Research Article

Dynamics of a System of Rational Higher-Order Difference Equation

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We focus on a system of a rational m -order difference equation $x_{n+1} = (x_{n-m+1})/(A + y_n y_{n-1} \cdots y_{n-m+1})$, $y_{n+1} = (y_{n-m+1})/(B + x_n x_{n-1} \cdots x_{n-m+1})$, $n = 0, 1, \dots$, where $A, B, x_0, x_{-1}, \dots, x_{-m+1}, y_0, y_{-1}, \dots, y_{-m+1} \in (0, \infty)$. We investigate the dynamical behavior of positive solution for the system.

1. Introduction

In 2011, Kurbanli et al. [1] studied the behavior of positive solutions of the system of rational difference equations

$$\begin{aligned}x_{n+1} &= \frac{x_{n-1}}{y_n x_{n-1} + 1}, \\ y_{n+1} &= \frac{y_{n-1}}{x_n y_{n-1} + 1},\end{aligned}\quad (1)$$

where the initial conditions are arbitrary nonnegative real numbers.

In the same year, Kurbanli [2] studied the behavior of solutions of the system of rational difference equations

$$\begin{aligned}x_{n+1} &= \frac{x_{n-1}}{y_n x_{n-1} - 1}, \\ y_{n+1} &= \frac{y_{n-1}}{x_n y_{n-1} - 1}, \\ z_{n+1} &= \frac{z_{n-1}}{y_n z_{n-1} - 1},\end{aligned}\quad (2)$$

where the initial conditions are arbitrary real numbers. Moreover, Kurbanli [3] studied the behavior of the solutions of the difference equation system

$$\begin{aligned}x_{n+1} &= \frac{x_{n-1}}{y_n x_{n-1} - 1}, \\ y_{n+1} &= \frac{y_{n-1}}{x_n y_{n-1} - 1}, \\ z_{n+1} &= \frac{1}{y_n z_n},\end{aligned}\quad (3)$$

where $x_0, x_{-1}, y_0, y_{-1}, z_0, z_{-1} \in \mathbb{R}$ such that $y_0 x_{-1} \neq 1$, $x_0 y_{-1} \neq 1$ and $y_0 z_0 \neq 1$.

In [4], Liu et al. gave more results of the solution of the system (2) including a new and simple expression of z_n and the asymptotical behavior of the solution.

In [5], Stević showed that the system of difference equations

$$x_{n+1} = \frac{ax_{n-1}}{by_n x_{n-1} + c}, \quad y_{n+1} = \frac{\alpha y_{n-1}}{\beta x_n y_{n-1} + \gamma}, \quad n = 0, 1, \dots, \quad (4)$$

can be solved.

In 2012, Gu and Ding [6] derived two canonical state space forms from multiple-input multiple-output systems described by difference equations.

The system of two nonlinear difference equations

$$\begin{aligned} x_{n+1} &= A + \frac{y_n}{x_{n-p}}, & y_{n+1} &= A + \frac{x_n}{y_{n-q}}, \\ n &= 0, 1, \dots, \end{aligned} \quad (5)$$

was studied by Papaschinopoulos and Schinas [7], where $p, q \in \mathbb{N}$.

Moreover, the system of rational difference equations

$$\begin{aligned} x_{n+1} &= \frac{x_n}{a + cy_n}, & y_{n+1} &= \frac{y_n}{b + dx_n}, \\ n &= 0, 1, \dots, \end{aligned} \quad (6)$$

was studied by Clark et al. [8, 9], where $a, b, c, d \in (0, \infty)$ and $x_0, y_0 \in [0, \infty)$.

Liu et al. [10] studied the behavior of a system of rational difference equations

$$\begin{aligned} x_{n+1} &= \frac{x_{n-1}}{y_n x_{n-1} - 1}, & y_{n+1} &= \frac{y_{n-1}}{x_n y_{n-1} - 1}, \\ z_{n+1} &= \frac{1}{x_n z_{n-1}}, & n &= 0, 1, \dots, \end{aligned} \quad (7)$$

where the initial conditions are nonzero real numbers.

