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Research Article

Existence and Global Asymptotical Stability of Periodic Solution for the T-Periodic Logistic System with Time-Varying Generating Operators and T_0-Periodic Impulsive Perturbations on Banach Spaces

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This paper studies the existence and global asymptotical stability of periodic PC-mild solution for the T-periodic logistic system with time-varying generating operators and T_0-periodic impulsive perturbations on Banach spaces. Two sufficient conditions that guarantee the exponential stability of the impulsive evolution operator corresponding to homogenous well-posed T-periodic system with time-varying generating operators and T_0-periodic impulsive perturbations are given. It is shown that the system have a unique periodic PC-mild solution which is globally asymptotically stable when T and T_0 are rational dependent and its period must be nT_0 for some $n \in N$. At last, an example is given for demonstration.

1. Introduction

For modeling the dynamics of an ecological system, Cushing [1] pointed out that it is necessary and important to consider models with periodic ecological parameters or perturbations which might be quite naturally exposed (e.g., those due to seasonal effects of wheatear, food supply, mating habits, etc.). The periodic solution theory of dynamic equations has been developed over the last decades. We refer the readers to [2–6] for infinite dimensional cases, and to [1, 7–9]

for finite dimensional cases. Especially, there are many results of periodic solutions (such as existence and stability) for impulsive periodic systems on finite dimensional spaces (see [7–9]). There are also some relative results of periodic solutions for periodic systems with time-varying generating operators on infinite dimensional spaces (see [5, 6, 10–12]).

On the other hand, the ecological system is often deeply perturbed by human exploit activities such as planting and harvesting. Usually, these activities are considered continuously by adding some items to [13–15], whereas this is not how things stand. It is often the case that planting and harvesting of the species are seasonal or occur in regular pulses. These perturbations may also naturally be periodic, for example, a fisherman may go fishing at the same time once a day or once a week. Systems with short-term perturbations are often naturally described by impulsive differential equations, which are found in almost every domain of applied sciences. For the basic theory on impulsive differential equations on finite dimensional spaces, the reader can refer to Lakshmikantham's book and Yang's book (see [9, 16]). For the basic theory on impulsive differential equations on infinite dimensional spaces, the reader can refer to Ahmed's paper, Liu's paper, and Xiang's papers (see [17–19]).

In this paper, we will study the following generalized Logistic system with impulsive perturbations:

$$\begin{gathered}\frac{\partial}{\partial t}x(t,y) = A(y,t,D)x(t,y) + f(t,y), \quad y \in \Omega,\ t > 0,\ t \neq \tau_k,\ k \in Z_0^+,\\ x(t,y) = 0, \quad y \in \partial\Omega,\ t > 0,\\ \Delta x(t,y) = B_k x(t,y) + c_k, \quad y \in \Omega,\ t = \pi_k,\ k \in Z_0^+,\end{gathered} \tag{1.1}$$

where $x(t,y)$ denotes the population number of isolated species at time t and location y, Ω is an open-bounded domain in R^2, and $\partial\Omega$ is smooth enough. The operator

$$A(y,t,D) \equiv \sum_{1\le i,j\le 2} a_{ij}(y,t)D_iD_j + \sum_{1\le i\le 2} b_i(y,t)D_i + c(y,t), \quad (y,t) \in \big(\Omega \times [0,+\infty)\big), \tag{1.2}$$

where all the coefficients are smooth functions enough and D_i denotes the spatial derivative with respect to y_i. f is related to the periodic change of the resources maintaining the evolution of the population. Time sequence $0 = \tau_0 < \tau_1 < \cdots < \tau_k \cdots$ and $\tau_k \to \infty$ as $k \to \infty$, $\Delta x(\tau_k, y) = x(\tau_k^+, y) - x(\tau_k, y) = B_k x(\tau_k, y) + c_k$ denote mutation of the isolate species at time τ_k.

Suppose the first equation of (1.1) is T-periodic and the third equation of (1.1) is T_0-periodic, that is, $A(y,t+T,D) = A(y,t,D)$, $f(t+T,y) = f(t,y)$, $t \ge 0$, and T_0 is the least-positive constant such that there are δ τ_ks in the interval $(0,T_0)$ and $\tau_{k+\delta} = \tau_k + T_0$, $B_{k+\delta} = B_k$, $c_{k+\delta} = c_k$, $k \in Z_0^+$.

The first equation of (1.1) describes the variation of the population number x of the species in T-periodically changing environment. The second equation of (1.1) shows that the species is isolated. The third equation of (1.1) reflects the possibility of impulsive effects on the population. As we assumed, these impulsive perturbations are T_0-periodic. Naturally, this period is distinct from T, the period of the change of environment. Even when we want to carry out the perturbations according to the period T, we cannot do it since we do not know T exactly. Thus, it is interesting how the dynamics of the first equation of (1.1) are affected by the periodic changing of environment together with the periodic impulsive perturbations.

Define $A(t)(y) = A(y,t,D)$, $x(\cdot)(y) = x(\cdot,y)$, $f(\cdot)(y) = f(\cdot,y)$, (1.1) can be abstracted impulsive periodic evolution equations of the form

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + f(t), \quad t \neq \tau_k, \ (T\text{-periodic}) \\ \Delta x(t) &= B_k x(t) + c_k, \quad t = \tau_k, \ (T_0\text{-periodic}) \end{aligned} \tag{1.3}$$

on the Banach space X. It is obvious that the investigation of (1.3) cannot only be used to discuss (1.1) but also provide a foundation for study of the general impulsive periodic systems.

Assume that T-periodic evolution equation of the form $\dot{x}(t) = A(t)x(t),\ x \in X,\ t > 0$, is well posed, that is, there exists a T-periodic evolutionary process $\{U(t,\theta),\ t \geq \theta \geq 0\}$ which satisfies, among other things, the conditions (1)–(5) of Definition 2.1 which follows. Once the evolution equations $\dot{x}(t) = A(t)x(t),\ x \in X,\ t > 0$, and $\dot{x}(t) = A(t)x(t) + f(t),\ x \in X,\ t > 0$, are well posed, the asymptotic behavior of solutions at infinity, such as stability and periodicity, is of particular interest, which has been a central topic discussed for the past decade. We refer the reader to the books [20, 21], and the surveys [22], and the references therein for more complete information on the subject. Because (1.3) can be used to describe more social and natural phenomena, it is naturally needed to study the stability and periodicity of solutions for (1.3).

