Research Article

Nearly Quadratic n -Derivations on Non-Archimedean Banach Algebras

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Let $n > 1$ be an integer, let A be an algebra, and X be an A -module. A quadratic function $D : A \rightarrow X$ is called a quadratic n -derivation if $D(\prod_{i=1}^n a_i) = D(a_1)a_2^2 \cdots a_n^2 + a_1^2 D(a_2)a_3^2 \cdots a_n^2 + \cdots + a_1^2 a_2^2 \cdots a_{n-1}^2 D(a_n)$ for all $a_1, \dots, a_n \in A$. We investigate the Hyers-Ulam stability of quadratic n -derivations from non-Archimedean Banach algebras into non-Archimedean Banach modules by using the Banach fixed point theorem.

1. Introduction

A functional equation (ξ) is stable if any function g satisfying the equation (ξ) approximately is near to a true solution of (ξ) .

The stability of functional equations was first introduced by Ulam [1] in 1964. In 1941, Hyers [2] gave a first affirmative answer to the question of Ulam for Banach spaces. In 1978, Th. M. Rassias [3] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences $\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$, ($\epsilon > 0, p \in [0, 1)$). In 1994, a generalization of Th. M. Rassias theorem was obtained by Găvruta [4], who replaced the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$ (see also [5–7]).

Every solution of the following functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \quad (1.1)$$

is said to be a quadratic function [8]. It is well known that a mapping f between real vector spaces is quadratic mapping if and only if there exists a unique symmetric biadditive mapping B_1 such that $f(x) = B_1(x, x)$ for all x . The biadditive mapping B_1 is given by $B_1(x, y) = (1/4)(f(x+y) - f(x-y))$.

The stability problem of the quadratic functional equation was proved by Skof [9] for mappings $f : A \rightarrow B$, where A is a normed space and B is a Banach space (see also [10, 11]). Let A be an algebra and let X be a A -bimodule. A quadratic function $D : A \rightarrow X$ is called a quadratic n -derivation if

$$D\left(\prod_{i=1}^n a_i\right) = D(a_1)a_2^2 \cdots a_n^2 + a_1^2 D(a_2)a_3^2 \cdots a_n^2 + \cdots + a_1^2 a_2^2 \cdots a_{n-1}^2 D(a_n) \quad (1.2)$$

for all $a_1, \dots, a_n \in A$. Recently, Gordji and Ghobadipour [12] introduced the quadratic derivations on Banach algebras. Indeed, they investigated the Hyers-Ulam-Aoki-Rassias stability and Ulam-Gavruta-Rassias type stability of quadratic derivations on Banach algebras.

More recently, Gordji et al. [13] investigated the Hyers-Ulam stability and the superstability of higher ring derivations on non-Archimedean Banach algebras (see also [12–32]). In this paper we investigate the Hyers-Ulam stability of quadratic n -derivations from non-Archimedean Banach algebras into non-Archimedean Banach modules by using the weighted space method (see [33]).

2. Preliminaries

Let us recall that a non-Archimedean field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0, \infty)$ such that $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$, and $|r+s| \leq \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$. An example of a non-Archimedean valuation is the mapping $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$. This valuation is called trivial (see [34]).

Definition 2.1. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|\cdot|$. A function $\|\cdot\| : X \rightarrow \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

$$(NA_1) \quad \|x\| = 0 \text{ if and only if } x = 0;$$

$$(NA_2) \quad \|rx\| = |r|\|x\| \text{ for all } r \in \mathbb{K} \text{ and } x \in X;$$

$$(NA_3) \quad \|x + y\| \leq \max\{\|x\|, \|y\|\} \text{ for all } x, y \in X \text{ (the strong triangle inequality).}$$

In 1897, Hensel [35] introduced a normed space which does not have the Archimedean property. It turned out that non-Archimedean spaces have many nice applications. The most important examples of non-Archimedean spaces are p -adic numbers. Let p be a prime number. For any nonzero rational number $x = (a/b)p^{n_x}$ such that a and b are integers not divisible by p , define the p -adic absolute value $|x|_p := p^{-n_x}$. Then $|\cdot|_p$ is a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to $|\cdot|_p$ is denoted by $\mathbb{Q}_p$ which is called the p -adic number field.

