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Research Article

Int-Soft Filters of *BE*-Algebras

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The notion of int-soft filters of a *BE*-algebra is introduced, and related properties are investigated. Characterization of an int-soft filter is discussed. The problem of classifying int-soft filters by their γ-inclusive filter is solved.

1. Introduction

In 1966, Imai and Iséki [1] and Iséki [2] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. As a generalization of a BCK-algebra, H. S. Kim and Y. H. Kim [3] introduced the notion of a *BE*-algebra and investigated several properties. In [4], Ahn and So introduced the notion of ideals in *BE*-algebras. They gave several descriptions of ideals in *BE*-algebras.

Various problems in system identification involve characteristics which are essentially nonprobabilistic in nature [5]. In response to this situation Zadeh [6] introduced *fuzzy set theory* as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [7]. To solve complicated problem in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [8]. Maji et al. [9] and Molodtsov [8] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [8] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [9] described the application of soft set theory to a decision-making problem. Maji et al. [10] also studied several operations on the theory of soft sets. Chen et al. [11] presented a new definition of soft set parametrization reduction and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. Çağman et al. [12] introduced fuzzy parameterized (FP) soft sets and their related properties. They proposed a decision-making method based on FP-soft set theory and provided an example which shows that the method can be successfully applied to the problems that contain uncertainties. Feng [13] considered the application of soft rough approximations in multicriteria group decision-making problems. Aktaş and Çağman [14] studied the basic concepts of soft set theory and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups.

In this paper, we introduce the notion of int-soft filter of a *BE*-algebra and investigate its properties. We

consider characterization of an int-soft filter and solve the problem of classifying int-soft subalgebras by their γ-inclusive filters. We provide conditions for a soft set to be an int-soft filter. We make a new int-soft filter from old one.

2. Preliminaries

Let $K(\tau)$ be the class of all algebras of type $\tau = (2,0)$. By a BE-algebra we mean a system $(X;*,1) \in K(\tau)$ in which the following axioms hold (see [3]):

$$(\forall x \in X) \quad (x * x = 1), \tag{1}$$

$$(\forall x \in X) \quad (x * 1 = 1), \tag{2}$$

$$(\forall x \in X) \quad (1 * x = x), \tag{3}$$

$$(\forall x, y, z \in X) \quad (x * (y * z) = y * (x * z)). \quad (\text{exchange}) \tag{4}$$

A relation "$\leq$" on a BE-algebra X is defined by

$$(\forall x, y \in X) \quad (x \leq y \Longleftrightarrow x * y = 1). \tag{5}$$

A BE-algebra $(X;*,1)$ is said to be *transitive* (see [4]) if it satisfies

$$(\forall x, y, z \in X) \quad (y * z \leq (x * y) * (x * z)). \tag{6}$$

A BE-algebra $(X;*,1)$ is said to be *self-distributive* (see [3]) if it satisfies

$$(\forall x, y, z \in X) \quad (x * (y * z) = (x * y) * (x * z)). \tag{7}$$

Every self-distributive BE-algebra $(X;*,1)$ satisfies the following properties:

$$(\forall x, y, z \in X) \quad (x \leq y \Longrightarrow z * x \leq z * y, \ y * z \leq x * z), \tag{8}$$

$$(\forall x, y \in X) \quad (x * (x * y) = x * y), \tag{9}$$

$$(\forall x, y, z \in X) \quad (x * y \leq (z * x) * (z * y)), \tag{10}$$

$$(\forall x, y, z \in X) \quad ((x * y) * (x * z) \leq x * (y * z)). \tag{11}$$

Note that every self distributive BE-algebra is transitive, but the converse is not true in general (see [4]).

Definition 1 (see [3]). Let $(X;*,1)$ be a BE-algebra and let F be a nonempty subset of X. Then F is a filter of X if

(F1) $1 \in F$,

(F2) $(\forall x, y \in X)$ $(x * y, x \in F \Rightarrow y \in F)$.

A soft set theory is introduced by Molodtsov [8]. In what follows, let U be an initial universe set and X a set of parameters. Let $\mathscr{P}(U)$ denote the power set of U and $A, B, C, \ldots \subseteq X$.

Definition 2 (see [8]). A soft set $(\tilde{f}, A)$ of X over U is defined to be the set of ordered pairs:

$$(\tilde{f}, A) := \{(x, \tilde{f}(x)) : x \in X, \tilde{f}(x) \in \mathscr{P}(U)\}, \tag{12}$$

where $\tilde{f} : X \to \mathscr{P}(U)$ such that $\tilde{f}(x) = \emptyset$ if $x \notin A$.

For a soft set $(\tilde{f}, A)$ of X and a subset γ of U, the γ-*inclusive set* of $(\tilde{f}, A)$, denoted by $i_A(\tilde{f};\gamma)$, is defined to be the set

$$i_A(\tilde{f};\gamma) := \{x \in A \mid \gamma \subseteq \tilde{f}(x)\}. \tag{13}$$

For any soft sets $(\tilde{f}, X)$ and $(\tilde{g}, X)$ of X, we call $(\tilde{f}, X)$ a *soft subset* of $(\tilde{g}, X)$, denoted by $(\tilde{f}, X)\tilde{\subseteq}(\tilde{g}, X)$, if $\tilde{f}(x) \subseteq \tilde{g}(x)$ for all $x \in X$. The *soft union* of $(\tilde{f}, X)$ and $(\tilde{g}, X)$, denoted by $(\tilde{f}, X)\tilde{\cup}(\tilde{g}, X)$, is defined to be the soft set $(\tilde{f}\,\tilde{\cup}\,\tilde{g}, X)$ of X over U in which $\tilde{f}\,\tilde{\cup}\,\tilde{g}$ is defined by

$$(\tilde{f}\,\tilde{\cup}\,\tilde{g})(x) = \tilde{f}(x) \cup \tilde{g}(x) \quad \forall x \in M. \tag{14}$$

The *soft intersection* of $(\tilde{f}, X)$ and $(\tilde{g}, X)$, denoted by $(\tilde{f}, X)\,\tilde{\cap}\,(\tilde{g}, X)$, is defined to be the soft set $(\tilde{f}\,\tilde{\cap}\,\tilde{g}, M)$ of X over U in which $\tilde{f}\,\tilde{\cap}\,\tilde{g}$ is defined by

$$(\tilde{f}\,\tilde{\cap}\,\tilde{g})(x) = \tilde{f}(x) \cap \tilde{g}(x) \quad \forall x \in S. \tag{15}$$

3. Int-Soft Filters

In what follows, we take a BE-algebra X, as a set of parameters unless otherwise specified.

