Research Article

Adaptive Fault-Tolerant Control for Flight Systems with Input Saturation and Model Mismatch

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A novel scheme for fault-tolerant control is proposed in this paper, in which model reference adaptive control method is incorporated with control allocation to cope with simultaneous actuator failures, input saturation, and model mismatch in the flight system. In order to reduce performance degradation caused by actuator failures, the proposed scheme redistributes the control signal to healthy actuators and updates the weighting matrix based on actuator effectiveness. Because of saturation errors resulting from actuator constraints and model mismatch caused by abnormal changes in the system, the original reference model may not be appropriate. Under this circumstance, an adaptive reference model which can also provide satisfactory performance is designed. Simulations of a flight control example are given to illustrate the effectiveness of the proposed scheme.

1. Introduction

Actuator failures may adversely affect the stability and performance of flight systems. Fault-tolerant control (FTC) has been studied as a method for accommodating failures. In the past decades, a number of approaches have been proposed for FTC, including sliding mode control-based designs [1, 2], variable structure control-based designs [3], learning-based approaches [4], and adaptive control-based designs [5].

The availability of actuator redundancy is an important element of FTC. Control allocation (CA) is an approach that can effectively manage redundancy by distributing virtual control law requirements to redundant actuators in the most efficient manner [6]. There is an extensive literature on CA approaches and applications. Enns presented the linear and quadratic programming approaches in detail in [7]. Härkegård and Glad compared control allocation with optimal control for solving actuator redundancy in [8]. Several CA approaches taking into account actuator limits have been discussed [9–11]. Kishore et al. gave an algorithm that updated the weighting matrix to deal with actuator limits

in [9]. However, in the event of failure, only depending on updating the weighting matrix sometimes may be invalid.

In addition, flight systems may experience other disruptive factors such as model mismatch. Model mismatch can be caused by icing or damage to the aircraft. Joshi et al. first considered this problem in [12] and further developed his work in [13]. Although the previous work has achieved satisfactory tracking performance, none of them considered actuator limits.

In this paper, we develop a novel scheme combining CA with model reference adaptive control (MRAC) for flight systems with simultaneous actuator failures, model mismatch, and input saturation. On the basis of the existing literature, this paper proposes an improved control allocation (ICA) method that updates the weighting matrix based on actuator effectiveness. In addition to updating the weighting matrix, this paper employs the positive μ -modification method [14] to impose constraints on actuators. MRAC can guarantee the closed-loop stability of the system and reduce the error caused by actuator failures. Because of input saturation and model mismatch, the original reference model may not

be appropriate. We have designed an adaptive reference model that can provide satisfactory performance under these circumstances. Compared with the traditional CA methods, simulation results show that the new scheme provides better tracking performance. The control structure is shown in Figure 1. This paper focuses on the tracking control on the single system. In the recent study, the coordination control of multiagent systems has attracted considerable attention. Several approaches and applications have been developed in [15–17]. We will explore the FTC of multiple flight systems in our future study.

This paper is organized as follows. Section 2 presents the problem statement. Section 3 gives a detailed description of the novel adaptive fault-tolerant control scheme. Simulation results of a flight control example are given in Section 4. Section 5 is concluding remarks.

2. Problem Statement

Consider a linear time-invariant system in the presence of model mismatch and actuator failures described by

$$\begin{aligned}\dot{x}(t) &= (A + \delta_a) x(t) + B_u \Delta u(t), \\ y(t) &= Cx(t),\end{aligned}\quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the actual control input, $y(t) \in \mathcal{R}^p$ is the output, and A , B_u , C are constant matrices of appropriate dimensions. The matrix $\delta_a \in \mathcal{R}^{n \times n}$ denotes unknown parameter deviations or model mismatch caused by icing, modeling errors, or damage. The actuator effectiveness matrix $\Delta \in \mathcal{R}^{m \times m}$ is a piecewise constant uncertain matrix which can be expressed as $\Delta = \text{diag}(\delta_1, \delta_2, \dots, \delta_m)$, where $0 \leq \delta_i \leq 1$, $i = 1, 2, \dots, m$, indicates the effectiveness of the i th actuator. For example, $\delta_i = 1$ indicates that the i th actuator is working well; $\delta_i = 0$ implies that the i th actuator has damaged completely; $0 < \delta_i < 1$ means that the i th actuator has partially lost its effectiveness.