In 2012, Zhang et al. [11] studied the solutions, stability character, and asymptotic behavior of the system of a rational third-order difference equation

$$\begin{aligned} x_{n+1} &= \frac{x_{n-2}}{A + y_n y_{n-1} y_{n-2}}, & y_{n+1} &= \frac{y_{n-2}}{B + x_n x_{n-1} x_{n-2}}, \\ n &= 0, 1, \dots, \end{aligned} \quad (8)$$

where $A, B, x_0, x_{-1}, x_{-2}, y_0, y_{-1}, y_{-2} \in (0, \infty)$.

In this paper, we studied the solutions, stability character, and asymptotic behavior of the system of a rational m -order difference equation

$$\begin{aligned} x_{n+1} &= \frac{x_{n-m+1}}{A + y_n y_{n-1} \cdots y_{n-m+1}}, \\ y_{n+1} &= \frac{y_{n-m+1}}{B + x_n x_{n-1} \cdots x_{n-m+1}}, & n &= 0, 1, \dots, \end{aligned} \quad (9)$$

where $A, B, x_0, x_{-1}, \dots, x_{-m+1}, y_0, y_{-1}, \dots, y_{-m+1} \in (0, \infty)$.

2. Preliminaries

Let $m \in \mathbb{N}$ and let $f : I_x^m \times I_y^m \rightarrow I_x$ and $g : I_x^m \times I_y^m \rightarrow I_y$ be continuously differentiable functions, where I_x and I_y are intervals in $\mathbb{R}$.

For any $(x_0, y_0), (x_{-1}, y_{-1}), \dots, (x_{-m+1}, y_{-m+1}) \in I_x \times I_y$, the system of difference equations

$$\begin{aligned} x_{n+1} &= f(x_n, x_{n-1}, \dots, x_{n-m+1}, y_n, y_{n-1}, \dots, y_{n-m+1}), \\ y_{n+1} &= g(x_n, x_{n-1}, \dots, x_{n-m+1}, y_n, y_{n-1}, \dots, y_{n-m+1}), \\ n &= 0, 1, \dots, \end{aligned} \quad (10)$$

has a unique solution $\{(x_n, y_n)\}_{n=-m+1}^{\infty}$.

Definition 1. A point $(\bar{x}, \bar{y}) \in I_x \times I_y$ is called an equilibrium point of the system (10) if $\bar{x} = f(\bar{x}, \bar{x}, \dots, \bar{x}, \bar{y}, \bar{y}, \dots, \bar{y})$ and $\bar{y} = g(\bar{x}, \bar{x}, \dots, \bar{x}, \bar{y}, \bar{y}, \dots, \bar{y})$.

Definition 2. The linearized system of the system (10) about the equilibrium $(\bar{x}, \bar{y})$ is the system of linear difference equations

$$\begin{aligned} x_{n+1} &= \sum_{i=0}^{m-1} \left(\frac{\partial f(\bar{x}, \bar{x}, \dots, \bar{x}, \bar{y}, \bar{y}, \dots, \bar{y})}{\partial x_{n-i}} x_{n-i} \right. \\ &\quad \left. + \frac{\partial f(\bar{x}, \bar{x}, \dots, \bar{x}, \bar{y}, \bar{y}, \dots, \bar{y})}{\partial y_{n-i}} y_{n-i} \right), \\ y_{n+1} &= \sum_{i=0}^{m-1} \left(\frac{\partial g(\bar{x}, \bar{x}, \dots, \bar{x}, \bar{y}, \bar{y}, \dots, \bar{y})}{\partial x_{n-i}} x_{n-i} \right. \\ &\quad \left. + \frac{\partial g(\bar{x}, \bar{x}, \dots, \bar{x}, \bar{y}, \bar{y}, \dots, \bar{y})}{\partial y_{n-i}} y_{n-i} \right). \end{aligned} \quad (11)$$