Definition 3 (see [15]). A soft set $(\tilde{f}, X)$ of X is called an *int-soft subalgebra* of X if it satisfies

$$(\forall x, y \in X) \quad (\tilde{f}(x * y) \supseteq \tilde{f}(x) \cap \tilde{f}(y)). \tag{16}$$

Definition 4. A soft set $(\tilde{f}, X)$ of X over U is called an *int-soft filter* of X if it satisfies

$$(\forall x \in X) \quad (\tilde{f}(1) \supseteq \tilde{f}(x)), \tag{17}$$

$$(\forall x, y \in X) \quad (\tilde{f}(x * y) \cap \tilde{f}(x) \subseteq \tilde{f}(y)). \tag{18}$$

Example 5. Let $E = X$ be the set of parameters where $X = \{1, a, b, c\}$ is a BE-algebra with the following Cayley table:

$$\begin{array}{c|cccc} * & 1 & a & b & c \\ \hline 1 & 1 & a & b & c \\ a & 1 & 1 & a & a \\ b & 1 & 1 & 1 & a \\ c & 1 & a & a & 1 \end{array}. \tag{19}$$

Let $(\tilde{f}, X)$ be a soft set of X over U defined as follows:

$$\tilde{f} : X \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \gamma_2 & \text{if } x \in \{1, c\} \\ \gamma_1 & \text{if } x \in \{a, b\}, \end{cases} \tag{20}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. It is easy to check that $(\tilde{f}, X)$ is an int-soft filter of X.

Proposition 6. *Every int-soft filter $(\tilde{f}, X)$ of X over U satisfies the following properties:*

(i) $(\forall x, y \in X)$ $(x \leq y \Rightarrow \tilde{f}(x) \subseteq \tilde{f}(y))$,

(ii) $(\forall x, y, z \in X)$ $(\tilde{f}(x * z) \supseteq \tilde{f}(x * (y * z)) \cap \tilde{f}(y))$.

Proof. (i) Let $x, y \in X$ be such that $x \leq y$. Then $x * y = 1$. It follows from (17) and (18) that

$$\tilde{f}(x) = \tilde{f}(1) \cap \tilde{f}(x) = \tilde{f}(x * y) \cap \tilde{f}(x) \subseteq \tilde{f}(y). \quad (21)$$

(ii) Using (18) and (4), we obtain

$$\begin{aligned} \tilde{f}(x * z) &\supseteq \tilde{f}(y * (x * z)) \cap \tilde{f}(y) \\ &= \tilde{f}(x * (y * z)) \cap \tilde{f}(y) \end{aligned} \quad (22)$$

for all $x, y, z \in X$. □

We provide conditions for a soft set to be an int-soft filter.

Theorem 7. *If a soft set $(\tilde{f}, X)$ of X over U satisfies* (17) *and Proposition 6(ii), then it is an int-soft filter of X.*

Proof. Taking $x := 1$ in Proposition 6(ii) and using (3), we have

$$\begin{aligned} \tilde{f}(z) &= \tilde{f}(1 * z) \supseteq \tilde{f}(1 * (y * z)) \cap \tilde{f}(y) \\ &= \tilde{f}(y * z) \cap \tilde{f}(y) \end{aligned} \quad (23)$$

for all $y, z \in X$. Hence $(\tilde{f}, X)$ is an int-soft filter of X. □

Corollary 8. *Let $(\tilde{f}, X)$ be a soft set of X over U. Then $(\tilde{f}, X)$ is an int-soft filter of X over U if and only if it satisfies* (17) *and Proposition 6(ii).*

Lemma 9. *Every int-soft filter $(\tilde{f}, X)$ of X over U satisfies the following inclusion:*

$$(\forall a, x \in X) \quad \left(\tilde{f}(a) \subseteq \tilde{f}((a * x) * x)\right). \quad (24)$$

Proof. If we take $y = (a * x) * x$ and $x = a$ in (18), then

$$\begin{aligned} \tilde{f}((a * x) * x) &\supseteq \tilde{f}(a * ((a * x) * x)) \cap \tilde{f}(a) \\ &= \tilde{f}((a * x) * (a * x)) \cap \tilde{f}(a) \\ &= \tilde{f}(1) \cap \tilde{f}(a) = \tilde{f}(a) \end{aligned} \quad (25)$$

by using (4), (1), and (17). □

Theorem 10. *A soft set $(\tilde{f}, X)$ of X over U is an int-soft filter of X over U if and only if it satisfies the following conditions:*

(i) $(\forall x, y \in X)$ $(\tilde{f}(y * x) \supseteq \tilde{f}(x))$,

(ii) $(\forall x, a, b \in X)$ $(\tilde{f}((a * (b * x)) * x) \supseteq \tilde{f}(a) \cap \tilde{f}(b))$.

Proof. Assume that $(\tilde{f}, X)$ is an int-soft filter of X over U. Using (18), (4), (1), (2), and (17), we get

$$\tilde{f}(y * x) \supseteq \tilde{f}(x * (y * x)) \cap \tilde{f}(x) = \tilde{f}(1) \cap \tilde{f}(x) = \tilde{f}(x) \quad (26)$$

for all $x, y \in X$. Using Proposition 6(ii) and Lemma 9, we have

$$\begin{aligned} \tilde{f}((a * (b * x)) * x) &\supseteq \tilde{f}((a * (b * x)) * (b * x)) \cap \tilde{f}(b) \\ &\supseteq \tilde{f}(a) \cap \tilde{f}(b) \end{aligned} \quad (27)$$

for any $x, a, b \in X$.