Assumption 1. Each element of the control input $u(t)$ is bounded by a constant such that

$$|u_i| \leq u_{\max_i} \quad \forall t > 0, i = 1, 2, \dots, m. \quad (2)$$

Assumption 2. There is sufficient actuator redundancy in the system so that the input matrix B_u can be factorized into a full column rank matrix $B_v \in \mathcal{R}^{n \times p}$ and a full row rank matrix $B \in \mathcal{R}^{p \times m}$; that is,

$$B_u = B_v B. \quad (3)$$

Assumption 3. $(A, B_u \Delta)$ is controllable.

Assumption 4. For the nominal system with $\Delta = I$ and $\delta_a = 0$, there exist gains $K_1 \in \mathcal{R}^{n \times p}$ and $K_2 \in \mathcal{R}^{p \times p}$ satisfying the following matching conditions:

$$A + B_v K_1^T = A_m; \quad B_v K_2 = B_m, \quad (4)$$

where $A_m \in \mathcal{R}^{n \times n}$ is Hurwitz and $B_m \in \mathcal{R}^{p \times p}$.

FIGURE 1: The overall control structure.

Based on Assumption 1, the static actuator model can be defined as follows [14]:

$$\begin{aligned}u_i(t) &= u_{\max_i} \text{sat} \left(\frac{u_{c_i}(t)}{u_{\max_i}} \right) \\ &= \begin{cases} u_{c_i}(t), & |u_{c_i}(t)| \leq u_{\max_i}, \\ u_{\max_i} \text{sgn}(u_{c_i}(t)), & |u_{c_i}(t)| > u_{\max_i}, \end{cases} \quad i = 1, \dots, m, \end{aligned} \quad (5)$$

where $u_{c_i}(t)$ denotes the i th element of the commanded control input $u_c(t)$. Based on the positive μ -modification method [14], the commanded control input $u_c(t)$ can be obtained by

$$u_c(t) = u_a(t) + \mu \tilde{u}_c(t), \quad (6)$$

where $\mu \in \mathcal{R}^{m \times m}$ is a diagonal matrix of design constants $\mu_1, \dots, \mu_m \geq 0$, $u_a(t)$ is the adaptive control input, and $\tilde{u}_c(t) = [\tilde{u}_{c_1}(t), \dots, \tilde{u}_{c_m}(t)]^T$, $\tilde{u}_{c_i}(t) = u_{\max_i}^{\delta_i} \text{sat}(u_{c_i}(t)/u_{\max_i}^{\delta_i}) - u_{c_i}(t)$, $i = 1, \dots, m$.

In addition, the saturation error can be defined as

$$\tilde{u}(t) = u(t) - u_c(t) \quad (7)$$

which may lead to performance degradation or even system instability.

Based on Assumption 4, we can define the conventional reference model as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \quad (8)$$

where $x_m(t) \in \mathcal{R}^n$ is the reference model state and $r(t) \in \mathcal{R}^p$ is a bounded reference input. Optimal and robust control theory can be employed to design the gain matrices K_1 , K_2 . However, due to model mismatch δ_a , $(A + \delta_a + B_v K_1^T)$ will not have the same structure as A_m for any K_1 . Therefore, stability and asymptotic tracking performance can no longer be guaranteed.

In response, this paper designs an adaptive reference model while taking saturation error $\tilde{u}$ and model mismatch δ_a into consideration. The control objective is to develop a novel adaptive fault-tolerant control scheme, for the faulty system given by (1) with simultaneous actuator failures, model mismatch, and input saturation, to achieve the desired tracking performance.

3. Adaptive Fault-Tolerant Control

In order to achieve the desired control objective, we propose an adaptive fault-tolerant control scheme in this section. The scheme mainly incorporates an improved control allocation method and an adaptive virtual controller design.

3.1. Improved Control Allocation Method. Substituting (6) and (7) into (1) yields the following closed-loop system dynamics:

$$\dot{x}(t) = (A + \delta_a) x(t) + B_f u_a(t) + B_f \tilde{u}_a(t), \quad (9)$$

where

$$B_f = B_u \Delta \tilde{u}_a(t) = \mu \tilde{u}_c(t) + \tilde{u}(t). \quad (10)$$

Based on Assumption 2, we can get

$$B_f u_a(t) = B_v B \Delta u_a(t) = B_v v(t), \quad (11)$$

where $v(t) \in \mathfrak{R}^p$ is a virtual control input that represents the total control effort produced by actuators.

