated over time. They have zero means and variances σ_u^2 and σ_v^2, respectively.

Here a peculiar feature of the supply behavior of the higher education institutions in Turkey needs to be noted. Even at low performance levels, many of these institutions do end up supplying services. The level of these services at time t depends on the level of these services at $t = 0$, and the growth rate of these services reflecting roughly the growth of student population in the system. Let, at the minimal performance levels, and in the absence of stochastic shocks, S_t have the value of A, which grows at a rate of g over time. Thus, at $P_t = 1$ and $P_{t-1} = 1$, $S_t = A(1+g)^t \Rightarrow \ln S_t = t.\ln A(1+g) = \beta_0$ (by the argument presented in the subsection on supply behavior below, the effects of prices have been left out).

To theorize about the movements over time (i.e., the dynamic trajectory) of service performance, we will make two reasonable assumptions. First, the relative strength (or magnitude) of the demand compared to the supply provides an impetus for performance to be adjusted upwards over time. Second, productivity growth contributes to performance improvements over time. Taking these factors into account, we formulate the following adjustment dynamic for performance.

$$\frac{P_{t+1}}{P_t} = \left(\frac{D_t}{S_t}\right)^K (1+\delta)^t, \tag{2.5}$$

where k is the coefficient of adjustment and δ is a productivity growth at t.

Taking the logarithmic transformation of both sides, we get

$$\ln P_{t+1} = \ln P_t + k(\ln D_t - \ln S_t) + t \ln(1+\delta). \tag{2.6}$$

We will call this the dynamic adjustment equation. Substituting the functional expressions (forms) for $\ln D_t$ and $\ln S_t$ specified above, setting the values of M_t, π_t^r, P_t, and P_{t-1} to their average values M^{avr}, π^{ravr}, P^{avr}, and $P^{\text{avr}-1}$, and rearranging the terms in the equation, we get

$$\begin{aligned} &\ln P_{t+1} + (k\beta_1 - 1)\ln P_t + k\beta_2 \ln P_{t-1} \\ &\quad = k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}}) \\ &\qquad + k(\alpha_1 \varepsilon t + u_t - v_t) - [k \ln A(1+g)]t + [\ln(1+\delta)]t, \end{aligned} \tag{2.7}$$

which is a second-order stochastic difference equation, the solution of which is provided in the appendix.

The solution in the appendix shows that the intertemporal equilibrium performance P^* is

$$\begin{aligned} P^* = \exp\Bigg\{ & \frac{k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})}{k(\beta_1 + \beta_2)} \\ & + \frac{(k\ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1+\beta_2))^2} - \frac{k\ln A(1+g) - \ln(1+\delta)}{k(\beta_1+\beta_2)} t \\ & + \frac{\lambda_1}{\lambda_1 - \lambda_2} \sum_{j=0}^{\infty} \lambda_1^j z_{t-j} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \sum_{j=0}^{\infty} \lambda_2^j z_{t-j} \Bigg\}, \end{aligned} \tag{2.8}$$

where $z_t = k(\alpha_1 \varepsilon_t + u_t - v_t)$,

$$\begin{aligned} \lambda_1 \lambda_2 &= k\beta_2, \\ \lambda_1 + \lambda_2 &= 1 - k\beta_1. \end{aligned} \tag{2.9}$$

In case where λ_1 and λ_2 are conjugate complex numbers, that is, $\lambda_1, \lambda_2 = h \pm vi = r(\cos\theta \pm i\sin\theta)$, the intertemporal equilibrium performance is

$$\begin{aligned} P^* = \exp\Bigg\{ & \frac{k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})}{k(\beta_1 + \beta_2)} \\ & + \frac{(k\ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1+\beta_2))^2} - \frac{k\ln A(1+g) - \ln(1+\delta)}{k(\beta_1+\beta_2)} t \\ & + \sum_{j=0}^{\infty} r^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j} \Bigg\}, \end{aligned} \tag{2.10}$$

where r is the absolute value of the complex number, and $\sin\theta = v/r$ and $\cos\theta = h/r$.