Conversely, let $(\tilde{f}, X)$ be a soft set of X over U satisfying conditions (i) and (ii). If we take $y := x$ in (i), then $\tilde{f}(1) = \tilde{f}(x * x) \supseteq \tilde{f}(x)$ for all $x \in X$. Using (ii), we obtain

$$\begin{aligned} \tilde{f}(y) &= \tilde{f}(1 * y) = \tilde{f}(((x * y) * (x * y)) * y) \\ &\supseteq \tilde{f}(x * y) \cap \tilde{f}(x) \end{aligned} \quad (28)$$

for all $x, y \in X$. Hence $(\tilde{f}, X)$ is an int-soft filter over U. □

Proposition 11. *Let $(\tilde{f}, X)$ be a soft set of X over U. Then $(\tilde{f}, X)$ is an int-soft filter of X over U if and only if it satisfies*

$$(\forall x, y, z \in X) \quad \left(z \leq x * y \Longrightarrow \tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(z)\right). \quad (29)$$

Proof. Assume that $(\tilde{f}, X)$ is an int-soft filter of X over U. Let $x, y, z \in X$ be such that $z \leq x * y$. By Proposition 6(i) and (18), we have

$$\tilde{f}(y) \supseteq \tilde{f}(x * y) \cap \tilde{f}(x) \supseteq \tilde{f}(z) \cap \tilde{f}(x). \quad (30)$$

Conversely, suppose that $(\tilde{f}, X)$ satisfies (29). By (2), we $x \leq x * 1 = 1$. Hence $\tilde{f}(1) \supseteq \tilde{f}(x)$ for all $x \in X$ by (29). Thus (17) is valid. Using (1) and (4), we obtain $x \leq (x * y) * y$ for all $x, y \in X$. By (29), we get $\tilde{f}(y) \supseteq \tilde{f}(x * y) \cap \tilde{f}(x)$. Hence (18) holds. Therefore $(\tilde{f}, X)$ is an int-soft filter of X. □

As a generalization of Proposition 11, we have the following results.

Theorem 12. *If a soft set $(\tilde{f}, X)$ of X over U is an int-soft filter of X over U, then*

$$\prod_{i=1}^{n} w_i * x = 1 \Longrightarrow \tilde{f}(x) \supseteq \bigcap_{i=1}^{n} \tilde{f}(w_i) \quad (31)$$

for all $x, w_1, \ldots, w_n \in X$, where

$$\prod_{i=1}^{n} w_i * x = w_n * (w_{n-1} * (\cdots (w_1 * x) \cdots)). \quad (32)$$

Proof. The proof is by induction on n. Let $(\tilde{f}, X)$ be an int-soft filter of X over U. By Proposition 6(i) and (29), we know that condition (31) is valid for $n = 1, 2$. Assume that $(\tilde{f}, X)$ satisfies condition (31) for $n = k$; that is,

$$\prod_{i=1}^{k} w_i * x = 1 \Longrightarrow \tilde{f}(x) \supseteq \bigcap_{i=1}^{k} \tilde{f}(w_i) \tag{33}$$

for all $x, w_1, \ldots, w_k \in X$. Suppose that $\prod_{i=1}^{k+1} w_i * x = 1$ for all $x, w_1, \ldots, w_k, w_{k+1} \in X$. Then

$$\tilde{f}(w_1 * x) \supseteq \bigcap_{i=2}^{k+1} \tilde{f}(w_i). \tag{34}$$

Since $(\tilde{f}, X)$ is an int-soft filter of X, it follows from (18) that

$$\begin{aligned} \tilde{f}(x) &\supseteq \tilde{f}(w_1 * x) \cap \tilde{f}(w_1) \\ &\supseteq \left(\bigcap_{i=2}^{k+1} \tilde{f}(w_i)\right) \cap \tilde{f}(w_1) \\ &= \bigcap_{i=1}^{k+1} \tilde{f}(w_i). \end{aligned} \tag{35}$$

This completes the proof. □

Now we consider the converse of Theorem 12.

Theorem 13. *Let $(\tilde{f}, X)$ be a soft set of X over U satisfying* (31). *Then $(\tilde{f}, X)$ is an int-soft filter of X over U.*

Proof. Let $x, y, z \in X$ be such that $z \leq x * y$. Then $z * (x * y) = 1$, and so $\tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(z)$ by (31). It follows from Proposition 11 that $(\tilde{f}, X)$ is an int-soft filter of X over U. □

Theorem 14. *A soft set $(\tilde{f}, X)$ of X over U is an int-soft filter of X over U if and only if the γ-inclusive set $i_X(\tilde{f}; \gamma)$ is a filter of X for all $\gamma \in \mathscr{P}(U)$ with $i_X(\tilde{f}; \gamma) \neq \emptyset$.*

The filter $i_X(\tilde{f}; \gamma)$ in Theorem 14 is called the *inclusive filter* of X.

Proof. Assume that $(\tilde{f}, X)$ is an int-soft filter over U. Let $x, y \in X$ and $\gamma \in \mathscr{P}(U)$ be such that $x * y \in i_X(\tilde{f}; \gamma)$ and $x \in i_X(\tilde{f}; \gamma)$. Then $\gamma \subseteq \tilde{f}(x)$ and $\gamma \subseteq \tilde{f}(x * y)$. It follows from (17) and (18) that $\gamma \subseteq \tilde{f}(x) \subseteq \tilde{f}(1)$ and $\gamma \subseteq \tilde{f}(x * y) \cap \tilde{f}(x) \subseteq \tilde{f}(y)$ for all $x, y \in X$. Hence $1 \in i_X(\tilde{f}; \gamma)$ and $y \in i_X(\tilde{f}; \gamma)$. Thus $i_X(\tilde{f}, X)$ is a filter of X.