As distinct from conventional CA problem, the matrix B_f incorporates fault information. Generally, CA can be formulated as an optimization problem as follows [1]:

$$\min_{u_a(t)} u_a^T W u_a, \quad \text{subject to (11),} \quad (12)$$

where $W = \text{diag}(w_1, w_2, \dots, w_m)$ is a positive definite weighting matrix and the scalar w_i represents the weight of the i th actuator.

In contrast to the preceding CA results, the weighting matrix W can be updated online based on actuator effectiveness in this paper. Suppose that the i th actuator has partially lost effectiveness; that is, $\Delta = \text{diag}(1, \dots, 1, \delta_i, 1, \dots, 1)$, $0 < \delta_i < 1$. Let the i th diagonal element of W be perturbed from w_i to $\kappa_i^2 w_i$, where the scalar κ_i can be obtained by $\kappa_i = 1/\delta_i$. Then the new weighting matrix becomes $\widehat{W} = \mathfrak{W} W \mathfrak{W}$, where $\mathfrak{W} = \text{diag}(1, \dots, 1, \kappa_i, 1, \dots, 1)$. Consequently, the solution to (11) can be formulated as

$$u_a(t) = \widehat{W}^{-1} B_f^T (B_f \widehat{W}^{-1} B_f^T)^{-1} v(t). \quad (13)$$

Usually the initial value of the weighting matrix W is selected based on the priority of actuators. Once actuator failures occur, W will be updated. As $\delta_i \rightarrow 0$, $\kappa_i \rightarrow \infty$, the associated component u_i in (12) is therefore weighted heavily since w_i becomes large. If the i th actuator has damaged completely, the value of w_i will be infinity. Equation (13) is able to distribute the virtual control signal to the remaining physical actuators based on actuator effectiveness. However, due to the failure, only depending on the adaption of W to avoid saturation sometimes may be invalid. Therefore, this paper employs a positive μ -modification method to ensure that all control inputs lie within the bounds.

The following lemma gives the algorithm of the positive μ -modification method.

Lemma 5 (see [14]). If $\mu_1, \dots, \mu_m \geq 0$, then the solution to (6) is given by a convex combination of $u_a(t)$ and $u_{\max}^\delta \text{sat}(u_a(t)/u_{\max}^\delta)$ for all $t > 0$:

$$u_c(t) = (I_q + \mu)^{-1} \left(u_a(t) + \mu u_{\max}^\delta \text{sat} \left(u_{\max}^{-1} u_a(t) \right) \right) \quad (14)$$

which can also be represented component-wise in the following form:

$$u_{q_i}(t) = \begin{cases} u_{a_i}(t), & |u_{a_i}(t)| \leq u_{\max_i}^{\delta_i}, \\ \frac{1}{1 + \mu_i} (u_{a_i}(t) + \mu_i u_{\max_i}^{\delta_i}), & u_{a_i}(t) > u_{\max_i}^{\delta_i}, \\ \frac{1}{1 + \mu_i} (u_{a_i}(t) - \mu_i u_{\max_i}^{\delta_i}), & u_{a_i}(t) < -u_{\max_i}^{\delta_i}, \end{cases} \quad i = 1, \dots, m. \quad (15)$$

Remark 6. Detailed discussion of the above lemma can be found in [14].

3.2. Adaptive Virtual Controller Design. The adaptive controller is designed as

$$v(t) = \widehat{K}_1 x(t) + \widehat{K}_2 r(t), \quad (16)$$

where $\widehat{K}_1$ and $\widehat{K}_2$ are the estimates of K_1 and K_2 , respectively.

Substituting (11) and (16) into (9) yields the closed-loop system

$$\begin{aligned} \dot{x} &= (A + B_v K_1 + \delta_a) x + B_v K_2 r \\ &\quad + B_v \widehat{K}_1 x + B_v \widehat{K}_2 x + B_v B_f \tilde{u}_a, \end{aligned} \quad (17)$$

where $\widetilde{K}_1 = \widehat{K}_1 - K_1$, $\widetilde{K}_2 = \widehat{K}_2 - K_2$.