Definition 2.2. Let X be a nonempty set and let $d : X \times X \rightarrow [0, \infty)$ satisfy the following properties:

- (D₁) $d(x, y) = 0$ if and only if $x = y$,
- (D₂) $d(x, y) = d(y, x)$ (symmetry),
- (D₃) $d(x, z) \leq \max\{d(x, y), d(y, z)\}$ (strong triangle inequality),

for all $x, y, z \in X$. Then (X, d) is called a non-Archimedean metric space. (X, d) is called a non-Archimedean complete metric space if every d -Cauchy sequence in X is d -convergent.

Theorem 2.3 (Non-Archimedean Banach Contraction Principle). *Let (X, d) be a non-Archimedean complete metric space and let $T : X \rightarrow X$ be a contraction; that is, there exists $\alpha \in [0, 1)$ such that*

$$d(Tx, Ty) \leq \alpha d(x, y), \quad \forall x, y \in X. \quad (2.1)$$

Then there exists a unique element $a \in X$ such that $Ta = a$. Moreover, $a = \lim_{n \rightarrow \infty} T^n x$, and

$$d(a, x) \leq d(x, Tx), \quad \forall x \in X. \quad (2.2)$$

Proof. A similar argument as Archimedean case can be applied to show that T has a unique element $a \in X$ such that $Ta = a$ and $a = \lim_{n \rightarrow \infty} T^n x$. It follows from strong triangle inequality that for all $x \in X$ and for each $n \in \mathbb{N}$, we have

$$\begin{aligned} d(T^n x, x) &\leq \max\{d(T(x), x), \dots, d(T^n(x), T^{n-1}(x))\} \\ &\leq \max\{d(T(x), x), \dots, \alpha^{n-1} d(T(x), x)\} \\ &= d(T(x), x). \end{aligned} \quad (2.3)$$

□

3. Main Results

In this section A denotes a non-Archimedean Banach algebra over a non-Archimedean field $\mathbb{K}$ and X is a non-Archimedean Banach A -module.

Theorem 3.1. *Let $\varphi : A \times A \rightarrow [0, \infty)$, $\psi : A \times \dots \times A \rightarrow [0, \infty)$ be functions. Let $f : A \rightarrow X$ be a given mapping such that $f(0) = 0$,*

$$\|f(x + y) + f(x - y) - 2f(x) - 2f(y)\| \leq \varphi(x, y) \quad (3.1)$$

and that

$$\left\| f\left(\prod_{i=1}^n x_i\right) - f(x_1)x_2^2 \cdots x_n^2 - x_1^2 f(x_2)x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f(x_n) \right\| \leq \psi(x_1, \dots, x_n) \quad (3.2)$$

for all $x_1, \dots, x_n, x, y \in A$. Suppose that there exist a natural number $k \in \mathbb{K}$ and $L, K \in (0, 1)$, such that

$$|k|^2 \varphi(k^{-1}x, k^{-1}y) \leq L\varphi(x, y), \quad |k|^2 \varphi(k^{-1}x_1, \dots, k^{-1}x_n) \leq K\varphi(x_1, \dots, x_n) \quad (3.3)$$

for all $x_1, \dots, x_n, x, y \in A$. Then there exists a unique quadratic n -derivation h from A into X such that

$$\|f(x) - h(x)\| \leq \frac{L\Phi(x)}{|k|^2} \quad (3.4)$$

for all $x \in A$, where

$$\Phi(x) = \max\{\varphi(0, 0), \varphi(x, x), \varphi(2x, x), \dots, \varphi((k-1)x, x)\} \quad (x \in A). \quad (3.5)$$

