Research Article

Sliding Mode Control for Markovian Switching Singular Systems with Time-Varying Delays and Nonlinear Perturbations

Guowei Yang,^{1,2} Yonggui Kao,² and Wei Li²

¹ College of Information Engineering, Nanchang Hangkong University, Nanchang, Jiangxi 330063, China

² Department of Mathematics, Harbin Institute of Technology, Weihai, Shandong 264209, China

Correspondence should be addressed to Yonggui Kao; kaoyonggui@sina.com

Received 13 October 2012; Revised 28 December 2012; Accepted 28 December 2012

Academic Editor: Her-Terng Yau

Copyright © 2013 Guowei Yang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper is devoted to investigating sliding mode control (SMC) for Markovian switching singular systems with time-varying delays and nonlinear perturbations. The sliding mode controller is designed to guarantee that the nonlinear singular system is stochastically admissible and its trajectory can reach the sliding surface in finite time. By using Lyapunov functional method, some criteria on stochastically admissible are established in the form of linear matrix inequalities (LMIs). A numerical example is presented to illustrate the effectiveness and efficiency of the obtained results.

1. Introduction

The sliding mode control (SMC) theory has made rapid progress since it was proposed by Utkin [1]. As an effective robust control strategy, SMC has been successfully applied to a wide variety of practical engineering systems such as robot manipulators, aircrafts, underwater vehicles, spacecrafts, flexible space structures, electrical motors, power systems, and automotive engines [2]. The SMC system has various attractive features such as fast response, good transient performance, and insensitiveness to the uncertainties on the sliding surface [2]. These advantages provide more freedom in designing the controllers for the system models which can be easily modified by introducing virtual disturbances to satisfy some requirements. The SMC strategy has been successfully applied to many kinds of systems, such as uncertain time-delay systems and Markovian jump system [3–14].

Singular systems, also referred to as descriptor systems, generalized state-space systems, differential-algebraic systems, or semistate systems, are more appropriate to describe the behavior of some practical systems, such as economic systems, power systems, and circuits systems, because singular systems mix up dynamic equations and static equations. Basic control theory for singular systems has been widely

studied, such as stability and stabilization [14–18], H_∞ control problem [19–22], and optimal control [23] and filtering problem [24–26]. Xu et al. have designed an integral sliding mode controller for singular stochastic hybrid systems [27]. They put up with new sufficient conditions in terms of strict LMIs, which guarantees stochastic stability of the sliding mode dynamics.

In practice, many physical systems may happen to have abrupt variations in their structure, due to random failures or repair of components, sudden environmental disturbances, changing subsystem interconnections, and abrupt variations in the operating points of a nonlinear plant. Therefore, Markovian jump systems have received increasing attention, see [28–33] and the references therein. Wu et al. [28] have probed sliding mode control with bounded H_2 gain performance of Markovian jump singular time-delay systems. Kao et al. [29] have investigated delay-dependent robust exponential stability of Markovian jumping reaction-diffusion Cohen-Grossberg neural networks with mixed delays. Zhang and Boukas have discussed mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probability. To the authors' best knowledge, sliding mode control for a class of Markovian switching singular systems with time-varying delays and nonlinear perturbations has not been properly investigated.

Especially few consider exponential stabilization for this kind of nonlinear singular systems by sliding mode control.

Motivated by the above discussion, we consider exponential stabilization for Markovian switching nonlinear singular systems via sliding mode control. First, we develop two lemmas. Based on these lemmas, delay-dependent sufficient condition on exponential stabilization for singular time-varying delay systems is given in terms of nonstrict LMIs. Some specified matrices are introduced and the non-strict LMIs are translated into strict LMIs which are easy to check by MATLAB LMI toolbox. Second, a sliding surface is derived using an equivalent control approach. A sliding mode controller is developed to drive the systems to the sliding surface in finite time and maintain a sliding motion thereafter. Finally, a numerical example is provided to show the effectiveness of the proposed result.

Notations. $(\Xi, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions. $L^P_{\mathcal{F}_0}$ is the family of all $\mathcal{F}_0$ -measurable $C([- \tau, 0]; R^n)$ valued random variables $\xi = \xi(\theta) : -\tau \leq \theta \leq 0$ such that $\sup_{-\tau \leq \theta \leq 0} \mathcal{E} \|\xi(\theta)\|_2^2 < \infty$, where $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. R^n and $R^{n \times m}$ denote, respectively, the n -dimensional Euclidean space and the set of $n \times m$ real matrices. The superscript T denotes the transpose, and the notation X, Y (resp., $X > Y$) where X and Y are symmetric matrices means that $X - Y$ is positive semi-definite (resp., positive definite). L^2 stands for the space of square integral vector functions. $\|\cdot\|$ will refer to the Euclidean vector norm, and $*$ represents the symmetric form of matrix.

