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Research Article

A Numerical Model for Railroad Freight Car-to-Car End Impact

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A numerical model based on Lagrange-D'Alembert principle is proposed for car-to-car end impact in this paper. In the numerical model, the friction forces are treated by using local linearization model when solving the differential equations. A computer program has been developed for the numerical model based on Runge-Kutta fourth-order method. The results are compared with the Multibody Dynamics/Kinematics software SIMPACK results and they are close. The ladings' relative displacement to struck car and the relative displacement between two ladings get larger as impact speed increases. There is no displacement between two ladings when the contact surfaces have the same friction coefficient.

1. Introduction

The freight damage incurred during railroad transportation is a serious economic and safety problem. The railroad freight car's dynamic characteristic leads to most of the freight damage. The dynamics of railroad car and freight damage can be divided into two groups:

(1) during the marshalling operation in train yard, the car-to-car end impacts from coupling cause high car and lading acceleration;

(2) the car-body vibrations come from track irregularities and some extra forces, such as the wind, and so forth.

Most of the damage is attributed to car-to-car end impacts in the marshalling yard, so more focus is given on it when working out the load support and load securement method. Railroad freight car impact tests are always carried out for checking if the method can ensure transportation safety and no damage to the ladings. A railroad freight car impact test usually needs a lot of work to do; it needs much workforce, material resources, and financial support. Most of the time, carrying out an impact test will lead to disorder and break-off

transportation. Compared with impact test, numerical simulation is a more economical and faster method of investigating the effect on ladings when coupled.

Car-to-car end impact is a special multibody dynamic problem between railroad freight cars. Investigation into the multibody dynamic has been carried out in the works [1–9]. Later, mathematical models are derived in [10] for studying the effect of impact on packaging. At the same time, numerical methods need to be developed for solving mathematical models. Euler tangent method, Newmark-β method, Wilson-θ method, and Runge-Kutta fourth-order method are developed and widely applied in solving mathematical models [11–17]. Runge-Kutta fourth-order method means that the truncation error per step is $O(h^5)$. It is an important numerical method used extensively in engineering problems for solving first-order differential equations.

Draft gear is the most important component of a freight car during impact. Its performance is investigated by mechanics dynamics software in [18–21]. The draft gear's characteristic is analyzed under different impact speeds. The force versus draft gear travel of the Chinese MT-2 under impact speed of 5~8 km/h is given by simulation and test.

In the paper, the second-order differential equations of the car-to-car end impact are converted to first-order differential equations and solved by using Runge-Kutta 4th order method.

2. Draft Gear Interaction Process

Railroad car-to-car end impacts usually occur in train yard, and most of the time the struck car is static when coupled with the striking car. The draft gear is an important component for reducing freight and car damage during car-to-car end impacts.

MT-2 friction draft gear is widely used in the class 70 t universal freight cars in China. This draft gear is composed by springs and friction mechanism; when it is compressed, part of kinetic energy is converted to friction energy and part of kinetic energy is converted to potential energy. MT-2 draft gear has different force versus travel characteristic curves when loading and unloading. In Figure 1, the irreversible force versus draft gear travel characteristic curve is shown [22].

As shown in Figure 1, Δx is draft gear travel and Δv is speed difference between striking car and struck car. $\Delta v > 0$ means draft gear loading process and $\Delta v < 0$ means unloading process. Figure 1 shows that the resistant force in loading process is larger than unloading process. In the numerical calculation program, draft gear force versus travel characteristic curve is based on the test results, and the force is calculated by linear interpolation. MT-2 draft gear force versus travel characteristic curves under impact speeds of 5 km/h, 6 km/h, 7 km/h, and 8 km/h are presented in the appendix.

3. Car-to-Car End Impact Dynamic Models

3.1. Railroad Freight Car Impact System

Railroad freight car impact test is using a striking car with a certain speed running to a static struck car and collides. The longitudinal status of the ladings and struck car is mainly observed during impact for checking the loading support and loading secure method. The method must ensure transportation safety and no lading damage.