Proof. By induction on i , one can show that for all $x \in A$ and $i \geq 2$,

$$\|f(ix) - i^2 f(x)\| \leq \max\{\varphi(0, 0), \varphi(x, x), \varphi(2x, x), \dots, \varphi((i-1)x, x)\}. \quad (3.6)$$

Let $x = y$ in (3.1). Then

$$\|f(2x) - 2^2 f(x)\| \leq \max\{\varphi(0, 0), \varphi(x, x)\} \quad (x \in A). \quad (3.7)$$

This proves (3.6) for $i = 2$. Let (3.6) hold for $i = 1, 2, \dots, j$. Replacing x by jx and y by x in (3.1) for all $x \in A$, we get

$$\|f((j+1)x) + f((j-1)x) - 2f(jx) - 2f(x)\| \leq \max\{\varphi(0, 0), \varphi(jx, x)\} \quad (3.8)$$

for all $x \in A$. Since

$$\begin{aligned} f((j+1)x) + f((j-1)x) - 2f(jx) - 2f(x) &= f((j+1)x) - (j+1)^2 f(x) \\ &\quad + f((j-1)x) - (j-1)^2 f(x) - 2[f(jx) - j^2 f(x)] \end{aligned} \quad (3.9)$$

for all $x \in A$, it follows from induction hypothesis and (3.8) that for all $x \in A$,

$$\begin{aligned} \|f((j+1)x) - (j+1)^2 f(x)\| &\leq \max\{\|f((j+1)x) + f((j-1)x) - 2f(jx) - 2f(x)\|, \\ &\quad \|f((j-1)x) - (j-1)^2 f(x)\|, |2|\|j^2 f(x) - f(jx)\|\} \\ &\leq \max\{\varphi(0, 0), \varphi(x, x), \varphi(2x, x), \dots, \varphi((j)x, x)\}. \end{aligned} \quad (3.10)$$

This proves (3.6) for all $i \geq 2$. In particular

$$\|f(kx) - k^2 f(x)\| \leq \Phi(x) \quad (x \in A). \quad (3.11)$$

Replacing x by $k^{-1}x$ in (3.11), we get

$$\|f(x) - k^2 f(k^{-1}x)\| \leq \Phi(k^{-1}x) \leq \frac{L}{|k|^2} \Phi(x) \quad (3.12)$$

for all $x \in A$. Let Ω be the set of all functions $u : A \rightarrow X$. We define the metric d on Ω as follows:

$$d(u, v) = \sup_{x \in A} D(x), \quad (3.13)$$

where $D(x) = (\|u(x) - v(x)\|)/\Phi(x)$ if $\Phi(x) \neq 0$ and $D(x) = \|u(x) - v(x)\|$ if $\Phi(x) = 0$. One has the operator $J : \Omega \rightarrow \Omega$ by $J(u)(x) = k^2 u(k^{-1}x)$. Then J is strictly contractive on Ω ; in fact, if

$$\|u(x) - v(x)\| \leq \alpha \Phi(x) \quad (x \in A), \quad (3.14)$$

then by (3.3),

$$\begin{aligned} \|J(u)(x) - J(v)(x)\| &= |k|^2 \|u(k^{-1}x) - v(k^{-1}x)\| \\ &\leq \alpha |k|^2 \Phi(k^{-1}x) \leq L \alpha \Phi(x), \quad (x \in A). \end{aligned} \quad (3.15)$$

It follows that

$$d(J(u), J(v)) \leq L d(u, v) \quad (u, v \in \Omega). \quad (3.16)$$

Hence J is a contractive with Lipschitz constant L . By Theorem 2.3, J has a unique fixed point $h : A \rightarrow X$ and

$$h(x) = \lim_{m \rightarrow \infty} J^m(f(x)) = \lim_{m \rightarrow \infty} k^{2m} f(k^{-m}x) \quad (3.17)$$

for all $x \in A$.