2. System Description and Definitions

Consider the following singular system with time-varying delays, nonlinear perturbations, and Markovian switching:

$$\begin{aligned} E(r(t)) \dot{x}(t) &= A(r(t))x(t) + A_d(r(t))x(t - \tau(t)) \\ &\quad + B(r(t))u(t) + f(x(t), t, r(t)), \quad (1) \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \end{aligned}$$

where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the control input; $f(x(t), t, r(t)) \in R^n$ represents the system nonlinearity and any model uncertainties in the systems including external disturbances with Markovian switching; $\phi(t) \in L^P_{\mathcal{F}_0}([- \tau, 0]; R^n)$ is a compatible vector valued continuous function. $A(r(t))$, $A_d(r(t))$, and $B(r(t))$ are real constant matrices with appropriate dimensions. The matrix $E(r(t)) \in R^{n \times n}$ may be singular, and we assume that $0 < \text{rank}(E(r(t))) = r \leq n$. $\tau(t)$ denotes the time-varying delay and satisfies

$$0 \leq \tau(t) \leq \tau, \dot{\tau}(t) \leq d < 1. \quad (2)$$

Let $\{r(t), t \geq 0\}$ be a continuous-time discrete-state Markovian process with right continuous trajectories taking

value in a finite set $\mathbb{S} = \{1, 2, \dots, N\}$ with transition probability matrix $\Pi = (r_{ij})$, $(i, j \in \mathbb{S})$,

$$P\{r(t + \Delta) = j \mid r(t) = i\} = \begin{cases} r_{ij} \times \Delta + o(\Delta), & i \neq j \\ 1 + r_{ii} \times \Delta + o(\Delta), & i = j, \end{cases} \quad (3)$$

where $\Delta > 0$ and $\lim_{\Delta \rightarrow 0} (o(\Delta))/\Delta = 0$, $r_{ij} > 0$ is the transition rate from i to j if $i \neq j$ and $r_{ii} = -\sum_{j=1, j \neq i}^N r_{ij}$.

The nominal Markovian jump singular and time-delay system of system (1) are as follows:

$$E(r(t)) \dot{x}(t) = A(r(t))x(t), \quad (4)$$

$$E(r(t)) \dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t - \tau(t)). \quad (5)$$

The initial Markovian jump singular system in (1) is assumed to be

$$x(0, \gamma_0) = \phi(0). \quad (6)$$

Recall that the Markovian process $\{r(t), t \geq 0\}$ takes value in a finite set $\mathbb{S} = \{1, 2, \dots, N\}$. For simplicity, we write $r(t) = i \in \mathbb{S}$, $E(i) = E_i$, $A(i) = A_i$, $A_d(i) = A_{di}$, $B(r(t)) = B_i$, $f(x(t), t, r(t)) = f_i(x(t), t)$.

Definition 1. (i) The system (4) is said to be regular if $\det(sE_i - A_i) \neq 0$ for every $i \in \mathbb{S}$.

(ii) The system (4) is said to impulse free if $\deg(\det(sE_i - A_i)) = \text{rank}(E_i)$ for every $i \in \mathbb{S}$.

Definition 2. For a given scalar $\tau > 0$, the Markovian jump singular delay system (5) is said to be regular and impulse free for any time delay $\tau(t)$ satisfying $0 \leq \tau(t) \leq \tau$, if the system (4) and the system $E(r(t))\dot{X}(t) = (A(r(t)) + A_d(r(t)))X(t)$ are all regular and impulse free.

The system (5) is said to be stochastically stable if for any $x_0 \in R^n$ and $r_0 \in \mathbb{S}$ there exists a scalar $\bar{M}(x_0, r_0) > 0$ such that $\lim_{t \rightarrow \infty} \mathcal{E}(\int_0^t x^T(s, x_0, r_0)x(s, x_0, r_0)ds \mid x_0, r_0) \leq \bar{M}(x_0, r_0)$, where $x(s, x_0, r_0)$ denotes the solution of system (5) at time t under the initial condition x_0 and r_0 .

The system (5) is said to be stochastically admissible if it is regular, impulse free, and stochastically stable.

We will assume the followings to be valid.

Assumption 1. $B(r(t))$ is full-rank: $\text{rank}(B(r(t))) = m$.

Assumption 2. The perturbation term $f(x(t), t, r(t))$ is Lipschitz, continuous and satisfies the following matching conditions:

$$f(x(t), t, r(t)) = B(r(t))\bar{f}(x(t), t, r(t)), \quad (7)$$

with $\bar{f}(x(t), t, r(t)) \in R^m$ bounded by

$$\|\bar{f}(x(t), t, r(t))\| \leq \epsilon_i \|x(t)\|, \quad (8)$$

where $\epsilon > 0$ is a constant.

Lemma 3 will support the non-strict LMI to be translated into strict LMI.

Lemma 3 (see [34]). Let $X \in R^{n \times n}$ be symmetric such that $E_L^T X E_L > 0$ and $T \in R^{(n-r) \times (n-r)}$ nonsingular. Then, $XE + M^T T S^T$ is nonsingular and its inverse is expressed as

$$(XE + M^T T S^T)^{-1} = \mathbb{X}E^T + S\mathbb{T}M, \quad (9)$$

where $\mathbb{X}$ is symmetric and $\mathbb{T}$ is a singular matrix with

$$E_R^T \mathbb{X} E_R = (E_L^T X E_L)^{-1}, \quad \mathbb{T} = (S^T S)^{-1} T^{-1} (M M^T)^{-1}, \quad (10)$$

where M and S are any matrices with full row rank and satisfy $ME = 0$ and $ES = 0$, respectively; E is decomposed as $E = E_L E_R^T$ with $E_L \in R^{n \times r}$ and $E_R \in R^{n \times r}$ are of full column rank.