The aim of this paper is to study the existence and global asymptotical stability of periodic PC-mild solution of (1.3) without compactness condition. We will show that (i) if $\gamma = T/T_0$ is rational, that is, T and T_0 are rational dependent, then (1.3) may have a unique periodic PC-mild solution which is globally asymptotically stable and (ii) if $x(t,\overline{x})$ is a periodic PC-mild solution of (1.3) with $x(0) = \overline{x}$, then its period must be nT_0 for some $n \in N$.

This paper is organized as follows. In Section 2, the properties of the impulsive evolution operator are collected, two sufficient conditions that guarantee the exponential stability of the impulsive evolution operator are given. In Section 3, the existence of periodic PC-mild solution which is globally asymptotically stable for (1.3) is obtained. At last, the abstract results are applied to a special case of (1.1). This work not only provides the theory basis for managing some single species but is also fundamental for further discussion on the existence and stability of periodic solution for nonlinear impulsive periodic system with time-varying generating operators on infinite dimensional spaces.

2. Exponential stability of impulsive evolution operator

Let X be a Banach space, $\pounds(X)$ denotes the space of linear operators on X; $\pounds_b(X)$ denotes the space of bounded linear operators on X. Let $\pounds_b(X)$ be the Banach space with the usual supremum norm. Denote γ is rational, let $\gamma = p/q$, $p,q \in N$, p, q are relatively prime. Denote $\widetilde{T} = pT_0 = qT$, $\widetilde{D} = \{\tau_1,\ldots,\tau_{p\delta}\} \subset [0,\widetilde{T}]$ and define $\mathrm{PC}([0,\widetilde{T};X) \equiv \{x : [0,\widetilde{T}] \to X \mid x$ is continuous at $t \in [0,\widetilde{T}] \setminus \widetilde{D}$, x is continuous from left and has right-hand limits at $t \in \widetilde{D}\}$, and $\mathrm{PC}^1([0,\widetilde{T};X) \equiv \{x \in \mathrm{PC}([0,\widetilde{T};X) \mid \dot{x} \in \mathrm{PC}([0,\widetilde{T};X)\}$. Set

$$\|x\|_{\mathrm{PC}} = \max\left\{\sup_{t\in[0,\widetilde{T}]}\|x(t+0)\|, \sup_{t\in[0,\widetilde{T}]}\|x(t-0)\|\right\}, \qquad \|x\|_{\mathrm{PC}^1} = \|x\|_{\mathrm{PC}} + \|\dot{x}\|_{\mathrm{PC}}. \tag{2.1}$$

It can be seen that endowed with the norm $\|\cdot\|_{\mathrm{PC}}$ ($\|\cdot\|_{\mathrm{PC}^1}$), $\mathrm{PC}([0,\widetilde{T};X)$ ($\mathrm{PC}^1([0,\widetilde{T};X)$) is a Banach space.

In order to investigate periodic solution, introduce the following two spaces:

$$L_T^1([0,+\infty);X) \equiv \left\{ f : [0,+\infty) \longrightarrow X \mid f(t) = f(t+T),\ \int_0^T \|f(t)\| dt < \infty \right\},$$
$$\mathrm{PC}_{\tilde{T}}([0,+\infty);X) \equiv \{ x \in \mathrm{PC}([0,+\infty);X) \mid x(t) = x(t+\tilde{T}),\ t \geq 0 \}. \tag{2.2}$$

Set

$$\|f\|_{L_T^1} = \int_0^T \|f(t)\| dt < \infty, \qquad \|x\|_{\mathrm{PC}_{\tilde{T}}} = \max\left\{ \sup_{t\in[0,\tilde{T}]} \|x(t+0)\|, \sup_{t\in[0,\tilde{T}]} \|x(t-0)\| \right\}. \tag{2.3}$$

It can be seen that endowed with the norm $\|\cdot\|_{L_T^1}$ ($\|\cdot\|_{\mathrm{PC}_{\tilde{T}}}$), $L_T^1([0,+\infty);X)$ ($\mathrm{PC}_{\tilde{T}}([0,+\infty);X)$) is a Banach space.

The following notation will be used throughout the paper, we recall these concepts in the following definitions.

Definition 2.1. A family of bounded linear operators $\{U(t,\theta),\ t \geq \theta \geq 0\}$ from a Banach space X to itself is called strongly continuous evolutionary process if the following conditions (1)–(4) are satisfied:

(1) $U(t,t) = I, t \geq 0$,

(2) $U(t,r)U(r,\theta) = U(t,\theta),\ t \geq r \geq \theta \geq 0$,

(3) the map $(t,\theta) \to U(t,\theta)x$ is continuous for every fixed $x \in X$,

(4) $\|U(t,\theta)\| \leq Me^{\omega(t-\theta)}$ for some $M \geq 1$, $\omega \in R$ independent of $t \geq \theta \geq 0$,

further, if

(5) $U(t+T,\theta+T) = U(t,\theta)$ for all $t \geq \theta \geq 0$.

Then the strongly continuous evolutionary process $\{U(t,\theta),\ t \geq \theta \geq 0\}$ is called T-periodic.

Definition 2.2. The evolutionary process $\{U(t,\theta),\ t \geq \theta \geq 0\}$ is called exponentially bounded if

$$\varpi(U) = \inf\left\{ \omega \in R : \exists M \geq 1 \text{ with } \|U(t,\theta)\| \leq Me^{\omega(t-\theta)} \text{ for } t \geq \theta \geq 0 \right\} < \infty. \tag{2.4}$$

Definition 2.3. The linear equation

$$\dot{x}(t) = A(t)x(t), \quad x \in X,\ t \geq 0 \tag{2.5}$$

is said to be well posed if there exists an evolutionary process $\{U(t,\theta),\ t \geq \theta \geq 0\}$, which satisfies the conditions (1)–(4) in Definition 2.1, such that for every $\theta \geq 0$ and $x \in D(A(\theta))$, the function $x(t) = U(t,\theta)\overline{x}$ is the uniquely determined classical solution of (2.5) satisfying $x(\theta) = \overline{x}$.

Definition 2.4. The function $x \in X$ is said to be a mild solution to the linear equation (2.5) if and only if

$$x(t) = U(t,\theta)x(\theta), \quad t \geq \theta \geq 0. \tag{2.6}$$

We introduce the following assumption (H1):

(H1.1) $A(t) : D(A(t)) \to X$, $t \geq 0$, is a family of linear unbounded operators on X, $A(t)$ is T-periodic, that is, for $t \geq 0$, $A(t+T) = A(t)$;

(H1.2) the linear equation $\dot{x}(t) = A(t)x(t),\ x \in X,\ t \geq 0$, is well posed;

(H1.3) there exists $\delta \in N$ such that $\tau_{k+\delta} = \tau_k + T_0$;

(H1.4) for each $k \in Z_0^+$, $B_k \in \pounds_b(X)$ and $B_{k+\delta} = B_k$.