Therefore

$$\begin{aligned} &\|h(x+y) + h(x-y) - 2h(x) - 2h(y)\| \\ &= \lim_{m \rightarrow \infty} |k|^{2m} \|f(k^{-m}(x+y)) + f(k^{-m}(x-y)) - 2f(k^{-m}x) - 2f(k^{-m}y)\| \\ &\leq \lim_{m \rightarrow \infty} |k|^{2m} \varphi(k^{-m}x, k^{-m}y) \\ &\leq \lim_{m \rightarrow \infty} L^m \varphi(x, y) = 0 \end{aligned} \quad (3.18)$$

for all $x, y \in A$. This shows that h is quadratic. It follows from Theorem 2.3 that

$$d(f, h) \leq d(J(f), f), \quad (3.19)$$

that is,

$$\|f(x) - h(x)\| \leq \frac{L\Phi(x)}{|k|^2} \quad (x \in A). \quad (3.20)$$

Replacing x_i by $k^{-m}x_i$, $i = 1, \dots, n$ in (3.2), we get

$$\begin{aligned} & \left\| f\left(\prod_{i=1}^n k^{-mn}x_i\right) - f(k^{-m}x_1)k^{-2m(n-1)}x_2^2 \cdots x_n^2 \right. \\ & \quad \left. - k^{-2m(n-1)}x_1^2 f(k^{-m}x_2)x_3^2 \cdots k^{-2m(n-1)}x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f(k^{-m}x_n) \right\| \\ & \leq \psi(k^{-m}x_1, \dots, k^{-m}x_n), \end{aligned} \quad (3.21)$$

and so

$$\begin{aligned} & |k|^{2mn} \left\| f\left(\prod_{i=1}^n k^{-mn}x_i\right) - f(k^{-m}x_1)k^{-2m(n-1)}x_2^2 \cdots x_n^2 \right. \\ & \quad \left. - k^{-2m(n-1)}x_1^2 f(k^{-m}x_2)x_3^2 \cdots x_n^2 - \cdots - k^{-2m(n-1)}x_1^2 \cdots x_{n-1}^2 f(k^{-m}x_n) \right\| \\ & = \left\| 2^{2mn} f\left(\prod_{i=1}^n k^{-mn}x_i\right) - k^{2m} f(k^{-m}x_1)x_2^2 \cdots x_n^2 \right. \\ & \quad \left. - x_1^2 k^{2m} f(k^{-m}x_2)x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 k^{2m} f(k^{-m}x_n) \right\| \\ & \leq |k|^{2mn} \psi(k^{-m}x_1, \dots, k^{-m}x_n) \leq |k|^{2mn} \frac{K^m}{|k|^{2m}} \psi(x_1, \dots, x_n) \end{aligned} \quad (3.22)$$

for all $x_1, \dots, x_n \in A$ and each $m \in \mathbb{N}$. By taking $m \rightarrow \infty$, we have

$$h\left(\prod_{i=1}^n x_i\right) = h(x_1)x_2^2 \cdots x_n^2 + x_1^2 h(x_2)x_3^2 \cdots x_n^2 + \cdots - x_1^2 \cdots x_{n-1}^2 h(x_n) \quad (3.23)$$

for all $x_1, \dots, x_n \in A$. □

In the following corollaries we will assume that A is a non-Archimedean Banach algebra over $\mathbb{K} = \mathbb{Q}_p$ the field of p -adic numbers, where $p > 2$ is a prime number.