Definition 3. An equilibrium point $(\bar{x}, \bar{y})$ of the system (10) is said to be stable relative to $I_x \times I_y$ if for every $\epsilon > 0$, there exists $\delta > 0$ such that for any $(x_0, y_0), (x_{-1}, y_{-1}), \dots, (x_{-m+1}, y_{-m+1}) \in I_x \times I_y$, with

$$\max \left\{ \sum_{i=-m+1}^0 |x_i - \bar{x}|, \sum_{i=-m+1}^0 |y_i - \bar{y}| \right\} < \delta. \quad (12)$$

One has $\max\{|x_n - \bar{x}|, |y_n - \bar{y}|\} < \epsilon$ for all $n \geq -m + 1$.

Definition 4. An equilibrium point $(\bar{x}, \bar{y})$ of the system (10) is called an attractor relative to $I_x \times I_y$ if for all $(x_0, y_0), (x_{-1}, y_{-1}), \dots, (x_{-m+1}, y_{-m+1}) \in I_x \times I_y$, one has $\lim_{n \rightarrow \infty} x_n = \bar{x}$ and $\lim_{n \rightarrow \infty} y_n = \bar{y}$.

Definition 5. An equilibrium point $(\bar{x}, \bar{y})$ of the system (10) is said to be asymptotically stable relative to $I_x \times I_y$ if it is stable, and it is also an attractor.

Definition 6. An equilibrium point $(\bar{x}, \bar{y})$ of the system (10) is said to be unstable if it is not stable.

Theorem 7 (see [12]). Let $X(n+1) = F(X(n))$, $n = 0, 1, \dots$, be a system of difference equations and let $\bar{X}$ be the equilibrium point of the system. If all eigenvalues of the Jacobian matrix evaluated at $\bar{X}$ lie inside the open unit disk, then $\bar{X}$ is asymptotically stable. If one of them has a modulus greater than one, then $\bar{X}$ is unstable.

Theorem 8 (see [13]). Let $X(n+1) = F(X(n))$, $n = 0, 1, \dots$, be a system of difference equations and let $\bar{X}$ be the equilibrium point of the system. Assume that the characteristic polynomial of the system about $\bar{X}$ is $a_0 \lambda^n + a_1 \lambda^{n-1} + \dots + a_{n-1} \lambda + a_n$ where $a_i \in \mathbb{R}$ for all i and $a_0 > 0$. Then all roots of the characteristic equation lie inside the open unit disk if and only if $\Delta_k > 0$ for

all positive integer $k \leq n$, where Δ_k is the principal minor of order k of the $n \times n$ matrix

$$\Delta_n = \begin{pmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{pmatrix}. \quad (13)$$

3. Results

We note that

- (i) if $A < 1$ and $B < 1$ then the system (9) has equilibrium $(0, 0)$ and $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$;
- (ii) if $A = 1$ and $B < 1$ then the system (9) has equilibrium $(0, 0)$ and $(\sqrt[m]{1-B}, 0)$;
- (iii) if $A < 1$ and $B = 1$ then the system (9) has equilibrium $(0, 0)$ and $(0, \sqrt[m]{1-A})$;
- (iv) if $A > 1$ and $B > 1$ then $(0, 0)$ is the unique equilibrium point of the system (9).

