Research Article

Existence of Periodic Solutions for Shunting Inhibitory Cellular Neural Networks with Neutral Delays

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This paper considers the existence of periodic solutions for shunting inhibitory cellular neural networks (SICNNs) with neutral delays. By applying the theory of abstract continuation theorem of k -set contractive operator and some analysis technique, a new result on the existence of periodic solutions is obtained.

1. Introduction

It is well known that SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. The dynamical behaviors of SICNNs with delays have been widely investigated in recent years. A large number of important results on the dynamical behaviors of SICNNs have been established and successfully applied to signal processing, pattern recognition, associative memories, and so on. In particular, there exist many results on the existence and stability of periodic and almost periodic solutions for SICNNs with delays. We refer the reader to [1–4] and references cited therein.

On the other hand, the stability analysis of various neutral delay-differential systems has drawn much research attention [5–7]. The theory of neutral delay-differential systems is of both theoretical and practical interest. For a large class of electrical networks containing lossless transmission lines, the describing equations can be reduced to neutral delay-differential equations; such networks arise in high speed computers where nearly lossless transmission lines are used to interconnect switching circuits. Also, the neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar.

Motivated by the above, we consider shunting inhibitory cellular neural networks with neutral delays described by

$$\begin{aligned} \dot{x}_{ij}(t) = & -a_{ij}(t)x_{ij}(t) \\ & - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t)f_{ij}(x_{kl}(t - \tau_{kl}(t)))x_{ij}(t) \\ & + b_{ij}(t)\dot{x}_{ij}(t - \sigma_{ij}(t)) + I_{ij}(t), \end{aligned} \quad (1)$$

where $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, C_{ij} is the cell at the (i, j) position of the lattice and the r -neighborhood $N_r(i, j)$ of C_{ij} is given as

$$\begin{aligned} N_r(i, j) = & \{C_{kl} : \max(|k - i|, |l - j|) \\ & \leq r, 1 \leq k \leq m, 1 \leq l \leq n\}. \end{aligned} \quad (2)$$

$x_{ij}(t)$ is the activity of the cell, C_{ij} , $a_{ij}(t) \geq 0$ is the passive decay rate of the cell activity, $I_{ij}(t)$ is the external input to C_{ij} , $C_{ij}^{kl} \geq 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij} , and the activity function $f_{ij}(x_{kl}(t))$ is a continuous function representing the output or firing rate of cell C_{kl} .

By using the Lyapunov functions and linear matrix inequality approach (LMI), most authors studied the asymptotic stability or exponential stability of the equilibrium point (see [7–10]). However, few papers have been published on the existence of periodic solutions or almost periodic solutions for neutral type neural networks ([11–14]). Thus, it is worthwhile to study the existence of periodic solutions for neutral type neural networks with delays. And to the author's best knowledge, there is no published paper considering the existence of periodic solutions for SICNNs with neutral delays.

The initial conditions of (1) are of the form

$$\begin{aligned} x_{ij}(t) &= \varphi_{ij}(s), \quad s \in (-\infty, 0], \\ i &= 1, \dots, m, \quad j = 1, \dots, n, \end{aligned} \quad (3)$$

where $\varphi_{ij} \in C((-\infty, 0], R)$.

For the sake of convenience, we denote

$$\begin{aligned} \Omega_1 &= \{1, 2, \dots, m\}, & \Omega_2 &= \{1, 2, \dots, n\}, \\ \bar{h} &= \max_{t \in [0, \omega]} |h(t)|, & \underline{h} &= \min_{t \in [0, \omega]} |h(t)|, \\ \tilde{h} &= \frac{1}{\omega} \int_0^\omega h(s) ds, \\ \|h\|_2 &= \left(\int_0^\omega |h(t)|^2 dt \right)^{1/2}, & \|\dot{h}\|_2 &= \left(\int_0^\omega |\dot{h}(t)|^2 dt \right)^{1/2}, \end{aligned} \quad (4)$$

where h is a continuous ω -periodic function.

The main purpose of this paper is to obtain some sufficient conditions for the existence of the periodic solutions for system (1). By applying the theory of abstract continuation theorem of k -set contractive operator and some analysis technique, we derive some new sufficient conditions ensuring the existence of periodic solutions for system (1). Moreover, numerical examples are provided to illustrate the feasibility of the new results.

2. Conditions and Lemma

Throughout this paper, for $i \in \Omega_1, j \in \Omega_2$, we assume that

- (H₁) $a_{ij}(t), C_{ij}^{kl}(t), b_{ij}(t), I_{ij}(t), \tau_{ij}(t)$ are continuous ω -periodic functions;
- (H₂) function $\sigma_{ij}(t)$ is nonnegative, bounded, ω -periodic, and continuously differential defined on R^+ and $\inf_{t \in R^+} \{1 - \dot{\sigma}_{ij}(t)\} > 0$, where $\dot{\sigma}_{ij}(t)$ expresses the derivative of $\sigma_{ij}(t)$;
- (H₃) there exists positive constant α_{ij} such that $|f_{ij}(u)| \leq \alpha_{ij}$, for all $u, v \in R$.