Corollary 3.2. Let $r < 1$ and let ε be δ be positive real numbers. Suppose that $f : A \rightarrow X$ is a mapping such that

$$\begin{aligned} & \|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \leq \varepsilon \|x\|^r \|y\|^r, \\ & \left\| f\left(\prod_{i=1}^n x_i\right) - f(x_1)x_2^2 \cdots x_n^2 - x_1^2 f(x_2)x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f(x_n) \right\| \\ & \leq \delta \max\{\|x_1\|^r, \dots, \|x_n\|^r\} \end{aligned} \quad (3.24)$$

for all $x_1, \dots, x_n, x, y \in A$. Then there exists a unique quadratic n -derivation h from A into X such that

$$\|f(x) - h(x)\| \leq \varepsilon p^{2r} \|x\|^{2r} \quad (3.25)$$

for all $x \in A$.

Proof. By (3.24), $f(0) = 0$. Let $\varphi(x, y) = \varepsilon \|x\|^r \|y\|^r$ and $\psi(x_1, \dots, x_n) = \delta \max\{\|x_1\|^r, \dots, \|x_n\|^r\}$ for all $x_1, \dots, x_n, x, y \in A$. Then

$$|p|^2 \varphi(p^{-1}x, p^{-1}y) = p^{2r-2} \varphi(x, y), \quad |p|^2 \psi(p^{-1}x_1, \dots, p^{-1}x_n) = p^{r-2} \psi(x_1, \dots, x_n) \quad (3.26)$$

for all $x_1, \dots, x_n, x, y \in A$.

Moreover,

$$\Phi(x) = \max\{\varphi(0, 0), \varphi(x, x), \varphi(2x, x), \dots, \varphi((p-1)x, x)\} = \varepsilon \|x\|^{2r} \quad (x \in A). \quad (3.27)$$

Put $L = p^{2r-2}$ and $K = p^{r-2}$ in Theorem 3.1. Then there exists a unique quadratic n -derivation h from A into X such that

$$\|f(x) - h(x)\| \leq \varepsilon p^{2r} \|x\|^{2r} \quad (3.28)$$

for all $x \in A$. □

Similarly, we can prove the following result.

Corollary 3.3. Let $r < 2$ and let ε be δ be positive real numbers. Suppose that $f : A \rightarrow X$ is a mapping such that

$$\begin{aligned} & \|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \leq \varepsilon \max\{\|x\|^r, \|y\|^r\}, \\ & \left\| f\left(\prod_{i=1}^n x_i\right) - f(x_1)x_2^2 \cdots x_n^2 - x_1^2 f(x_2)x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f(x_n) \right\| \\ & \leq \delta \max\{\|x_1\|^r, \dots, \|x_n\|^r\} \end{aligned} \quad (3.29)$$

for all $x_1, \dots, x_n, x, y \in A$. Then there exists a unique quadratic n -derivation h from A into X such that

$$\|f(x) - h(x)\| \leq \varepsilon p^r \|x\|^r \quad (3.30)$$

for all $x \in A$.

Remark 3.4. We can use similar arguments to obtain corollaries like Corollaries 3.2 and 3.3, when $r > 1$ and $r > 2$.

By using the same technique of proving Theorem 3.1, we can prove the following result.

Remark 3.5. Let $\varphi : A \times A \rightarrow [0, \infty)$, $\psi : A \times \dots \times A \rightarrow [0, \infty)$ be functions. Let $f : A \rightarrow X$ be a given mapping such that $f(0) = 0$,

$$\|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \leq \varphi(x, y) \quad (3.31)$$

and that

$$\left\| f\left(\prod_{i=1}^n x_i\right) - f(x_1)x_2^2 \cdots x_n^2 - x_1^2 f(x_2)x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f(x_n) \right\| \leq \psi(x_1, \dots, x_n) \quad (3.32)$$

for all $x_1, \dots, x_n, x, y \in A$. Suppose that there exist a natural number $k \in \mathbb{K}$ and $L, K \in (0, 1)$, such that

$$\varphi(kx, y) \leq |k|^2 L \varphi(x, y), \quad \psi(kx_1, \dots, kx_n) \leq |k|^2 K \psi(x_1, \dots, x_n) \quad (3.33)$$

for all $x_1, \dots, x_n, x, y \in A$. Then there exists a unique quadratic n -derivation d from A into X such that

$$\|f(x) - d(x)\| \leq |k|^2 L \Phi(x) \quad (3.34)$$

for all $x \in A$, where

$$\Phi(x) = \max \left\{ \varphi(0, 0), \varphi(x, x), \varphi\left(\frac{x}{2}, x\right), \dots, \varphi\left(\frac{x}{(k-1)}, x\right) \right\} \quad (x \in A). \quad (3.35)$$