The assumptions in models are

(1) the wind acting on the striking car and struck car, and the rolling resistance between wheel and rail are neglected;

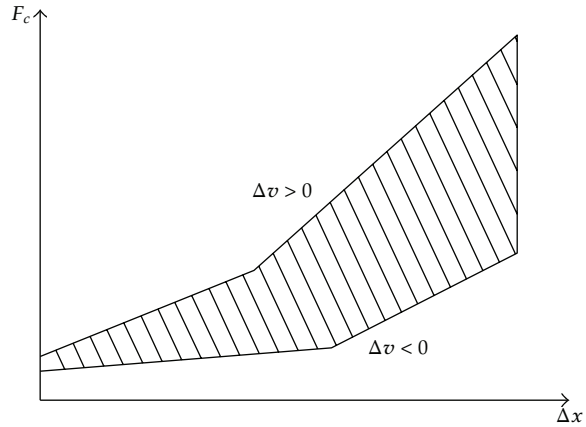

Figure 1: Draft gear's force and travel characteristic.

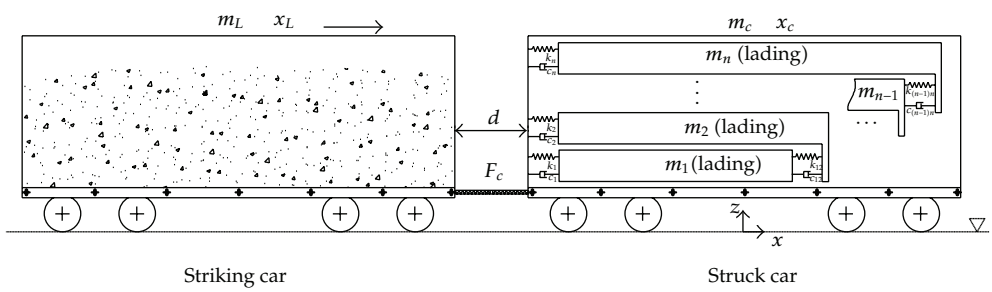

Figure 2: Car-to-car end impact dynamic system.

(2) the car-body deformations during impact are neglected;

(3) the car-body vertical bounce, yaw, pitch, and sway vibrations are neglected;

(4) no lateral forces between ladings.

Sometimes more than one lading are loaded on freight car. There are longitudinal forces between ladings and car, between adjacent ladings. Figure 2 shows the railroad freight car-to-car end impact dynamic system, where m, x represent mass and displacement, L represents striking car, c represents struck car, $1, 2 \ldots, n$ represent ladings, k_n, c_n are the stiffness and damping coefficients between ladings and car, $k_{(n-1)n}$, $c_{(n-1)n}$ are the stiffness and damping coefficients between ladings n and $(n-1)$, F_c is draft gear force, and d is distance between two cars.

3.2. Longitudinal Dynamic Equations

The striking car, struck car and ladings in the impact dynamic system are treated as mass elements. According to the assumptions in Section 3.1, the external forces, constraint forces and inertial forces are an equivalent static system based on Lagrange-D'Alembert principle. Then, universal longitudinal dynamic equations can be derived for each mass element.

External force acting on the striking car is the draft gear force. So the differential equation for striking car is

$$m_L \ddot{x}_L + F_c = 0, \tag{3.1}$$

where m_L is gross weight of the striking car and F_c is calculated by the following equation:

$$F_c = \begin{cases} F_1(\Delta x) & \Delta v > 0, \\ F_2(\Delta x) & \Delta v < 0. \end{cases} \tag{3.2}$$

The acceleration and velocity initial values of the striking car are $\ddot{x}_L = 0$, $\dot{x}_L = v_L$; the initial value of the draft gear force is $F_c = 0$.

The external forces acting on the struck car are the draft gear force and forces between ladings and car,

$$m_c \ddot{x}_c + \sum_{i=1}^{n} c_i(\dot{x}_c - \dot{x}_i) + \sum_{i=1}^{n} k_i(x_c - x_i) - F_c = 0, \tag{3.3}$$

where m_c is the struck car tare weight and the acceleration and velocity initial values of struck car are $\ddot{x}_c = 0$, $\dot{x}_c = 0$.