By (H1.1) and (H1.2), $\{U(t,\theta),\ t \geq \theta \geq 0\}$ is T-periodic strongly continuous evolutionary process.

Under (H1), consider

$$\begin{aligned} \dot{x}(t) &= A(t)x(t), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t), \quad t = \tau_k, \end{aligned} \tag{2.7}$$

and Cauchy problem

$$\begin{aligned} \dot{x}(t) &= A(t)x(t), \quad t \in [0,\tilde{T}] \setminus \tilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k), \quad k = 1,2,\ldots,p\delta, \\ x(0) &= \overline{x}. \end{aligned} \tag{2.8}$$

For every $\overline{x} \in X$, $D(A(t))$, $t \geq 0$, is an invariant subspace of B_k; and step by step, one can verify that the Cauchy problem (2.7) has a unique classical solution $x \in \mathrm{PC}^1([0,\tilde{T}];X)$ represented by $x(t) = \Phi(t,0)\overline{x}$, where

$$\Phi(\cdot,\cdot) : \Delta = \{(t,\theta) \in [0,\tilde{T}] \times [0,\tilde{T}] \mid 0 \leq \theta \leq t \leq \tilde{T}\} \longrightarrow \pounds(X), \tag{2.9}$$

given by

$$\Phi(t,\theta) = \begin{cases} U(t,\theta), & \tau_{k-1} \leq \theta \leq t \leq \tau_k, \\ U(t,\tau_k^+)(I+B_k)U(\tau_k,\theta), & \tau_{k-1} \leq \theta < \tau_k < t \leq \tau_{k+1}, \\ U(t,\tau_k^+)\left[\prod\limits_{\theta<\tau_j<t}(I+B_j)U(\tau_j,\tau_{j-1}^+)\right](I+B_i)U(\tau_i,\theta), & \tau_{i-1} \leq \theta < \tau_i \leq \cdots < \tau_k < t \leq \tau_{k+1}. \end{cases} \tag{2.10}$$

The operator $\{\Phi(t,\theta),\ (t,\theta) \in \Delta\}$ is called impulsive evolution operator associated with $\{U(t,\theta),\ (t,\theta) \in \Delta\}$ and $\{B_k;\tau_k\}_{k=1}^{\infty}$.

The following lemma on the properties of the impulsive evolution operator $\{\Phi(t,\theta),\ (t,\theta) \in \Delta\}$ associated with $\{U(t,\theta),\ (t,\theta) \in \Delta\}$ and $\{B_k;\tau_k\}_{k=1}^{\infty}$ are widely used in this paper.

Lemma 2.5. *Assumption (H1) holds. The impulsive evolution operator $\{\Phi(t,\theta),\ (t,\theta) \in \Delta\}$ has the following properties:*

(1) *for $0 \leq \theta \leq t \leq \tilde{T}$, $\Phi(t,\theta) \in \pounds_b(X)$, there exists $M_{\tilde{T}} > 0$ such that $\sup_{0\leq\theta\leq t\leq\tilde{T}}\|\Phi(t,\theta)\| \leq M_{\tilde{T}}$,*

(2) *for $0 \leq \theta < r < t \leq \tilde{T}$, $r \neq \tau_k$, $\Phi(t,\theta) = \Phi(t,r)\Phi(r,\theta)$,*

(3) *for* $0 \le \theta \le t \le \tilde{T}$, $n \in Z^+$, $\Phi(t + n\tilde{T}, \theta + n\tilde{T}) = \Phi(t, \theta)$,

(4) *for* $0 \le \theta \le t \le \tilde{T}$, $n \in Z^+$, $\Phi(t + n\tilde{T}, 0) = \Phi(t, 0)[\Phi(\tilde{T}, 0)]^n$,

(5) *for* $0 \le \theta < t$, *there exits* $M \ge 1$, $\omega \in R$ *such that*

$$\|\Phi(t,\theta)\| \le M \exp\left\{\omega(t-\theta) + \sum_{\theta \le \tau_n < t} \ln\left(M\|I + B_n\|\right)\right\}. \tag{2.11}$$

Proof. For (1)–(4), the reader can see [23, Lemma 2.7]. We only need to verify (5). Without loss of generality, for $\tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}$,

$$\begin{aligned}
\|\Phi(t,\theta)\| &= \|U(t,\tau_k^+)\|\|I + B_k\|\|U(\tau_k,\tau_{k-1}^+)\| \cdots \|I + B_i\|\|U(\tau_i,\theta)\| \\
&\le Me^{\omega(t-\tau_k^+)}\left[\prod_{n=i+1}^{k}\|I + B_n\|Me^{(\tau_n-\tau_{n-1}^+)}\right]\|I + B_i\|Me^{\omega(\tau_i-\theta)} \\
&\le M\exp\left\{\omega(t-\theta) + \sum_{\theta \le \tau_n < t}\ln\left(M\|I + B_n\|\right)\right\}.
\end{aligned} \tag{2.12}$$

This completes the proof. □

In order to study the asymptotical properties of periodic solution, it is necessary to discuss the exponential stability of the impulsive evolution operator $\{\Phi(t,\theta),\ t \ge \theta \ge 0\}$. We first give the definition of exponentially stable for $\{\Phi(t,\theta),\ t \ge \theta \ge 0\}$.

Definition 2.6. $\{\Phi(t,\theta),\ t \ge \theta \ge 0\}$ is called exponentially stable if there exist $K \ge 0$ and $\nu > 0$ such that

$$\|\Phi(t,\theta)\| \le Ke^{-\nu(t-\theta)}, \quad t > \theta \ge 0. \tag{2.13}$$

Assumption (H2): $\{U(t,\theta),\ t \ge \theta \ge 0\}$ is exponentially stable, that is, there exist $K_0 > 0$ and $\nu_0 > 0$ such that

$$\|U(t,\theta)\| \le K_0e^{-\nu_0(t-\theta)}, \quad t > \theta \ge 0. \tag{2.14}$$

Next, two sufficient conditions that guarantee the impulsive evolution operator $\{\Phi(t,\theta),\ t \ge \theta \ge 0\}$ with rational γ is exponentially stable are given.