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References

- [1] S. M. Ulam, *Problems in Modern Mathematics*, chapter 6, John Wiley & Sons, New York, NY, USA, 2nd edition, 1964.
- [2] D. H. Hyers, "On the stability of the linear functional equation," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 27, pp. 222–224, 1941.
- [3] T. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proceedings of the American Mathematical Society*, vol. 72, no. 2, pp. 297–300, 1978.
- [4] P. Găvruta, "A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings," *Journal of Mathematical Analysis and Applications*, vol. 184, no. 3, pp. 431–436, 1994.
- [5] R. Badora, "Report of Meeting: The Thirty-fourth International Symposium on Functional Equations, June 10 to 19, 1996, Wisła-Jawornik, Poland," *Aequationes Mathematicae*, vol. 53, no. 1-2, pp. 162–205, 1997.
- [6] H. Khodaei and T. M. Rassias, "Approximately generalized additive functions in several variables," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 1, pp. 22–41, 2010.
- [7] J. Tabor, "Remark 20, In Report on the 34th ISFE," *Aequationes Mathematicae*, vol. 53, pp. 194–196, 1997.
- [8] J. Aczél and J. Dhombres, *Functional Equations in Several Variables*, vol. 31 of *Encyclopedia of Mathematics and its Applications*, Cambridge University Press, Cambridge, UK, 1989.
- [9] F. Skof, "Local properties and approximation of operators," *Rendiconti del Seminario Matematico e Fisico di Milano*, vol. 53, pp. 113–129, 1983.
- [10] P. W. Cholewa, "Remarks on the stability of functional equations," *Aequationes Mathematicae*, vol. 27, no. 1-2, pp. 76–86, 1984.
- [11] St. Czerwik, "On the stability of the quadratic mapping in normed spaces," *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, vol. 62, pp. 59–64, 1992.
- [12] M. Eshaghi Gordji and N. Ghobadipour, "Hyers-Ulam-Aoki-Rassias stability and Ulam-Gavruta-Rassias stability of quadratic homomorphisms and quadratic derivations on Banach Algebras," in *Functional Equations, Difference Inequalities*, vol. and Ulam Stability Notions (F.U.N.) of *Mathematics Research Developments*, NOVA Publishers, 2010.
- [13] M. Eshaghi Gordji, M. B. Ghaemi, and B. Alizadeh, "A fixed point method for perturbation of higher ring derivations in non-Archimedean Banach algebras," *International Journal of Geometric Methods in Modern Physics*, vol. 8, no. 7, pp. 1611–1625, 2011.
- [14] E. H. Lee, I.-S. Chang, and Y.-S. Jung, "On stability of the functional equations having relation with a multiplicative derivation," *Bulletin of the Korean Mathematical Society*, vol. 44, no. 1, pp. 185–194, 2007.
- [15] H.-M. Kim and I.-S. Chang, "Stability of the functional equations related to a multiplicative derivation," *Journal of Applied Mathematics & Computing A*, vol. 11, no. 1-2, pp. 413–421, 2003.
- [16] M. Eshaghi Gordji, "Nearly ring homomorphisms and nearly ring derivations on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2010, Article ID 393247, 12 pages, 2010.
- [17] M. Eshaghi Gordji and Z. Alizadeh, "Stability and superstability of ring homomorphisms on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2011, Article ID 123656, 10 pages, 2011.
- [18] M. Eshaghi Gordji, M. B. Ghaemi, and B. Alizadeh, "A fixed point approach to superstability of generalized derivations on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2011, Article ID 587097, 9 pages, 2011.
- [19] M. E. Gordji and H. Khodaei, *Stability of Functional Equations*, LAP LAMBERT Academic Publishing, 2010.
- [20] M. Eshaghi Gordji, H. Khodaei, and R. Khodabakhsh, "General quartic-cubic-quadratic functional equation in non-Archimedean normed spaces," *University of Bucharest: Scientific Bulletin A*, vol. 72, no. 3, pp. 69–84, 2010.
- [21] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of cubic and quartic functional equations in non-Archimedean spaces," *Acta Applicandae Mathematicae*, vol. 110, no. 3, pp. 1321–1329, 2010.
- [22] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of a mixed type cubic-quartic functional equation in non-Archimedean spaces," *Applied Mathematics Letters*, vol. 23, no. 10, pp. 1198–1202, 2010.
- [23] M. Eshaghi Gordji, M. B. Savadkouhi, and M. Bidkham, "Stability of a mixed type additive and quadratic functional equation in non-Archimedean spaces," *Journal of Computational Analysis and Applications*, vol. 12, no. 2, pp. 454–462, 2010.
- [24] J. M. Rassias, "On approximation of approximately linear mappings by linear mappings," *Journal of Functional Analysis*, vol. 46, no. 1, pp. 126–130, 1982.