External forces acting on lading i are the forces between lading i and car, between adjacent ladings,

$$\begin{aligned} &m_i \ddot{x}_i - c_i(\dot{x}_c - \dot{x}_i) - k_i(x_c - x_i) - c_{(i-1)i}(\dot{x}_{i-1} - \dot{x}_i) - k_{(i-1)i}(x_{i-1} - x_i) \\ &\qquad + c_{i(i+1)}(\dot{x}_i - \dot{x}_{i+1}) + k_{i(i+1)}(x_i - x_{i+1}) = 0, \quad i \in [2, n-1], \\ &m_1 \ddot{x}_1 - c_1(\dot{x}_c - \dot{x}_1) - k_1(x_c - x_1) + c_{12}(\dot{x}_1 - \dot{x}_2) \\ &\qquad + k_{12}(x_1 - x_2) = 0, \quad i = 1, \\ &m_n \ddot{x}_n - c_n(\dot{x}_c - \dot{x}_n) - k_n(x_c - x_n) - c_{(n-1)n}(\dot{x}_{n-1} - \dot{x}_n) \\ &\qquad - k_{(n-1)n}(x_{n-1} - x_n) = 0, \quad i = n, \end{aligned} \tag{3.4}$$

where the acceleration and velocity initial values of lading i are $\ddot{x}_i = 0$, $\dot{x}_i = 0$.

4. Double-Stack Loading Impact Models

Double-stack loading method in gondola car is proposed as an example for analyzing longitudinal relation between ladings and car. It shows that the numerical method applied in railroad freight car-to-car end impact simulation. In the model, the striking car and struck car are the same type and have the same draft gears.

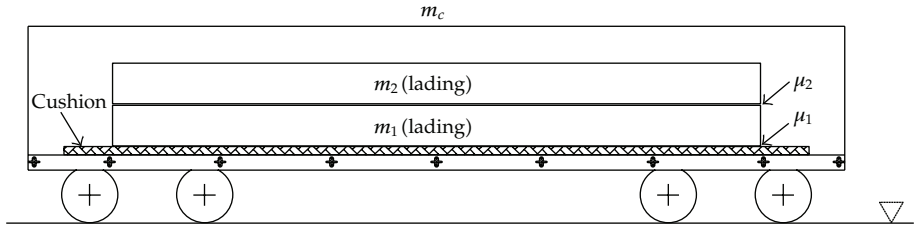

Figure 3: Load support and load securement method.

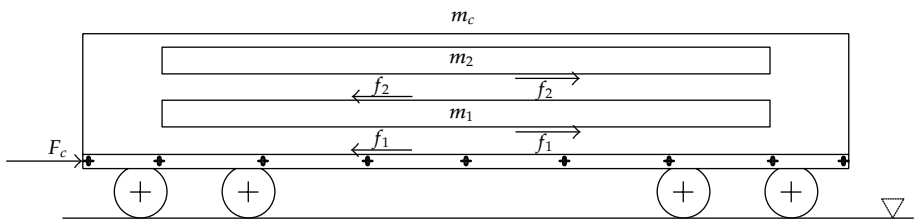

Figure 4: Force analysis.

4.1. Load Support and Load Securement Method and Force Analysis

Figure 3 shows the double-stack loading in a 70 t class gondola car. The securement method is using friction cushion to enlarge friction force.

The above struck car system includes three mass elements that are the struck car, 1st lading and 2nd lading. Forces acting on struck car are draft gear force and friction force from 1st lading; forces acting on the 1st lading are friction forces from struck car and the 2nd lading; force acting on the 2nd lading is friction force from 1st lading. The force analysis is illustrated in Figure 4.

4.2. Dynamic Equations of Motion

The universal longitudinal dynamic equations in Section 3.2 can be rewritten based on the force analysis

$$\begin{aligned} \text{striking car}&: \ m_L\ddot{x}_L = -F_c, \\ \text{struck car}&: m_c\ddot{x}_c = F_c - f_1, \\ \text{1st lading}&: \ m_1\ddot{x}_1 = f_1 - f_2, \\ \text{2nd lading}&: m_2\ddot{x}_2 = f_2. \end{aligned} \tag{4.1}$$

As striking car and struck car have the same draft gear type, so the travel of one draft gear is given as

$$\Delta x = \frac{(x_L - x_c)}{2}. \tag{4.2}$$

The speed difference between striking car and struck car is given as

$$\Delta v = \dot{x}_L - \dot{x}_c. \tag{4.3}$$

The model has two one-dimensional friction elements: one is between 1st lading and car-body and the other is between 1st lading and 2nd lading. The one-dimensional friction element's friction force direction is dependent on the direction of the relative sliding velocity. Figure 5(a) shows that the direction of friction force changes abruptly as the direction of relative velocity changes. More calculation time is needed near the zero-point, and even the differential equations cannot be integrated at zero-point. The friction element is treated by using a local linearization model [23], which uses a parameter v_0 called switching speed. Figure 5(b) illustrates linear relationship between friction force and relative velocity, and the friction force is given as

$$f = \begin{cases} \mu mg & \Delta v > v_0, \\ \dfrac{\Delta v}{v_0}\mu mg & |\Delta v| \le v_0, \\ -\mu mg & \Delta v < -v_0. \end{cases} \tag{4.4}$$

