Research Article

Permanence for a Discrete Ratio-Dependent Predator-Prey System with Holling Type III Functional Response and Feedback Controls

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A new set of sufficient conditions for the permanence of a ratio-dependent predator-prey system with Holling type III functional response and feedback controls are obtained. The result shows that feedback control variables have no influence on the persistent property of the system, thus improving and supplementing the main result of Yang (2008).

1. Introduction

The aim of this paper is to investigate the permanent property of the following discrete ratio-dependent predator-prey system with Holling type III and feedback controls:

$$\begin{aligned} x(n+1) &= x(n) \exp \left\{ a(n) - b(n)x(n) - \frac{c(n)x(n)y(n)}{x^2(n) + m^2(n)y^2(n)} - e_1(n)u_1(n) \right\}, \\ y(n+1) &= y(n) \exp \left\{ -d(n) + \frac{f(n)x^2(n)}{x^2(n) + m^2(n)y^2(n)} - e_2(n)u_2(n) \right\}, \\ \Delta u_1(n) &= -\eta_1(n)u_1(n) + q_1(n)x(n), \\ \Delta u_2(n) &= -\eta_2(n)u_2(n) + q_2(n)y(n), \end{aligned} \quad (1)$$

where $x(t)$ and $y(t)$ are the densities of the prey population and predator population at time t , respectively, for $i = 1, 2$, $\{a(n)\}$, $\{b(n)\}$, $\{c(n)\}$, $\{d(n)\}$, $\{f(n)\}$, $\{m(n)\}$, $\{e_i(n)\}$, $\{\eta_i(n)\}$, and $q_i(n)$ are all bounded nonnegative sequences such that

$$\begin{aligned} 0 < a^L \leq a(n) \leq a^U, \quad 0 < b^L \leq b(n) \leq b^U, \\ 0 < c^L \leq c(n) \leq c^U, \\ 0 < d^L \leq d(n) \leq d^U, \quad 0 < f^L \leq f(n) \leq f^U, \\ 0 < m^L \leq m(n) \leq m^U, \\ 0 < e_i^L \leq e_i(n) \leq e_i^U, \quad 0 < \eta_i^L \leq \eta_i(n) \leq \eta_i^U < 1, \\ 0 < q_i^L \leq q_i(n) \leq q_i^U. \end{aligned} \quad (2)$$

Here, for any bounded sequence $\{a(n)\}$, $a^L = \inf_{n \in \mathbb{N}} \{a(n)\}$, $a^U = \sup_{n \in \mathbb{N}} \{a(n)\}$.

By the biological meaning, we will focus our discussion on the positive solutions of system (1). So, we consider (1) together with the following initial conditions:

$$\begin{aligned} x(0) &> 0, \quad y(0) > 0, \\ u_i(0) &> 0, \quad i = 1, 2. \end{aligned} \quad (3)$$

It is not difficult to see that the solutions of (1)–(3) are well defined and satisfy

$$\begin{aligned} x(n) > 0, \quad y(n) > 0, \quad u_i(n) > 0, \\ i = 1, 2, \quad \text{for } n \in N. \end{aligned} \quad (4)$$

Recently, Yang [1] proposed and studied the permanence of system (1). Set

$$\begin{aligned} M_1 &= \frac{\exp\{a^U - 1\}}{b^L}, \quad M_2 = \frac{f^U M_1 \exp\{f^U - d^L\}}{2d^L m^L}, \\ W_1 &= \frac{q_1^U M_1}{\eta_1^L}, \quad W_2 = \frac{q_2^U M_2}{\eta_2^L}. \end{aligned} \quad (5)$$

Using the comparison theorem of difference equation, Yang obtained the following result.

Theorem A (see [1]). *Assume that*

$$a^L - \frac{c^U}{2m^L} - e_1^U W_1 > 0, \quad (A_1)$$

$$f^L - d^U - e_2^U W_2 > 0, \quad (A_2)$$

hold; then system (1) is permanent.

Theorem A shows that feedback control variables play important roles in the persistent property of the system (1). But the question is whether or not the feedback control variables have influence on the permanence of the system. On the other hand, as was pointed out by Fan and Wang [2], “if we use the method of comparison theorem, then the additional condition, in some extent, is necessary. But for the system itself, this condition may not necessary.[sic]” In [2], by establishing a new difference inequality, Fan and Wang showed that feedback control has no influence on the permanence of a single species discrete model. Their success motivated us to consider the persistent property of system (1). Indeed, in this paper, we will apply the analysis technique of Fan and Wang [2] to establish sufficient conditions, which is independent of feedback control variables, to ensure the permanence of the system. We finally obtain the following main results.