Lemma 4 (see [35]). There exists symmetric matrix X such that

$$\begin{bmatrix} P_1 + X & Q_1 \\ Q_1^T & R_1 \end{bmatrix} < 0, \quad \begin{bmatrix} P_2 - X & Q_2 \\ Q_2^T & R_2 \end{bmatrix} < 0 \quad (11)$$

if and only if

$$\begin{bmatrix} P_1 + P_2 & Q_1 & Q_2 \\ * & R_1 & 0 \\ * & * & R_2 \end{bmatrix} < 0. \quad (12)$$

Lemma 5. Let $Q = Q^T$, S , $R = R^T$ be matrices of appropriate dimensions, then $R < 0$, $Q - SR^{-1}S^T < 0$ is equivalent to

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} < 0. \quad (13)$$

Lemma 6. Let $X \in R^n$, $Y \in R^n$, and $Q > 0$. Then we have $X^T Y + Y^T X \leq Y^T Q Y + X^T Q^{-1} X$.

We give the following result for the stochastic admissibility of the system (4) without proof, and readers are referred to [36] for detailed proof.

Lemma 7. The Markovian jump singular system (4) is stochastically admissible if and only if there exists matrices P_i , $i = 1, 2, \dots, N$ such that

$$\begin{aligned} E_i^T P_i &= P_i^T E_i \geq 0 \\ \sum_{j=1}^N r_{ij} E_i^T P_j + P_i^T A_i + A_i^T P_i &< 0. \end{aligned} \quad (14)$$

Lemma 8. For a prescribed scalars $\tau > 0$, d , and any time delay $\tau(t)$ satisfying $0 \leq \tau(t) \leq \tau$, the Markovian jump singular time-delay system (5) is stochastically admissible, if there exist symmetric positive-definite matrices Q , R and nonsingular matrix P_i for every $i = 1, 2, \dots, N$, such that

$$E_i^T P_i = P_i^T E_i \geq 0, \quad (15)$$

$$\Phi_i = \begin{bmatrix} \Phi_i^{11} & P_i^T A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\frac{1}{\tau}R \end{bmatrix} < 0, \quad (16)$$

where

$$\Phi_i^{11} = P_i^T A_i + A_i^T P_i + Q + \tau R + \sum_{j=1}^N r_{ij} E_j^T P_j. \quad (17)$$

Proof. First to prove the system $E(r(t))\dot{x}(t) = [A(r(t)) + A_d(r(t))]x(t)$ is regular and impulse free.

From (16), it is easy to know $\Phi_i^{11} < 0$ and

$$\begin{bmatrix} \Phi_i^{11} & P_i^T A_{di} \\ * & -(1-d)Q \end{bmatrix} < 0. \quad (18)$$

By pre- and postmultiplying (18) by $[I, I]$ and $[I, I]^T$, we get

$$P_i^T (A_i + A_{di}) + (A_i + A_{di})^T P_i + dQ + \tau R + \sum_{j=1}^N r_{ij} E_j^T P_j < 0, \quad (19)$$

where Q and R are symmetric positive-definite matrices, we have

$$P_i^T (A_i + A_{di}) + (A_i + A_{di})^T P_i + \sum_{j=1}^N r_{ij} E_j^T P_j < 0, \quad (20)$$

$$\sum_{j=1}^N r_{ij} E_i^T P_j + P_i^T A_i + A_i^T P_i < 0. \quad (21)$$

Based on Lemma 7, (15) and (20) show that the system $E(r(t))\dot{x}(t) = [A(r(t)) + A_d(r(t))]x(t)$ is regular and impulse free, and (15) and (21) ensure that the system $E(r(t))\dot{X}(t) = A(r(t))X(t)$ is regular and impulse free. Hence, according to Definition 2, the system (5) is regular and impulse free for any delay $\tau(t)$ satisfying $0 \leq \tau(t) \leq \tau$.

Second, to prove the system (5) is stochastically stable. Take a functional candidate for the system as follows:

$$\begin{aligned} V(x(t), r(t)) &= X^T(t) E^T(r(t)) P(r(t)) X(t) \\ &+ \int_{t-\tau(t)}^t X^T(s) Q X(s) ds \\ &+ \int_{-\tau}^0 \int_{t+\theta}^t X^T(s) R X(s) ds d\theta. \end{aligned} \quad (22)$$

Then, let $\mathcal{L}$ be the weak infinitesimal generator of the random process $x(t)$, $r(t)$, and for each $i \in \mathbb{S}$, we have

$$\begin{aligned} \mathcal{L}V(x(t), r(t) = i) &= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \{ \mathcal{E}[V(x(t+\Delta), r(t+\Delta)) | x(t), \\ &r(t) = i] - V(x(t), r(t) = i) \} \end{aligned}$$

$$\begin{aligned}
&\leq 2x^T(t) P_i^T E_i \dot{x}(t) + x^T(t) \left(\sum_{j=1}^N r_{ij} E_j^T P_j \right) x(t) \\
&\quad + x^T(t) Q x(t) \\
&\quad - (1 - \dot{\tau}(t)) x^T(t - \tau(t)) Q x(t - \tau(t)) \\
&\quad + \tau \left(x^T(t) R x(t) \right) - \int_{t-\tau}^t x^T(s) R x(s) ds \\
&\leq 2x^T(t) P_i^T A_i x(t) \\
&\quad + 2x^T(t) P_i^T A_{di} x(t - \tau(t)) \\
&\quad + x^T(t) \left(\sum_{j=1}^N r_{ij} E_j^T P_j \right) x(t) + x^T(t) Q x(t) \\
&\quad - (1 - d) x^T(t - \tau(t)) Q x(t - \tau(t)) \\
&\quad + \tau x^T(t) R x(t) - \left[\int_{t-\tau}^t x^T(s) ds \right] \\
&\quad \times \frac{R}{\tau} \left[\int_{t-\tau}^t x(s) ds \right] \\
&= \xi(t) \Phi_i \xi^T(t),
\end{aligned} \tag{23}$$

where $\xi(t) = [x^T(t), x^T(t - \tau(t)), \int_{t-\tau}^t x^T(s) ds]$.