Lemma 2.7. *Suppose* γ *is rational and (H1) and (H2) hold. There exists* $0 < \lambda < \nu_0$ *such that*

$$\prod_{k=1}^{\delta}\left(K_0\|I + B_k\|\right)^{\gamma}e^{-\lambda T} < 1. \tag{2.15}$$

Then $\{\Phi(t,\theta),\ t \ge \theta \ge 0\}$ *is exponentially stable.*

Proof. Since γ is rational, let $\gamma = p/q$, $p, q \in N$ and p, q are relatively prime. Let $\widetilde{T} = pT_0$ $(= qT)$, then (2.7) is $\widetilde{T}$-periodic. Without loss of generality, for $\tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}$, by (5) of Lemma 2.5, we have

$$\|\Phi(t,\theta)\| \le K_0 e^{-(\nu_0-\lambda)(t-\theta)}\left(\prod_{\theta\le\tau_k<t}(K_0\|I+B_k\|)e^{-\lambda(t-\theta)}\right). \tag{2.16}$$

Suppose $t \in (n\widetilde{T}, (n+1)\widetilde{T}]$ and let $b = \max_{s\in[0,\widetilde{T}]}\Pi_{0<\tau_k<s}\{K_0\|I+B_k\|\}$. Then

$$\begin{aligned}
\prod_{\theta\le\tau_k<t}(K_0\|I+B_k\|)e^{-\lambda(t-\theta)} &\le \prod_{0\le\tau_k<n\widetilde{T}}(K_0\|I+B_k\|)e^{-\lambda n\widetilde{T}}\cdot\prod_{n\widetilde{T}\le\tau_k<t}(K_0\|I+B_k\|)e^{-\lambda(t-n\widetilde{T})}e^{\lambda\theta}\\
&\le \prod_{0\le\tau_k<npT_0}(K_0\|I+B_k\|)e^{-\lambda nqT}be^{\lambda\theta}\\
&\le \prod_{k=1}^{\delta}(K_0\|I+B_k\|)^{np}e^{-\lambda nqT}be^{\lambda\theta}\\
&= \left[\prod_{k=1}^{\delta}(K_0\|I+B_k\|)^{\gamma}e^{-\lambda T}\right]^{nq}be^{\lambda\theta}\\
&< be^{\lambda\theta}.
\end{aligned} \tag{2.17}$$

Let $K = K_0be^{\lambda\theta} > 0$ and $\nu = \nu_0 - \lambda > 0$, then we obtain $\|\Phi(t,\theta)\| \le Ke^{-\nu(t-\theta)},\ t > \theta \ge 0$. □

Remark 2.8. An exponentially bounded evolutionary process $\{U(t,\theta),\ t \ge \theta \ge 0\}$ is exponential stability if and only if for some $1 \le p < \infty$ and all $x \in X$ and $t > \theta \ge 0$, there is a constant $\kappa > 0$ such that

$$\int_\theta^\infty \|U(t,\theta)x\|^p dt \le \kappa^p\|x\|^p. \tag{2.18}$$

Lemma 2.9. *Assumption (H1) holds. Suppose*

$$0 < \mu_1 = \inf_{k=1,2,\ldots,\delta}(\tau_k - \tau_{k-1}) \le \sup_{k=1,2,\ldots,\delta}(\tau_k - \tau_{k-1}) = \mu_2 < \infty. \tag{2.19}$$

If there exists $\alpha > 0$ such that

$$\omega + \frac{1}{\mu}\ln\left(M\|I+B_k\|\right) \le -\alpha < 0, \quad k = 1,2,\ldots,\delta, \tag{2.20}$$

where

$$\mu = \begin{cases}\mu_1, & \alpha+\omega < 0,\\ \mu_2, & \alpha+\omega \ge 0,\end{cases} \tag{2.21}$$

then $\{\Phi(t,\theta),\ t > \theta \ge 0\}$ is exponentially stable.

Proof. It comes from (2.20) that

$$\ln\left(M\|I+B_k\|\right) \le -\mu(\alpha+\omega) < 0, \quad k = 1,2,\ldots,\delta. \tag{2.22}$$

Further,

$$\sum_{\theta\le\tau_k<t}\ln\left(M\|I+B_k\|\right) \le -\sum_{\theta\le\tau_k<t}\mu(\alpha+\omega) = -\mu(\alpha+\omega)N(\theta,t), \tag{2.23}$$

where $N(\theta,t)$ is denoted the number of impulsive points in $[\theta,t)$.

For $\tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}$, by (2.19), we obtain the following two inequalities:

$$\begin{gathered} t-\theta \ge (\tau_k - \tau_{k-1}) + \cdots + (\tau_{i+1} - \tau_i) \ge (N(\theta,t)-1)\mu_1, \\ t-\theta \le (\tau_{k+1} - \tau_k) + (\tau_k - \tau_{k-1}) + \cdots + (\tau_{i+1} - \tau_i) + (\tau_i - \tau_{i-1}) \le (N(\theta,t)+1)\mu_2. \end{gathered} \tag{2.24}$$

This implies that

$$\mu_1(N(\theta,t)-1) \le t-\theta \le \mu_2(N(\theta,t)+1), \tag{2.25}$$

that is,

$$\frac{1}{\mu_2}(t-\theta) - 1 \le N(\theta,t) \le \frac{1}{\mu_1}(t-\theta) + 1. \tag{2.26}$$

Then

$$\begin{aligned} -\mu(\alpha+\omega)N(\theta,t) &\le \begin{cases} -\mu_1(\alpha+\omega)\left[\dfrac{1}{\mu_1}(t-\theta)+1\right] = -(\alpha+\omega)(t-\theta) - \mu_1(\alpha+\omega), & \alpha+\omega < 0. \\ -\mu_2(\alpha+\omega)\left[\dfrac{1}{\mu_2}(t-\theta)-1\right] = -(\alpha+\omega)(t-\theta) + \mu_2(\alpha+\omega), & \alpha+\omega \ge 0. \end{cases} \\ &= -(\alpha+\omega)(t-\theta) + \mu|\alpha+\omega|. \end{aligned} \tag{2.27}$$

Thus, we obtain

$$\omega(t-\theta) + \sum_{\theta \le \tau_k < t} \ln\left(M\|I+B_k\|\right) \le -\alpha(t-\theta) + \mu|\alpha+\omega|. \tag{2.28}$$

By (5) of Lemma 2.5, let $K = Me^{\mu|\alpha+\omega|} > 0$, $\nu = \alpha > 0$, $\|\Phi(t,\theta)\| \le Ke^{-\nu(t-\theta)}$, $t > \theta \ge 0$. □

3. Periodic solution and global asymptotical stability

Assumption (H3): $f \in L_T^1([0,+\infty);X)$. For each $k \in Z_0^+$, $\delta \in N$, and $c_k \in X$, $c_{k+\delta} = c_k$.

