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Research Article

The Mathematical Study of Pest Management Strategy

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The theory of impulsive state feedback control is used to establish a mathematical model in the pest management strategy. Then, the qualitative analysis of the mathematical model was provided. Here, a successor function in the geometry theory of differential equations is used to prove the sufficient conditions for uniqueness of the 1-periodic solution. It proved the orbital asymptotic stability of the periodic solution. In addition, numerical analysis is used to discuss the application significance of the mathematical model in the pest management strategy.

1. Introduction

Impulse is an interference in the thing at a short time in the course of its development. It is a method of external control. This kind of method is widely used in biological control, prevention of epidemic, cancer cells of chemotherapeutics, and so on. We use impulsive differential equation to reflect the method of external control. We can use impulsive differential equation to describe some biological phenomena in population ecology. There are mainly two kinds of impulsive differential equation. One kind is fixed times impulsive differential equation, and the other kind is differential system with state impulses. In the recent thirty years, many authors have studied the impulsive differential equation [1–5]. They obtained some theories of impulsive differential equation; particularly the theory of fixed times impulsive differential equation is widely used in population ecology. Many authors have studied the dynamics of predator-prey models with impulsive control strategies [6–12].

Pest management is a focus which people are concerned with. Because the technological revolutions have recently hit the industrial world and the experience and lessons are

accumulated, the ideology and strategy of pest management have changed a lot. Pest management changes from chemical control to integrated control. It is fully integrated into the development of agriculture and forestry sustainability.

The study of pest management strategy has good application value and significant agriculture production. In the past few decades, many authors have made a lot of research and discussion it [13]. There are two major methods of pest management. The first is chemical control. It means that the main method to control the amount of pests is spraying insecticide. But its drawback is that it will cause pollution to the environment. In addition, spray insecticide will kill natural enemies and other beneficial organisms. Although this can control pest, it had a negative impact. The second is biological control, which means that the method to control the amount of pests is culturing the natural enemies of pests. Because the biological control can avoid the environmental pollution, many scholars studied biological control. Some people put forward the integrated control method (IPM) by combining chemical control and biological control. Thus we not only can use the fast speed of chemical control, but also can use biological control to avoid the environmental pollution. In the process of pest management, we see culturing the natural enemies of pests or insecticide spraying as an instant action, and this action is not regular. This action is decided by the number of pests; when the amount of pests reached a critical value, we spray insecticide or release the natural enemies of pests at the instant of that time; here, the critical value is called economic threshold or ET. Here, the instant action of culturing the natural enemies of pests or insecticide spraying is impulsive control as we said before, so we use differential system with state impulses to describe integrated control method (IPM) in pest management.

In recent years, the application of differential system with state impulses in integrated pest management has been greatly developed. Tang used differential system with state impulses in pest management [14, 15]. They established a system with state impulses:

$$\begin{aligned} \frac{dx}{dt} &= x(a - by) \\ \frac{dy}{dt} &= y(cx - d) \end{aligned} \qquad x < x_1 \tag{1.1}$$

$$\Delta x = -px, \quad \Delta y = h, \quad x = x_1.$$

Here x is the densities of the pest, y is the densities of natural enemies of the pest, x_1 is the critical value of economic, a is the growth rate of the pest, b is the trapping rate of natural enemies of the pest, c is the absorption rate of natural enemies of the pest, d is the death rate of natural enemies of the pest, $0 < p < 1$ is the rate of killed pest by spraying insecticide, and h is the amount of natural enemies of the pest that we released; they are all positive numbers. This system is a spatial model; we can get the explicit solution of it. For system (1.1), the stability and existence of 1-periodic solution and the existence of 2-periodic solution all can be gotten by using comparison principle to transform the system into difference equation.

System (1.1) considered a two-species predator-prey model (Lotka-Volterra) that there is not density dependence for the continuous process of pulse points; this disagrees with practical significance. In order to be closer to the actual, Zeng at [16, 17] made the system

(1.1) to be that there is density dependence for the continuous process of pulse points. This can reflect the practical situation; the model is as follows:

$$\begin{aligned} \frac{dx}{dt} &= x(a - rx - by) \\ \frac{dy}{dt} &= y(cx - d) \end{aligned} \quad x < x_1$$
$$\Delta x = -px, \quad \Delta y = h, \quad x = x_1. \tag{1.2}$$

Here r is the density-dependent coefficient of the pest. They use tectonic Lambert-W function and comparison principle to get the condition of the existence of 1-periodic solution.

For system (1.2), it did not consider the influence of spraying insecticide on natural enemies. In order to reflect the actual more accurately, we introduced the rate of killed natural enemies by spraying insecticide ($0 < q < 1$) at the foundation on system (1.2); then we get the model:

$$\begin{aligned} \frac{dx}{dt} &= x(a - rx - by) \\ \frac{dy}{dt} &= y(cx - d) \end{aligned} \quad x < x_1,$$
$$\begin{aligned} \Delta x &= -px \\ \Delta y &= -qy + h \end{aligned} \quad x = x_1. \tag{1.3}$$

The significance of parameters is the same as the aforementioned.

The remainder of this paper is organized as follows. In Section 3 we use the successor function about geometry theory of semicontinuous dynamical systems to get the condition of existence and stability of 1-periodic solution for system (1.3). Section 4 combined with numerical simulations gives the application for system (1.3) in pest management.

2. Preliminaries

Definition 2.1. For the state impulse differential equation

$$\begin{aligned} &\frac{dx}{dt} = f(x,y), \quad \frac{dy}{dt} = g(x,y), \quad (x,y) \notin M\{x,y\}, \\ &\Delta x = \alpha(x,y), \quad \Delta y = \beta(x,y), \quad (x,y) \in M\{x,y\}. \end{aligned} \tag{2.1}$$

Here $M\{x,y\}$, $N\{x,y\}$, and $R_2(x,y)$ are lines or curves on the plane, $M\{x,y\}$ is the pulse set, and $N\{x,y\}$ is the phase set. We describe a dynamical system made by the solution maps of system (2.1) as a semicontinuous dynamical system, which is denoted as (Ω, f, φ, M). The initial mapping point p is not in the pulse set, $P \in \Omega = R_2 - M\{x,y\}$, φ is a continuous mapping, $\varphi(M) = N$, and φ is known as pulse mapping.

Definition 2.2. $f(P,t)$ is the semicontinuous dynamical system mapping described by system (2.1) at $\Omega \to \Omega$; $f(P,t)$ is a mapping in itself. It includes two parts:

(1) differential equation

$$\frac{dx}{dt} = f(x,y), \qquad \frac{dy}{dt} = g(x,y). \tag{2.2}$$

The Poincare mapping $\pi(P,t)$ is the mapping of (2.2) at the initial mapping point P; if $f(P,t) \cap M(x,y) = 0$, then the semicontinuous dynamical system mapping at the initial mapping point P is $f(P,t) = \pi(P,t)$.

(2) If there is a T_1, then $f(P,T_1) = Q_1 \in M\{x,y\}$; pulse mapping is

$$\varphi(Q_1) = \varphi(f(P,T_1)) = P_1 \in N, \tag{2.3}$$

and if $f(P_1,t) \cap M\{x,y\} = 0$, then the semicontinuous dynamical system mapping at the initial mapping point P is $f(P,t) = \pi(P,T_1) + \pi(P_1,t-T_1)$.