Theorem B. *Assume that*

$$a^L - \frac{c^U}{2m^L} > 0, \quad (H_1)$$

$$f^L - d^U > 0, \quad (H_2)$$

hold; then system (1) is permanent.

Comparing with Theorem A, it is easy to see that (H_i) in Theorem B are weaker than (A_i) in Theorem A ($i = 1, 2$) and feedback control variables have no influence on the permanent property of system (1), so our results improve the main results in [1]. For more works on this direction, one could refer to [3–10] and the references cited therein.

The remaining part of this paper is organized as follows. In Section 2, we will introduce several lemmas. The permanence of system (1) is then studied in Section 3. In Section 4, a suitable example together with its numerical simulations shows the feasibility of our results.

2. Preliminaries

In this section, we will introduce several useful lemmas.

Lemma 1 (see [11]). *Assume that $\{x(n)\}$ satisfies*

$$x(n+1) \geq x(n) \exp\{a(n) - b(n)x(n)\}, \quad n \geq N_0, \quad (6)$$

$\limsup_{n \rightarrow +\infty} x(n) \leq x^$ and $x(N_0) > 0$, where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants and $N_0 \in N$. Then*

$$\liminf_{n \rightarrow +\infty} x(n) \geq \min \left\{ \frac{a^L}{b^U} \exp\{a^L - b^U x^*\}, \frac{a^L}{b^U} \right\}. \quad (7)$$

Lemma 2 (see [2]). *Assume that $A > 0$ and $y(0) > 0$. Suppose that*

$$y(n+1) \leq Ay(n) + B(n), \quad n = 1, 2, \dots \quad (8)$$

Then, for any integer $k \leq n$,

$$y(n) \leq A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1). \quad (9)$$

In particular, if $A < 1$ and B is bounded above with respect to M , then

$$\limsup_{n \rightarrow +\infty} y(n) \leq \frac{M}{1-A}. \quad (10)$$

Lemma 3 (see [2]). *Assume that $A > 0$ and $y(0) > 0$. Suppose that*

$$y(n+1) \geq Ay(n) + B(n), \quad n = 1, 2, \dots \quad (11)$$

Then, for any integer $k \leq n$,

$$y(n) \geq A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1). \quad (12)$$

In particular, if $A < 1$ and B is bounded above with respect to m^ , then*

$$\liminf_{n \rightarrow +\infty} y(n) \geq \frac{m^*}{1-A}. \quad (13)$$

3. Permanence

In this section, we detail the proof of our main result by several lemmas. The following lemma is a direct conclusion of [1].

Lemma 4. *There exist two positive constants M_1 and W_1 such that*

$$\begin{aligned} \limsup_{n \rightarrow +\infty} x(n) &\leq M_1, & \limsup_{n \rightarrow +\infty} y(n) &\leq M_2, \\ \limsup_{n \rightarrow +\infty} u_1(n) &\leq W_1, & \limsup_{n \rightarrow +\infty} u_2(n) &\leq W_2, \end{aligned} \quad (14)$$

where M_i and W_i ($i = 1, 2$) are defined in (5).

Lemma 5. *Assume*

$$a^L - \frac{c^U}{2m^L} > 0, \quad (H'_1)$$

holds then there exist two positive constants m_1 and w_1 such that

$$\liminf_{n \rightarrow +\infty} x(n) \geq m_1, \quad \liminf_{n \rightarrow +\infty} u_1(n) \geq w_1, \quad (15)$$

where m_1 and w_1 are defined in the proof.