Under the (H1) and (H3), consider the following system

$$\begin{gathered} \dot{x}(t) = A(t)x(t) + f(t), \quad t \ne \tau_k, \\ \Delta x(t) = B_k x(t) + c_k, \quad t = \tau_k, \end{gathered} \tag{3.1}$$

and Cauchy problem

$$\begin{gathered} \dot{x}(t) = A(t)x(t) + f(t), \quad t \in [0,\tilde{T}] \setminus \tilde{D}, \\ \Delta x(\tau_k) = B_k x(\tau_k) + c_k, \quad k = 1,2,\ldots,p\delta, \\ x(0) = \overline{x}. \end{gathered} \tag{3.2}$$

Now we list the PC-mild solution of Cauchy problem (3.2) and $\tilde{T}$-periodic PC-mild solution of (3.1).

Definition 3.1. For every $\overline{x} \in X$, $f \in L^1([0,+\infty);X)$, the function $x \in \mathrm{PC}([0,\widetilde{T}];X)$, given by

$$x(t) = \Phi(t,0)\overline{x} + \int_0^t \Phi(t,\theta)f(\theta)d\theta + \sum_{0\le\tau_k<t} \Phi(t,\tau_k^+)c_k \tag{3.3}$$

for $t \in [0,\widetilde{T}]$, is said to be a PC-mild solution of the Cauchy problem (3.2).

Definition 3.2. A function $x \in \mathrm{PC}([0,+\infty);X)$ is said to be a $\widetilde{T}$-periodic PC-mild solution of (3.1) if it is a PC-mild solution of Cauchy problem (3.2) corresponding to some $\overline{x}$ and $x(t+\widetilde{T}) = x(t)$ for $t \ge 0$.

In the sequel, we show the existence and global asymptotical stability of $\widetilde{T}$-periodic PC-mild solution of (3.1).

Theorem 3.3. *Assumptions (H1) and (H3) hold. Suppose* $\|\Phi(T,0)\| = l < 1$, *then* (3.1) *has a unique* $\widetilde{T}$*-periodic PC-mild solution* $x_{\widetilde{T}}(t) \equiv (P(f,c_k))(t)$, $t \ge 0$, *given by*

$$x_{\widetilde{T}}(t) = \Phi(t,0)\left[I - \Phi(\widetilde{T},0)\right]^{-1}z + \int_0^t \Phi(t,\theta)f(\theta)d\theta + \sum_{0\le\tau_k<t} \Phi(t,\tau_k^+)c_k \equiv \left(P(f,c_k)\right)(t), \tag{3.4}$$

where

$$z = \int_0^{\widetilde{T}} \Phi(\widetilde{T},\theta)f(\theta)d\theta + \sum_{0\le\tau_k<\widetilde{T}} \Phi(\widetilde{T},\tau_k^+)c_k. \tag{3.5}$$

Further, operator

$$P : L_T^1([0,+\infty);X) \times X^\delta \longrightarrow PC_{\widetilde{T}}([0,+\infty);X) \tag{3.6}$$

is a bounded linear operator, that is, there exists $\widetilde{B} > 0$ *such that*

$$\|P(f,c_k)\|_{PC_{\widetilde{T}}} = \|x_{\widetilde{T}}\|_{PC_{\widetilde{T}}} \le \widetilde{B}\left(q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta}\|c_k\|\right), \tag{3.7}$$

where B *is independent on* f *and* c_k.

Proof. By (1) and (4) of Lemma 2.5, $\|\Phi(\widetilde{T},0)\| = \|\Phi(T,0)\|^q = l^q < 1$, operator $[I - \Phi(\widetilde{T},0)]^{-1}$ exists, is bounded and

$$\left[I - \Phi(\widetilde{T},0)\right]^{-1} = \sum_{n=0}^{\infty}\Phi(n\widetilde{T},0). \tag{3.8}$$

Thus,

$$\left\|\left[I - \Phi(\widetilde{T},0)\right]^{-1}\right\| \le \frac{1}{1 - \|\Phi(\widetilde{T},0)\|} = \frac{1}{1-l^q}. \tag{3.9}$$

For

$$z = \int_0^{\tilde{T}} \Phi(\tilde{T},\theta) f(\theta) d\theta + \sum_{0\le\tau_k<\tilde{T}} \Phi(\tilde{T},\tau_k^+) c_k, \tag{3.10}$$

operator equation $[I - \Phi(\tilde{T},0)]\overline{x} = z$ has a unique solution $\overline{x} = [I - \Phi(\tilde{T},0)]^{-1} z$.

Let $x(0) = \overline{x} = [I - \Phi(\tilde{T},0)]^{-1} z$, consider the Cauchy problem

$$\begin{gathered} \dot{x}(t) = A(t)x(t) + f(t), \quad t \in [0,\tilde{T}] \setminus \tilde{D}, \\ \Delta x(\tau_k) = B_k x(\tau_k) + c_k, \quad k = 1,2,\ldots,p\delta, \\ x(0) = \left[I - \Phi(\tilde{T},0)\right]^{-1} z. \end{gathered} \tag{3.11}$$

It has a PC-mild solution $x_{\tilde{T}}(\cdot)$ given by

$$x_{\tilde{T}}(t) = \Phi(t,0)\left[I - \Phi(\tilde{T},0)\right]^{-1} z + \int_0^t \Phi(t,\theta) f(\theta) d\theta + \sum_{0\le\tau_k<t} \Phi(t,\tau_k^+) c_k \equiv \left(P(f,c_k)\right)(t). \tag{3.12}$$

By (2), (3), and (4) of Lemma 2.5, one can easily verify that $x_{\tilde{T}}(\cdot)$ is just the unique $\tilde{T}$-periodic PC-mild solution of (3.1).

Obviously, operator $P : L_T^1([0,+\infty);X) \times X^\delta \to \mathrm{PC}_{\tilde{T}}([0,+\infty);X)$ is linear.