4.3. Solution Methodology

For using Runge-Kutta fourth-order method, second-order differential equations need to be rewritten in the form of first-order differential equations. In general, to solve first-order differential equation $\dot{y} = f(x, y)$, Runge-Kutta fourth-order method is given as

$$\begin{aligned} y_{i+1} &= y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4), \\ K_1 &= f(x_i, y_i), \\ K_2 &= f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_1\right), \\ K_3 &= f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_2\right), \\ K_4 &= f(x_i + h, y_i + hK_3), \end{aligned} \tag{4.5}$$

where h is the step size.

Let $x_{dL} = \dot{x}_L$ in the striking car's second-order differential equation, then the reduced-order differential equations are given as

$$\begin{aligned} \dot{x}_{dL} &= -\frac{F_c}{m_L}, \\ \dot{x}_L &= x_{dL}. \end{aligned} \tag{4.6}$$

Table 1: Numerical model parameters.

Case	m_L/t	m_c/t	m_1/t	m_2/t	μ_1	μ_2	h/s
1	92.5	22.5	30	30	0.4	0.3	0.00001
2	92.5	22.5	30	30	0.4	0.4	0.00001
3	92.5	22.5	30	20	0.4	0.3	0.00001

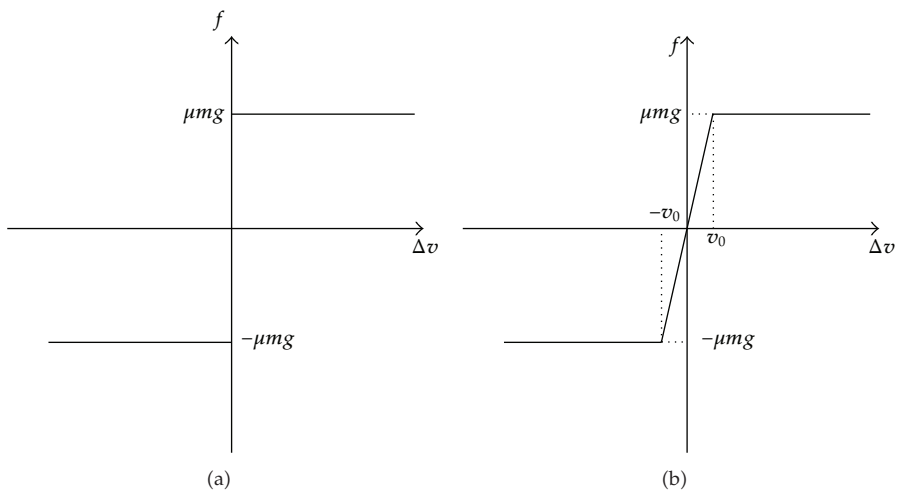

Figure 5: One-dimensional friction element model.

In the same way, the second-order differential equations of other mass elements are given as,

$$\begin{aligned}
&\text{Struck car}\begin{cases}\dot{x}_{dc} = \dfrac{F_c - f_1}{m_c},\\ \dot{x}_c = x_{dc},\end{cases}\\
&\text{1st lading}\begin{cases}\dot{x}_{d1} = \dfrac{f_1 - f_2}{m_1},\\ \dot{x}_1 = x_{d1},\end{cases}\\
&\text{2nd lading}\begin{cases}\dot{x}_{d2} = \dfrac{f_2}{m_2},\\ \dot{x}_2 = x_{d2}.\end{cases}
\end{aligned} \tag{4.7}$$

4.4. Numerical Results and Discussion

The type of draft gear is MT-2 in the dynamic models; the impact speeds are 5 km/h, 6 km/h, 7 km/h, and 8 km/h. The simulation is for studying relative displacement of 1st lading and 2nd lading, which are secured by the friction cushion. Case 1 only has friction cushion between 1st lading and car floor; Case 2 has friction cushion between 1st lading and car floor, and between two ladings; the friction cushion in Case 3 is the same as Case 1, but two ladings have different weight. The parameters in numerical model are given in Table 1.