With (16), it is easy to know that there exists a scalar $\mu > 0$, such that

$$\mathcal{L}V(x(t), r(t) = i) \leq -\mu x^T(t) x(t). \tag{24}$$

By Dynkin's formula, we get

$$\begin{aligned}
&\mathcal{E} \{V((x(t), r(t)) | x(0), r(0)) - V(x(0), r(0))\} \\
&= \mathcal{E} \left\{ \int_0^t LV(x(s), r(s)) ds \mid x(0), r(0) \right\} \\
&\leq -\mu \mathcal{E} \left\{ \int_0^t x^T(s) x(s) ds \mid x(0), r(0) \right\},
\end{aligned} \tag{25}$$

and this means

$$\mathcal{E} \left(\int_0^t x^T(s) x(s) ds \mid x(0), r(0) \right) \leq \frac{1}{\mu} V(x(0), r(0)). \tag{26}$$

Therefore by Definition 2, the Markovian jump singular system (5) is stochastically stable. This completes the proof. $\square$

3. Sliding Motion Stability Analysis

3.1. Sliding Surface Design. SMC design involves two basic steps: sliding surface design and controller design. For every

$i \in \mathbb{S}$, integral sliding surface with delay and Markovian switching is considered as follows:

$$\begin{aligned}
S(t) &= G_i E_i x(t) - \int_0^t G_i (A_i + B_i K_i) x(\theta) d\theta \\
&\quad - \int_0^t G_i A_{di} x(\theta - \tau(\theta)) d\theta,
\end{aligned} \tag{27}$$

where $K_i \in \mathbf{R}^{m \times n}$ is real matrix to be designed and $G_i \in \mathbf{R}^{m \times n}$ is designed to satisfy that $G_i B_i$ is nonsingular. According to SMC theory, when the system trajectories reach onto the sliding surface, it follows that $S(t) = 0$ and $\dot{S}(t) = 0$.

Therefore, by $\dot{S}(t) = 0$, we get the equivalent control as

$$u_{eqi}(t) = K_i x(t) - \bar{f}(x(t), t, i). \tag{28}$$

Substituting $u_{eq}(r(t)) = K(r(t))x(t) - \bar{f}(x(t), t, r(t))$ into (1), we obtain the following sliding mode dynamics:

$$\begin{aligned}
E(r(t)) \dot{x}(t) &= [A(r(t)) + B(r(t)) K(r(t))] x(t) \\
&\quad + A_d(r(t)) x(t - \tau(t)).
\end{aligned} \tag{29}$$

3.2. Sliding Mode Dynamics Analysis. In this section, we pay attention to establishing LMI conditions to check the stochastic admissibility of the system (29). Based on Lemmas 4, it is easy to get the sufficient condition provided in the following theorem.

Theorem 9. *Given scalars τ and d , for any delays $\tau(t)$ satisfying (2), the system (29) is stochastically admissible if there exist nonsingular matrices P_i and symmetric positive-definite matrix Q and R such that*

$$E_i^T P_i = P_i^T E_i \geq 0, \tag{30}$$

$$\begin{bmatrix} \Phi_i^{11} & P_i^T A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\frac{1}{\tau}R \end{bmatrix} < 0, \tag{31}$$

where

$$\begin{aligned}
\Phi_i^{11} &= P_i^T (A_i + B_i K_i) + (A_i + B_i K_i)^T P_i \\
&\quad + Q + \tau R + \sum_{j=1}^N r_{ij} E_j^T P_j.
\end{aligned} \tag{32}$$

Remark 10. Note that the conditions in Theorem 9 are not strict LMI conditions due to matrix equality constraint of (30). According to Lemma 3 and Theorem 9, the strict LMI conditions are given as follows.

Theorem 11. *Given scalars τ and d , for any delays $\tau(t)$ satisfying (2), the system (29) is stochastically admissible if there exist symmetric positive-definite matrices $\mathbb{X}$, Q , Y_i , H , R , and symmetric matrix $\mathbb{T} \in \mathbf{R}^{(n-r) \times (n-r)}$, matrix $\mathbb{L}_i \in \mathbf{R}^{m \times n}$,*

and any matrices with full row rank M_i, S_i satisfying $M_i E_i = 0, E_i S_i = 0$, respectively, such that

$$\begin{bmatrix} \Gamma_i^{11} & A_{di} & N_1 \\ * & -(1-d)Q & 0 \\ * & * & N_2 \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} Q + \tau R - H & 0 & N_3 \\ * & -\frac{1}{\tau}R & 0 \\ * & * & N_4 \end{bmatrix} < 0, \quad (34)$$

$$\begin{bmatrix} -Y_i & 0 & E_i \otimes + M_i^T \mathbb{T} S_i^T & 0 \\ * & -H & 0 & H \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (35)$$