For $t \in [0,\tilde{T}]$,

$$\begin{aligned} \left\|P(f,c_k)(t)\right\| = \left\|x_{\tilde{T}}(t)\right\| &\le \left(\left\|\Phi(t,0)\right\| \left\|\left[I - \Phi(\tilde{T},0)\right]^{-1}\right\| + 1\right) \\ &\quad \times \left(\int_0^{\tilde{T}} \left\|\Phi(\tilde{T},\theta)\right\| \left\|f(\theta)\right\| d\theta + \sum_{0\le\tau_k<\tilde{T}} \left\|\Phi(\tilde{T},\tau_k^+)\right\| \left\|c_k\right\|\right) \\ &\le M_{\tilde{T}}\left(\frac{M_{\tilde{T}}}{1-l^q} + 1\right)\left(q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta}\left\|c_k\right\|\right). \end{aligned} \tag{3.13}$$

Let $\tilde{B} = M_{\tilde{T}}(M_{\tilde{T}}/(1-l^q) + 1)$, one can obtain the estimation immediately. □

Theorem 3.4. *Assumptions (H1) and (H3) hold. Suppose* $\{\Phi(t,\theta),\ t \ge \theta \ge 0\}$ *is exponentially stable, then* (3.1) *has a unique* $\tilde{T}$*-periodic PC-mild solution* $x_{\tilde{T}}(\cdot,\overline{x})$ *given by*

$$x_{\tilde{T}}(t,\overline{x}) = \Phi(t,0)\overline{x} + \int_0^t \Phi(t,\theta) f(\theta) d\theta + \sum_{0\le\tau_k<t} \Phi(t,\tau_k^+) c_k, \tag{3.14}$$

where

$$\overline{x} = \left[I - \Phi(\tilde{T},0)\right]^{-1} z, \quad z = \int_0^{\tilde{T}} \Phi(\tilde{T},\theta) f(\theta) d\theta + \sum_{0\le\tau_k<\tilde{T}} \Phi(\tilde{T},\tau_k^+) c_k, \tag{3.15}$$

and there exists $\widehat{B} > 0$ *such that*

$$\left\|x_{\tilde{T}}(t,\overline{x})\right\| \le \widehat{B}\left(q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta}\left\|c_k\right\|\right), \tag{3.16}$$

where $\widehat{B} > 0$ *is independent on* f *and* c_k.

Further, for arbitrary $x_0 \in X$, the PC-mild solution of the Cauchy problem (3.2) *corresponding to the initial value x_0 satisfies the following inequality:*

$$\left\| x(t,x_0) - x_{\tilde{T}}(t,\overline{x}) \right\| \le B_1 B_2 e^{-\nu t}, \tag{3.17}$$

where $x_{\tilde{T}}(\cdot,\overline{x})$ is the $\tilde{T}$-periodic PC-mild solution of (3.1), *$B_1 > 0$ is independent on x_0, f, and c_k; $B_2 = \|x_0\| + q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta}\|c_k\|$.*

Proof. Consider the operator series $S = \sum_{n=0}^{\infty}[\Phi(\tilde{T},0)]^n$. By (4) of Lemma 2.5 and the stability of $\{\Phi(\cdot,\cdot)\}$, we have

$$\left\| [\Phi(\tilde{T},0)]^n \right\| = \left\| \Phi(n\tilde{T},0) \right\| \le Ke^{-\nu n\tilde{T}} \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{3.18}$$

Thus, we obtain

$$\|S\| \le \sum_{n=0}^{\infty} \left\| [\Phi(\tilde{T},0)]^n \right\| \le \sum_{n=0}^{\infty} Ke^{-\nu n\tilde{T}}. \tag{3.19}$$

Obviously, the series $\sum_{n=0}^{\infty} Ke^{-\nu n\tilde{T}}$ is convergent, thus operator $S \in \pounds_b(X)$. It comes from

$$\left[I - \Phi(\tilde{T},0)\right] S = S\left[I - \Phi(\tilde{T},0)\right] = I \tag{3.20}$$

that

$$S = \left[I - \Phi(\tilde{T},0)\right]^{-1} \in \pounds_b(X). \tag{3.21}$$

Similar proof in Theorem 3.3, it is not difficult to verify that (3.1) has a unique $\tilde{T}$-periodic PC-mild solution $x_{\tilde{T}}(\cdot,\overline{x})$ and $x_{\tilde{T}}(\cdot,\overline{x})$ can be given by (3.14) and (3.15).

Next, verify the estimation (3.16). In fact, for $t \in [0,\tilde{T}]$, we have

$$\left\| x_{\tilde{T}}(t,\overline{x}) \right\| \le \left\| \Phi(t,0) \right\| \|\overline{x}\| + \int_0^t \left\| \Phi(t,\theta) \right\| \left\| f(\theta) \right\| d\theta + \sum_{0 \le \tau_k < t} \left\| \Phi(t,\tau_k^+) \right\| \left\| c_k \right\|. \tag{3.22}$$

On the other hand,

$$\begin{aligned} \|\overline{x}\| &\le \left\| \left[I - \Phi(\tilde{T},0)\right]^{-1} \right\| \left[\int_0^{\tilde{T}} \left\| \Phi(\tilde{T},\theta) \right\| \left\| f(\theta) \right\| d\theta + \sum_{0 \le \tau_k < \tilde{T}} \left\| \Phi(\tilde{T},\tau_k^+) \right\| \left\| c_k \right\| \right] \\ &\le \|S\| \left[\int_0^{\tilde{T}} Ke^{-\nu(\tilde{T}-\theta)} \left\| f(\theta) \right\| d\theta + Ke^{-\nu(\tilde{T}-\tau_k^+)} p \sum_{k=1}^{\delta} \left\| c_k \right\| \right] \\ &\le K\|S\| \left(q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta} \left\| c_k \right\| \right). \end{aligned} \tag{3.23}$$

Let $\widehat{B} = K(K\|S\| + 1)$, one can obtain (3.16) immediately.