A virtual model is built in Multibody Dynamics/Kinematics software SIMPACK which has the same parameters as Case 1 to validate the model and numerical method. It

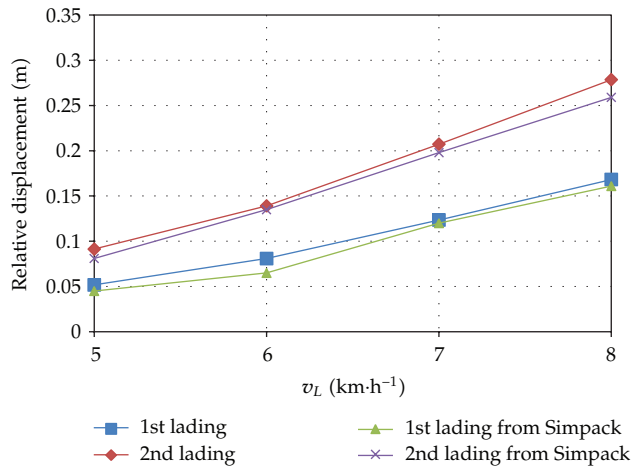

Figure 6: Results under $m1 = 30\,\text{t}$, $m_2 = 30\,\text{t}$, $\mu_1 = 0.4$, and $\mu_2 = 0.3$.

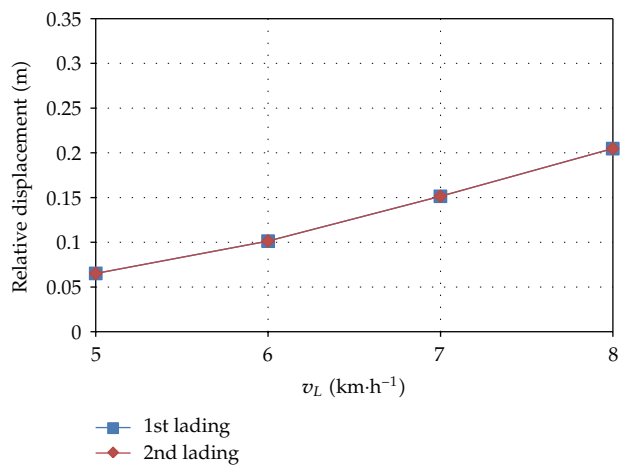

Figure 7: Results under $m_1 = 30\,\text{t}$, $m_2 = 30\,\text{t}$, $\mu_1 = 0.4$, and $\mu_2 = 0.4$.

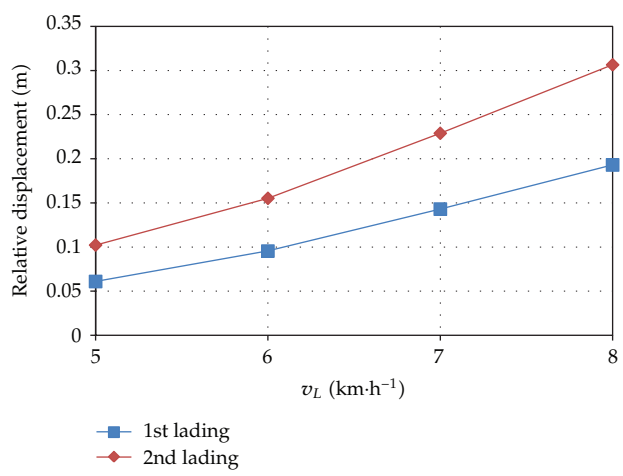

Figure 8: Results under $m_1 = 30\,\text{t}$, $m_2 = 20\,\text{t}$, $\mu_1 = 0.4$, and $\mu_2 = 0.3$.