(3) At the situation of (2), if $f(P_1,t) \cap M\{x,y\} \neq 0$, and having a T_2 made $f(P_1,T_2) = Q_2 \in M\{x,y\}$, then

$$f(P,t) = \pi(P,T_1) + f(P_1,t-T_1) = \pi(P,T_1) + \pi(P_1,T_2) + f(P_2,t-T_1-T_2). \tag{2.4}$$

(4) For repeated superior surface, $f(P_1,t) \cap M\{x,y\} \neq 0$; then we have $f(P_1,T_1) = \sum_{k=1} \pi(P_k,T_k) + f(P_k,t)$

$$f(P_1,t) = \begin{cases} f(P_1,t) & 0 \le t < T_1 \\ \pi(P_1,T_1) + f(P_2,t-T_1) & T_1 \le t < T_1 + T_2 \\ \pi(P_1,T_1) + \pi(P_2,T_2) + f(P_3,t-T_1-T_2) & T_1 + T_2 \le t < T_1 + T_2 + T_3 \\ \vdots & \\ \sum_{k=1}^{n} \pi(P_k\ ,T_k) + f\left(P_{k+1}, t - \sum_{k=1}^{n} T_k\right) & \sum_{k=1}^{n} T_k \le t \le \sum_{k=1}^{n+1} T_k \\ \vdots & \end{cases} \tag{2.5}$$

Property 1. The mapping of the semi-continuous dynamical system:

(1) $f(P,0) = P$; (2) $f(f(P,t_1),t_2) = f(P,t_1+t_2)$; (3) $f(P,t)$ is continuously at initial mapping point P.

Definition 2.3. If the periodic solution Γ_0 of system (2.1) does not intersect with pulse set $M\{x,y\}$, then Γ_0 is also the periodic solution for system (2.1).

Definition 2.4. When there is a point P at phase set N and a T_1, make $f(P,T_1) = Q_1 \in M\{x,y\}$; it also has $\varphi(Q_1) = \varphi(f(P,T_1)) = P \in N$; then $f(P,T_1)$ is said to be 1-periodic solution.

Definition 2.5. Successor function: let L be a coordinate axis defined at N, the origin point is intersection point of line $x = (1-p)x_1$ with x axis, and the positive direction *consistent with* the positive direction of y axis, an arbitrary point $x \in N$, $l(x)$ is the coordinate of x at N,

$l(x) \in C^0$; if there exists a $t_1 \in R^+$, making $\pi(x,t_1) \in M, x^+ = \varphi(\pi(x,t_1)) \in N$, then $F(x)$ is the successor function of point x; here $F(x) = l(x^+) - l(x)$.

Lemma 2.6. *The successor function $F(x)$ is continuous.*

In fact, successor function $F(x)$ is that continuous solution $\pi(x,t_1)$ of differential equation compound with continuous functions $I(x)$ and $F(x)$ is a complex function of two continuous functions, so $F(x)$ is continuous.

Lemma 2.7. *Let continuous dynamical system be as (X,Π); if there are two points x_1 and x_2 at phase set, making $F(x_1)\cdot F(x_2) < 0$, then there must exist a point P between x_1 and x_2 such that $F(P) = 0$; thus there must exist 1-periodic solution by point P.*

Lemma 2.8 (Poincaré's criterion). *The T-periodic solution $x = \phi(t), y = \varphi(t)$ of system*

$$\begin{aligned} &\frac{dx}{dt} = f(x,y), \quad \frac{dy}{dt} = g(x,y), \quad \Phi(x,y) \neq 0, \\ &\Delta x = \alpha(x,y), \quad \Delta y = \beta(x,y), \quad \Phi(x,y) = 0 \end{aligned} \tag{2.6}$$

is orbitally asymptotically stable if the multiplier μ_2 satisfies the condition $|\mu_2| < 1$, where

$$u_2 = \prod_{k=1}^{q} \Delta_k \exp\left[\int_0^T \left(\frac{\partial f}{\partial x}(\phi(t),\varphi(t)) + \frac{\partial g}{\partial y}(\phi(t),\varphi(t))\right)dt\right], \tag{2.7}$$

$$\begin{aligned} \Delta_k = &\left(f_+\left(\left(\frac{\partial \beta}{\partial y}\right)\cdot\left(\frac{\partial \Phi}{\partial x}\right) - \left(\frac{\partial \beta}{\partial x}\right)\cdot\left(\frac{\partial \Phi}{\partial y}\right) + \partial\Phi/\partial x\right)\right. \\ &\left. + g_+\left(\left(\frac{\partial \alpha}{\partial x}\right)\cdot\left(\frac{\partial \Phi}{\partial y}\right) - \left(\frac{\partial \alpha}{\partial y}\right)\cdot\left(\frac{\partial \Phi}{\partial x}\right) + \partial\Phi/\partial y\right)\right) \\ &\times\left(f\left(\frac{\partial \Phi}{\partial x}\right) + g\left(\frac{\partial \Phi}{\partial y}\right)\right)^{-1}. \end{aligned} \tag{2.8}$$

Here f, g, $\partial\alpha/\partial x$, $\partial\alpha/\partial y$, $\partial\beta/\partial x$, $\partial\beta/\partial y$, $\partial\Phi/\partial x$, $\partial\Phi/\partial y$ are calculated for the point $(\phi(\tau_k),\varphi(\tau_k))$,

$$f_+ = f(\phi(\tau_k^+),\varphi(\tau_k^+)), \qquad g_+ = g(\phi(\tau_k^+),\varphi(\tau_k^+)). \tag{2.9}$$

3. The Stability and Existence of 1-Periodic Solution to Pest Management Model with Impulsive State Control

Statement 3.1. At system (1.3), if $p = q = h = 0$, then we get Lotka-Volterra predator-prey model:

$$\begin{aligned} \frac{dx}{dt} &= x(a - rx - by), \\ \frac{dy}{dt} &= y(cx - d). \end{aligned} \tag{3.1}$$

When $0 < ac - rd < r^2 d/4c$, system has stable focus $E(d/c, (ac - rd)/bc)$. This stable focus is asymptotically stable. When $p > 0$, $q > 0$, $h > 0$, we get system (1.3).

Statement 3.2. The intersection point of pulse set $x = x_1$ and isoclines $a - rx - by = 0$ is denoted by $H(H_x, H_y)$; then there exists a trajectory of system Γ_1 that tangency with $x = x_1$ to $H(H_x, H_y)$, and the phase point of H at phase set $x = (1-p)x_1$ is denoted by $H_1(H_{1x}, H_{1y})$.