Conversely, suppose that $i_X(\tilde{f}; \gamma)$ is a filter of X for all $\gamma \in \mathscr{P}(U)$ with $i_X(\tilde{f}; \gamma) \neq \emptyset$. For any $x \in X$, let $\tilde{f}(x) = \gamma$. Then $x \in i_X(\tilde{f}; \gamma)$. Since $i_X(\tilde{f}; \gamma)$ is a filter of X, we have $1 \in i_X(\tilde{f}; \gamma)$ and so $\tilde{f}(x) = \gamma \subseteq \tilde{f}(1)$. For any $x, y \in X$, let $\tilde{f}(x * y) = \gamma_{x*y}$ and $\tilde{f}(x) = \gamma_x$. Take $\gamma = \gamma_{x*y} \cap \gamma_x$. Then $x * y \in i_X(\tilde{f}; \gamma)$ and $x \in i_X(\tilde{f}; \gamma)$ which imply that $y \in i_X(\tilde{f}; \gamma)$. Hence

$$\tilde{f}(y) \supseteq \gamma = \gamma_{x*y} \cap \gamma_x = \tilde{f}(x * y) \cap \tilde{f}(x). \tag{36}$$

Thus $(\tilde{f}, X)$ is an int-soft filter of X over U. □

We make a new int-soft filter from old one.

Theorem 15. *Let $(\tilde{f}, X) \in S(U)$ and define a soft set $(\tilde{f}^*, X)$ of X over U by*

$$\tilde{f}^* : X \longrightarrow \mathscr{P}(U), \quad x \longmapsto \begin{cases} \tilde{f}(x) & \text{if } x \in i_X(\tilde{f}; \gamma), \\ \emptyset & \text{otherwise}, \end{cases} \tag{37}$$

where γ is a nonempty subset of U. If $(\tilde{f}, X)$ is an int-soft filter of X, then so is $(\tilde{f}^*, X)$.

Proof. Assume that $(\tilde{f}, X)$ is an int-soft filter of X. Then $i_X(\tilde{f}; \gamma)$ $(\neq \emptyset)$ is a filter of X over U for all $\gamma \subseteq U$ by Theorem 14. Hence $1 \in i_X(\tilde{f}; \gamma)$, and so $\tilde{f}^*(1) = \tilde{f}(1) \supseteq \tilde{f}(x) \supseteq \tilde{f}^*(x)$ for all $x \in X$. Let $x, y \in X$. If $x * y \in i_X(\tilde{f}; \gamma)$ and $x \in i_X(\tilde{f}; \gamma)$, then $y \in i_X(\tilde{f}; \gamma)$. Hence

$$\tilde{f}^*(y) = \tilde{f}(y) \supseteq \tilde{f}(x * y) \cap \tilde{f}(x) = \tilde{f}^*(x * y) \cap \tilde{f}^*(x). \tag{38}$$

If $x * y \notin i_X(\tilde{f}; \gamma)$ or $x \notin i_X(\tilde{f}; \gamma)$, then $\tilde{f}^*(x * y) = \emptyset$ or $\tilde{f}^*(x) = \emptyset$. Thus

$$\tilde{f}^*(y) \supseteq \emptyset = \tilde{f}^*(x * y) \cap \tilde{f}^*(x). \tag{39}$$

Therefore $(\tilde{f}^*, X)$ is an int-soft filter of X. □

For two elements a and b of X, consider a soft set $(\tilde{f}_a^b, X)$ over U where

$$\tilde{f}_a^b : X \longrightarrow \mathscr{P}(U), \quad x \longmapsto \begin{cases} \gamma_1 & \text{if } a * (b * x) = 1, \\ \gamma_2 & \text{otherwise}, \end{cases} \tag{40}$$

where γ_1 and γ_2 are subsets of U with $\gamma_2 \subsetneq \gamma_1$. In the following example, we know that there exists $a, b \in X$ such that $(\tilde{f}_a^b, X)$ is not an int-soft filter of X.

Example 16. Consider the *BE*-algebra $X = \{1, a, b, c, d, 0\}$ which is given in Example 19. Then $(\tilde{f}_1^a, X)$ is not an int-soft filter of X over U since

$$\tilde{f}_1^a(a * b) \cap \tilde{f}_1^a(a) \nsubseteq \tilde{f}_1^a(b). \tag{41}$$

Now we provide a condition for the soft set $(\tilde{f}_a^b, X)$ to be an int-soft filter of X over U for all $a, b \in X$.

Theorem 17. *If X is self distributive, then the soft set $(\tilde{f}_a^b, X)$ is an int-soft filter of X over U for all $a, b \in X$.*

Proof. Let $a, b \in X$. Obviously, $\tilde{f}_a^b(1) \supseteq \tilde{f}_a^b(x)$ for all $x \in X$. Let $x, y \in X$ be such that $a * (b * (x * y)) \neq 1$ or $a * (b * x) \neq 1$. Then $\tilde{f}_a^b(x * y) = \gamma_2$ or $\tilde{f}_a^b(x) = \gamma_2$. Hence

$$\tilde{f}_a^b(x * y) \cap \tilde{f}_a^b(x) = \gamma_2 \subseteq \tilde{f}_a^b(y). \tag{42}$$

Assume that $a * (b * (x * y)) = 1$ and $a * (b * x) = 1$. Then

$$\begin{aligned} 1 &= a * (b * (x * y)) \\ &= a * ((b * x) * (b * y)) \\ &= (a * (b * x)) * (a * (b * y)) \\ &= 1 * (a * (b * y)) \\ &= a * (b * y), \end{aligned} \tag{43}$$

and so $\tilde{f}_a^b(x * y) \cap \tilde{f}_a^b(x) = \gamma_1 = \tilde{f}_a^b(y)$. Therefore $(\tilde{f}_a^b, X)$ is an int-soft filter of X over U for all $a, b \in X$. □

Theorem 18. *If $(\tilde{f}, X)$ and $(\tilde{g}, X)$ are int-soft filters of X, then the soft intersection $(\tilde{f}, X) \tilde{\cap} (\tilde{g}, X)$ of $(\tilde{f}, X)$ and $(\tilde{g}, X)$ is an int-soft filter of X.*