To determine the levels of intertemporal equilibrium performances associated with the low income and high income levels, and to determine whether they remain stable over time, we need to empirically estimate the parameters involved. This is done in the next section.

3. Empirical analysis

(a) *The sample*. Data for this study was gathered using a questionnaire including questions about demand, supply, incomes, prices, and performances in the higher educational services in Turkey. The questionnaire was distributed to the students in two consecutive time periods. In the first period 100 students were asked to respond to the relevant questions. 66 useable questionnaires were returned giving a response rate of 66 percent, which was considered satisfactory for subsequent analysis. Some responses with considerable missing information were excluded. Each question (item) was rated on a seven-point Likert scale anchored at the numeral 1 representing the lowest score that can be assigned, and at the numeral 7 representing the highest. The same procedure has been repeated to obtain the data for the second period. The supply-side information is obtained from the educational institutions.

(b) *Estimation of the parameters*. To estimate the parameters involved, we formulate the following regression equations:

$$\begin{aligned} \ln D_t &= \alpha_0 + \alpha_1 \ln M_t + \alpha_2 \ln \pi_t^r + u_{t^*}, \\ \ln S_t &= \beta_0 + \beta_1 \ln P_t + \beta_2 \ln P_{t-1} + \beta_3 \ln \pi_t^r + v_t, \end{aligned} \tag{3.1}$$

where $u_{t^*} = u_t + \alpha_1 \varepsilon_t$ and v_t are disturbance terms.

(i) *Demand*. Since the prices under consideration are fairly close to one another, the relative prices are close to one, thus $\ln P_t^r$ is close to zero, and as such, it drops out of the equation. This supposition and an additional assumption that minimal income induces minimal demand imply that $\alpha_0 = 0$. We will estimate demand for two cases, namely for the case with low incomes and for the case with high incomes. The regression-results are as follows:

$$\begin{aligned} &\text{The low income-case: } \ln D_t = 0.762 \ln M_t \, (2.017) \\ &R^2 = 0.40. \ t\text{-statistic is given in parentheses.} \\ &\text{The high income-case: } \ln D_t = 0.762 \ln M_t \, (10.407) \\ &R_2 = 0.87. \ t\text{-statistic is given in parentheses.} \end{aligned} \tag{3.2}$$

Thus,

$$\begin{aligned} \alpha_0 &= 0, \\ \alpha_1 &= 0.762, \\ \alpha_2 &= 0. \end{aligned} \tag{3.3}$$

(ii) *Supply*. Supply is largely determined by central bureaucratic authorities whose decisions are based on certain criteria, such as the adequacy and quality of physical infrastructure and human resources, rather than prices. Thus, prices could be conveniently left out of the supply function. P_t^r drops out of the log-linear formulation of the supply equation. To estimate the other parameters of the supply equation, we asked officials of the relevant institutions questions, the answers of which were designed to give the values of the elasticities of supply with respect to the present and past d performances. The answers indicate that a 1% increase in the past performance would increase the quantity supplied by about 0.25%, but a 1% increase in the present performance would increase the quantity supplied by about 0.75%. However, by virtue of the enrollment constraints placed by the Higher Education Council, what the institutions under examination could supply was 90% of what they were willing to supply. Thus,

$$\begin{aligned} \beta_1 &= 0.9^* 0.75 = 0.675, \\ \beta_2 &= 0.9^* 0.25 = 0.225. \end{aligned} \tag{3.4}$$

The value of A is normalized to 1.

(iii) *The coefficient of adjustment* (k). For simplicity, we will assume that P_{t+1}/P_t is proportional to the ratio of demand to supply, and hence, $k = 1$.