- [25] J. M. Rassias, "On approximation of approximately linear mappings by linear mappings," *Bulletin des Sciences Mathématiques*, vol. 108, no. 4, pp. 445–446, 1984.
- [26] J. M. Rassias, "Solution of a problem of Ulam," *Journal of Approximation Theory*, vol. 57, no. 3, pp. 268–273, 1989.
- [27] G. Z. Eskandani, H. Vaezi, and Y. N. Dehghan, "Stability of a mixed additive and quadratic functional equation in non-Archimedean Banach modules," *Taiwanese Journal of Mathematics*, vol. 14, no. 4, pp. 1309–1324, 2010.
- [28] T. Z. Xu, J. M. Rassias, and W. X. Xu, "Stability of a general mixed additive-cubic functional equation in non-Archimedean fuzzy normed spaces," *Journal of Mathematical Physics*, vol. 51, no. 9, Article ID 093508, 19 pages, 2010.
- [29] T. Z. Xu, J. M. Rassias, and W. X. Xu, "Intuitionistic fuzzy stability of a general mixed additive-cubic equation," *Journal of Mathematical Physics*, vol. 51, no. 6, Article ID 063519, 21 pages, 2010.
- [30] T. Z. Xu, J. M. Rassias, and W. X. Xu, "On the stability of a general mixed additive-cubic functional equation in random normed spaces," *Journal of Inequalities and Applications*, vol. 2010, Article ID 328473, 16 pages, 2010.
- [31] T. Z. Xu, J. M. Rassias, and W. X. Xu, "A fixed point approach to the stability of a general mixed AQCC-functional equation in non-Archimedean normed spaces," *Discrete Dynamics in Nature and Society*, vol. 2010, Article ID 812545, 24 pages, 2010.
- [32] T. Z. Xu, "Stability of multi-Jensen mappings in non-Archimedean normed spaces," *Journal of Mathematical Physics*, vol. 53, Article ID 10.1063/1.3684746, 9 pages, 2012.
- [33] P. Găvruta and L. Găvruta, "A new method for the generalized Hyer-Ulam-Rassias stability," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 2, pp. 11–18, 2010.
- [34] L. M. Arriola and W. A. Beyer, "Stability of the Cauchy functional equation over p -adic fields," *Real Analysis Exchange*, vol. 31, no. 1, pp. 125–132, 2005.
- [35] K. Hensel, "Über eine neue Begründung der Theorie der algebraischen Zahlen," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 6, pp. 83–88, 1897.