Proof. For any $x \in X$, we have

$$\left(\tilde{f} \tilde{\cap} \tilde{g}\right)(1) = \tilde{f}(1) \cap \tilde{g}(1) \supseteq \tilde{f}(x) \cap \tilde{g}(x) = \left(\tilde{f} \tilde{\cap} \tilde{g}\right)(x). \tag{44}$$

Let $x, y \in X$. Then

$$\begin{aligned} \left(\tilde{f} \tilde{\cap} \tilde{g}\right)(y) &= \tilde{f}(y) \cap \tilde{g}(y) \\ &\supseteq \left(\tilde{f}(x * y) \cap \tilde{f}(x)\right) \cap \left(\tilde{g}(x * y) \cap \tilde{g}(x)\right) \\ &= \left(\tilde{f}(x * y) \cap \tilde{g}(x * y)\right) \cap \left(\tilde{f}(x) \cap \tilde{g}(x)\right) \\ &= \left(\tilde{f} \tilde{\cap} \tilde{g}\right)(x * y) \cap \left(\tilde{f} \tilde{\cap} \tilde{g}\right)(x). \end{aligned} \tag{45}$$

Hence $(\tilde{f}, X) \tilde{\cap} (\tilde{g}, X)$ is an int-soft filter of X. □

The following example shows that the soft union of int-soft filters of X may not be an int-soft filter of X.

Example 19. Let $E = X$ be the set of parameters and $U = X$ the initial universe set, where $X = \{1, a, b, c, d, 0\}$ is a BE-algebra with the following Cayley table (see [4]):

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

(46)

Let $(\tilde{f}, X)$ and $(\tilde{g}, X)$ be soft sets of X over U defined, respectively, as follows:

$$\begin{aligned} \tilde{f} : X &\longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \gamma_3 & \text{if } x \in \{1, c\} \\ \gamma_1 & \text{if } x \in \{a, b, d, 0\}, \end{cases} \\ \tilde{g} : X &\longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \gamma_4 & \text{if } x \in \{1, a, b\} \\ \gamma_2 & \text{if } x \in \{c, d, 0\}, \end{cases} \end{aligned} \tag{47}$$

where $\gamma_1, \gamma_2, \gamma_3$, and γ_4 are subsets of U with $\gamma_1 \subsetneq \gamma_2 \subsetneq \gamma_3 \subsetneq \gamma_4$. It is easy to check that $(\tilde{f}, X)$ and $(\tilde{g}, X)$ are int-soft filters of X over U. But $(\tilde{f}, X) \tilde{\cup} (\tilde{g}, X) = (\tilde{f} \tilde{\cup} \tilde{g}, X)$ is not an int-soft filter of X over U, since

$$\begin{aligned} &\left(\tilde{f} \tilde{\cup} \tilde{g}\right)(c * d) \cap \left(\tilde{f} \tilde{\cup} \tilde{g}\right)(c) \\ &\quad = \left(\tilde{f} \tilde{\cup} \tilde{g}\right)(a) \cap \left(\tilde{f} \tilde{\cup} \tilde{g}\right)(c) \\ &\quad = \left(\tilde{f}(a) \cup \tilde{g}(a)\right) \cap \left(\tilde{f}(c) \cup \tilde{g}(c)\right) \\ &\quad = \gamma_4 \cap \gamma_3 = \gamma_3 \nsubseteq \gamma_2 = \gamma_1 \cup \gamma_2 \\ &\quad = \tilde{f}(d) \cup \tilde{g}(d) = \left(\tilde{f} \tilde{\cup} \tilde{g}\right)(d). \end{aligned} \tag{48}$$

Theorem 20. *Let $(\tilde{f}, X)$ be an int-soft filter of X. Let γ_1 and γ_2 be subsets of U such that $\gamma_1 \subsetneq \gamma_2$. If the γ_1-inclusive set of $(\tilde{f}, X)$ is equal to the γ_2-inclusive set of $(\tilde{f}, X)$, then there is no $x \in X$ such that $\gamma_1 \subseteq \tilde{f}(x) \subsetneq \gamma_2$.*

Proof. The proof is straightforward. □

The converse of Theorem 20 is not true in general as seen in the following example.

Example 21. Let $E = X$ be the set of parameters and $U = X$ the initial universe set where $X = \{1, a, b, c\}$ is a BE-algebra as in Example 5. Consider a soft set $(\tilde{f}, X)$ of X over U which is given by

$$\tilde{f} : X \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} X & \text{if } x = 1, \\ \{1, a\} & \text{if } x \in \{a, b\}, \\ \{1, a, c\} & \text{if } x = c. \end{cases} \tag{49}$$

Then $(\tilde{f}, X)$ is an int-soft filter of X. The γ-inclusive sets of $(\tilde{f}, X)$ are described as follows:

$$i_X\left(\tilde{f}; \gamma\right) = \begin{cases} X & \text{if } \gamma \in \{\emptyset, \{1\}, \{a\}, \{1, a\}\} \\ \{1, c\} & \text{if } \gamma \in \{\{c\}, \{a, c\}, \{1, a, c\}\}, \\ \{1\} & \text{if } \gamma \in \dfrac{\mathscr{P}(U)}{\{\emptyset, \{1\}, \{a\}, \{1, a\}, \{c\}, \{a, c\}, \{1, a, c\}\}}. \end{cases} \tag{50}$$

If we take $\gamma_1 = \{1, c\}$ and $\gamma_2 = \{1, b, c\}$, then $\gamma_1 \subsetneq \gamma_2$ and there is no $x \in X$ such that $\gamma_1 \subseteq \tilde{f}(x) \subsetneq \gamma_2$. But $i_X(\tilde{f}; \gamma_1) = \{1, c\} \neq \{1\} = i_X(\tilde{f}; \gamma_2)$.