Proof. According to Lemma 4, for any $\varepsilon > 0$ small enough, there exists enough large $N_1 > 0$, such that, for $n \geq N_1$,

$$x(n) \leq M_1 + \varepsilon, \quad u_1(n) \leq W_1 + \varepsilon. \quad (16)$$

Thus, it follows from (16) and the first equation of system (1) that

$$\begin{aligned} x(n+1) &\geq x(n) \\ &\times \exp \left\{ a^L - b^U(M_1 + \varepsilon) - \frac{c^U}{2m^L} - e_1^U(W_1 + \varepsilon) \right\} \\ &\geq x(n) \exp \left\{ -b^U(M_1 + \varepsilon) - \frac{c^U}{2m^L} - e_1^U(W_1 + \varepsilon) \right\} \\ &\stackrel{\Delta}{=} x(n) \exp \{D_{1\varepsilon}\}, \end{aligned} \quad (17)$$

for $n \geq N_1$, where $D_{1\varepsilon} = -b^U(M_1 + \varepsilon) - (c^U/2m^L) - e_1^U(W_1 + \varepsilon) < 0$. For any integer $k \leq n$, it follows from (17) that

$$\prod_{j=n-k}^{n-1} \frac{x(j+1)}{x(j)} \geq \prod_{j=n-k}^{n-1} \exp \{D_{1\varepsilon}\} = \exp \{D_{1\varepsilon}k\}. \quad (18)$$

Thus

$$x(n-k) \leq x(n) \exp \{-D_{1\varepsilon}k\}. \quad (19)$$

From the third equation of system (1), we have

$$\begin{aligned} u_1(n+1) &= (1 - \eta_1(n)) u_1(n) + q_1(n) x(n) \\ &\leq (1 - \eta_1^L) u_1(n) + q_1^U x(n) \\ &\stackrel{\Delta}{=} A_1 u_1(n) + B_1(n), \end{aligned} \quad (20)$$

where $A_1 = 1 - \eta_1^L$ and $B_1(n) = q_1^U x(n)$. Then, for any $k \leq n$, according to Lemma 2, (19), and (20)

$$\begin{aligned} u_1(n) &\leq A_1^k u_1(n-k) + \sum_{i=0}^{k-1} A_1^i B_1(n-i-1) \\ &= A_1^k u_1(n-k) + \sum_{i=0}^{k-1} A_1^i q_1^U x(n-i-1) \\ &\leq A_1^k u_1(n-k) + q_1^U x(n) \sum_{i=0}^{k-1} A_1^i \exp \{-D_{1\varepsilon}(i+1)\}. \end{aligned} \quad (21)$$

Note that $0 < \eta_1^L < 1$; hence $0 < A_1 < 1$. Therefore,

$$0 \leq A_1^k u_1(n-k) \leq A_1^k (W_1 + \varepsilon) \longrightarrow 0, \quad \text{as } k \longrightarrow \infty. \quad (22)$$

Then, there exists a positive integer $N_2 > N_1$ such that, for any positive solution of system (1), $e_1^U A_1^{N_2} (W_1 + \varepsilon) < (1/2)(a^L - c^U/2m^L)$ for all $n \geq N_2$. In fact, we could choose $N_2 = \ln P_1 / \ln A_1 + 1$, where $P_1 = (2m^L a^L - c^U)/4e_1^U m^L (W_1 + \varepsilon)$. Fixing N_2 , for $n \geq N_1 + N_2$, we get

$$\begin{aligned} u_1(n) &\leq A_1^{N_2} u_1(n - N_2) \\ &\quad + q_1^U x(n) \sum_{i=0}^{N_2-1} A_1^i \exp \{-D_{1\varepsilon}(i+1)\} \\ &\leq A_1^{N_2} (W_1 + \varepsilon) + q_1^U x(n) \sum_{i=0}^{N_2-1} A_1^i \exp \{-D_{1\varepsilon}(i+1)\} \\ &\stackrel{\Delta}{=} A_1^{N_2} (W_1 + \varepsilon) + G_{1\varepsilon} x(n), \end{aligned} \quad (23)$$

where $G_{1\varepsilon} = q_1^U \sum_{i=0}^{N_2-1} A_1^i \exp \{-D_{1\varepsilon}(i+1)\}$.