Theorem 9. Let (x_n, y_n) be positive solution of the system (9). For all nonnegative integer k , one has

$$0 \leq x_n \leq \begin{cases} \frac{x_{-m+1}}{A^{k+1}}, & n = mk + 1; \\ \frac{x_{-m+2}}{A^{k+1}}, & n = mk + 2; \\ \vdots \\ \frac{x_0}{A^{k+1}}, & n = mk + m, \end{cases} \quad (14)$$

$$0 \leq y_n \leq \begin{cases} \frac{y_{-m+1}}{B^{k+1}}, & n = mk + 1; \\ \frac{y_{-m+2}}{B^{k+1}}, & n = mk + 2; \\ \vdots \\ \frac{y_0}{B^{k+1}}, & n = mk + m. \end{cases}$$

Proof. Obviously, they are true for $k = 0$. Suppose that they are true for $k = l$. Then

$$x_n = \begin{cases} x_{m(l+1)+1} \leq \frac{x_{m(l+1)-m+1}}{A} = \frac{1}{A} x_{ml+1} \leq \frac{1}{A} \left(\frac{x_{-m+1}}{A^{l+1}} \right), & n = m(l+1) + 1; \\ x_{m(l+1)+2} \leq \frac{x_{m(l+1)-m+2}}{A} = \frac{1}{A} x_{ml+2} \leq \frac{1}{A} \left(\frac{x_{-m+2}}{A^{l+1}} \right), & n = m(l+1) + 2; \\ \vdots \\ x_{m(l+1)+m} \leq \frac{x_{m(l+1)}}{A} = \frac{1}{A} x_{ml+m} \leq \frac{1}{A} \left(\frac{x_0}{A^{l+1}} \right), & n = m(l+1) + m, \end{cases} \quad (15)$$

$$y_n = \begin{cases} y_{m(l+1)+1} \leq \frac{y_{m(l+1)-m+1}}{B} = \frac{1}{B} y_{ml+1} \leq \frac{1}{B} \left(\frac{y_{-m+1}}{B^{l+1}} \right), & n = m(l+1) + 1; \\ y_{m(l+1)+2} \leq \frac{y_{m(l+1)-m+2}}{B} = \frac{1}{B} y_{ml+2} \leq \frac{1}{B} \left(\frac{y_{-m+2}}{B^{l+1}} \right), & n = m(l+1) + 2; \\ \vdots \\ y_{m(l+1)+m} \leq \frac{y_{m(l+1)}}{B} = \frac{1}{B} y_{ml+m} \leq \frac{1}{B} \left(\frac{y_0}{B^{l+1}} \right), & n = m(l+1) + m. \end{cases}$$

Thus, they are true for $k = l + 1$.

By the mathematical induction, this proof is completed. $\square$

Corollary 10. Let (x_n, y_n) be positive solution of the system (9). If $A > 1$ and $B > 1$, then the sequence $\{(x_n, y_n)\}$ converges exponentially to the equilibrium point $(0, 0)$.

Theorem 11. Let $A > 1$ and $B > 1$. Then the equilibrium point $(0, 0)$ of the system (9) is asymptotically stable.

Proof. The linearized system of the system (9) about the equilibrium $(0, 0)$ is

$$\Phi_{n+1} = D\Phi_n, \quad (16)$$

where

$$\Phi_n = \begin{pmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ \vdots \\ x_{n-m+1} \\ y_n \\ y_{n-1} \\ y_{n-2} \\ \vdots \\ y_{n-m+1} \end{pmatrix},$$

$$D = \begin{pmatrix} 0 & 0 & \cdots & 0 & \frac{1}{A} & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{B} \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (17)$$

The characteristic equation of the system (16) is

$$\left(\lambda^m - \frac{1}{A}\right)\left(\lambda^m - \frac{1}{B}\right) = 0. \quad (18)$$

Thus, $|\lambda| < 1$. By Theorem 7, the equilibrium point $(0, 0)$ is asymptotically stable. $\square$

Theorem 12. *Let $A < 1$ and $B < 1$. Then both the equilibrium points $(0, 0)$ and $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$ of the system (9) are unstable.*

Proof. We note by the characteristic equation (18) that $|\lambda| > 1$ and then, by Theorem 7, the equilibrium point $(0, 0)$ is unstable.