In order to study (1), we should make some preparations.

Let E be a Banach space. For a bounded subset $A \subset E$, let

$$\begin{aligned} \alpha_E(A) &= \inf \left\{ \delta > 0 : \text{there is a finite number of subsets} \right. \\ &\quad \left. A_i \subset A \text{ such that } A = \bigcup_i A_i \text{ and } \text{diam}(A_i) \leq \delta \right\} \end{aligned} \quad (5)$$

denote the (Kuratowski) measure of noncompactness, where $\text{diam}(A_i)$ denotes the diameter of the set A_i . Let X, Y be two Banach spaces and Ω be a bounded open subset of X . A continuous and bounded map $N : \bar{\Omega} \rightarrow Y$ is called k -set contractive if for any bounded set $S \subset \Omega$ we have $\alpha_Y(N(S)) \leq k\alpha_X(S)$, where k is a constant. In addition, for a Fredholm operator $L : X \rightarrow Y$ with index zero, according to [15], we define

$$\begin{aligned} l(L) &= \sup \{a \geq 0 : a\alpha_X(S) \leq \alpha_Y(L(S)), \\ &\quad \forall \text{ bounded subset } S \subset X\}. \end{aligned} \quad (6)$$

Lemma 1 (see [16]). *Let $L : X \rightarrow Y$ be a Fredholm operator with index zero, and let $r \in Y$ be a fixed point. Suppose that $N : \Omega \rightarrow Y$ is a k -set contractive with $k < l(L)$, where $\Omega \subset X$ is bounded, open, and symmetric about $0 \in \Omega$. Furthermore, we suppose that*

- (1) $Lx \neq \lambda Nx + \lambda r$ for $x \in \partial\Omega, \lambda \in (0, 1)$,
- (2) $[QN(x) + Qr, x] \cdot [QN(-x) + Qr, x] < 0$ for $x \in \partial\Omega \cap \text{Ker } L$, where $[\cdot, \cdot]$ is some bilinear form on $Y \times X$ and Q is the project of Y onto $\text{Coker}(L)$.

Then there is a $x \in \bar{\Omega}$ such that $Lx - Nx = r$.

In order to use Lemma 1 to establish the existence of periodic solutions of (1), we set

$$\begin{aligned} Y = C_\omega &= \{x \in C(R, R) : x(t + \omega) = x(t), \\ x &= (x_{11}, \dots, x_{1n}, \dots, x_{i1}, \dots, \\ &\quad x_{in}, \dots, x_{m1}, \dots, x_{mn})^T\} \end{aligned} \quad (7)$$

with the norm defined by $\|x\| = \sum_{i=1}^m \sum_{j=1}^n |x_{ij}|_0$, where $|x_{ij}|_0 = \max_{t \in [0, \omega]} |x_{ij}(t)|$ and

$$\begin{aligned} X &= C_\omega^1 \\ &= \{x \in C^1(R, R) : x(t + \omega) = x(t), \\ x &= (x_{11}, \dots, x_{1n}, \dots, x_{i1}, \dots, \\ &\quad x_{in}, \dots, x_{m1}, \dots, x_{mn})^T\} \end{aligned} \quad (8)$$

with the norm defined by $\|x\|_1 = \max\{\|x\|, \|\dot{x}\|\}$, where $\|\dot{x}\| = \sum_{i=1}^m \sum_{j=1}^n |\dot{x}_{ij}|_0$, $|\dot{x}_{ij}|_0 = \max_{t \in [0, \omega]} |\dot{x}_{ij}(t)|$. Then X, Y are Banach spaces.

Let $L : X \rightarrow Y$ defined by

$$Lx = \frac{dx}{dt} = (\dot{x}_{11}, \dots, \dot{x}_{1n}, \dots, \dot{x}_{i1}, \dots, \dot{x}_{in}, \dots, \dot{x}_{m1}, \dots, \dot{x}_{mn})^T \quad (9)$$

and $N : X \rightarrow Y$ defined by the following:

$$\begin{aligned} Nx_{ij} = & -a_{ij}(t) x_{ij}(t) \\ & - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) \\ & + b_{ij}(t) \dot{x}_{ij}(t - \sigma_{ij}(t)), \end{aligned} \quad (10)$$

for $i \in \Omega_1, j \in \Omega_2$.