Theorem 3.3. *When pulse set is $x_1 = d/c$, then there exists a point M at phase set $x = (1-p)(d/c)$; make $F(M) = 0$, and then system* (1.3) *has a 1-periodic solution.*

Proof. Let pulse set be $x_1 = d/c$, phase set is $x = (1-p)x_1$, the intersection point of pulse set $x = (1-p)x_1$ and isoclines $a - rx - by = 0$ is denoted by $A(A_x, A_y) = A((1-p)(d/c), (ac - rd(1-p))/bc)$, A_x is the x coordinate of A, A_y is the y coordinate of A, there exists a trajectory L_1 at initial point A of system, its tangency with $x = (1-p)x_1$ at A and intersects with $x = x_1$ at $C_1(d/c, C_{1y})$, that pulse to $x = (1-p)x_1$, phase point is $A_1(A_{1x}, A_{1y}) = A((1-p)(d/c), (1-q)C_{1y} + h)$, it is called the successor point of A. From Definition 2.5, we get the successor faction is $F(A) = A_{1y} - A_y$ of A.

(i) If $F(A) = A_{1y} - A_y < 0$, (see Figure 1), there exists a point $S : S_y \gg 0$ at $x = (1-p)x_1$, the trajectory L_2 over S intersects with $x = (1-p)x_1$, the intersection point is denoted by B, let $0 < B_y \ll 1$, trajectory L_2 intersects with $x = x_1$ at point C_2, that is pulsed to $x = (1-p)x_1$, phase point is $B_1(B_{1x}, B_{1y})$, so $F(B) = B_{1y} - B_y > 0$, so there must exist a point M at phase set $x = (1-p)x_1$, it satisfies $B_y < M_y < A_y$, that make $F(M) = 0$, from Lemma 2.7, we get that system (1.3) has a 1-periodic solution.

(ii) If $F(A) = A_{1y} - A_y > 0$, (see Figure 2), then there exists a trajectory L_3 which can sufficiently approach trajectory L_1 to make the intersection point $A' : 0 < A'_y - A_y \ll 1$ of trajectory L_3 and $x = (1-p)x_1$. It means that point A' can sufficiently approach point A. The trajectory L_3 over S intersects with $x = x_1$ at point C_3, that pulse to $x = (1-p)x_1$, phase point is $A'_1(A'_{1x}, A'_{1y})$, and point B_1 satisfies $0 < A_{1y} - A'_{1y} \ll 1$. That means A'_1 can sufficiently approach point A_1; then $F(A') = A'_{1y} - A'_y > 0$. At the same time, there exists a point $S : S_y \gg 0$ at $x = (1-p)x_1$, the trajectory L_2 over S intersects with $x = x_1$, the intersection point is denoted by C_2, that is pulsed to $x = (1-p)x_1$, phase point is $S_1(S_{1x}, S_{1y})$, then $F(S) = S_{1y} - S_y < 0$. So, there must exist a point M at phase set $x = (1-p)x_1$, it satisfy $A'_y < M_y < S_y$, that makes $F(M) = 0$, from Lemma 2.7, we get that system (1.3) has a 1-periodic solution. This completes the proof. □

Theorem 3.4. *When pulse set is $0 < x_1 < d/c$, there exists a point M at phase set $x = (1-p)x_1$; make $F(M) = 0$, and then system* (1.3) *has a 1-periodic solution.*

Proof. Let pulse set is $0 < x_1 < d/c$, phase set be $x = (1-p)x_1$, the intersection point of pulse set $x = (1-p)x_1$ and isoclines $a - rx - by = 0$ is denoted by $A(A_x, A_y)$, and there exists a trajectory L_1 at initial point A of system, its tangency with $x = (1-p)x_1$ at A and intersects with $x = x_1$ at $C_1(C_{1x}, C_{1y})$, that is pulsed to $x = (1-p)x_1$, phase point is $A_1(A_{1x}, A_{1y})$, it is called the successor point of A. From Definition 2.5, we get the successor function $F(A) = A_{1y} - A_y$ of A. Here, we make the discussion similar to the proof of Theorem 3.3; then we get that system (1.3) has a 1-periodic solution whether at $F(A) = A_{1y} - A_y < 0$ (see Figure 3) or at $F(A) = A_{1y} - A_y > 0$, (see Figure 4). This completes the proof. □

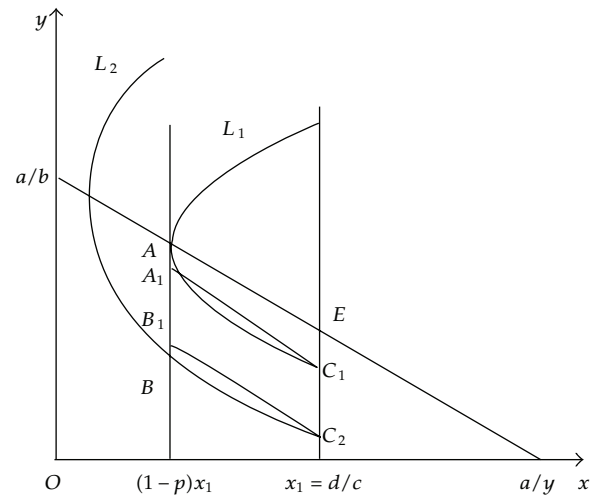

Figure 1

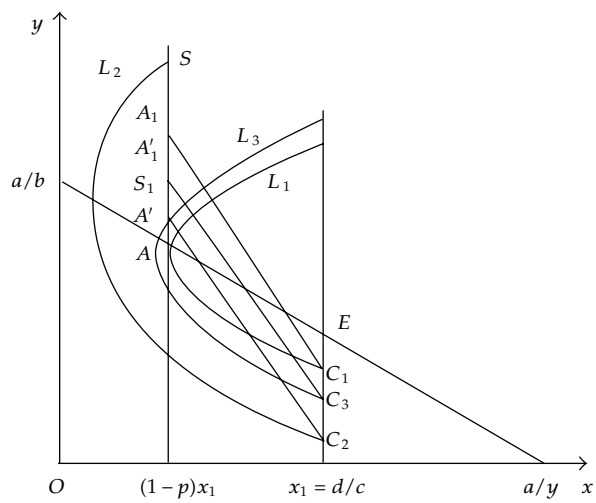

Figure 2

Statement 3.5. Let pulse set be $x = x_1(d/c < x_1 < a/r)$, and phase set is $x = (1-p)x_1$; we know that trajectory L_0 of system tangency with $x = x_1$ at $H(H_x, H_y)$; the negative semiorbits of point H are denoted by $R(H,t)$; here $t \leq 0$ the phase point of H at phase set $x = (1-p)x_1$ is denoted by $A_1(A_{1x}, A_{1y})$.