Given the empirical values of the parameters obtained above, we get,

$$\begin{aligned} \lambda_1 &= 0.162 + 0.445i, \\ \lambda_2 &= 0.162 - 0.445i. \end{aligned} \tag{3.5}$$

We will now consider a particularly interesting case where the student population growth is equal to the productivity growth, that is, $g = \delta$. With this assumption and with all the needed parameter values at hand, the intertemporal equilibrium performance for low-income and high-income cases, P^*_{low} and P^*_{high}, are

$$\begin{aligned} P^*_{\text{low}} &= \exp\left\{0.56 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j}\right\}, \\ P^*_{\text{high}} &= \exp\left\{1.50 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j}\right\}. \end{aligned} \tag{3.6}$$

For analytical convenience, we will carry out some of our analysis in terms of logarithmically transformed performance, $\ln P$, rather than P. Since ln function is an order-preserving transformation, analysis in terms of $\ln P$ and P will yield the same qualitative results; and the quantitative results could be transformed into one another. The expected value of the logarithmically transformed intertemporal equilibrium performances for low-income and high-income cases are

$$\begin{aligned} E(\ln P^*_{\text{low}}) &= 0.56 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}), \\ E(\ln P^*_{\text{high}}) &= 1.50 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}). \end{aligned} \tag{3.7}$$

Since, by virtue of the assumptions about ε_t, u_t, and v_t, $E(\varepsilon_t) = 0$, $E(u_t) = 0$, and $E(v_t) = 0$, $E(z_t) = k(\alpha_1 E(\varepsilon_t) + E(u_t) - E(v_t)) = 0$. Thus,

$$\begin{aligned} E(\ln P^*_{\text{low}}) &= 0.56, \\ E(\ln P^*_{\text{high}}) &= 1.50. \end{aligned} \tag{3.8}$$

In view of the logarithmically transformed performance scale of $\ln 1 = 0$ to $\ln 7 \cong 1.95$, an intertemporal equilibrium expected performance of 0.56 is low, and a performance of 1.5 is high. As proven in the appendix, these performance values are also stable over time in the particular sense that they have stationary distributions with constant means and variances. This indicates that low income and high income help lead to, respectively, stable low performance and stable high performance values over time.

The following section will formulate a stochastic subsidy policy, which will enable the sector to escape the low performance equilibria by helping the sector to reach a high-performance target, and which will stabilize the sector around that target.

4. Policy implications: an example of a stochastic subsidy policy

Suppose that the educational service providers in Turkey aim to reach a stable (sustainable) high-performance target in the presence of stochastic shocks. Consider a stochastic shock to income ($\ln M_{ts}$) in the magnitude of ε_t, which may have come, for instance, from an economic downturn (of the kind that took place in Turkey in 2001) leading to reductions in customers' incomes. Let us design the following demand side stochastic policy response (a subsidy rule):

$$\mathrm{SR}_t = \eta_0 + \eta_1 \varepsilon_t, \tag{4.1}$$

where η_0 and η_1 represent the nonstochastic and stochastic components, respectively. With this rule, the modified income level, in logarithmic form, will be defined as

$$\ln \widetilde{M}_t = \ln M_{ts} + \mathrm{SR}_t = \ln M_t + \eta_0 + (1+\eta_1)\varepsilon_t. \tag{4.2}$$

With the modified income level, the second order stochastic difference equation will be

$$\begin{aligned} &\ln P_{t+1} + (k\beta_1 - 1)\ln P_t + k(\beta_2)\ln P_{t-1} \\ &\quad = k(\alpha_0 + \alpha_1 \ln M^{\mathrm{avr}} + \alpha_1\eta_0 + (\alpha_2 - \beta_3)\ln \pi^{\mathrm{ravr}}) \\ &\qquad + k(\alpha_1(1+\eta_1)\varepsilon_t + u_t - v_t) - [k\ln A(1+g)]t + [\ln(1+\delta)]t. \end{aligned} \tag{4.3}$$

Reaching a stable (minimally varying) expected quality target in the presence of stochastic shocks turns out to be equivalent to minimizing the expected loss function of the following kind:

$$E[(P_t - P^{**})^2], \tag{4.4}$$

where P^{**} is the performance target. We decompose the expected loss function in the following manner. For a decomposition, though in a different context, see Sargent [46];

$$\begin{aligned} E[(P_t - P^{**})^2] &= E[((Pt - E(Pt)) + (E(Pt) - P^{**}))^2] \\ &= E((Pt - E(Pt))^2) + E((E(Pt) - P^{**})^2) \\ &\quad + 2E(Pt - E(Pt))(E(Pt) - P^{**}). \end{aligned} \tag{4.5}$$

Since $E(Pt) - P^{**}$ is not random and since $E(Pt - E(Pt)) = E(Pt) - E(Pt) = 0$, the decomposition will boil down to

$$E[(P_t - P^{**})^2] = E((Pt - E(Pt))^2) + (E(Pt) - P^{**})^2. \tag{4.6}$$

The first term represents the variance of performance and the second term denotes the "squared deviation" around P^{**}. Thus, minimizing expected loss is equivalent to minimizing the squared deviation, which requires that expected performance to be equal to the performance target, and minimizing the variance of performance, enabling the educational service provider to reach a stable (minimally varying) performance target.