Theorem 22. *Let $(\tilde{f}, X)$ be an int-soft filter of X. Let γ_1 and γ_2 be subsets of U such that $\gamma_1 \subsetneq \gamma_2$ and $\{\gamma_1, \gamma_2, \tilde{f}(x)\}$ is totally ordered by set inclusion for all $x \in X$. If there is no $x \in X$ such that $\gamma_1 \subseteq \tilde{f}(x) \subsetneq \gamma_2$, then the γ_1-inclusive set of $(\tilde{f}, X)$ is equal to the γ_2-inclusive set of $(\tilde{f}, X)$.*

Proof. Since $\gamma_1 \subsetneq \gamma_2$, we have $i_X(\tilde{f}; \gamma_2) \subseteq i_X(\tilde{f}; \gamma_1)$. If $x \in i_X(\tilde{f}; \gamma_1)$, then $\gamma_1 \subseteq \tilde{f}(x)$. Since $\{\gamma_1, \gamma_2, \tilde{f}(x) \mid x \in X\}$ is

totally ordered by set inclusion and there is no $x \in X$ such that $\gamma_1 \subseteq \tilde{f}(x) \subsetneq \gamma_2$, it follows that $\gamma_2 \subseteq \tilde{f}(x)$; that is, $x \in i_X(\tilde{f};\gamma_2)$. Therefore the γ_1-inclusive set of $(\tilde{f}, X)$ is equal to the γ_2-inclusive set of $(\tilde{f}, X)$. □

We have the following question.

Question. Given an int-soft filter $(\tilde{f}, X)$ of X, does any filter can be represented as a γ-inclusive set of $(\tilde{f}, X)$?

The following example shows that the answer to the question above is false.

Example 23. Let $E = X$ be the set of parameters and $U = X$ the initial universe set where $X = \{1, a, b, c\}$ is a *BE*-algebra as in Example 5. Consider a soft set $(\tilde{f}, X)$ of X over U which is given by

$$\tilde{f} : X \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \{1, c\} & \text{if } x = 1, \\ \{c\} & \text{if } x \in \{a, b, c\}. \end{cases} \tag{51}$$

Then $(\tilde{f}, X)$ is an int-soft filter of X. The γ-inclusive sets of $(\tilde{f}, X)$ are described as follows:

$$i_X(\tilde{f};\gamma) = \begin{cases} X & \text{if } \gamma \in \{\emptyset, \{c\}\}, \\ \{1\} & \text{if } \gamma \in \{\{1\}, \{1, c\}\}, \\ \emptyset & \text{otherwise.} \end{cases} \tag{52}$$

The filter $\{1, b\}$ cannot be a γ-inclusive set $i_X(\tilde{f};\gamma)$, since there is no $\gamma \subseteq U$ such that $i_X(\tilde{f};\gamma) = \{1, b\}$.

However, we have the following theorem.

Theorem 24. *Every filter of a BE-algebra can be represented as a γ-inclusive set of an int-soft filter.*

Proof. Let F be a filter of a *BE*-algebra X. For a subset γ of U, define a soft set $(\tilde{f}, X)$ over U by

$$\tilde{f} : X \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \gamma & \text{if } x \in F, \\ \emptyset & \text{if } x \notin F. \end{cases} \tag{53}$$

Obviously, $F = i_X(\tilde{f};\gamma)$. We now prove that $(\tilde{f}, X)$ is an int-soft filter of X. Since $1 \in F = i_X(\tilde{f};\gamma)$, we have $\tilde{f}(1) \supseteq \gamma \supseteq \tilde{f}(x)$ for all $x \in X$. Let $x, y \in X$. If $x * y \in F$ and $x \in F$, then $y \in F$ because F is a filter of X. Hence $\tilde{f}(x * y) = \tilde{f}(x) = \tilde{f}(y) = \gamma$, and so $\tilde{f}(x * y) \cap \tilde{f}(x) \subseteq \tilde{f}(y)$. If $x * y \notin F$ or $x \notin F$, then $\tilde{f}(x * y) = \emptyset$ or $\tilde{f}(x) = \emptyset$. Hence $\tilde{f}(x * y) \cap \tilde{f}(x) = \emptyset \subseteq \tilde{f}(y)$. Therefore $(\tilde{f}, X)$ is an int-soft filter of X. □

Note that if $E = X$ is a finite *BE*-algebra, then the number of filters of X over U is finite whereas the number of γ-inclusive sets of an int-soft filter of X over U appears to be infinite. But, since every γ-inclusive set is indeed a filter of X, not all these γ-inclusive sets are distinct. The next theorem characterizes this aspect.

Theorem 25. *Let $(\tilde{f}, X)$ be an int-soft filter of X over U and let $\gamma_1 \subsetneq \gamma_2 \subseteq U$ be such that $\{\gamma_1, \gamma_2, \tilde{f}(x)\}$ is a chain for all $x \in X$. Two γ-inclusive sets $i_X(\tilde{f};\gamma_1)$ and $i_X(\tilde{f};\gamma_2)$ are equal if and only if there is no $x \in X$ such that $\gamma_1 \subsetneq \tilde{f}(x) \subsetneq \gamma_2$.*

Proof. Let γ_1 and γ_2 be subsets of U such that $i_X(\tilde{f};\gamma_1) = i_X(\tilde{f};\gamma_2)$. Assume that there exists $x \in X$ such that $\gamma_1 \subsetneq \tilde{f}(x) \subsetneq \gamma_2$. Then $i_X(\tilde{f};\gamma_2)$ is a proper subset of $i_X(\tilde{f};\gamma_1)$, which contradicts the hypothesis.

Conversely, suppose that there is no $x \in X$ such that $\gamma_1 \subsetneq \tilde{f}(x) \subsetneq \gamma_2$. Obviously, $i_X(\tilde{f};\gamma_2) \subseteq i_X(\tilde{f};\gamma_1)$. If $x \in i_X(\tilde{f};\gamma_1)$, then $\gamma_1 \subseteq \tilde{f}(x)$. It follows from the assumption that $\gamma_2 \subseteq \tilde{f}(x)$; that is, $x \in i_X(\tilde{f};\gamma_2)$. Therefore $i_X(\tilde{f};\gamma_1) = i_X(\tilde{f};\gamma_2)$. □

Let $(\tilde{f}, X)$ be a soft set of X. For any $a, b \in X$ and $k \in \mathbb{N}$; consider the set

$$\tilde{f}\left[a^k; b\right] := \left\{x \in X \mid \tilde{f}\left(a^k * (b * x)\right) = \tilde{f}(1)\right\}, \tag{54}$$

where $\tilde{f}(a^k * x) = \tilde{f}(a * (a * (\cdots * (a * (a * x)) \cdots)))$ in which a appears k-times. Note that $a, b, 1 \in \tilde{f}[a^k; b]$ for all $a, b \in X$ and $k \in \mathbb{N}$.