Theorem 3.6. *When the pulse set is $d/c < x_1 < a/r$ and phase set is $0 < (1-p)x_1 < d/c$,*

(1) *if the negative semiorbits of point H are $R(H,t) \cap N = \phi$, then there exists a point M at $x = (1-p)x_1$ making $F(M) = 0$. It means that system* (1.3) *has a 1-periodic solution.*

(2) *The negative semiorbits of point H are $R(H,t)$. If for the first time it intersects with phase set at $A(A_x, A_y)$, the second time it intersects with phase set at $B(B_x, B_y)$, here $A_y > B_y$, when $A_{1y} - A_y > 0$ or $A_{1y} - B_y < 0$, there exists a point M at phase set $x = (1-p)x_1$, making $F(M) = 0$; it means that system* (1.3) *has a 1-periodic solution.*

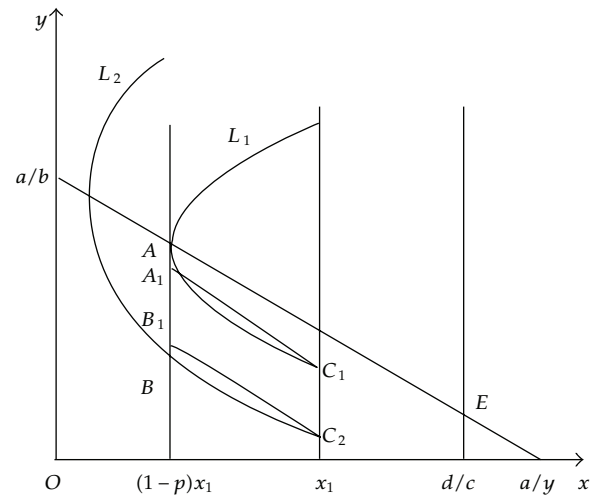

Figure 3

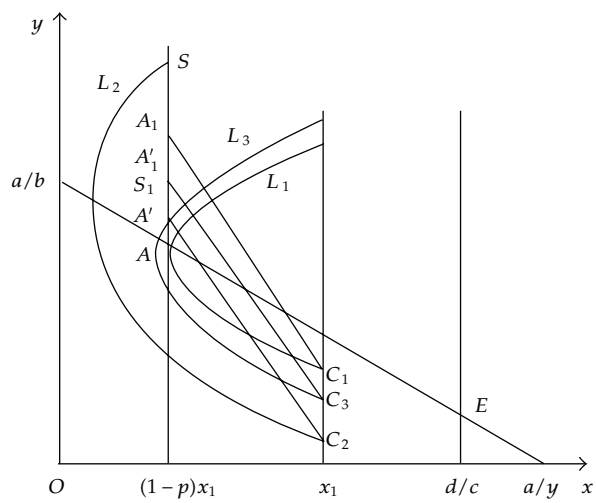

Figure 4

Proof. (1) If the negative semiorbits of point H are $R(H,t) \cap N = \phi$, there exists a trajectory L_1 at initial point A of system, that tangency with $x = (1-p)x_1$ to A and intersects with $x = x_1$ at $C_1(C_{1x}, C_{1y})$, that is pulsed to $x = (1-p)x_1$, phase point is $A_1(A_{1x}, A_{1y})$, it is called the successor point of A, the successor faction is $F(A) = A_y - A_{1y}$.

(i) If $f(A) = A_{1y} - A_y < 0$ (see Figure 5), there exists a point $S : S_y \gg 0$, the trajectory of system L_2 which cross the point S intersect with $x = (1-p)x_1$ at B, it make $0 < B_y \ll 1$, the trajectory L_2 of system intersect with $x = x_1$ at point C_2, that is pulse to $x = (1-p)x_1$, phase point is $B_1(B_{1x}, B_{1y})$, then $F(B) = B_{1y} - B_y > 0$, so there must exist a point M at phase set $x = (1-p)x_1$, it satisfies $B_y < M_y < A_y$, it makes $F(M) = 0$. From Lemma 2.7, we get that system (1.3) has a 1-periodic solution.

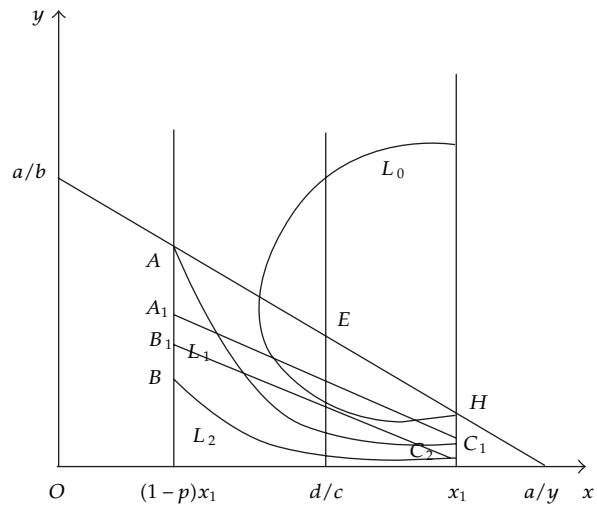

Figure 5

(ii) If $F(A) = A_{1y} - A_y > 0$ (see Figure 6), there exists a trajectory L_3 which can sufficiently approach trajectory L_1 to make the intersection point $A' : 0 < A'_y - A_y \ll 1$ of trajectory L_3 and $x = (1-p)x_1$; it means that point A' can sufficiently approach point A, the trajectory L_3 over S intersects with $x = x_1$ at point C_3, that is pulsed to $x = (1-p)x_1$, phase point is $A'_1(A'_{1x}, A'_{1y})$, point A'_1 satisfies $0 < A_{1y} - A'_{1y} \ll 1$, and that means A'_1 can sufficiently approach point A_1; then $F(A') = A'_{1y} - A'_y > 0$. At the same time, there exists a point $S : S_y \gg 0$ at $x = (1-p)x_1$, the trajectory L_2 over S intersects with $x = x_1$, the intersection point is denoted by C_2, that pulse to $x = (1-p)x_1$, phase point is $F_1(F_{1x}, F_{1y})$, and then $F(S) = S_{1y} - S_y < 0$, so there must exist a point M at phase set $x = (1-p)x_1$; it satisfies $A'_y < M_y < S_y$, making $F(M) = 0$. From Lemma 2.7, we get that system (1.3) has a 1-periodic solution.

(2) The negative semiorbits of point H are (H, t). If for the first time it intersects with phase set at $A''(A''_x, A''_y)$, the second time it intersects with phase set at $B''(B''_x, B''_y)$; here $A''_y > B''_y$; here we consider the phase point $A''_1(A''_{1x}, A''_{1y})$ of H at $x = (1-p)x_1$.

(i) If $F(A'') = A''_{1y} - A'' > 0$ (see Figure 7), then there exists a trajectory L_3 which can sufficiently approach trajectory L_1 to make the intersection point $D : 0 < D_y - A_y \ll 1$ of trajectory L_3 and $x = (1-p)x_1$. It means that point D can sufficiently approach point A, the trajectory L_3 intersect with $x = x_1$ at point C_3, that pulse to $x = (1-p)x_1$, phase point is $D_1(D_{1x}, D_{1y})$, it satisfies $D_1 : 0 < A_{1y} - D_{1y} \ll 1$; that means D_1 can sufficiently approach point A_1; then we have $f(D) = D_{1y} - D_y > 0$, there exists a point $S : S_y \gg 0$ at $x = (1-p)x_1$, the trajectory L_2 over S intersects with $x = x_1$, the intersection point is denoted by C_2, that pulse to $x = (1-p)x_1$, phase point is $F_1(F_{1x}, F_{1y})$, and then $F(S) = S_{1y} - S_y < 0$, so there must exist a point M at phase set $x = (1-p)x_1$; it satisfies $D_y < M_y < S_y$, making $F(M) = 0$. From Lemma 2.7, we get that system (1.3) has a 1-periodic solution.