It is easy to see that L is a Fredholm operator with index zero. Clearly, (1) has a ω -periodic solution if and only if $Lx = Nx + r$ for some $x \in X$, where

$$\begin{aligned} r := I(t) = & (I_{11}(t), \dots, I_{1n}(t), \dots, I_{i1}(t), \dots, \\ & I_{in}(t), \dots, I_{m1}(t), \dots, I_{mn}(t))^T. \end{aligned} \quad (11)$$

Lemma 2 (see [17]). *The differential operator L is a Fredholm operator with index zero and satisfies $l(L) \geq 1$.*

Lemma 3. *If $k = \max_{i \in \Omega_1, j \in \Omega_2} \{\bar{b}_{ij}\} < 1$, then $N : \bar{\Omega} \rightarrow Y$ is a k -set contractive map, where $\bar{\Omega} \subset X$.*

Proof. The proof of Lemma 3 is similar to Lemma 3 in [14] and will be omitted here. $\square$

Lemma 4 (see [13]). *Let $t_1, t_2 \in [0, \omega]$. If $x : [0, \omega] \rightarrow R$ is ω -periodic, then*

$$\begin{aligned} x(t) & \leq x(t_1) + \int_0^\omega |\dot{x}(s)| ds, \\ x(t) & \geq x(t_2) - \int_0^\omega |\dot{x}(s)| ds. \end{aligned} \quad (12)$$

Lemma 5 (see [18]). *Suppose that $\sigma \in X$ and $\dot{\sigma}(t) < 1$ for all $t \in [0, \omega]$, then the function $t - \sigma(t)$ has an inverse $\nu \in C(R, R)$ with $\nu(a + \omega) = \nu(a) + \omega$, for all $a \in R$.*

Throughout this paper, we assume that $\sigma_{ij} \in X$ and $\dot{\sigma}_{ij}(t) < 1$, $i \in \Omega_1, j \in \Omega_2$. So that $\sigma_{ij}(t)$ has a unique inverse, and we set $\nu_{ij}(t)$ to represent the inverse of function $t - \sigma_{ij}(t)$.

3. Main Results

Set

$$\begin{aligned} p_{ij}^2 &= \max_{t \in R} \left\{ \frac{1}{1 - \dot{\sigma}_{ij}(\nu_{ij}(t))} \right\}, \\ A &= \text{diag}(a_{11}, \dots, a_{1n}, \dots, a_{m1}, \dots, a_{mn}), \\ B &= \text{diag}(\bar{b}_{11}p_{11}, \dots, \bar{b}_{1n}p_{1n}, \dots, \bar{b}_{m1}p_{m1}, \dots, \bar{b}_{mn}p_{mn}), \\ D &= \text{diag}(1 - \bar{b}_{11}p_{11}, \dots, 1 - \bar{b}_{1n}p_{1n}, \dots, \\ & \quad 1 - \bar{b}_{m1}p_{m1}, \dots, 1 - \bar{b}_{mn}p_{mn}), \\ C &= \text{diag} \left(\sum_{C_{kl} \in N_r(i,j)} \bar{C}_{11}^{kl} \alpha_{11}, \dots, \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{1n}^{kl} \alpha_{1n}, \dots, \right. \\ & \quad \left. \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{m1}^{kl} \alpha_{m1}, \dots, \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{mn}^{kl} \alpha_{mn} \right), \\ I &= (\|I_{11}\|_2, \dots, \|I_{1n}\|_2, \dots, \|I_{m1}\|_2, \dots, \|I_{mn}\|_2)^T, \\ Z &= (\|x_{11}\|_2, \dots, \|x_{1n}\|_2, \dots, \|x_{m1}\|_2, \dots, \|x_{mn}\|_2)^T, \\ W &= (\|\dot{x}_{11}\|_2, \dots, \|\dot{x}_{1n}\|_2, \dots, \|\dot{x}_{m1}\|_2, \dots, \|\dot{x}_{mn}\|_2)^T. \end{aligned} \quad (13)$$

Theorem 6. *Suppose that (H_1) – (H_3) hold, furthermore, assume that*

$$(H_4) \min_{i \in \Omega_1, j \in \Omega_2} \{1 - \bar{b}_{ij}p_{ij}, 1 - \bar{b}_{ij}\} > 0 \text{ and } \max_{i \in \Omega_1, j \in \Omega_2} \{\bar{a}_{ij} - \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij}\} > 0,$$

$$(H_5) A - C - BD^{-1}(A + C) \text{ is nonsingular } M\text{-matrix, then system (1) has at least one } \omega\text{-periodic solution.}$$

Proof. We consider the operator equation $Lx = \lambda Nx + \lambda r$, $\lambda \in (0, 1)$, where L, N defined by (9) and (10), respectively. Corresponding to system (1), for $i \in \Omega_1, j \in \Omega_2$, we have

$$\begin{aligned} \dot{x}_{ij}(t) = & \lambda \left[-a_{ij}(t) x_{ij}(t) \right. \\ & - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) \\ & \left. + b_{ij}(t) \dot{x}_{ij}(t - \sigma_{ij}(t)) + I_{ij}(t) \right]. \end{aligned} \quad (14)$$