Proposition 26. *Let $(\tilde{f}, X)$ be a soft set of X over U such that condition* (17) *and $\tilde{f}(x * y) = \tilde{f}(x) \cup \tilde{f}(y)$ for all $x, y \in X$. For any $a, b \in X$ and $k \in \mathbb{N}$, if $x \in \tilde{f}[a^k; b]$, then $y * x \in \tilde{f}[a^k; b]$ for all $y \in X$.*

Proof. Assume that $x \in \tilde{f}[a^k; b]$. Then $\tilde{f}(a^k * (b * x)) = \tilde{f}(1)$, and so

$$\begin{aligned} \tilde{f}\left(a^k * (b * (y * x))\right) &= \tilde{f}\left(a^k * (y * (b * x))\right) \\ &= \tilde{f}\left(y * \left(a^k * (b * x)\right)\right) \\ &= \tilde{f}(y) \cup \tilde{f}\left(a^k * (b * x)\right) \\ &= \tilde{f}(y) \cup \tilde{f}(1) = \tilde{f}(1) \end{aligned} \tag{55}$$

for all $y \in X$ by the exchange property of the operation $*$. Hence $y * x \in \tilde{f}[a^k; b]$ for all $y \in X$. □

Proposition 27. *For any soft set $(\tilde{f}, X)$ of X, let $a \in X$ satisfy the following assertion:*

$$(\forall x \in X) \quad (a * x = 1). \tag{56}$$

Then $\tilde{f}[a^k; b] = X = \tilde{f}[b^k; a]$ for all $b \in X$ and $k \in \mathbb{N}$.

Proof. For any $x \in X$, we have

$$\begin{aligned} \tilde{f}\left(a^k * (b * x)\right) &= \tilde{f}\left(a^{k-1} * (a * (b * x))\right) \\ &= \tilde{f}\left(a^{k-1} * (b * (a * x))\right) \\ &= \tilde{f}\left(a^{k-1} * (b * 1)\right) \\ &= \tilde{f}(1), \end{aligned} \tag{57}$$

and so $x \in \tilde{f}[a^k; b]$. Similarly, $x \in \tilde{f}[b^k; a]$. □

Proposition 28. *Let X be a self distributive BE-algebra and let $(\tilde{f}, X)$ be an order-preserving soft set of X over U with the property* (17). *If $b \leq c$ in X, then $\tilde{f}[a^k; c] \subseteq \tilde{f}[a^k; b]$ for all $a \in X$ and $k \in \mathbb{N}$.*

Proof. Let $a, b, c, \in X$ be such that $b \leq c$. For any $k \in \mathbb{N}$, if $x \in \tilde{f}[a^k; c]$, then

$$\begin{aligned} \tilde{f}(1) &= \tilde{f}\left(a^k * (c * x)\right) = \tilde{f}\left(c * \left(a^k * x\right)\right) \\ &\subseteq \tilde{f}\left(b * \left(a^k * x\right)\right) = \tilde{f}\left(a^k * (b * x)\right) \end{aligned} \tag{58}$$

by (4) and (8), and so $\tilde{f}(a^k * (b * x)) = \tilde{f}(1)$. Thus $x \in \tilde{f}[a^k; b]$, which completes the proof. □

The following example shows that there exists a soft set $(\tilde{f}, X)$ of $X, a, b \in X$ and $k \in \mathbb{N}$ such that $\tilde{f}[a^k; b]$ is not a filter of X.

Example 29. Let $E = X$ be the set of parameters and $U = X$ the initial universe set where $X = \{1, a, b, c\}$ is a *BE*-algebra as in Example 5. Consider a soft set $(\tilde{f}, X)$ of X over U which is given by

$$\tilde{f} : X \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \gamma_2 & \text{if } x = 1, \\ \gamma_1 & \text{if } x \in \{a, b, c\}, \end{cases} \tag{59}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. Then it is a soft set of X over U. But $\tilde{f}[c; b] = \{x \in X \mid \tilde{f}(c * (b * x)) = \tilde{f}(1)\} = \{1, a, b\}$ is not a filter, since $a * c = a \in \tilde{f}[c; b]$ and $c \notin \tilde{f}[c; b]$.

We provide conditions for a set $\tilde{f}[a^k; b]$ to be a filter.

Theorem 30. *Let $(\tilde{f}, X)$ be a soft set over X. If X is a self distributive BE-algebra and $\tilde{f}$ is injective, then $\tilde{f}[a^k; b]$ is a filter of X for all $a, b \in X$ and $k \in \mathbb{N}$.*

Proof. Assume that X is a self distributive *BE*-algebra and $\tilde{f}$ is injective. Obviously, $1 \in \tilde{f}[a^k; b]$. Let $a, b, x, y \in X$ and $k \in \mathbb{N}$ be such that $x * y \in \tilde{f}[a^k; b]$ and $x \in \tilde{f}[a^k; b]$. Then $\tilde{f}(a^k * (b * x)) = \tilde{f}(1)$ which implies that $a^k * (b * x) = 1$ since $\tilde{f}$ is injective. Using (7), we have

$$\begin{aligned} \tilde{f}(1) &= \tilde{f}\left(a^k * (b * (x * y))\right) \\ &= \tilde{f}\left(a^{k-1} * (a * (b * (x * y)))\right) \\ &= \tilde{f}\left(a^{k-1} * (a * ((b * x) * (b * y)))\right) \\ &= \cdots \\ &= \tilde{f}\left(\left(a^k * (b * x)\right) * \left(a^k * (b * y)\right)\right) \\ &= \tilde{f}\left(1 * \left(a^k * (b * y)\right)\right) \\ &= \tilde{f}\left(a^k * (b * y)\right), \end{aligned} \tag{60}$$

which implies that $y \in \tilde{f}[a^k; b]$. Therefore $\tilde{f}[a^k; b]$ is a filter of X for all $a, b \in X$ and $k \in \mathbb{N}$. □