(ii) If $F(B'') = A''_{1y} - B''_y < 0$ (see Figure 8), there exists a point $S : S_y \gg 0$, the trajectory of system L_2 which crosses the point S intersects with $x = (1-p)x_1$ at G, it makes

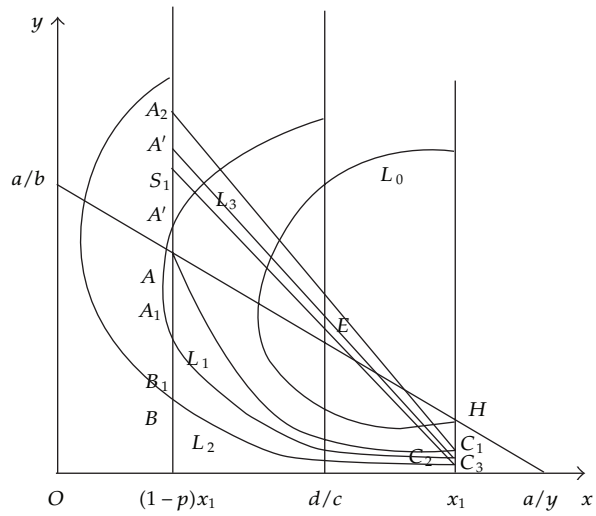

Figure 6

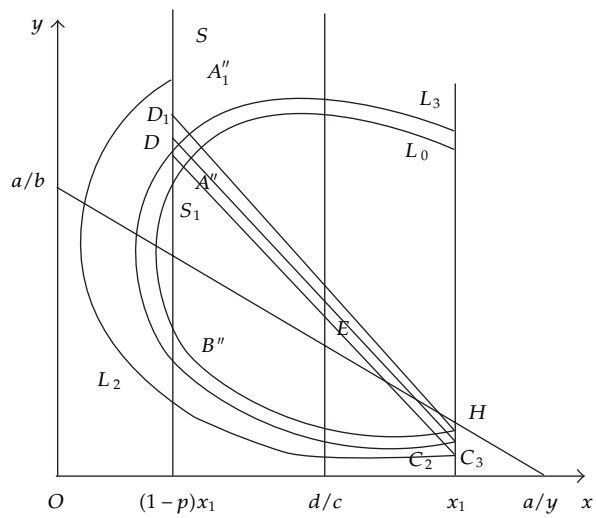

Figure 7

$0 < G_y \ll 1$, the trajectory of system L_2 intersects with $x = x_1$ at point C_2, that pulse to $x = (1-p)x_1$; phase point is $G_1(G_{1x}, G_{1y})$, and then $f(G) = G_{1y} - G_y > 0$, so there must exist a point M at phase set $x = (1-p)x_1$; it satisfies $G_y < M_y < B''_y$, making $F(M) = 0$. From Lemma 2.7, we get that system (1.3) has a 1-periodic solution. □

Statement 3.7. If $B_y < h < A_{1y} < A_y$, then system (1.3) has no 1-periodic solution.

If $d/c \leq (1-p)x_1 < x_1$, the periodic solution at the right of stable focus under the influence of impulsive control, then it does not have practical significance, so we did not discuss it.

Theorem 3.8. *If the condition* $0 < (bh - rpx_1 - bq\phi_0)/(a - rx_1 - b\phi_0) < 2$ *holds, then the 1-periodic solution* Γ_0 *which crosses the point* (x_1, φ_0) *of system* (1.3) *is orbitally asymptotically stable.*

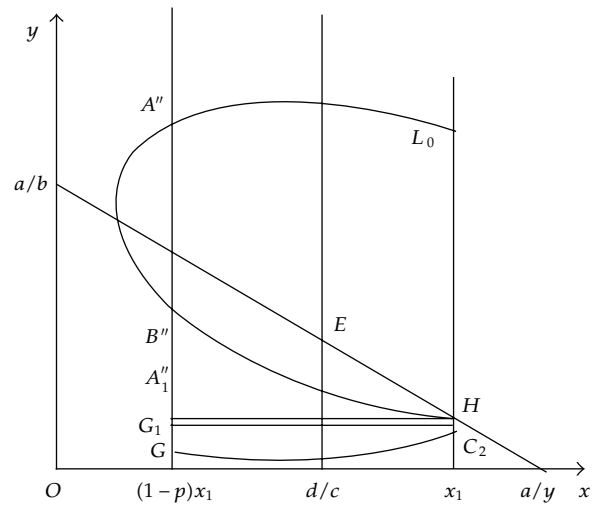

Figure 8

Proof. From system (1.3), we have

$$\frac{\partial f}{\partial x} = a - 2rx - by, \qquad \frac{\partial g}{\partial y} = cx - d, \qquad \frac{\partial \alpha}{\partial x} = -p, \qquad \frac{\partial \beta}{\partial y} = -q,$$

$$\Phi(x,y) = x - x_1, \qquad \frac{\partial \Phi}{\partial x} = 1, \qquad \frac{\partial \Phi}{\partial y} = 0, \qquad (\phi(T), \varphi(T)) = (x_1, \varphi_0),$$

$$\frac{\partial \alpha}{\partial y} = 0, \qquad \frac{\partial \beta}{\partial x} = 0,$$

$$(\phi(T^+), \varphi(T^+)) = ((1-p)x_1, (1-q)\varphi_0 + h),$$

$$\begin{aligned}
\Delta_2 = {} & \left(f_+\left(\left(\frac{\partial \beta}{\partial y}\right)\cdot\left(\frac{\partial \Phi}{\partial x}\right) - \left(\frac{\partial \beta}{\partial x}\right)\cdot\left(\frac{\partial \Phi}{\partial y}\right) + \left(\frac{\partial \Phi}{\partial x}\right)\right)\right. \\
& \left. + g_+\left((\partial\alpha/\partial x)\cdot\left(\frac{\partial \Phi}{\partial y}\right) - \left(\frac{\partial \alpha}{\partial y}\right)\cdot\left(\frac{\partial \Phi}{\partial x}\right) + \left(\frac{\partial \Phi}{\partial y}\right)\right)\right) \times \left(f\left(\frac{\partial \Phi}{\partial x}\right) + g\left(\frac{\partial \Phi}{\partial y}\right)\right)^{-1} \\
= {} & \frac{f_+\cdot(1-q)}{g} = \frac{(1-q)\cdot f_+(\phi(T^+)\cdot\varphi(T^+))}{f(\phi(T)\cdot\varphi(T))} \\
= {} & \frac{(1-p)x_1[a - r(1-p)x_1 - b((1-q)\varphi_0 + h)](1-q)}{x_1(a - rx_1 - b\varphi_0)} \\
= {} & \frac{(1-p)[a - r(1-p)x_1 - b((1-q)\varphi_0 + h)](1-q)}{a - rx_1 - b\varphi_0},
\end{aligned}$$