where

$$\begin{aligned} \Gamma_i^{11} &= A_i (\otimes E_i^T + S_i \mathbb{T} M_i) + (\otimes E_i^T + S_i \mathbb{T} M_i)^T A_i^T + B_i \mathbb{L}_i E_i^T \\ &\quad + (B_i \mathbb{L}_i E_i^T)^T + r_{ii} (\otimes E_i^T + S_i \mathbb{T} M_i)^T E_i^T + Y_i, \end{aligned}$$

$$N_1 = [B_i \mathbb{L}_i, M_i^T \mathbb{T} S_i^T], \quad N_2 = \text{diag}[-\infty, -\infty],$$

$$N_3 = [E_{1R}, \dots, E_{i-1,R}, E_{i+1,R}, \dots, E_{nR}],$$

$$\begin{aligned} N_4 &= \text{diag} \left[-\frac{1}{r_{i1}} E_{1R}^T \otimes E_{1R}, \dots, -\frac{1}{r_{i,i-1}} E_{i-1,R}^T \otimes E_{i-1,R}, \right. \\ &\quad \left. -\frac{1}{r_{i,i+1}} E_{i+1,R}^T \otimes E_{i+1,R}, \dots, -\frac{1}{r_{in}} E_{nR}^T \otimes E_{nR} \right], \end{aligned}$$

$$\mathbb{L}_i = K_i \otimes. \quad (36)$$

Proof. Let $P_i \triangleq X E_i + M_i^T \mathbb{T} S_i^T$ in Theorem 9. We can get

$$\begin{bmatrix} \Phi_i^{11} & (X E_i + M_i^T \mathbb{T} S_i^T)^T A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\frac{1}{\tau}R \end{bmatrix} < 0, \quad (37)$$

where

$$\begin{aligned} \Phi_i^{11} &= (X E_i + M_i^T \mathbb{T} S_i^T)^T (A_i + B_i K_i) \\ &\quad + (A_i + B_i K_i)^T (X E_i + M_i^T \mathbb{T} S_i^T) \\ &\quad + Q + \tau R + \sum_{j=1}^N r_{ij} E_j^T X E_j. \end{aligned} \quad (38)$$

Using Lemma 3, we get

$$(X E_i + M_i^T \mathbb{T} S_i^T)^{-1} = \otimes E_i^T + S_i \mathbb{T} M_i \triangleq L_i. \quad (39)$$

By pre- and postmultiplying (37) by $\text{diag}[L_i^T, I, I]$ and $\text{diag}[L_i, I, I]$, we get

$$\begin{bmatrix} \Psi_i'^{11} & A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\frac{1}{\tau}R \end{bmatrix} < 0, \quad (40)$$

where $\Psi_i'^{11} = (A_i + B_i K_i) L_i + L_i^T (A_i + B_i K_i)^T + L_i^T Q L_i + \tau L_i^T R L_i + \sum_{j=1}^N r_{ij} L_i^T E_j^T X E_j$.

In light of Lemma 4, there exists symmetric matrix Y_i such that

$$\begin{bmatrix} (A_i + B_i K_i) L_i + L_i^T (A_i + B_i K_i)^T + r_{ii} L_i^T E_i^T + Y_i & A_{di} \\ * & -(1-d)Q \end{bmatrix} < 0, \quad (41)$$

$$\begin{bmatrix} L_i^T Q L_i + \tau L_i^T R L_i + \sum_{i \neq j=1}^N r_{ij} L_i^T E_j^T X E_j L_i - Y_i & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0. \quad (42)$$

It is easy to know that

$$\begin{aligned} B_i K_i S_i \mathbb{T} M_i + (B_i K_i S_i \mathbb{T} M_i)^T &= B_i K_i \otimes \otimes^{-1} S_i \mathbb{T} M_i \\ &\quad + (B_i K_i \otimes \otimes^{-1} S_i \mathbb{T} M_i)^T \\ &\leq (B_i K_i \otimes) \otimes^{-1} (\otimes K_i^T B_i^T) \\ &\quad + (M_i^T \mathbb{T} S_i^T) \otimes^{-1} (S_i \mathbb{T} M_i). \end{aligned} \quad (43)$$

Using Shur's complement, the following inequality can ensure (41) as

$$\begin{bmatrix} A_i L_i + L_i^T A_i^T + B_i K_i \otimes E_i^T + (B_i K_i \otimes E_i^T)^T + r_{ii} L_i^T E_i^T + Y_i & A_{di} & N_1 \\ * & -(1-d)Q & 0 \\ * & * & N_2 \end{bmatrix} > 0, \quad (44)$$

where $N_1 = [B_i K_i \otimes, M_i^T \mathbb{T} S_i^T]$ and $N_2 = \text{diag}[-\infty, -\infty]$. Equation (42) is equivalent to

$$\begin{bmatrix} Q + \tau R + \sum_{i \neq j=1}^N r_{ij} E_j^T X E_j - P_i^T Y_i P_i & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0. \quad (45)$$

There exists a matrix $H^T = H > 0$ such that

$$\begin{bmatrix} Q + \tau R + \sum_{i \neq j=1}^N r_{ij} E_j^T X E_j - P_i^T Y_i P_i & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0. \quad (46)$$

$$< \begin{bmatrix} Q + \tau R + \sum_{i \neq j=1}^N r_{ij} E_j^T X E_j - H & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0$$

FIGURE 1: State trajectories of the open-loop system.