To find, for the special case where $g = \delta$, the parameters of the stochastic policy rule which minimize the expected loss function, let us incorporate the rule into the function,

$$E[(P_t - P^{**})^2] = \left\{ \sum_{j=0}^{\infty} 0.47^{2j} \left(\frac{\sin\theta(j+1)}{\sin\theta} \right)^2 \right\} [(\alpha_1^2(1+\eta_1)^2\sigma_\varepsilon^2 + \sigma_u^2 + \sigma_v^2)] + \left\{ \frac{\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln P^{\text{ravr}}}{\beta_1 + \beta_2} + \frac{\alpha_1\eta_0}{\beta_1+\beta_2} - P^{**} \right\}^2. \tag{4.7}$$

The values of η_0 and η_1 that minimize the expected loss function are

$$\eta_0 = \left\{ P^{**} - \frac{\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}}}{\beta_1 + \beta_2} \right\} \frac{(\beta_1+\beta_2)}{\alpha_1}, \qquad \eta_1 = -1. \tag{4.8}$$

For instance, the value of η_0, which will bring the low-income-induced performance above (0.56) to the level of a performance target $P^{**} = 1.5$, is calculated to be 1.11. This represents the nonstochastic component of the subsidy. The value of the stochastic component of the subsidy, $\eta_1 = -1$, implies that, for stabilization against the kind of negative (income-reducing) shock exemplified here, income should be increased by the magnitude of the stochastic shock.

The applicability of the model developed in this paper is not restricted to the case in Turkey. Inequality-induced outcomes in education are characteristics of many developing and even developed countries. Inequality patterns tend to evolve in response to a number of factors including changes in demand and technology. This model could serve as a frame of reference to account for such changes/developments in the education sectors in various countries. Among these developments are the information-technology-based changes, which have been transforming the supply and demand in education sectors across the world. Information technology has increased productivity and quality, reduced costs, and increased the expectations for higher income associated with information-intensive skills acquired through education. The stochastic subsidy policy proposed above could be reformulated to incorporate the effects of information technology. Our conjecture is that possibilities created by the information technology may, under certain conditions, well increase the effectiveness of the stochastic subsidy policy in reducing the degree of inequality in education.

5. Concluding remarks

The paper develops a model for the stochastic dynamics of economic inequality-induced outcomes in education, and explores the differences in service performance induced by unequal incomes. The paper formulates a stochastic subsidy policy that could help the sector to escape the low performance equilibria and that could stabilize the sector at relatively higher performance levels. The designed policy response is one among many other stochastic resolutions which could take the form of, for instance, a demand side policy or a supply side policy. The rich array of policies in question are worthy of future research.

Appendix

The solution for the second-order stochastic difference equation,

$$\begin{aligned}&\ln P_{t+1} + (k\beta_1 - 1)\ln P_t + k\beta_2 \ln P_{t-1}\\&\quad = k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}}) + k(\alpha_1 \varepsilon_t + u_t - v_t)\\&\qquad - [k \ln A(1+g)]t + [\ln(1+\delta)]t,\end{aligned} \tag{A.1}$$

has two components, namely, a particular solution and a complementary function. We will find these components for $\ln P_t$ and then take the antilog of $\ln P_t$ so as to find the solution for P_t.