$$\begin{aligned}
\mu_2 &= \Delta_2 \cdot \exp\left\{\int_0^T \left(\frac{\partial f}{\partial x}(\phi,\varphi) + \frac{\partial g}{\partial y}(\phi,\varphi)\right)dt\right)\right\} \\
&= \Delta_2 \cdot \exp\left\{\int_0^T (a - r\phi - b\varphi + c\phi - d - r\phi)dt\right\} \\
&= \Delta_2 \cdot \exp\left\{\int_{(1-p)x_1}^{x_1} \frac{1}{\phi(t)}d\phi(t) + \int_{(1-q)\varphi_0+h}^{\varphi_0} \frac{1}{\varphi(t)}d\varphi(t) - \int_0^T r\phi(t)dt\right\} \\
&= \Delta_2 \cdot \exp\left\{\ln\frac{1}{1-p} + \ln\frac{1}{(1-q)\varphi_0+h} - r\int_0^T \phi(t)dt\right\} \\
&= \Delta_2 \cdot \frac{1}{1-p} \cdot \frac{\varphi_0}{(1-q)\varphi_0+h} \cdot \exp\left\{-r\int_0^T \phi(t)dt\right\},
\end{aligned} \tag{3.2}$$

then

$$\begin{aligned}
u_2 &= \frac{(1-p)\left[a - r(1-p)x_1 - b\left((1-q)\varphi_0+h\right)\right](1-q)}{a - rx_1 - b\varphi_0} \cdot \frac{1}{1-p} \\
&\quad \cdot \frac{\varphi_0}{(1-q)\varphi_0+h} \cdot \exp\left\{-r\int_0^T \phi(t)dt\right\} \\
&= \frac{\left[a - r(1-p)x_1 - b\left((1-q)\varphi_0+h\right)\right](1-q)}{a - rx_1 - b\varphi_0} \\
&\quad \cdot \frac{\varphi_0}{(1-q)\varphi_0+h} \cdot \exp\left\{-r\int_0^T \phi(t)dt\right\} \\
&\le \frac{\left[a - r(1-p)x_1 - b\left((1-q)\varphi_0+h\right)\right](1-q)}{a - rx_1 - b\varphi_0} \cdot \frac{\varphi_0}{(1-q)\varphi_0} \cdot \exp\left\{-r\int_0^T \phi(t)dt\right\} \\
&= \frac{\left[a - r(1-p)x_1 - b\left((1-q)\varphi_0+h\right)\right]}{a - rx_1 - b\varphi_0} \cdot \exp\left\{-r\int_0^T \phi(t)dt\right\} \\
&\le \frac{\left[a - r(1-p)x_1 - b\left((1-q)\varphi_0+h\right)\right]}{a - rx_1 - b\varphi_0} \\
&= 1 - \frac{bh - rpx_1 - bq\varphi_0}{a - rx_1 - b\varphi_0}.
\end{aligned} \tag{3.3}$$

When $0 < (bh - rpx_1 - bq\phi_0)/(a - rx_1 - b\phi_0) < 2$, then $|u_2| < 1$, so the 1-periodic solution Γ_0 is orbitally asymptotically stable. □

Statement 3.9. If $p = q = 0$, $h > 0$, we get the pest management model of nonpollution; when the economic threshold of system (1.3) is $x = x_1$, then $y_1 = (a - rx_1)/b$, and it is called the key point of the natural enemies of pests. At the time that the number of the natural enemies of pests is less than y_1, we release the natural enemies of pests, so we have the following model:

$$\begin{aligned} \frac{dx}{dt} &= x(a - rx - by) \\ \frac{dy}{dt} &= y(cx - d) \end{aligned} \qquad y > y_1, \qquad \Delta y = h, \quad y = y_1. \tag{3.4}$$

here $0 < h < (r/b)x_1$ or $0 < h < ((a/b) - y_1)$ means the number of the natural enemies we released at one time; the significance of other parameters is the same as the aforementioned.

Theorem 3.10. (1) *If $x_1 \le d/c$, then the system* (3.4) *has a 1-periodic solution.*

(2) *If $x_1 > d/c$, then the system* (3.4) *has a 1-periodic solution or it has $x(t) \le x_1$ for any t.*

Theorem 3.11. *The 1-periodic solution of system* (3.4) *is orbitally asymptotically stable.*

4. Numerical Analysis and Biological Significance

In this part, we use numerical simulation to analyse the dynamical behavior and ecological significance of system (1.3). We fixed the coefficients of the system, then we get system (2.1):

$$\begin{aligned} \frac{dx}{dt} &= x(16 - x - 2y) \\ \frac{dy}{dt} &= y(2x - 10) \end{aligned} \qquad x < x_1 \qquad \Delta x = -px, \qquad \Delta y = -qy + h, \quad y = y_1. \tag{4.1}$$

Statement 4.1. If there is no impulse, then system has the unique positive equilibrium (5, 4.5) which is globally asymptotically stable.

Next, we consider the existence of 1-periodic solution for system (4.1) under the different values of pulse set x_1, the parameters p, q, h, and initial point (x_0, y_0).

Case 1. $p = 0.6$, $q = 0.4$, $h = 1$, $(x_0, y_0) = (3.77, 1.42)$, pulse set is $x_1 = 4.5 < 5$, see Figure 9, and the system has a 1-periodic solution.

Case 2. $p = 0.6$, $q = 0.4$, $h = 5$, $(x_0, y_0) = (3.77, 1.42)$, pulse set is $x_1 = 4.5 < 5$, see Figure 10, and the system has a 1-periodic solution.

Case 3. $p = 0.6$, $q = 0.4$, $h = 1$, $(x_0, y_0) = (3.77, 1.42)$, pulse set is $x_1 = 5$, see Figure 11, and the system has a 1-periodic solution.

Case 4. $p = 0.6$, $q = 0.4$, $h = 5$, $(x_0, y_0) = (3.77, 1.42)$, pulse set is $x_1 = 5$, see Figure 12, and the system has a 1-periodic solution.

Case 5. $p = 0.6$, $q = 0.4$, $h = 1$, $(x_0, y_0) = (3.77, 1.42)$, pulse set is $x_1 = 6 > 5$, phase set is $(1 - p)x_1 = 2.4 < 5$, see Figure 13, and the system has a 1-periodic solution.