Next, we consider the equilibrium point $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$. The linearized system of the system (9) about the equilibrium $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$ is

$$\Phi_{n+1} = G\Phi_n, \quad (19)$$

where

$$\Phi_n = \begin{pmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ \vdots \\ x_{n-m+1} \\ y_n \\ y_{n-1} \\ y_{n-2} \\ \vdots \\ y_{n-m+1} \end{pmatrix},$$

$$G = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 & \alpha & \alpha & \cdots & \alpha & \alpha \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & \beta & \cdots & \beta & \beta & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad (20)$$

in which

$$\alpha = -\sqrt[m]{(1-A)^{m-1}(1-B)}, \quad (21)$$

$$\beta = -\sqrt[m]{(1-A)(1-B)^{m-1}}.$$

The characteristic polynomial of the system (19) is

$$1 - \alpha\beta - \sum_{i=1}^m (i\alpha\beta\lambda^{i-1}) - 2\lambda^m - \sum_{i=1}^{m-1} (i\alpha\beta\lambda^{2m-(i+1)}) + \lambda^{2m}. \quad (22)$$

We note the characteristic polynomial $a_0\lambda^{2m} + a_1\lambda^{2m-1} + \cdots + a_{2m-1}\lambda + a_{2m}$ that $a_1 = 0$. Thus, we obtain that not all of $\Delta_k > 0$, $k = 1, 2, \dots, 2m$. By Theorems 7 and 8, the equilibrium point $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$ is unstable. $\square$

Theorem 13. *Let $A, B < 1$ and $\Omega_1 = (0, \sqrt[m]{1-B}) \times (0, \sqrt[m]{1-A})$, $\Omega_2 = (\sqrt[m]{1-B}, \infty) \times (0, \sqrt[m]{1-A})$. Assume that $\{(x_n, y_n)\}_{n=-m+1}^\infty$ satisfies the system (9). Then*

(i) *if $\{(x_n, y_n)\}_{n=-m+1}^0 \subseteq \Omega_1$, then $\{(x_n, y_n)\}_{n=-m+1}^\infty \subseteq \Omega_1$;*

(ii) *if $\{(x_n, y_n)\}_{n=-m+1}^0 \subseteq \Omega_2$, then $\{(x_n, y_n)\}_{n=-m+1}^\infty \subseteq \Omega_2$.*

Proof. (i) Assume that $\{(x_n, y_n)\}_{n=-m+1}^0 \subseteq \Omega_1$. Then, for any $i \in \{0, 1, \dots, m-1\}$,

$$x_{i+1} = \frac{x_{i-m+1}}{A + y_i y_{i-1} \cdots y_{i-m+1}} < \frac{x_{i-m+1}}{A + (\sqrt[m]{1-A})^m} = x_{i-m+1},$$

$$y_{i+1} = \frac{y_{i-m+1}}{B + x_i x_{i-1} \cdots x_{i-m+1}} > \frac{y_{i-m+1}}{B + (\sqrt[m]{1-B})^m} = y_{i-m+1}. \quad (23)$$

Then $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m) \in \Omega_1$.

Next, we suppose that $(x_k, y_k), (x_{k-1}, y_{k-1}), \dots, (x_{k-m+1}, y_{k-m+1}) \in \Omega_1$ where k is a positive integer. Then

$$\begin{aligned} x_{k+1} &= \frac{x_{k-m+1}}{A + y_k y_{k-1} \cdots y_{k-m+1}} < \frac{x_{k-m+1}}{A + \left(\sqrt[m]{1-A}\right)^m} = x_{k-m+1}, \\ y_{k+1} &= \frac{y_{k-m+1}}{B + x_k x_{k-1} \cdots x_{k-m+1}} > \frac{y_{k-m+1}}{B + \left(\sqrt[m]{1-B}\right)^m} = y_{k-m+1}. \end{aligned} \quad (24)$$

Then $(x_{k+1}, y_{k+1}) \in \Omega_1$.

By the mathematical induction, $\{(x_n, y_n)\}_{n=-m+1}^{\infty} \subseteq \Omega_1$.

(ii) This is similar to the proof of (i). $\square$

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