Suppose that $(x_{11}(t), \dots, x_{1n}(t), \dots, x_{m1}(t), \dots, x_{mn}(t))^T$ is a solution of system (14) for a parameter $\lambda \in (0, 1)$. Multiplying

both sides of system (14) by $x_{ij}(t)$ and integrating over $[0, \omega]$, from (H_3) and Cauchy-Schwarz inequality, we obtain

$$\begin{aligned}
& \underline{a}_{ij} \|x_{ij}\|_2^2 \\
& \leq \int_0^\omega a_{ij}(t) x_{ij}^2(t) dt \\
& = \int_0^\omega \left[- \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl}(t) f_{ij}(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) \right. \\
& \quad \left. + b_{ij}(t) \dot{x}_{ij}(t - \sigma_{ij}(t)) + I_{ij}(t) \right] x_{ij}(t) dt \\
& \leq \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \int_0^\omega x_{ij}^2(t) dt \\
& \quad + \bar{b}_{ij} \int_0^\omega |x_{ij}(t)| |\dot{x}_{ij}(t - \sigma_{ij}(t))| dt \\
& \quad + \int_0^\omega |x_{ij}(t)| |I_{ij}(t)| dt \\
& \leq \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \|x_{ij}\|_2^2 \\
& \quad + \bar{b}_{ij} \|x_{ij}\|_2 \left(\int_0^\omega |\dot{x}_{ij}(t - \sigma_{ij}(t))|^2 dt \right)^{1/2} \\
& \quad + \left(\int_0^\omega |I_{ij}(t)|^2 dt \right)^{1/2} \left(\int_0^\omega |x_{ij}(t)|^2 dt \right)^{1/2}, \tag{15}
\end{aligned}$$

and according to Lemma 5, we have

$$\begin{aligned}
& \left(\int_0^\omega |\dot{x}_{ij}(t - \sigma_{ij}(t))|^2 dt \right)^{1/2} \\
& = \left(\int_0^\omega \frac{1}{1 - \dot{\sigma}_{ij}(\nu_{ij}(s))} |\dot{x}_{ij}(s)|^2 ds \right)^{1/2} \tag{16} \\
& \leq p_{ij} \|\dot{x}_{ij}\|_2, \quad i \in \Omega_1, \quad j \in \Omega_2.
\end{aligned}$$

That is,

$$\begin{aligned}
\underline{a}_{ij} \|x_{ij}\|_2 & \leq \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \|x_{ij}\|_2 \\
& \quad + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 + \|I_{ij}\|_2, \quad i \in \Omega_1, \quad j \in \Omega_2. \tag{17}
\end{aligned}$$

From the above inequality, we obtain

$$\begin{aligned}
& \left(\underline{a}_{ij} - \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) \|x_{ij}\|_2 \leq \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 + \|I_{ij}\|_2, \tag{18} \\
& i \in \Omega_1, \quad j \in \Omega_2,
\end{aligned}$$

which gives

$$(A - C) Z \leq BW + I. \tag{19}$$

Multiplying both sides of (14) by $\dot{x}_{ij}(t)$ and integrating over $[0, \omega]$, for $i \in \Omega_1, j \in \Omega_2$, we have

$$\begin{aligned}
& \|\dot{x}_{ij}\|_2^2 \\
& = \lambda \int_0^\omega \dot{x}_{ij}(t) \left[- a_{ij}(t) x_{ij}(t) \right. \\
& \quad \left. - \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl}(t) f_{ij}(x_{kl}(t - \tau_{kl}(t))) \right. \\
& \quad \left. \times x_{ij}(t) + b_{ij}(t) \dot{x}_{ij} \right. \\
& \quad \left. \times (t - \sigma_{ij}(t)) + I_{ij}(t) \right] dt \\
& \leq \left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) \left(\int_0^\omega |\dot{x}_{ij}(t)|^2 dt \right)^{1/2} \\
& \quad \times \left(\int_0^\omega |x_{ij}(t)|^2 dt \right)^{1/2} + \bar{b}_{ij} \left(\int_0^\omega |\dot{x}_{ij}(t)|^2 dt \right)^{1/2} \\
& \quad \times \left(\int_0^\omega |\dot{x}_{ij}(t - \sigma_{ij}(t))|^2 dt \right)^{1/2} \\
& \quad + \left(\int_0^\omega |I_{ij}(t)|^2 dt \right)^{1/2} \left(\int_0^\omega |\dot{x}_{ij}(t)|^2 dt \right)^{1/2} \\
& \leq \left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) \|x_{ij}\|_2 \|\dot{x}_{ij}\|_2 \\
& \quad + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2^2 + \|\dot{x}_{ij}\|_2 \|I_{ij}\|_2. \tag{20}
\end{aligned}$$

Thus

$$\begin{aligned}
\|\dot{x}_{ij}\|_2 & \leq \left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) \|x_{ij}\|_2 \\
& \quad + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 + \|I_{ij}\|_2, \tag{21}
\end{aligned}$$

that is,

$$\begin{aligned}
(1 - \bar{b}_{ij} p_{ij}) \|\dot{x}_{ij}\|_2 & \leq \left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) \|x_{ij}\|_2 \\
& \quad + \|I_{ij}\|_2, \tag{22}
\end{aligned}$$

where $1 - \bar{b}_{ij}p_{ij} > 0$, $i \in \Omega_1$, $j \in \Omega_2$. Then we may rewrite (22) as

$$\begin{aligned} DW &\leq (A + C)Z + I \implies W \\ &\leq D^{-1}(A + C)Z + D^{-1}I. \end{aligned} \quad (23)$$