Substituting (23) into the first equation of system (1), we can get

$$\begin{aligned} x(n+1) &\geq x(n) \exp \left\{ a^L - b^U x(n) - \frac{c^U}{2m^L} - e_1^U u_1(n) \right\} \\ &\geq x(n) \exp \left\{ a^L - b^U x(n) - \frac{c^U}{2m^L} \right. \\ &\quad \left. - e_1^U (A_1^{N_2} (W_1 + \varepsilon) + G_{1\varepsilon} x(n)) \right\} \\ &= x(n) \exp \left\{ a^L - \frac{c^U}{2m^L} - e_1^U A_1^{N_2} (W_1 + \varepsilon) \right. \\ &\quad \left. - (b^U + e_1^U G_{1\varepsilon}) x(n) \right\} \\ &\geq x(n) \exp \left\{ \frac{1}{2} \left(a^L - \frac{c^U}{2m^L} \right) - (b^U + e_1^U G_{1\varepsilon}) x(n) \right\} \\ &\stackrel{\Delta}{=} x(n) \exp \{E_1 - E_{2\varepsilon} x(n)\}, \end{aligned} \quad (24)$$

where $E_1 = (1/2)(a^L - c^U/2m^L)$ and $E_{2\varepsilon} = b^U + e_1^U G_{1\varepsilon}$.

By applying Lemma 1 to (24), it immediately follows that

$$\liminf_{n \rightarrow +\infty} x(n) \geq \min \left\{ \frac{E_1}{E_{2\varepsilon}} \exp \{E_1 - E_{2\varepsilon} M_1\}, \frac{E_1}{E_{2\varepsilon}} \right\}. \quad (25)$$

Setting $\varepsilon \rightarrow 0$ in the above inequality, we obtain

$$\liminf_{n \rightarrow +\infty} x(n) \geq \min \left\{ \frac{E_1}{E_2} \exp \{E_1 - E_2 M_1\}, \frac{E_1}{E_2} \right\} \triangleq m_1. \quad (26)$$

It follows from (26) that there exists large enough $N_3 \geq N_1 + N_2$ such that

$$x(n) \geq \frac{m_1}{2}, \quad \forall n \geq N_3. \quad (27)$$

This together with the third equation of system (1) leads to

$$\Delta u_1(n) \geq -\eta_1(n) u_1(n) + \frac{m_1 q_1(n)}{2}, \quad \forall n \geq N_3. \quad (28)$$

Hence,

$$u_1(n+1) \geq (1 - \eta_1^U) u_1(n) + \frac{m_1 q_1^L}{2}, \quad \forall n \geq N_3. \quad (29)$$

By applying Lemma 3, it follows from (29) that

$$\liminf_{n \rightarrow +\infty} u_1(n) \geq \frac{m_1 q_1^L}{2\eta_1^U} \triangleq w_1. \quad (30)$$

This completes the proof of Lemma 5. $\square$

Lemma 6. Assume that

$$f^L - d^U > 0, \quad (H'_2)$$

holds; then there exist two positive constants m_2 and w_2 such that

$$\liminf_{n \rightarrow +\infty} y(n) \geq m_2, \quad \liminf_{n \rightarrow +\infty} u_2(n) \geq w_2, \quad (31)$$

where m_2 and w_2 are defined in the proof.

Proof. According to Lemmas 4 and 5, for any $\varepsilon > 0$ small enough, there exists enough large $N_4 > 0$, such that, for $n \geq N_4$,

$$y(n) \leq M_1 + \varepsilon, \quad x(n) \geq m_1 - \varepsilon, \quad u_2(n) \leq W_2 + \varepsilon. \quad (32)$$

Thus, it follows from (32) and the second equation of system (1) that

$$\begin{aligned} y(n+1) &= y(n) \exp \left\{ -d(n) + f(n) - \frac{f(n) m^2(n) y^2(n)}{x^2(n) + m^2(n) y^2(n)} \right. \\ &\quad \left. - e_2(n) u_2(n) \right\} \\ &\geq y(n) \exp \left\{ -d^U + f^L - \frac{f(n) m(n) y(n)}{2x(n)} - e_2^U(W_2 + \varepsilon) \right\} \\ &\geq y(n) \exp \left\{ -d^U + f^L - \frac{f^U m^U(M_1 + \varepsilon)}{2(m_1 - \varepsilon)} - e_2^U(W_2 + \varepsilon) \right\} \\ &\geq y(n) \exp \left\{ -d^U - \frac{f^U m^U(M_1 + \varepsilon)}{2(m_1 - \varepsilon)} - e_2^U(W_2 + \varepsilon) \right\} \\ &\triangleq y(n) \exp \{D_{2\varepsilon}\}, \end{aligned} \quad (33)$$

for $n \geq N_4$, where $D_{2\varepsilon} = -d^U - f^U m^U(M_1 + \varepsilon)/2(m_1 - \varepsilon) - e_2^U(W_2 + \varepsilon) < 0$. For any integer $k \leq n$, it follows from (33) that

$$\prod_{j=n-k}^{n-1} \frac{y(j+1)}{y(j)} \geq \prod_{j=n-k}^{n-1} \exp \{D_{2\varepsilon}\} = \exp \{D_{2\varepsilon} k\}. \quad (34)$$