(a) *Particular solution.* Letting $x_t = \ln P_t$, using the lag operator L (defined as $L^i P_t = P_{t-i}$, for $i = 1,2,3\ldots$), and rearranging the terms, we get the following form of the second-order stochastic difference equation above:

$$\begin{aligned}[1 - (1 - k\beta_1)L - (-k\beta_2)L^2]x_t &= k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})\\&\quad + k(\alpha_1 \varepsilon t + u_t - v_t) - [k \ln A(1+g)]t + [\ln(1+\delta)]t,\end{aligned} \tag{A.2}$$

which could be transformed into

$$\begin{aligned}(1 - \lambda_1 L)(1 - \lambda_2 L)x_t &= k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})\\&\quad + k(\alpha_1 \varepsilon t + u_t - v_t) - [k \ln A(1+g)]t + [\ln(1+\delta)]t,\end{aligned} \tag{A.3}$$

where

$$\begin{aligned}\lambda_1 \lambda_2 &= k\beta_2,\\ \lambda_1 + \lambda_2 &= 1 - k\beta_1.\end{aligned} \tag{A.4}$$

Thus, we get

$$x_t = (1 - \lambda_1 L)^{-1}(1 - \lambda_2 L)^{-1} \begin{Bmatrix} k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}}) \\ +k(\alpha_1 \varepsilon_t + u_t - v_t) - [k \ln A(1+g) - \ln(1+\delta)]t \end{Bmatrix}. \tag{A.5}$$

Using the properties of partial fractions,

$$(1 - \lambda_1 L)^{-1}(1 - \lambda_2 L)^{-1} = \frac{\lambda_1}{\lambda_1 - \lambda_2}(1 - \lambda_1 L)^{-1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - \lambda_2 L)^{-1}. \tag{A.6}$$

Thus,

$$x_t = \frac{\lambda_1}{\lambda_1 - \lambda_2}(1 - \lambda_1 L)^{-1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - \lambda_2 L)^{-1} \begin{Bmatrix} k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}} \\ +k(\alpha_1 \varepsilon_t + u_t - v_t) \\ -[k \ln A(1+g) - \ln(1+\delta)]t \end{Bmatrix}. \tag{A.7}$$

Using the properties of some series and lag operators and doing some algebraic manipulations, we get

$$\begin{aligned} x_t = \Bigg\{ & \frac{k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})}{k(\beta_1 + \beta_2)} \\ & + \frac{(k\ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1+\beta_2))^2} - \frac{k\ln A(1+g) - \ln(1+\delta)}{k(\beta_1+\beta_2)} t \\ & + \frac{\lambda_1}{\lambda_1 - \lambda_2} \sum_{j=0}^{\infty} \lambda_1^j z_{t-j} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \sum_{j=0}^{\infty} \lambda_2^j z_{t-j} \Bigg\}, \end{aligned} \tag{A.8}$$

where $z_t = k(\alpha_1 \varepsilon_t + u_t - v_t)$.

This is the parametric expression of $x_t = \ln P_t$ at the intertemporal equilibrium. Let P^* denote the intertemporal equilibrium performance. Thus,

$$\begin{aligned} P^* = \exp \Bigg\{ & \frac{k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})}{k(\beta_1 + \beta_2)} \\ & + \frac{(k\ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1+\beta_2))^2} - \frac{k\ln A(1+g) - \ln(1+\delta)}{k(\beta_1+\beta_2)} t \\ & + \frac{\lambda_1}{\lambda_1 - \lambda_2} \sum_{j=0}^{\infty} \lambda_1^j z_{t-j} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \sum_{j=0}^{\infty} \lambda_2^j z_{t-j} \Bigg\}. \end{aligned} \tag{A.9}$$

In case where λ_1 and λ_2 are conjugate complex numbers, that is, $\lambda_1, \lambda_2 = h \pm vi = r(\cos\theta \pm i\sin\theta)$, the intertemporal equilibrium performance is

$$\begin{aligned} P^* = \exp \Bigg\{ & \frac{k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})}{k(\beta_1 + \beta_2)} \\ & + \frac{(k\ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1+\beta_2))^2} - \frac{k\ln A(1+g) - \ln(1+\delta)}{k(\beta_1+\beta_2)} t \\ & + \sum_{j=0}^{\infty} r^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j} \Bigg\}, \end{aligned} \tag{A.10}$$

where r is the absolute value of the complex number, and $\sin\theta = v/r$ and $\cos\theta = h/r$.