if and only if

$$-P_i^T Y_i P_i < -H < 0. \quad (47)$$

The above inequality is equivalent to the following inequality:

$$\begin{bmatrix} -Y_i & L_i^T H \\ * & -H \end{bmatrix} < 0. \quad (48)$$

And the following inequality can guarantee that (48) is true:

$$\begin{bmatrix} -Y_i & 0 & L_i^T & 0 \\ * & -H & 0 & H \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0. \quad (49)$$

By Shur's complement, the right of (46) is equivalent to

$$\begin{bmatrix} Q + \tau R - H & 0 & N_3 \\ * & -\frac{1}{\tau} R & 0 \\ * & * & N_4 \end{bmatrix} < 0, \quad (50)$$

where $N_3 = [E_{1R}, \dots, E_{i-1,R}, E_{i+1,R}, \dots, E_{nR}]$ and $N_4 = \text{diag}[-(1/r_{i1})E_{1R}^T \otimes E_{1R}, \dots, -(1/r_{i,i-1})E_{i-1,R}^T \otimes E_{i-1,R}, -(1/r_{i,i+1})E_{i+1,R}^T \otimes E_{i+1,R}, \dots, -(1/r_{in})E_{nR}^T \otimes E_{nR}]$. E_i is decomposed as $E_i = E_{iL}E_{iR}^T$ with $E_{iL} \in \mathbf{R}^{n \times r}$ and $E_{iR} \in \mathbf{R}^{n \times r}$ are of full column rank. This completes the proof. $\square$

Remark 12. From the proof of the Theorem 11, it is not difficult to know that Theorem 11 is more easy to compute than Theorem 9.

3.3. Sliding Mode Control Design. After switching surface design, the next important part of sliding mode control is

FIGURE 2: Switching surface $s(t)$.

FIGURE 3: State trajectories of the closed-loop system.

to design a slide mode controller to guarantee the existence of a sliding mode. Now, we design an SMC law, by which the trajectories of singular system (1) can be driven onto the designed sliding surface $S(t) = 0$ in a finite time.

Theorem 13. With the constant matrix K_i mentioned in Theorem 11 and the integral sliding surface given by (29), the trajectory of the closed-loop system (1) can be driven onto the sliding surface in finite time with the control (51) as

$$u_i(t) = K_i x(t) - (\rho_i + \epsilon_i \|x(t)\|) \text{sign}(B_i^T G_i^T S(t)), \quad (51)$$

where ρ_i is a positive constant.

FIGURE 4: Controller input $u(t)$.

Proof. Choose G_i under the condition of $G_i B_i$ is nonsingular. Consider the following Lyapunov function:

$$V(S(t), t) = \frac{1}{2} S^T(t) S(t). \quad (52)$$

Due to (29), we have

$$dS = G_i B_i [-K_i x(t) + u_i(t) + \bar{f}_i] dt. \quad (53)$$

Differentiating $V_i(t)$ along the closed-loop trajectories and using (53), we have

$$\begin{aligned} \mathcal{L}V(S(t), t) &= S_i^T(t) dS_i(t) \\ &= S_i^T(t) G_i B_i [-K_i x(t) + u_i(t) + \bar{f}_i] \\ &= S_i^T(t) G_i B_i [-(\rho + \epsilon \|x(t)\|) \text{sign} \\ &\quad \times (B_i^T G_i^T S(t) + \bar{f}_i)] \\ &\leq -(\rho + \epsilon \|x(t)\|) \|B_i^T G_i^T S_i(t)\| \\ &\quad + \|\bar{f}_i\| \|B_i^T G_i^T S_i(t)\| \\ &\leq -\rho \|B_i^T G_i^T S_i(t)\| \\ &\leq -\bar{\rho} \|S_i(t)\| < 0, \quad \text{if } S_i(t) \neq 0, \end{aligned} \quad (54)$$

where $\bar{\rho}_i > \rho_i \sqrt{\lambda_{\min}(G_i B_i B_i^T G_i^T)}$. Then the state trajectory converges to the surface and is restricted to the surface for all subsequent time. This completes the proof. $\square$

4. Numerical Example

In this section, a numerical example is presented to illustrate the effectiveness of the main results in this paper.

Example 14. Let us consider the system (1) with Markovian process that governs that the mode switching has generator $\Pi = (r_{ij})$, $(i, j = 1, 2)$, $r_{12} = 0.25$, $r_{21} = 0.2$. The system data are as follows:

$$\begin{aligned} E_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, & A_1 &= \begin{bmatrix} -6.7 & 1.6 & 0 \\ 0 & -5 & 0 \\ 0.5 & 2.2 & 3.2 \end{bmatrix}, \\ A_{d1} &= \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, & B_1 &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \\ M_1 &= [0 \ 0 \ 1], & S_1 &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & E_{1R} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \\ E_2 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, & A_2 &= \begin{bmatrix} -5.7 & -4.2 & 0 \\ -3.7 & 4.8 & 1.3 \\ 0 & 2.4 & -6 \end{bmatrix}, \\ A_{d2} &= \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \\ M_2 &= [0 \ 1 \ 0], & S_2 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, & E_{2R} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \quad (55)$$

In addition, $\tau = 0.3$, $d = 0.3$, $\tau(t) = 0.3e^{-t}$ and $f_1(x) = 0.65B_1x(t) \sin x(t)$, $f_2(x) = 0.65B_2x(t) \sin x(t)$. Solve the LMI (33), (34), and (35) as follows:

$$\begin{aligned} K_1 &= [0.0963 \quad -0.0506 \quad -0.003] \\ K_2 &= [0.0953 \quad -0.0039 \quad 0.1793]. \end{aligned} \quad (56)$$