System (3.1) has a unique $\tilde{T}$-periodic PC-mild solution $x_{\tilde{T}}(\cdot,\overline{x})$ given by (3.14) and (3.15). The PC-mild solution of the Cauchy problem (3.2) corresponding to initial value x_0 can be given by (3.3) replacing $\overline{x}$ with x_0. Combining with (3.23), we obtain

$$\left\| x(t,x_0) - x_{\tilde{T}}(t,\overline{x}) \right\| \le Ke^{-\nu t} \left[\left\| x_0 \right\| + K\|S\| \left(q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta} \left\| c_k \right\| \right) \right]. \tag{3.24}$$

Let $B_1 = \max\{K, K^2\|S\|\} > 0$, $B_2 = \|x_0\| + q\|f\|_{L_T^1} + p\sum_{k=1}^{\delta}\|c_k\|$, one can obtain (3.17) immediately. □

Definition 3.5. The $\widetilde{T}$-periodic PC-mild solution $x_{\widetilde{T}}(\cdot,\overline{x})$ of (3.1) is said to be globally asymptotically stable in the sense that

$$\lim_{t\to+\infty}\left\|x(t,x_0)-x_{\widetilde{T}}(t,\overline{x})\right\|=0, \tag{3.25}$$

where $x(\cdot,x_0)$ is any PC-mild solution of the Cauchy problem (3.2) corresponding to initial value $x_0\in X$.

By Theorem 3.4 and Lemma 2.7 (Lemma 2.9), one can obtain the following results.

Corollary 3.6. *Assumptions of Lemma 2.7 (Lemma 2.9) and (H3) hold. System* (3.1) *has a unique $\widetilde{T}$-periodic PC-mild solution $x_{\widetilde{T}}(\cdot,\overline{x})$ which is globally asymptotically stable.*

Theorem 3.7. *If $x(t,\overline{x})$ is a periodic PC-mild solution of* (3.1)*, then its period must be nT_0 for some $n\in N$.*

Proof. Let $\widetilde{T}$ be the period of $x(t)=x(t,\overline{x})$. Then

$$x\left(\left(\widetilde{T}+t\right)\pm 0\right)=x(t\pm 0),\quad t\geq 0. \tag{3.26}$$

Clearly, $\widetilde{T}$ is not an impulsive moment, suppose there are s τ_ks in the interval $(0,\widetilde{T})$. Let $t=\tau_1$. We have

$$\begin{gathered}x\left(\widetilde{T}+\tau_1\right)=x\left(\tau_1\right),\\ x\left(\left(\widetilde{T}+\tau_1\right)^{+}\right)=x\left(\tau_1^{+}\right)=\left(I+B_1\right)x\left(\tau_1\right)+c_1=\left(I+B_1\right)x\left(\widetilde{T}+\tau_1\right)+c_1,\end{gathered} \tag{3.27}$$

which means that $\widetilde{T}+\tau_1$ is one of the impulsive moments. Clearly, there is no τ_ks in the interval $(\widetilde{T},\widetilde{T}+\tau_1)$. For otherwise, suppose $\bar{t}\in(\widetilde{T},\widetilde{T}+\tau_1)$ is an impulsive moment, then

$$x\left(\left(\widetilde{T}+\left(\bar{t}-\widetilde{T}\right)\right)^{+}\right)=x\left(\bar{t}^{+}\right)\neq x\left(\bar{t}\right)=x\left(\widetilde{T}+\left(\bar{t}-\widetilde{T}\right)\right)=x\left(\bar{t}-\widetilde{T}\right)=x\left(\left(\bar{t}-\widetilde{T}\right)^{+}\right), \tag{3.28}$$

which is a contradiction. Thus, $\widetilde{T}+\tau_1=\tau_{s+1}$, $B_{s+1}=B_1$, and $c_{s+1}=c_1$. Similarly, we have

$$\widetilde{T}+\tau_k=\tau_{s+k},\quad B_{s+k}=B_k,\quad c_{s+k}=c_k,\quad k\in Z_0^{+}. \tag{3.29}$$

Now we can claim that $s=n\delta$ for some $n\in N$. Otherwise, suppose $s=n\delta+j$ for some $n\in N\cup\{0\}$ and $1<j<\delta$. As a consequence, $nT_0<\widetilde{T}<(n+1)T_0$. By (3.29), we have

$$\begin{gathered}\widetilde{T}+\tau_k=\tau_{s+k}=\tau_{n\delta+j+k}=nT_0+\tau_{j+k},\\ B_k=B_{s+k}=B_{n\delta+j+k}=B_{j+k},\\ c_k=c_{s+k}=c_{n\delta+j+k}=c_{j+k},\quad k\in Z_0^{+},\end{gathered} \tag{3.30}$$

or

$$\left(\widetilde{T}-nT_0\right)+\tau_k=\tau_{j+k},\quad B_k=B_{j+k},\quad c_k=c_{j+k},\quad k\in Z_0^{+}. \tag{3.31}$$

Thus $\widetilde{T}-nT_0\in(0,T_0)$ is a period of the impulsive perturbations for the second equation of (3.1), which contradicts to that T_0 is the least-positive period. Therefore,

$$\widetilde{T}+\tau_1=\tau_{s+1}=\tau_{n\delta+1}=nT_0+\tau_1 \tag{3.32}$$

and $\widetilde{T}=nT_0$ for some $n\in N$. The proof is complete. □

4. Example

Consider a special case of (1.1)

$$\begin{aligned}
&\frac{\partial}{\partial t}x(t,y) = \big((\Delta - \lambda I) + Q(t)\big)x(t,y) + \sin\big(t + |y|\big),\\
&\qquad |y| = \sqrt{y_1^2 + y_2^2} \in \Omega, \quad t > 0,\ t \neq \tau_k,\ k \in Z_0^+,\\
&x(t,y) = 0, \quad y \in \partial\Omega,\ t > 0,\\
&\Delta x(t,y) = \begin{cases} 0.05Ix(t,y), & k = 3m-2,\\ -0.05Ix(t,y), & k = 3m-1, \quad y \in \Omega,\ t > 0,\ t = \tau_k = \dfrac{k+m-1}{4}\pi,\ k,m \in Z_0^+.\\ 0.05Ix(t,y), & k = 3m, \end{cases}
\end{aligned} \tag{4.1}$$

where $\Omega \subset R^2$ is bounded domain and $\partial\Omega \in C^2$, Δ is the Laplace operator in R^2, λ is a parameter. For each $t \in [0,2\pi]$, $Q(t) \in £_b(L_2(\Omega), L_2(\Omega))$, $\sup\{\|Q(t)\|, t \in [0,2\pi]\} < \infty$, and $Q(\cdot + 2\pi) = Q(\cdot)$.