(b) *Complementary function.* To find this component of the solution, we need to consider the following reduced form of the second-order difference equation:

$$\ln P_{t+1} + (k\beta_1 - 1)\ln P_t + k\beta_2 \ln P_{t-1} = 0. \tag{A.11}$$

A possible general solution could take the form $\ln P_t = Ay^t$. Hence, $\ln P_{t+1} = Ay^{t+1}$ and $\ln P_{t-1} = Ay^{t-1}$. Substituting these expressions into the reduced form of the second-order equation, we get

$$Ay^{t+1} + (k\beta_1 - 1)Ay^t + k\beta_2 Ay^{t-1} = 0. \tag{A.12}$$

Canceling the common factor $Ay^{t-1} \neq 0$,

$$y^2 + (k\beta_1 - 1)y + k\beta_2 = 0. \tag{A.13}$$

This quadratic equation could have at most two roots. Substituting $\beta_1 = 0.675$ and $\beta_2 = 0.225$, and $k = 1$ into the quadratic equation and solving it for the roots, we get

$$\begin{aligned} y_1 &= 0.162 + 0.445i, \\ y_2 &= 0.162 - 0.445i. \end{aligned} \tag{A.14}$$

Thus, the solution for the reduced equation is

$$A_1 y_1^t + A_2 y_2^t = A_1(0.162 + 0.445i)^t + A_2(0.162 - 0.445i)^t, \tag{A.15}$$

where A_1 and A_2 are nonzero constants. This could be shown to be equivalent to

$$0.47^t(A_3 \cos\theta t + A_4 \sin\theta t), \tag{A.16}$$

where $A_3 = A_1 + A_2$ and $A_4 = (A_1 - A_2)i$, $\sin\theta = 0.445/0.47$ and $\cos\theta = 0.162/0.47$.

(c) The general solution. The general solution for the equation is the sum of the two solutions obtained in (a) and (b),

$$\begin{aligned} \ln P^* = \Bigg\{ & \frac{k(\alpha_0 + \alpha_1 \ln M^{\text{avr}} + (\alpha_2 - \beta_3)\ln \pi^{\text{ravr}})}{k(\beta_1 + \beta_2)} \\ & + \frac{(k\ln A(1+g) - \ln(1+\delta))(1 - k\beta_2)}{(k(\beta_1+\beta_2))^2} - \frac{k\ln A(1+g) - \ln(1+\delta)}{k(\beta_1+\beta_2)} t \\ & + \sum_{j=0}^{\propto} r^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j} \Bigg\} \\ & + 0.47t(A_3 \cos\theta t + A_4 \sin\theta_t). \end{aligned} \tag{A.17}$$

In the paper, we analyze the case where $g = \delta$. Substituting the values of the parameters involved, we get, for this special case,

$$\begin{aligned} \ln P^*_{\text{low}} &= 0.56 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}) + 0.47^t(A_3 \cos\theta t + A_4 \sin\theta t), \\ \ln P^*_{\text{high}} &= 1.50 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}) + 0.47^t(A_3 \cos\theta t + A_4 \sin\theta t). \end{aligned} \tag{A.18}$$

The values of A_3 and A_4 could be obtained by specifying two initial conditions. However, for the purposes of our analysis, we do not need to know the values of those constants.

Since the absolute value of the complex number involved is 0.47, which is less than 1, as $t \to \propto$, $0.47^t(A_3 \cos\theta t + A_4 \sin\theta t)$ will converge toward zero, and hence the general

solution converges toward the particular solution,

$$\begin{aligned}
\ln P^*_{\text{low}} &= 0.56 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}), \quad \text{for the low-income case,} \\
\ln P^*_{\text{high}} &= 1.50 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}), \quad \text{for the high-income case.}
\end{aligned} \tag{A.19}$$

Thus,

$$\begin{aligned}
E(\ln P^*_{\text{low}}) &= 0.56 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}), \\
E(\ln P^*_{\text{high}}) &= 1.50 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} E(z_{t-j}).
\end{aligned} \tag{A.20}$$