Substituting (23) into (19), we get

$$\begin{aligned} (A - C)Z &\leq B(D^{-1}(A + C)Z + D^{-1}I) + I \\ \implies (A - C - BD^{-1}(A + C))Z & \\ &\leq BD^{-1}I + I. \end{aligned} \quad (24)$$

By (H₅), we have $Z \leq H^+ \triangleq (h_{11}^+, \dots, h_{1n}^+, \dots, h_{m1}^+, \dots, h_{mm}^+)^T$. That is,

$$\|x_{ij}\|_2 \leq h_{ij}^+, \quad i \in \Omega_1, \quad j \in \Omega_2, \quad (25)$$

where $H^+ = (A - C - BD^{-1}(A + C))^{-1}(BD^{-1}I + I)$. Substituting (25) into (23), we have $W \leq G^+ \triangleq (g_{11}^+, \dots, g_{1n}^+, \dots, g_{m1}^+, \dots, g_{mm}^+)^T$. That is,

$$\|\dot{x}_{ij}\|_2 \leq g_{ij}^+, \quad i \in \Omega_1, \quad j \in \Omega_2, \quad (26)$$

where $G^+ = D^{-1}(A + C)H^+ + D^{-1}I$.

Integrating both sides of (14), from 0 to ω , we obtain

$$\begin{aligned} 0 &= - \int_0^\omega a_{ij}(t) x_{ij}(t) dt \\ &+ \int_0^\omega \left[- \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) \right. \\ &\quad \left. + b_{ij}(t) \dot{x}_{ij}(t - \sigma_{ij}(t)) + I_{ij}(t) \right] dt. \end{aligned} \quad (27)$$

Using Cauchy-Schwarz inequality, we have

$$\begin{aligned} &\left| \int_0^\omega a_{ij}(t) x_{ij}(t) dt \right| \\ &\leq \sum_{C_{kl} \in N_r(i,j)} \int_0^\omega \bar{C}_{ij}^{kl} \alpha_{ij} |x_{ij}(t)| dt \\ &\quad + \int_0^\omega \bar{b}_{ij} |\dot{x}_{ij}(t - \sigma_{ij}(t))| dt + \int_0^\omega |I_{ij}(t)| dt \\ &\leq \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \|x_{ij}\|_2 \sqrt{\omega} \\ &\quad + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 \sqrt{\omega} + \|I_{ij}\|_2 \sqrt{\omega}. \end{aligned} \quad (28)$$

In view of (14), we have

$$\begin{aligned} \int_0^\omega |\dot{x}_{ij}(t)| dt &\leq \int_0^\omega \bar{a}_{ij} |x_{ij}(t)| dt \\ &\quad + \sum_{C_{kl} \in N_r(i,j)} \int_0^\omega \bar{C}_{ij}^{kl} \alpha_{ij} |x_{ij}(t)| dt \\ &\quad + \int_0^\omega \bar{b}_{ij} |\dot{x}_{ij}(t - \sigma_{ij}(t))| dt + \int_0^\omega |I_{ij}(t)| dt \\ &\leq \bar{a}_{ij} \|x_{ij}\|_2 \sqrt{\omega} \\ &\quad + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \|x_{ij}\|_2 \sqrt{\omega} \\ &\quad + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 \sqrt{\omega} + \|I_{ij}\|_2 \sqrt{\omega}. \end{aligned} \quad (29)$$

From Lemma 4, for any $t_1^{ij}, t_2^{ij} \in [0, \omega]$, $i \in \Omega_1$, $j \in \Omega_2$, we have

$$\begin{aligned} \int_0^\omega a_{ij}(t) x_{ij}(t) dt &\leq \int_0^\omega a_{ij}(t) x_{ij}(t_1^{ij}) dt \\ &\quad + \int_0^\omega a_{ij}(t) \left(\int_0^\omega |\dot{x}_{ij}(t)| dt \right) dt, \\ \int_0^\omega a_{ij}(t) x_{ij}(t) dt &\geq \int_0^\omega a_{ij}(t) x_{ij}(t_2^{ij}) dt \\ &\quad - \int_0^\omega a_{ij}(t) \left(\int_0^\omega |\dot{x}_{ij}(t)| dt \right) dt. \end{aligned} \quad (30)$$

Dividing by $\int_0^\omega a_{ij}(t) dt$ on the two sides of the above inequalities, we obtain that

$$\begin{aligned} x_{ij}(t_1^{ij}) &\geq \frac{1}{\int_0^\omega a_{ij}(t) dt} \int_0^\omega a_{ij}(t) x_{ij}(t) dt - \int_0^\omega |\dot{x}_{ij}(t)| dt, \\ x_{ij}(t_2^{ij}) &\leq \frac{1}{\int_0^\omega a_{ij}(t) dt} \int_0^\omega a_{ij}(t) x_{ij}(t) dt + \int_0^\omega |\dot{x}_{ij}(t)| dt. \end{aligned} \quad (31)$$