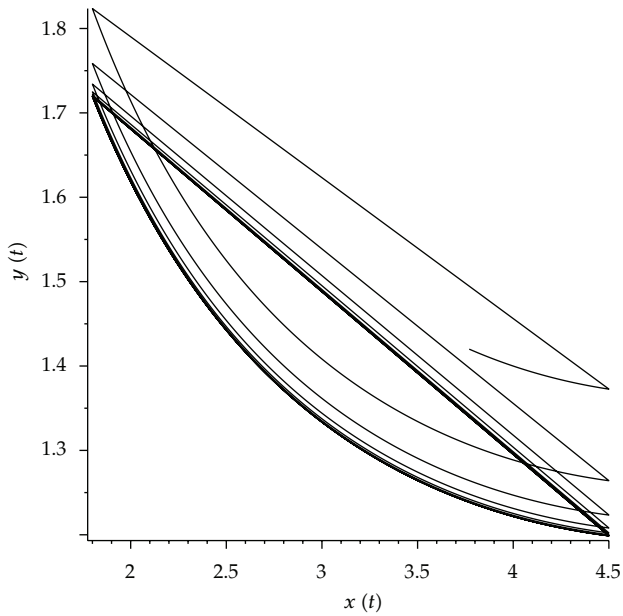

Figure 9

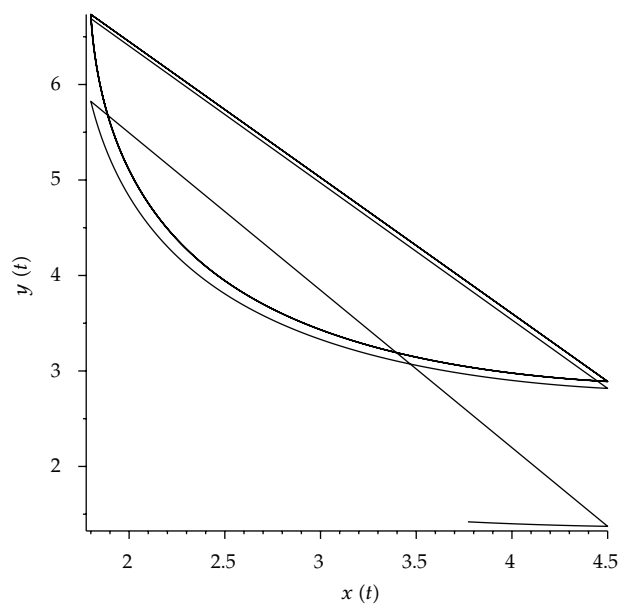

Figure 10

Case 6. $p = 0.6$, $q = 0.4$, $h = 5, (x_0, y_0) = (3.77, 1.42)$, pulse set is $x_1 = 6 > 5$, phase set is $(1-p)x_1 = 2.4 < 5$, see Figure 14, and the system has a 1-periodic solution.

Case 7. $p = 0.2$, $q = 0.5$, $h = 1.1, (x_0, y_0) = (6.44, 0.435)$, pulse set is $x_1 = 7 > 5$, phase set is $(1-p)x_1 = 5.6 > 5$, see Figure 15, and the system has a 1-periodic solution.

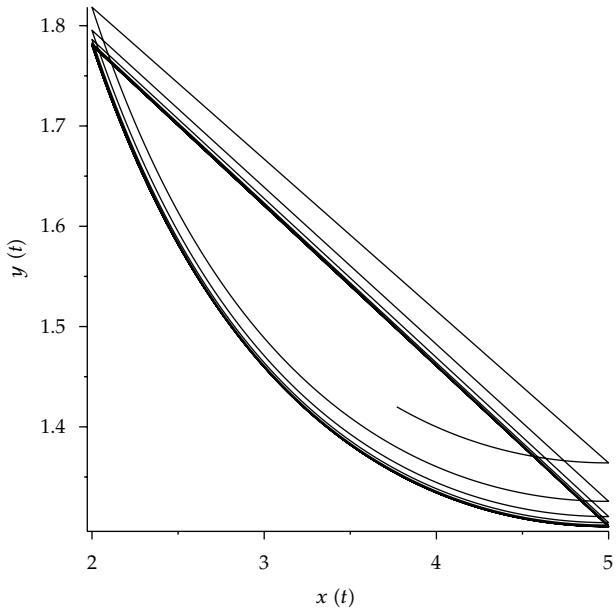

Figure 11

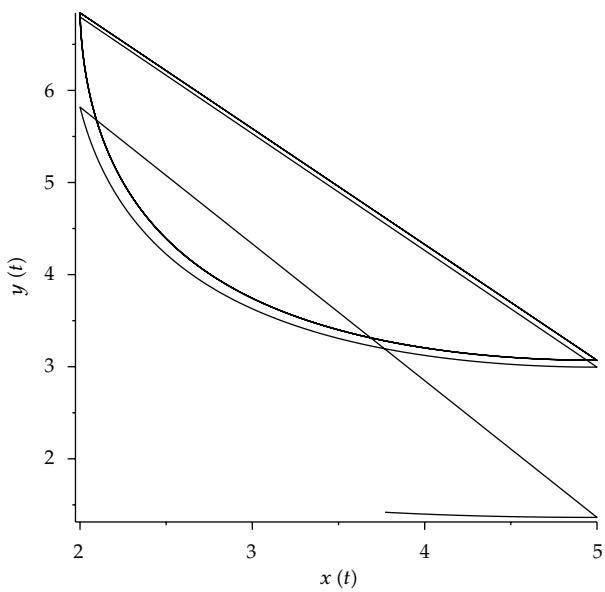

Figure 12

Case 8. $p = 0.2$, $q = 0.5$, $h = 12$, $(x_0, y_0) = (6.44, 0.435)$, pulse set is $x_1 = 7 > 5$, phase set is $(1-p)x_1 = 5.6 > 5$, see Figure 16, and the system has a 1-periodic solution.

Case 9. $p = 0.2, q = 0.5$, $h = 1.2$, $(x_0, y_0) = (6.44, 0.435)$, pulse set is $x_1 = 7 > 5$, phase set is $(1-p)x_1 = 5.6 > 5$, see Figure 17, and the system has no 1-periodic solution.

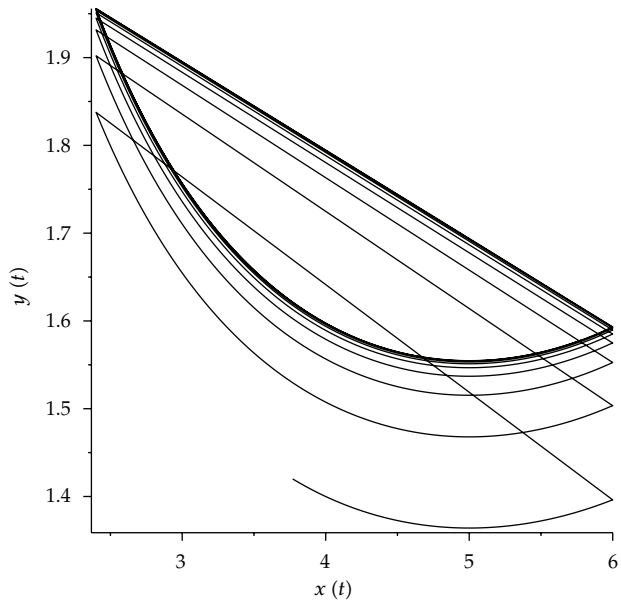

Figure 13

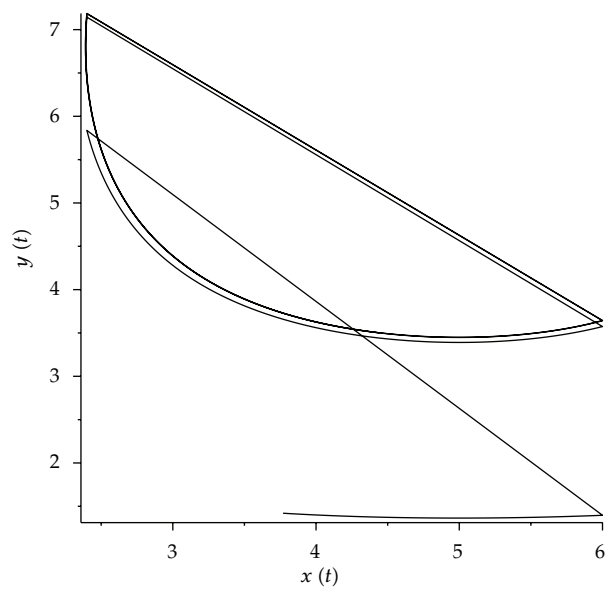

Figure 14

Case 10. $p = 0.2$, $q = 0.5$, $h = 11$, $(x_0, y_0) = (6.44, 0.435)$, pulse set is $x_1 = 7 > 5$, phase set is $(1 - p)x_1 = 5.6 > 5$, see Figure 18, and the system has no 1-periodic solution.