For $\det(G_i B_i) \neq 0$, we can choose G_1 and G_2 as

$$G_1 = [1.2 \quad 1.6 \quad 1.1]; \quad G_2 = [1.3 \quad 1.2 \quad 1.5]. \quad (57)$$

Thus the sliding surface function is

$$s_1(t) = \begin{cases} s_1(t) = [1.2, 1.6, 0] x(t) \\ \quad - \int_0^t [-7.2204, -3.8017, -3.5116] x(\theta) d\theta \\ \quad - \int_0^t [-0.24, -0.016, -0.11] x(\theta - \tau(\theta)) d\theta, & i = 1 \\ s_2(t) = [1.3, 0, 1.5] x(t) \\ \quad - \int_0^t [-11.4688, 3.8852, -6.7228] x(\theta) d\theta \\ \quad - \int_0^t [-0.13, -0.12, 0] x(\theta - \tau(\theta)) d\theta, & i = 2. \end{cases} \quad (58)$$

FIGURE 5: State trajectories of sliding mode.

Take $\rho_1 = \rho_2 = 0.64$, then the SMC law designed in (51) can be described as

$$u(t) = \begin{cases} u_1(t) = [0.0963, -0.0506, -0.003] x(t) \\ \quad -\rho(t) \operatorname{sign}(2.8s_1(t)), & i = 1 \\ u_2(t) = [0.0953, -0.0037, 0.1793] x(t) \\ \quad -\rho(t) \operatorname{sign}(4s_2(t)), & i = 2, \end{cases} \quad (59)$$

where $\rho(t) = 0.64 + 0.65\|x(t)\|$. The simulation results are given in Figures 1, 2, 3, 4, and 5, which show the validity of the proposed method.

Remark 15. Obviously, our results include Markovian switching and nonlinear perturbation effects, and this model can not be dealt with by the results of [6, 8, 10–14, 16, 18, 20, 22, 26, 27, 34, 37], which show that our results are new.

5. Conclusion

In this paper, the stochastically admissible using sliding mode control for singular system with time-varying delay and nonlinear perturbations is studied by LMI method. The sliding mode control is designed to ensure that the closed-loop system is stochastically admissible. A numerical example demonstrates the effectiveness of the method mentioned above.

Acknowledgments

The authors would like to thank the editors and the anonymous reviewers for their valuable comments and constructive suggestions. This research is supported by the Natural Science Foundation of Guangxi Autonomous Region (no. 2012GXNSFBA053003), the National Natural Science Foundations of China (60973048, 61272077, 60974025, 60673101,

60939003), National 863 Plan Project (2008 AA04Z401, 2009AA043404), the Natural Science Foundation of Shandong Province (no. Y2007G30), the Scientific and Technological Project of Shandong Province (no. 2007GG3WZ04016), the Science Foundation of Harbin Institute of Technology (Weihai) (HIT(WH) 200807), the Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT.NSRIF. 2001120), the China Postdoctoral Science Foundation (2010048 1000), and the Shandong Provincial Key Laboratory of Industrial Control Technique (Qingdao University).

References

- [1] V. I. Utkin, "Variable structure systems with sliding modes," *IEEE Transactions on Automatic Control*, vol. 22, no. 2, pp. 212–222, 1977.
- [2] V. I. Utkin, *Sliding Modes in Control and Optimization*, Communications and Control Engineering Series, Springer, Berlin, Germany, 1992.
- [3] Y. Xia, Z. Zhu, C. Li, H. Yang, and Q. Zhu, "Robust adaptive sliding mode control for uncertain discrete-time systems with time delay," *Journal of the Franklin Institute*, vol. 347, no. 1, pp. 339–357, 2010.
- [4] H. H. Choi, "LMI-based sliding surface design for integral sliding mode control of mismatched uncertain systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 4, pp. 736–742, 2007.
- [5] X. R. Han, E. Fridman, S. K. Spurgeon, and C. Edwards, "On the design of sliding-mode static-output-feedback controllers for systems with state delay," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3656–3664, 2009.
- [6] H. Yang, Y. Xia, and P. Shi, "Observer-based sliding mode control for a class of discrete systems via delta operator approach," *Journal of the Franklin Institute*, vol. 347, no. 7, pp. 1199–1213, 2010.
- [7] E. Fridman, F. Gouaisbaut, M. Dambrine, and J.-P. Richard, "Sliding mode control of systems with time-varying delays via descriptor approach," *International Journal of Systems Science*, vol. 34, no. 8–9, pp. 553–559, 2003.
- [8] Y. Xia and Y. Jia, "Robust sliding-mode control for uncertain time-delay systems: an LMI approach," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 1086–1092, 2003.
- [9] C.-C. Wen and C.-C. Cheng, "Design of sliding surface for mismatched uncertain systems to achieve asymptotical stability," *Journal of the Franklin Institute*, vol. 345, no. 8, pp. 926–941, 2008.
- [10] Y. Niu, D. W. C. Ho, and X. Wang, "Robust H_∞ control for nonlinear stochastic systems: a sliding-mode approach," *IEEE Transactions on Automatic Control*, vol. 53, no. 7, pp. 1695–1701, 2008.
- [11] H. Wang, Z.-Z. Han, Q.-Y. Xie, and W. Zhang, "Sliding mode control for chaotic systems based on LMI," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 4, pp. 1410–1417, 2009.
- [12] N. Cai, Y. Jing, and S. Zhang, "Modified projective synchronization of chaotic systems with disturbances via active sliding mode control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 6, pp. 1613–1620, 2010.