Let $\bar{t}_{ij}, \underline{t}_{ij} \in [0, \omega]$, $i \in \Omega_1$, $j \in \Omega_2$, such that $x_{ij}(\bar{t}_{ij}) = \max_{t \in [0, \omega]} x_{ij}(t)$, $x_{ij}(\underline{t}_{ij}) = \min_{t \in [0, \omega]} x_{ij}(t)$, then by (25)–(31), we have

$$\begin{aligned} x_{ij}(\bar{t}_{ij}) &\leq \frac{1}{\bar{a}_{ij} \omega} \int_0^\omega |a_{ij}(t) x_{ij}(t)| dt + \int_0^\omega |\dot{x}_{ij}(t)| dt \\ &\leq \frac{1}{\bar{a}_{ij} \sqrt{\omega}} \left(\sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \|x_{ij}\|_2 \right. \\ &\quad \left. + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 + \|I_{ij}\|_2 \right) \end{aligned}$$

$$\begin{aligned}
& + \sqrt{\omega} \left[\left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) \|x_{ij}\|_2 \right. \\
& \quad \left. + \bar{b}_{ij} p_{ij} \|\dot{x}_{ij}\|_2 + \|I_{ij}\|_2 \right] \\
& \leq \frac{1}{a_{ij} \sqrt{\omega}} \left(\sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} h_{ij}^+ \right. \\
& \quad \left. + \bar{b}_{ij} p_{ij} g_{ij}^+ + \|I_{ij}\|_2 \right) \\
& + \sqrt{\omega} \left[\left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) h_{ij}^+ \right. \\
& \quad \left. + \bar{b}_{ij} p_{ij} g_{ij}^+ + \|I_{ij}\|_2 \right] \triangleq M_{ij}^{(1)}.
\end{aligned} \tag{32}$$

Similarly,

$$\begin{aligned}
x_{ij}(t_{ij}) & \geq - \left[\frac{1}{a_{ij} \omega} \int_0^\omega |a_{ij}(t) x_{ij}(t)| dt + \int_0^\omega |\dot{x}_{ij}(t)| dt \right] \\
& \geq - \left\{ \frac{1}{a_{ij} \sqrt{\omega}} \left(\sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} h_{ij}^+ \right. \right. \\
& \quad \left. \left. + \bar{b}_{ij} p_{ij} g_{ij}^+ + \|I_{ij}\|_2 \right) \right. \\
& \quad \left. + \sqrt{\omega} \left[\left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) h_{ij}^+ \right. \right. \\
& \quad \left. \left. + \bar{b}_{ij} p_{ij} g_{ij}^+ + \|I_{ij}\|_2 \right] \right\} = -M_{ij}^{(1)}.
\end{aligned} \tag{33}$$

Thus, for all $i \in \Omega_1, j \in \Omega_2$, $|x_{ij}(t)| \leq M_{ij}^{(1)}$, which yields that $|x_{ij}|_0 \leq M_{ij}^{(1)}$. Again from (14), we get

$$\begin{aligned}
|\dot{x}_{ij}|_0 & \leq \bar{a}_{ij} |x_{ij}|_0 + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} |x_{ij}|_0 + \bar{b}_{ij} |\dot{x}_{ij}|_0 + \bar{I}_{ij} \\
& \leq \left(\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij} \right) M_{ij}^{(1)} + \bar{b}_{ij} |\dot{x}_{ij}|_0 + \bar{I}_{ij},
\end{aligned} \tag{34}$$

which gives $|\dot{x}_{ij}|_0 \leq ((\bar{a}_{ij} + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij}) M_{ij}^{(1)} + \bar{I}_{ij}) / (1 - \bar{b}_{ij}) \triangleq M_{ij}^{(2)}$.

Denote $M^{(1)} = \sum_{i=1}^m \sum_{j=1}^n M_{ij}^{(1)}$, $M^{(2)} = \sum_{i=1}^m \sum_{j=1}^n M_{ij}^{(2)}$, $M^{(3)} = \max_{i \in \Omega_1, j \in \Omega_2} \{|\bar{I}_{ij}| / (\bar{a}_{ij} - \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij})\}$. Let $M > \max\{M^{(1)}, M^{(2)}, M^{(3)}\}$, $\Omega = \{x \in C_\omega^1 : \|x\|_1 < M\}$, and define a bounded bilinear form $[\cdot, \cdot]$ on $C_\omega \times C_\omega^1$ by $[y, x] = \int_0^\omega y(t)x(t)dt$. Also we define $Q : \gamma \rightarrow \text{Coker}(L)$ by $\gamma \rightarrow \int_0^\omega \gamma(t)dt$. Obviously, $\{x \mid x \in \text{Ker } L \cap \partial\Omega\} = \{x \mid x \equiv M \text{ or } x \equiv -M\}$. Without loss of generality, we may assume that $x \equiv M$. Let $P_{ij} = -\bar{a}_{ij}M - \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} f_{ij}(M)M + \bar{I}_{ij}$, $Q_{ij} = \bar{a}_{ij}M + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} f_{ij}(-M)M + \bar{I}_{ij}$, $i \in \Omega_1, j \in \Omega_2$. Thus

$$\begin{aligned}
& [QN(x) + Q(r), x] [QN(-x) + Q(r), x] \\
& = M^2 \omega^2 \cdot \begin{pmatrix} P_{11} Q_{11} \\ \vdots \\ P_{mn} Q_{mn} \end{pmatrix}.
\end{aligned} \tag{35}$$