Theorem 31. *Let X be a self distributive BE-algebra. Let $(\tilde{f}, X)$ be a soft set of X over U satisfying condition* (17) *and*

$$(\forall x, y \in X) \quad \left(\tilde{f}(x * y) = \tilde{f}(x) \cap \tilde{f}(y)\right). \tag{61}$$

Then $\tilde{f}[a^k; b]$ is a filter of X for all $a, b \in X$ and $k \in \mathbb{N}$.

Proof. Let $a, b \in X$ and $k \in \mathbb{N}$. Obviously, $1 \in \tilde{f}[a^k; b]$. Let $x, y \in X$ be such that $x * y \in \tilde{f}[a^k; b]$ and $x \in \tilde{f}[a^k; b]$. Then $\tilde{f}(a^k * (b * x)) = \tilde{f}(1)$, which implies from (61) and (17) that

$$\begin{aligned} \tilde{f}(1) &= \tilde{f}\left(a^k * (b * (x * y))\right) \\ &= \tilde{f}\left(a^{k-1} * (a * (b * (x * y)))\right) \\ &= \tilde{f}\left(a^{k-1} * (a * ((b * x) * (b * y)))\right) \\ &= \cdots \\ &= \tilde{f}\left(\left(a^k * (b * x)\right) * \left(a^k * (b * y)\right)\right) \\ &= \tilde{f}\left(a^k * (b * x)\right) \cap \tilde{f}\left(a^k * (b * y)\right) \\ &= \tilde{f}(1) \cap \tilde{f}\left(a^k * (b * y)\right) \\ &= \tilde{f}\left(a^k * (b * y)\right). \end{aligned} \tag{62}$$

Hence $y \in \tilde{f}[a^k; b]$ and therefore $\tilde{f}[a^k; b]$ is a filter of X for all $a, b \in X$ and $k \in \mathbb{N}$. □

Proposition 32. *Let $(\tilde{f}, X)$ be a soft set of X over U in which $\tilde{f}$ is injective. If F is a filter of X, then the following assertion is valid:*

$$(\forall a, b \in F) \ (\forall k \in \mathbb{N}) \quad \left(\tilde{f}\left[a^k; b\right] \subseteq F\right). \tag{63}$$

Proof. Assume that F is a filter of X over U and let $a, b \in F$ and $k \in \mathbb{N}$. If $x \in \tilde{f}[a^k; b]$, then

$$\tilde{f}\left(a * \left(a^{k-1} * (b * x)\right)\right) = \tilde{f}\left(a^k * (b * x)\right) = \tilde{f}(1) \tag{64}$$

and so $a * (a^{k-1} * (b * x)) = 1 \in F$ since $\tilde{f}$ is injective. Since F is a filter of X, it follows from (F2) that $a^{k-1} * (b * x) \in F$. Continuing this process, we obtain $b * x \in F$ and so $x \in F$. Therefore $\tilde{f}[a^k; b] \subseteq F$ for all $a, b \in F$ and $k \in \mathbb{N}$. □

Theorem 33. *Let $(\tilde{f}, X)$ be a soft set of X. For any subset F of X, if condition* (63) *holds, then F is a filter of X.*

Proof. Suppose that condition (63) holds. Note that $1 \in \tilde{f}[a^k; b] \subseteq F$. Let $x, y \in X$ be such that $x * y \in F$ and $x \in F$. Then

$$\begin{aligned}\tilde{f}\left(x^k * ((x * y) * y)\right) &= \tilde{f}\left(x^{k-1} * (x * ((x * y) * y))\right) \\ &= \tilde{f}\left(x^{k-1} * ((x * y) * (x * y))\right) \\ &= \tilde{f}\left(x^{k-1} * 1\right) = \tilde{f}(1),\end{aligned} \tag{65}$$

and thus $y \in \tilde{f}[x^k; b] \subseteq F$ where $b = x * y$. Therefore F is a filter of X. □

Theorem 34. *Let $(\tilde{f}, X)$ be a soft set of X. If F is a filter of X, then*

$$(\forall k \in \mathbb{N}) \quad \left(F = \cup\left\{\tilde{f}\left[a^k; b\right] \mid a, b \in F\right\}\right). \tag{66}$$

Proof. Let F be a filter of X. By Proposition 32, the inclusion

$$\cup\left\{\tilde{f}\left[a^k; b\right] \mid a, b \in F\right\} \subseteq F \tag{67}$$

holds. Let $x \in F$. Since $x \in \tilde{f}[1^k; x]$ for all $k \in \mathbb{N}$, it follows that

$$F \subseteq \cup\left\{\tilde{f}\left[1^k; x\right] \mid x \in F\right\} \subseteq \cup\left\{\tilde{f}\left[a^k; b\right] \mid a, b \in F\right\}. \tag{68}$$

This completes the proof. □

4. Conclusion

Using the notion of int-soft sets, we have introduced the concept of int-soft filters in *BE*-algebras and investigated related properties. We have considered characterization of an int-soft filter and solved the problem of classifying int-soft filters by their inclusive filters. We have provided conditions for a soft set to be an int-soft filter. We have made a new int-soft filter from the old one. We have considered the soft intersection of int-soft filters and have shown that the soft union of int-soft filters is not an int-soft filter by providing a counterexample.

Work is ongoing. Some important issues for future work are

(1) to develop strategies for obtaining more valuable results,

(2) to study the soft set application in ideal theory of *BE*-algebras,

(3) to apply these notions and results for studying related notions in other (soft) algebraic structures,

(4) to study generalizations of soft set application in ideal and filter theory of *BE*-algebras.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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