Set $X = L_2(\Omega)$, for fixed $\lambda \geq 1$, $D(A_\lambda) = H^2(\Omega) \cap H_0^1(\Omega)$. Define operator $\mathcal{B}_\lambda x = (\Delta - \lambda I)x$, $x \in D(A_\lambda)$. By [24, Theorem 2.5], $\mathcal{B}_\lambda$ is just the infinitesimal generator of a contraction C_0-semigroup in $L_2(\Omega)$, that is, $\|T_\lambda(t)\| \leq 1$. Obviously, for $\lambda > 1$, $\mathcal{B}_\lambda + I = \Delta - (\lambda - 1)I$ can generate a exponentially stable C_0-semigroup in $L_2(\Omega)$ and $\|T_\lambda(t)\| \leq e^{-(\lambda-1)t}$.

Define operator series

$$\begin{aligned}
&U_{\lambda,0}(t,\theta) = T_\lambda(t-\theta),\\
&U_{\lambda,n}(t,\theta)x = \int_\theta^t T_\lambda(t-\tau)Q(\tau)U_{\lambda,n-1}(\tau,\theta)x\,d\tau, \quad 0 \leq \theta \leq t \leq 2\pi,\ n = 1,2,\ldots.
\end{aligned} \tag{4.2}$$

One can easily verify the following results:

(i) the evolution operator $U_\lambda(t,\theta) = \sum_{n=0}^{\infty}\|U_{\lambda,n}(t,\theta)\|$, $0 \leq \theta \leq t \leq 2\pi$ is uniformly convergence;

(ii) $U_\lambda(t,\theta)$ is the unique solution of the integral equation

$$U_\lambda(t,\theta)x = T_\lambda(t-\theta)x + \int_\theta^t T_\lambda(t-\tau)Q(\tau)U_\lambda(\tau,\theta)x\,d\tau; \tag{4.3}$$

(iii) $U_\lambda(t,\theta)$ satisfies:

(1) $U_\lambda(t,t) = I$, $t \geq 0$,

(2) $U_\lambda(t,\tau)U_\lambda(\tau,\theta) = U_\lambda(t,\theta)$, $t \geq \tau \geq \theta \geq 0$,

(3) for every fixed $x \in X$, $(t,\theta) \to U_\lambda(t,\theta)x$ is strongly continuous,

(4) $\|U_\lambda(t,\theta)\| \leq e^{-(\lambda-1-\|Q\|)(t-\theta)}$, $t \geq \theta \geq 0$,

(5) $U_\lambda(t+2\pi,\theta+2\pi) = U_\lambda(t,\theta)$, $t \geq \theta \geq 0$.

Thus, linear equation $\dot{x}(t) = A_\lambda(t)x(t),\ x \in X,\ t \geq 0$ is well posed, where $A_\lambda(t) \equiv \Delta - (\lambda - 1)I + Q(t)$. Obviously, for $\lambda > 1 + \|Q\|$, $\mathcal{A}_\lambda(t) \equiv \Delta - (\lambda - 1 - \|Q\|)I + Q(t)$ can determine an exponentially stable 2π-periodic strongly continuous evolutionary process $\{\mathcal{U}_\lambda(\cdot,\cdot)\}$ in $L_2(\Omega)$.

Define $x(\cdot)(y) = x(\cdot, y)$, $f(\cdot)(y) = \sin(\cdot + |y|)$, then (4.1) can be abstracted

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}_\lambda(t)x(t) + f(t), \quad t > 0,\ t \neq \tau_k,\ k \in Z_0^+, \\ \Delta x(t) &= B_k x(t), \quad t = \tau_k,\ k \in Z_0^+, \end{aligned} \tag{4.4}$$

where

$$B_k = \begin{cases} 0.05I, & k = 3m - 2, \\ -0.05I, & k = 3m - 2, \\ 0.05I, & k = 3m, \end{cases} \tag{4.5}$$

and $\tau_k = ((k + m - 1)/4)\pi$, $k, m \in N$.

Obviously, $\mathcal{A}_\lambda(\cdot + 2\pi) = \mathcal{A}_\lambda(\cdot)$, $\tau_{k+3} = \tau_k + \pi$, $B_{k+3} = B_k$ and $f(\cdot + 2\pi)(y) = \sin(\cdot + |y| + 2\pi) = \sin(\cdot + |y|) = f(\cdot)(y)$, that is, $f(\cdot + 2\pi) = f(\cdot)$. For $\lambda > 1 + \|Q\| + \overline{\lambda}$, where $\overline{\lambda} > \ln\left[(1.05)^2 \times 0.95\right]^2/2\pi \approx 0.0147$, by Lemma 2.7, $\{\Phi_\lambda(\cdot,\cdot)\}$ is exponentially stable. Now, all the assumptions are met in Theorem 3.4. Thus (4.4) has a unique 2π-periodic PC-mild solution $x_{2\pi}(\cdot, y) \in \mathrm{PC}_{2\pi}([0 + \infty); L_2(\Omega))$ which is globally asymptotically stable.

That is, suppose $x(\cdot, y)$ is the PC-mild solution of the following initial-boundary value problem:

$$\begin{aligned} &\frac{\partial}{\partial t}x(t,y) = \big((\Delta - \lambda I) + Q(t)\big)x(t,y) + \sin\big(t + |y|\big), \\ &\qquad |y| = \sqrt{y_1^2 + y_2^2} \in \Omega, \quad t > 0,\ t \neq \tau_k,\ k \in Z_0^+, \\ &x(t,y) = 0, \quad y \in \partial\Omega,\ t > 0, \\ &x(0,y) = \cos|y| \in L^2(\Omega), \\ &\Delta x(t,y) = \begin{cases} 0.05Ix(t,y), & k = 3m - 2, \\ -0.05Ix(t,y), & k = 3m - 1, \\ 0.05Ix(t,y), & k = 3m, \end{cases} \quad y \in \Omega,\ t > 0,\ t = \tau_k = \frac{k + m - 1}{4}\pi,\ k, m \in Z_0^+. \end{aligned} \tag{4.6}$$

Then, for $\lambda > 1 + \|Q\| + \overline{\lambda}$, where $\overline{\lambda} > 0.0147$,

$$\left\|x(t,y) - x_{2\pi}(t,y)\right\|_{L^2(\Omega)} = \left(\int_\Omega \left|x(t,y) - x_{2\pi}(t,y)\right|^2 dy\right)^{1/2} \longrightarrow 0 \quad \text{as } t \longrightarrow +\infty. \tag{4.7}$$

From the above discussion, it is not difficult to find that a suitable parameter λ chosen by human, which will guarantee the model (4.1) has a unique 2π-periodic PC-mild solution which is globally asymptotically stable. That is, we can use a biological approach to maintain the balance of a single, isolated species or eradicate pests. It provides us a reliable method for managing the single and isolated species in the nature.

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