Considering assumption (H_4) , we know $M > \max_{i \in \Omega_1, j \in \Omega_2} \{|\bar{I}_{ij}| / (\bar{a}_{ij} - \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij})\} > 0$, and it follows that $\bar{a}_{ij}M > \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \alpha_{ij}M + \bar{I}_{ij} > \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} |f_{ij}(M)|M + \bar{I}_{ij}$. So we have $[QN(x_{ij}) + Q(r_{ij}), x_{ij}] [QN(-x_{ij}) + Q(r_{ij}), x_{ij}] < 0$, for all $i \in \Omega_1, j \in \Omega_2$.

Therefore, by using Lemma 1, we obtain that (1) has at least one ω -periodic solution. The proof is complete. $\square$

4. Examples

In this section, we give two examples to demonstrate the results obtained in previous sections.

Consider the following shunting inhibitory cellular neural networks with neutral delays:

$$\begin{aligned}
\dot{x}_{ij}(t) & = -a_{ij}(t) x_{ij}(t) \\
& \quad - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) \\
& \quad + b_{ij}(t) \dot{x}_{ij}(t - \sigma_{ij}(t)) + I_{ij}(t),
\end{aligned} \tag{36}$$

where $i = 1, \dots, m, j = 1, \dots, n$.

Example 1. Let $m = n = 2$, $f_{ij}(u) = \tanh u = (e^u - e^{-u}) / (e^u + e^{-u})$. We take

$$\begin{aligned}
(a_{ij})_{3 \times 3} & = \begin{pmatrix} 4 + 0.2 \sin 3t & 5 + 0.1 \cos t \\ 6 + \sin \sqrt{2}t & 5 - \cos t \end{pmatrix}, \\
(C_{ij})_{3 \times 3} & = \begin{pmatrix} 0.3 - 0.1 \sin t & 1 + 0.2 \cos t \\ 0.6 |\sin t| & 0.3 - 0.2 \sin t \end{pmatrix},
\end{aligned}$$

$$\begin{aligned}
(b_{ij})_{3 \times 3} &= \begin{pmatrix} 0.03 \sin t & 0.1 \\ 0.15 & 0.1 \sin 2t \end{pmatrix}, \\
(I_{ij})_{3 \times 3} &= \begin{pmatrix} 1.6 & \frac{\cos t + \sin t}{3} \\ \tan \frac{t}{2} & 0.5 \end{pmatrix}, \\
(\tau_{ij})_{3 \times 3} &= \begin{pmatrix} \sin t \cos t & 0.25 \cos t \\ 7 \sin t & \sin 0.1t \end{pmatrix}, \\
(\sigma_{ij})_{3 \times 3} &= \begin{pmatrix} 1 + \sin 0.5t & 0.8 \cos t \\ 0.3 \sin t & \sin 0.2t \end{pmatrix}.
\end{aligned} \tag{37}$$

Obviously, (H_1) – (H_3) hold and

$$\begin{aligned}
\sum_{C_{kl} \in N_1(1,1)} \overline{C_{11}^{kl}} &= \sum_{C_{kl} \in N_1(1,2)} \overline{C_{12}^{kl}} \\
&= \sum_{C_{kl} \in N_1(2,1)} \overline{C_{21}^{kl}} \\
&= \sum_{C_{kl} \in N_1(2,2)} \overline{C_{22}^{kl}} = 2.7,
\end{aligned} \tag{38}$$

$p_{11} = 1.414$, $p_{12} = 2.236$, $p_{21} = 1.195$, and $p_{22} = 1.118$.
Let $\alpha_{ij} = 1$, we have $A = \text{diag}(3.8, 4.9, 5, 4)$,

$$B = \text{diag}(0.042, 0.224, 0.179, 0.112),$$

$$C = \text{diag}(2.7, 2.7, 2.7, 2.7),$$

$$D = \text{diag}(0.958, 0.776, 0.821, 0.888),$$

$$A - C - BD^{-1}(A + C) = \text{diag}(0.815, 0.006, 0.621, 0.455). \tag{39}$$

It is easy to see that (H_4) and (H_5) hold. Therefore, all conditions of Theorem 6 are satisfied, then system (36) has at least one 2π -periodic solution.