Since, by virtue of the assumptions about ε_t, u_t, and v_t, $E(\varepsilon_t) = 0$, $E(u_t) = 0$, and $E(v_t) = 0$, $E(z_t) = k(\alpha_1 E(\varepsilon_t) + E(u_t) - E(v_t)) = 0$. Thus,

$$\begin{aligned}
E(\ln P^*_{\text{low}}) &= 0.56, \\
E(\ln P^*_{\text{high}}) &= 1.50,
\end{aligned} \tag{A.21}$$

which are nothing but the intertemporal expected equilibrium performances in low-income and high-income cases, respectively. Note that ε_t, u_t, and v_t are uncorrelated over time, and so is z_t. They have zero covariances. Thus, the variance (V) of $\ln P^*$ is

$$\begin{aligned}
V(\ln P^*_{\text{low}}) &= V\left\{0.56 + \sum_{j=0}^{\propto} 0.47^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j}\right\}, \\
V(\ln P^*_{\text{low}}) &= \left\{\sum_{j=0}^{\propto} 0.47^{2j} \left(\frac{\sin\theta(j+1)}{\sin\theta}\right)^2\right\} V(z_{t-j}), \\
V(\ln P^*_{\text{low}}) &= \left\{\sum_{j=0}^{\propto} 0.47^{2j} \left(\frac{\sin\theta(j+1)}{\sin\theta}\right)^2\right\} [(\alpha_1^2(1+\eta_1)^2\sigma_\varepsilon^2 + \sigma_u^2 + \sigma_v^2)],
\end{aligned} \tag{A.22}$$

which is constant. (Please note the value of $\sin\theta$ specified above.) It is straightforward to show that the variance in the case of high income is the same as well. Taking the limits of mean and variance as $t \to \propto$,

$$\begin{aligned}
&\lim_{t\to\propto} E(\ln P^*_{\text{low}}) = 0.56, \\
&\lim_{t\to\propto} E(\ln Ph^*_{\text{high}}) = 1.50, \\
&\lim_{t\to\propto} V(\ln P^*_{\text{low}}) = \lim_{t\to\propto} V(\ln P^*_{\text{high}}) \\
&\qquad = \left\{\sum_{j=0}^{\propto} 0.47^{2j} \left(\frac{\sin\theta(j+1)}{\sin\theta}\right)^2\right\} [(\alpha_1^2(1+\eta_1)^2\sigma_\varepsilon^2 + \sigma_u^2 + \sigma_v^2)].
\end{aligned} \tag{A.23}$$

Thus, logarithmically transformed intertemporal performances in low-income and high-income cases have stationary distributions in the sense that they have constant means and variances.

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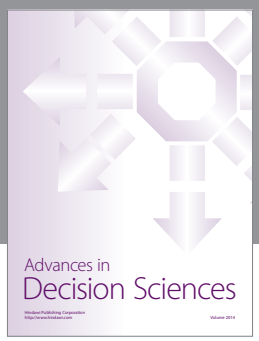
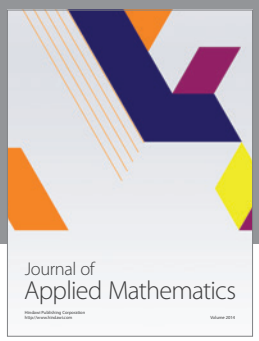
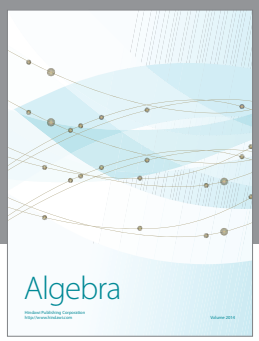
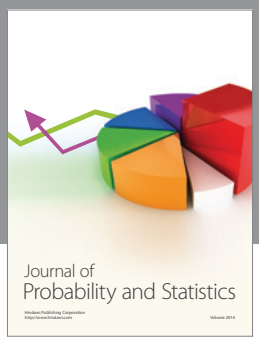
Ahmet Kara: Department of Economics, Fatih University, 34500 Büyükçekmece, Istanbul, Turkey
Email address: econs@fatih.edu.tr









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