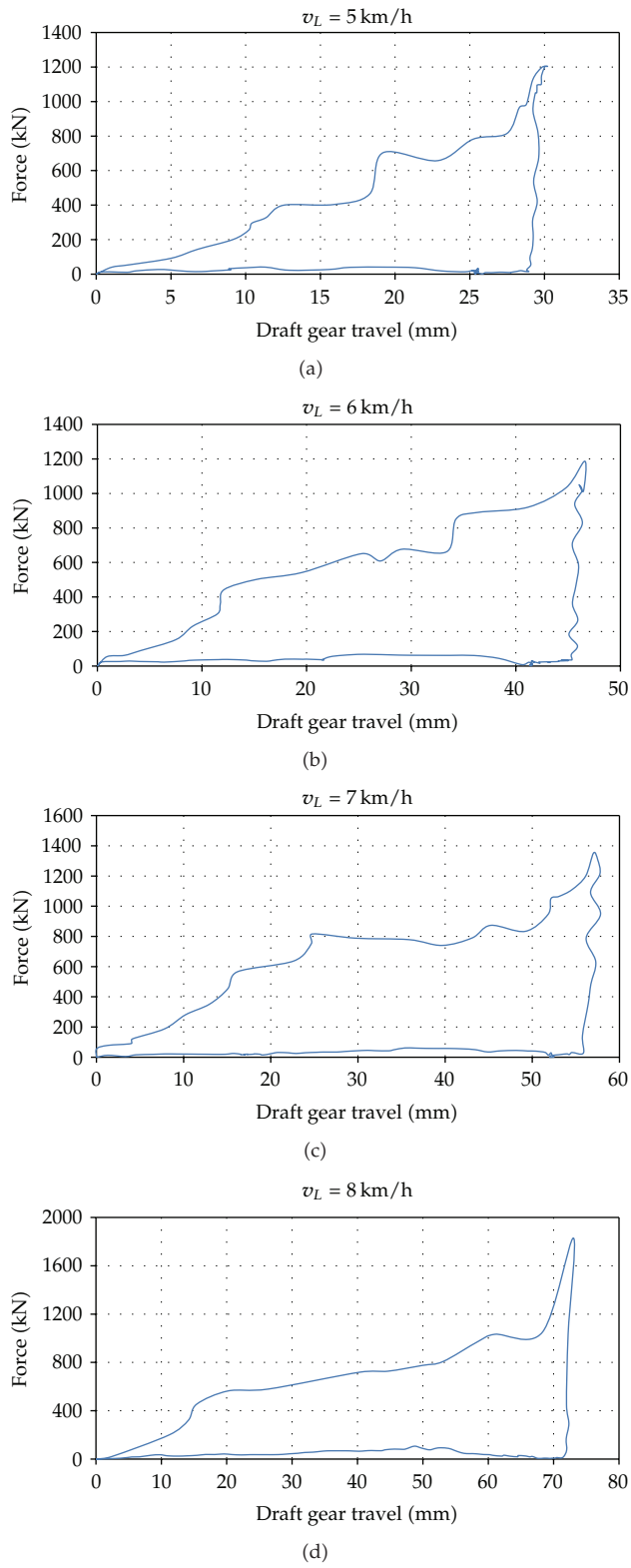

Figure 9: Draft gear force and travel characteristic curves. v_L is the velocity of striking car just before impact.

can be observed from Figure 6 that there is an excellent agreement between the results from Runge-Kutta fourth-order method and SIMPACK. The displacement between ladings and car-body and between 1st lading and 2nd lading increased with impact speeds.

Figure 7 shows the displacement between ladings and car-body under Case 2 versus different impact speeds. The displacement between ladings and car-body increased with impact speeds, but the displacement between 1st lading and 2nd lading is zero as two surfaces have the same friction coefficient.

In Case 3, the weight of 2nd lading is less than that in Case 1, so the gross weight of struck car reduced. The displacement between ladings and car-body versus impact speeds are illustrated in Figure 8. The displacements between ladings and car-body are increased compared with Case 1 under the same impact speed.

5. Conclusion

In this paper, railroad freight car-to-car end impact system and the influence on ladings are considered. To derive differential equations of the system motion, forces acting in the system are analyzed and Lagrange-D'Alembert principle is used. The obtained solution of the differential equations by Runge-Kutta fourth-order method is close to the results from SIMPACK. Based on numerical results from double-stack model, it is concluded that the higher the impact speed, the larger the ladings' relative displacement to struck car. The lower weight the ladings, the larger the ladings' relative displacement to struck car. There is no displacement between two ladings if they have the same friction coefficient between struck car and 1st lading and between 1st lading and 2nd lading.

Appendix

See Figures 9(a)–9(d).

Acknowledgment

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