Thus

$$y(n-k) \leq y(n) \exp \{-D_{2\varepsilon} k\}. \quad (35)$$

From the fourth equation of system (1), we can get

$$\begin{aligned} u_2(n+1) &= (1 - \eta_2(n)) u_2(n) + q_2(n) y(n) \\ &\leq (1 - \eta_2^L) u_2(n) + q_2^U y(n) \\ &\triangleq A_2 u_2(n) + B_2(n), \end{aligned} \quad (36)$$

where $A_2 = 1 - \eta_2^L$ and $B_2(n) = q_2^U y(n)$. Then, for any $k \leq n$, according to Lemma 2, (35) and (36), that

$$\begin{aligned} u_2(n) &\leq A_2^k u_2(n-k) + \sum_{i=0}^{k-1} A_2^i B_2(n-i-1) \\ &= A_2^k u_2(n-k) + \sum_{i=0}^{k-1} A_2^i q_2^U y(n-i-1) \\ &\leq A_2^k u_2(n-k) + q_2^U y(n) \sum_{i=0}^{k-1} A_2^i \exp \{-D_{2\varepsilon}(i+1)\}. \end{aligned} \quad (37)$$

Note that $0 < \eta_2^L < 1$; hence $0 < A_2 < 1$. Therefore,

$$0 \leq A_2^k u_2(n-k) \leq A_2^k (W_2 + \varepsilon) \longrightarrow 0, \quad \text{as } k \longrightarrow \infty. \quad (38)$$

Then, there exists a positive integer $N_5 > N_4$ such that, for any positive solution of system (1), $e_2^U A_2^{N_5} (W_2 + \varepsilon) < (1/2)(f^L - d^U)$ for all $n \geq N_5$. In fact, we could choose $N_5 = \ln P_2 / \ln A_2 + 1$, where $P_2 = (f^L - d^U) / 2e_2^U (W_2 + \varepsilon)$. Fixing N_5 , for $n \geq N_4 + N_5$, we get

$$\begin{aligned} u_2(n) &\leq A_2^{N_5} u_2(n - N_5) + q_2^U y(n) \sum_{i=0}^{N_5-1} A_2^i \exp\{-D_{2\varepsilon}(i+1)\} \\ &\leq A_2^{N_5} (W_2 + \varepsilon) + q_2^U y(n) \sum_{i=0}^{N_5-1} A_2^i \exp\{-D_{2\varepsilon}(i+1)\} \\ &\triangleq A_2^{N_5} (W_2 + \varepsilon) + G_{2\varepsilon} y(n), \end{aligned} \quad (39)$$

where $G_{2\varepsilon} = q_2^U \sum_{i=0}^{N_5-1} A_2^i \exp\{-D_{2\varepsilon}(i+1)\}$.

Substituting (39) into the second equation of system (1), we can get

$$\begin{aligned} y(n+1) &\geq y(n) \\ &\quad \times \exp\left\{-d^U + f^L - \frac{f(n)m(n)y(n)}{2x(n)} - e_2^U u_2(n)\right\} \\ &\geq y(n) \exp\left\{-d^U + f^L - \frac{f^U m^U y(n)}{2(m_1 - \varepsilon)}\right. \\ &\quad \left.- e_2^U (A_2^{N_5} (W_2 + \varepsilon) + G_{2\varepsilon} y(n))\right\} \\ &= y(n) \exp\left\{-d^U + f^L - e_2^U A_2^{N_5} (W_2 + \varepsilon)\right. \\ &\quad \left.- \left(\frac{f^U m^U}{2(m_1 - \varepsilon)} + e_2^U G_{2\varepsilon}\right) y(n)\right\} \\ &\geq y(n) \exp\left\{\frac{1}{2}(-d^U + f^L)\right. \\ &\quad \left.- \left(\frac{f^U m^U}{2(m_1 - \varepsilon)} + e_2^U G_{2\varepsilon}\right) y(n)\right\} \\ &\triangleq y(n) \exp\{E_3 - E_{4\varepsilon} y(n)\}, \end{aligned} \quad (40)$$

for all $n \geq N_5$, where $E_3 = (1/2)(-d^U + f^L)$ and $E_{4\varepsilon} = f^U m^U / 2(m_1 - \varepsilon) + e_2^U G_{2\varepsilon}$.