- [13] Y. Niu, D. W. C. Ho, and X. Wang, "Sliding mode control for Itô stochastic systems with Markovian switching," *Automatica*, vol. 43, no. 10, pp. 1784–1790, 2007.
- [14] L. Wu, X. Yao, and W. X. Zheng, "Generalized H_2 fault detection for two-dimensional Markovian jump systems," *Automatica*, vol. 48, no. 8, pp. 1741–1750, 2012.
- [15] W. Xiang and F. Chen, "An adaptive sliding mode control scheme for a class of chaotic systems with mismatched perturbations and input nonlinearities," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 1, pp. 1–9, 2011.
- [16] E. K. Boukas, S. Xu, and J. Lam, "On stability and stabilizability of singular stochastic systems with delays," *Journal of Optimization Theory and Applications*, vol. 127, no. 2, pp. 249–262, 2005.
- [17] Z. Wu, H. Su, J. Chu, and W. Zhou, "Improved result on stability analysis of discrete stochastic neural networks with time delay," *Physics Letters A*, vol. 373, no. 17, pp. 1546–1552, 2009.
- [18] Y. Xia, E.-K. Boukas, P. Shi, and J. Zhang, "Stability and stabilization of continuous-time singular hybrid systems," *Automatica*, vol. 45, no. 6, pp. 1504–1509, 2009.
- [19] S. Wo, Y. Zou, M. Sheng, and S. Xu, "Robust control for discrete-time singular large-scale systems with parameter uncertainty," *Journal of the Franklin Institute*, vol. 344, no. 2, pp. 97–106, 2007.
- [20] J. Raouf and E. K. Boukas, "Stochastic output feedback controller for singular Markovian jump systems with discontinuities," *IET Control Theory & Applications*, vol. 3, no. 1, pp. 68–78, 2009.
- [21] J. H. Kim, "New design method on memoryless H_∞ control for singular systems with delayed state and control using LMI," *Journal of the Franklin Institute*, vol. 342, no. 3, pp. 321–327, 2005.
- [22] S. Ma, C. Zhang, and Z. Cheng, "Delay-dependent robust H_∞ control for uncertain discrete-time singular systems with time-delays," *Journal of Computational and Applied Mathematics*, vol. 217, no. 1, pp. 194–211, 2008.
- [23] S. Zhou and W. X. Zheng, "Robust H_∞ control of delayed singular systems with linear fractional parametric uncertainties," *Journal of the Franklin Institute*, vol. 346, no. 2, pp. 147–158, 2009.
- [24] W. Zhou and J. Fang, "Delay-dependent robust H_∞ admissibility and stabilization for uncertain singular system with Markovian jumping parameters," *Circuits, Systems, and Signal Processing*, vol. 28, no. 3, pp. 433–450, 2009.
- [25] N. Kumaresan and P. Balasubramaniam, "Optimal control for stochastic nonlinear singular system using neural networks," *Computers & Mathematics with Applications*, vol. 56, no. 9, pp. 2145–2154, 2008.
- [26] S. Ma and E. K. Boukas, "Robust H_∞ filtering for uncertain discrete Markov jump singular systems with mode-dependent time delay," *IET Control Theory & Applications*, vol. 3, no. 3, pp. 351–361, 2009.
- [27] S. Xu, J. Lam, and Y. Zou, " H_∞ filtering for singular systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 12, pp. 2217–2222, 2003.
- [28] L. Wu, X. Su, and P. Shi, "Sliding mode control with bounded L_2 gain performance of Markovian jump singular time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1929–1933, 2012.
- [29] Y.-G. Kao, J.-F. Guo, C.-H. Wang, and X.-Q. Sun, "Delay-dependent robust exponential stability of Markovian jumping reaction-diffusion Cohen-Grossberg neural networks with mixed delays," *Journal of the Franklin Institute*, vol. 349, no. 6, pp. 1972–1988, 2012.
- [30] L. Zhang and J. Lam, "Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions," *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1695–1701, 2010.
- [31] L. Zhang and E.-K. Boukas, "Mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 6, pp. 1462–1467, 2009.
- [32] L. Zhang, " H_∞ estimation for discrete-time piecewise homogeneous Markov jump linear systems," *Automatica*, vol. 45, no. 11, pp. 2570–2576, 2009.
- [33] L. Zhang, E.-K. Boukas, and J. Lam, "Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2458–2464, 2008.
- [34] L. Wu and D. W. C. Ho, "Sliding mode control of singular stochastic hybrid systems," *Automatica*, vol. 46, no. 4, pp. 779–783, 2010.
- [35] K. Gu, "A further refinement of discretized Lyapunov functional method for the stability of time-delay systems," *International Journal of Control*, vol. 74, no. 10, pp. 967–976, 2001.
- [36] E.-K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Communications and Control Engineering Series, Springer, Berlin, Germany, 2008.
- [37] D. Yue and Q.-L. Han, "Robust H_∞ filter design of uncertain descriptor systems with discrete and distributed delays," *IEEE Transactions on Signal Processing*, vol. 52, no. 11, pp. 3200–3212, 2004.