Example 2. Let $m = n = 3$, $f_{ij}(u) = (1/2) \sin u$. We take

$$\begin{aligned}
(a_{ij})_{3 \times 3} &= \begin{pmatrix} 4 + \sin 3t & 5 + \cos t & 6 - \sin t \\ 5 + \sin 3t & 6 + \sin 2t & 4 + \sin t \\ 6 + \cos 5t & 4 + \sin t & 5 + \cos t \end{pmatrix}, \\
(C_{ij})_{3 \times 3} &= \begin{pmatrix} 0.1 |\cos 3t| & 0.2 |\cos t| & 0.1 |\sin t| \\ 0.2 |\sin 2t| & 0.3 |\sin t| & 0.4 |\cos t| \\ 0.1 |\cos 5t| & 0.1 |\sin t| & 0.2 |\sin t| \end{pmatrix}, \\
(b_{ij})_{3 \times 3} &= \begin{pmatrix} 0.01 \sin t & 0.02 \cos 5t & 0.01 \sin 2t \\ 0.02 \sin 3t & 0.03 \sin 2t & 0.01 \sin 5t \\ 0.03 \cos 3t & 0.01 \sin t & 0.02 \cos 3t \end{pmatrix},
\end{aligned}$$

$$\begin{aligned}
(I_{ij})_{3 \times 3} &= \begin{pmatrix} 2 + \sin t & \cos t & \sin t \\ \sin t & \cos t & 1 + \cos t \\ \cos t & \sin 2t & \sin t + \cos t \end{pmatrix}, \\
(\tau_{ij})_{3 \times 3} &= \begin{pmatrix} 0.1 \sin t & 0.5 \cos t & 0.6 \sin t \\ 0.8 \sin t & 0.1 \sin t & 0.4 \sin t \\ 0.5 \cos t & 0.15 \sin t & 0.55 \cos t \end{pmatrix}, \\
(\sigma_{ij})_{3 \times 3} &= \begin{pmatrix} 0.6 \cos t & 0.2 \cos t & 0.15 \sin t \\ 0.2 \sin t & 0.75 \sin t & 0.4 \cos 2t \\ 0.8 \cos t & 0.9 \sin t & 0.75 \sin t \end{pmatrix}.
\end{aligned} \tag{40}$$

Obviously, (H_1) – (H_3) hold and

$$\begin{aligned}
\sum_{C_{kl} \in N_1(1,1)} \overline{C_{11}^{kl}} &= 0.8, & \sum_{C_{kl} \in N_1(1,2)} \overline{C_{12}^{kl}} &= 1.3, \\
\sum_{C_{kl} \in N_1(1,3)} \overline{C_{13}^{kl}} &= 1, \\
\sum_{C_{kl} \in N_1(2,1)} \overline{C_{21}^{kl}} &= 1, & \sum_{C_{kl} \in N_1(2,2)} \overline{C_{22}^{kl}} &= 1.7, \\
\sum_{C_{kl} \in N_1(2,3)} \overline{C_{23}^{kl}} &= 1.3, \\
\sum_{C_{kl} \in N_1(3,1)} \overline{C_{31}^{kl}} &= 0.7, & \sum_{C_{kl} \in N_1(3,2)} \overline{C_{32}^{kl}} &= 1.3, \\
\sum_{C_{kl} \in N_1(3,3)} \overline{C_{33}^{kl}} &= 1,
\end{aligned} \tag{41}$$

$p_{11} = 1.581$, $p_{12} = 1.118$, $p_{13} = 1.085$, $p_{21} = 1.118$, $p_{22} = 2$, $p_{23} = 2.236$, $p_{31} = 2.236$, $p_{32} = 3.162$, and $p_{33} = 2$.

Let $\alpha_{ij} = 1/2$, we have $A = \text{diag}(3, 4, 5, 4, 5, 3, 5, 3, 4)$,

$$\begin{aligned}
B &= \text{diag}(0.016, 0.022, 0.011, 0.022, 0.06, \\
&\quad 0.022, 0.067, 0.032, 0.04),
\end{aligned}$$

$$C = \text{diag}(0.4, 0.65, 0.5, 0.5, 0.85, 0.65, 0.35, 0.65, 0.5),$$

$$\begin{aligned}
D &= \text{diag}(0.984, 0.978, 0.989, 0.978, 0.94, \\
&\quad 0.978, 0.933, 0.968, 0.96),
\end{aligned} \tag{42}$$

$$\begin{aligned}
A - C - BD^{-1}(A + C) \\
&= \text{diag}(2.545, 3.245, 4.439, 3.399, 3.777, \\
&\quad 2.268, 4.266, 2.229, 3.313).
\end{aligned}$$

It is easy to see that (H_4) and (H_5) hold. Therefore, all conditions of Theorem 6 are satisfied, then system (36) has at least one 2π -periodic solution.

5. Conclusion

In this paper, a class of neutral type shunting inhibitory cellular neural networks is investigated by means of an abstract

continuous theorem of k -set contractive operator and some analysis technique. Sufficient conditions are derived to ensure the existence of periodic solutions. To the best of our knowledge, the results presented here have not appeared in the related literature. Also, illustrative examples are given to demonstrate the effectiveness of the results obtained.

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