By applying Lemma 1 to (40), it immediately follows that

$$\liminf_{n \rightarrow +\infty} y(n) \geq \min\left\{\frac{E_3}{E_{4\varepsilon}} \exp\{E_3 - E_{4\varepsilon} M_2\}, \frac{E_3}{E_{4\varepsilon}}\right\}. \quad (41)$$

Setting $\varepsilon \rightarrow 0$ in the above inequality, we obtain

$$\liminf_{n \rightarrow +\infty} y(n) \geq \min\left\{\frac{E_3}{E_4} \exp\{E_3 - E_4 M_1\}, \frac{E_3}{E_4}\right\} \triangleq m_2. \quad (42)$$

It follows from (42) that there exists large enough $N_6 \geq N_4 + N_5$ such that

$$y(n) \geq \frac{m_2}{2}, \quad \forall n \geq N_6. \quad (43)$$

This together with the fourth equation of system (1) leads to

$$\Delta u_2(n) \geq -\eta_2(n) u_2(n) + \frac{m_2 q_2(n)}{2}, \quad \forall n \geq N_6. \quad (44)$$

Hence,

$$u_2(n+1) \geq (1 - \eta_2^U) u_2(n) + \frac{m_2 q_2^L}{2}, \quad \forall n \geq N_6. \quad (45)$$

By applying Lemma 3, it follows from (45) that

$$\liminf_{n \rightarrow +\infty} u_2(n) \geq \frac{m_2 q_2^L}{2\eta_2^U} \triangleq w_2. \quad (46)$$

This completes the proof of Lemma 6. $\square$

Lemmas 4–6 show that the conclusion of Theorem B holds.

4. Example and Numerical Simulation

Consider the following system:

$$\begin{aligned} x(n+1) &= x(n) \exp\left\{1.32 + 0.32 \sin(\sqrt{17}n)\right. \\ &\quad \left.- (5.3 + 0.2 \sin(n)) x(n)\right. \\ &\quad \left.- \frac{(0.75 + 0.25 \sin(\sqrt{7}n)) x(n) y(n)}{x^2(n) + (0.8 + 0.2 \cos(\sqrt{11}n))^2 y^2(n)}\right. \\ &\quad \left.- (2 + 0.1 \cos(\sqrt{13}n)) u_1(n)\right\}, \\ y(n+1) &= y(n) \exp\left\{-(0.125 + 0.025 \cos(\sqrt{5}n))\right. \\ &\quad \left.+ \frac{(0.25 + 0.05 \sin(\sqrt{3}n)) x^2(n)}{x^2(n) + (0.8 + 0.2 \cos(\sqrt{11}n))^2 y^2(n)}\right. \\ &\quad \left.- (1.5 + 0.5 \sin(n)) u_2(n)\right\}. \end{aligned}$$

$$\Delta u_1(n) = -(0.09 + 0.01 \sin(\sqrt{7}n)) u_1(n)$$

$$+ (0.3 + 0.2 \cos(n)) x(n),$$

$$\Delta u_2(n) = -(0.06 + 0.04 \cos(\sqrt{11}n)) u_2(n)$$

$$+ (0.1 + 0.05 \sin(n)) y(n).$$

(47)

FIGURE 1: Dynamic behavior of the system (47) with the initial condition $(x(0), y(0), u_1(0), u_2(0)) = (0.2, 0.4, 0.1, 0.14)^T$ and $(0.4, 0.5, 0.21, 0.3)^T$, respectively.

In this case, we have

$$\begin{aligned} a^L - \frac{c^U}{2m^L} &\approx 0.1667 > 0, \\ f^L - d^U &= 0.05 > 0. \end{aligned} \quad (48)$$

Equation (48) means that all conditions of Theorem B are satisfied in system (47). Thus, the system (47) is permanent. Our numerical simulation supports our result (see Figure 1). However,

$$a^L - \frac{c^U}{2m^L} - e_1^U W_1 \approx -4.7139 < 0, \quad (49)$$

that is to say, (A_1) does not hold and we could not obtain the result of the permanence from Theorem A. Thus our results improve the main results in [1].

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