From the numerical analyses, we know that it is better to use comprehensive control including chemistry control and biological technique according to different values of economic threshold to pest. When the pulse set is below or equal to the number of the pests of

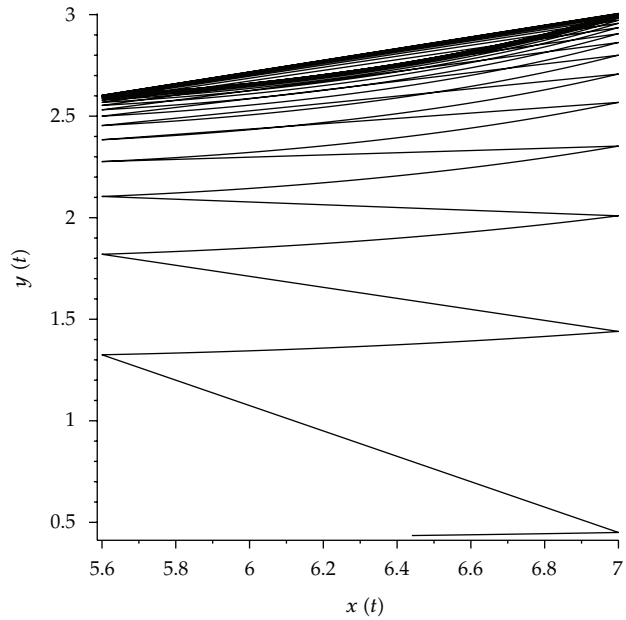

Figure 15

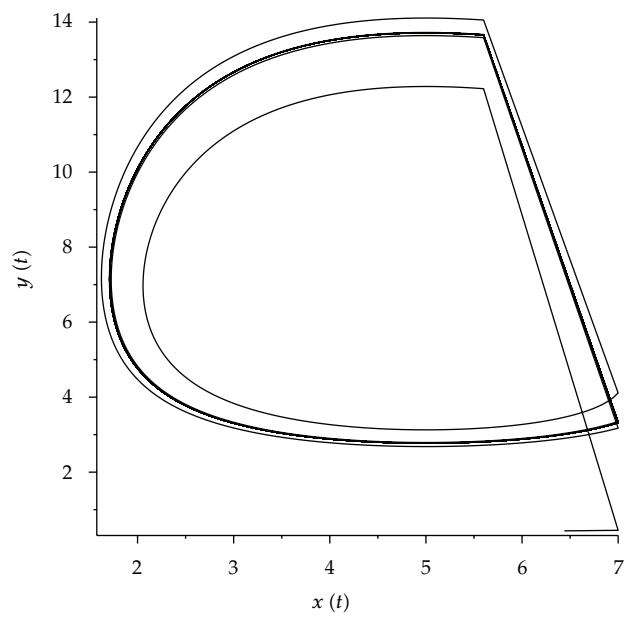

Figure 16

the system at the equilibrium state without pulse, the system has a 1-periodic solution, which is consistent with the proof of the theorem. When the pulse set is more than the number of the pests of the system at the equilibrium state without pulse and the phase set is less than the number of the pests of the system at the equilibrium state without pulse, the system has a 1-periodic solution, which is consistent with the proof of the theorem. When the pulse set

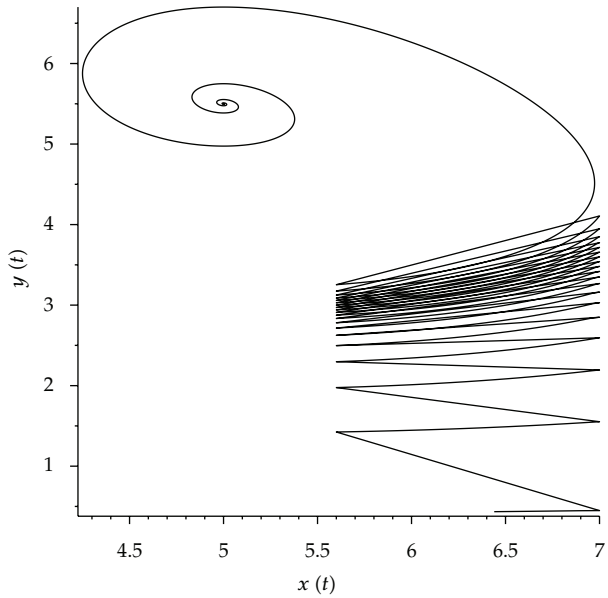

Figure 17

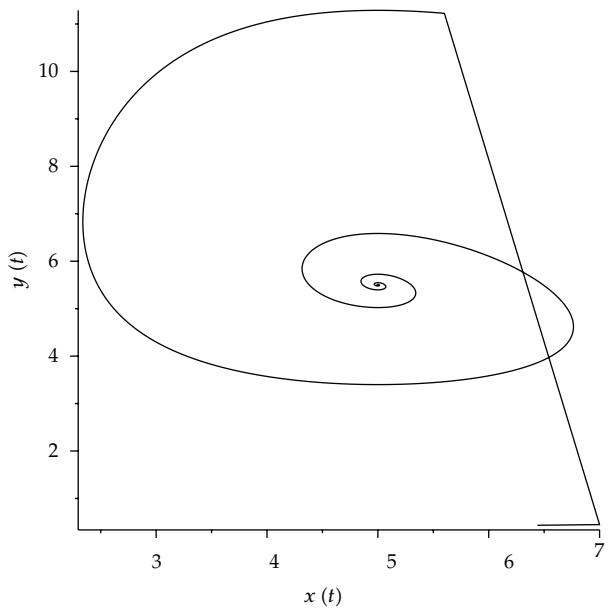

Figure 18

is more than the number of the pests of the system at the equilibrium state without pulse and the phase set is more than the number of the pests of the system at the equilibrium state without pulse, the 1-periodic solution of system may not necessarily exist; we must consider different kinds of the number of the natural enemies we released; then the 1-periodic solution exists and has different periods, which is consistent with the proof of the theorem. So we

take different release strategies according to different growth periods of the crop. In order to decide how to control the number of the natural enemies we released, the control strategy with impulsive state needs observing and recording the number of the pests and the natural enemies. In theory, we can predict the cycle time without repeated measurements, which can save a lot of manpower and material resources. The model in this paper is closer to the reality than the model that there is no density dependence for the continuous process of pulse points; it is also closer to the reality than the model that did not consider the influence of natural enemies of spraying insecticide.

Acknowledgments

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