In order to compensate for the saturation error $\tilde{u}_a$, we design an adaptive compensation controller $\widehat{K}_3 \in \mathfrak{R}^{p \times p}$. The initial value of $\widehat{K}_3$ is defined as K_3 and satisfies

$$K_2 K_3 = I. \quad (18)$$

Using (4) and (18), the closed-loop system in (17) can be modified as

$$\dot{x} = (A_m + \delta_a) x + B_m (r + K_3 B_f \tilde{u}_a) + B_v \widetilde{K}_1 x + B_v \widetilde{K}_2 r. \quad (19)$$

The modified closed-loop system dynamics lead to consideration of the following adaptive reference model system:

$$\dot{x}_m = A_M x_m + B_m (r + \widehat{K}_3 B_f \tilde{u}_a), \quad (20)$$

where A_M is a time-varying matrix incorporating the model mismatch estimate $\widehat{\delta}_a$, denoted as

$$A_M = A_m + \widehat{\delta}_a. \quad (21)$$

Following [18], the system matrix A_M can be described as an affine function of a parameter vector $\rho \in \mathfrak{R}^{n_p}$ that lies in a convex polytope $\mathcal{S}$ having vertices ρ^j , $j = 1, 2, \dots, n_v$; that is,

$$A_M[\rho(t)] = A_m + \sum_{i=1}^{n_p} \rho_i(t) A_{M_i}, \quad (22)$$

where A_{M_i} are constant matrices and $\rho_i(t) \in [\rho_{i_{\min}}, \rho_{i_{\max}}]$. Suppose that $A_M(\rho^j)$ denotes the value of the reference model system matrix at vertex ρ^j and there exist positive definite matrices $P = P^T, Q = Q^T \in \mathfrak{R}^{n \times n}$ satisfying

$$A_M(\rho^j)^T P + P A_M(\rho^j) < -Q, \quad j = 1, \dots, n_v. \quad (23)$$

Based on (23), we can get the following Lyapunov inequality for all $\rho(t) \in \mathcal{S}$:

$$A_M[\rho(t)]^T P + P A_M[\rho(t)] < -Q, \quad j = 1, \dots, n_v. \quad (24)$$

Thus the autonomous part of the adaptive reference model in (20) is exponentially stable for all $\rho(t) \in \mathcal{S}$ [18].

Note that this paper only considers the case that the true value of model mismatch lies within the stable region Figure 6. The model mismatch δ_a is assumed to satisfy

$$(A_m + \delta_a)^T P + P (A_m + \delta_a) < -Q. \quad (25)$$

Subtracting (20) from (19) yields the following error dynamics:

$$\dot{e} = A_M e + B_v \bar{K}_1 x + B_v \bar{K}_2 r - \bar{\delta}_a x - B_m \bar{K}_3 B_f \bar{u}_a, \quad (26)$$

where $e = x - x_m$ and $\bar{\delta}_a = \hat{\delta}_a - \delta_a$ denote the tracking error and model mismatch estimate error, respectively.

The following theorem gives the adaption and parameter estimation laws which ensure that $e(t) \rightarrow 0$ and all signals remain bounded. We will first assume that the initial conditions satisfy Theorem 4.1 in [14].

Theorem 7. *For the system given by (1), the adaptive controller (16) with gain adaption laws*

$$\begin{aligned} \dot{\hat{K}}_1 &= -\Gamma_1 B_v^T P e x^T, \\ \dot{\hat{K}}_2 &= -\Gamma_2 B_v^T P e r^T \end{aligned} \quad (27)$$

and parameter estimation laws

$$\begin{aligned} \dot{\hat{K}}_3 &= \Gamma_3 B_m^T P e (B_f \bar{u}_a)^T, \\ \dot{\hat{\delta}}_a &= \Gamma_a P e x^T, \end{aligned} \quad (28)$$

where $\Gamma_1, \Gamma_2, \Gamma_3 \in \mathfrak{R}^{p \times p}$ and $\Gamma_a \in \mathfrak{R}^{n \times n}$ are positive definite matrices and P is determined by (24), guarantees that all the closed-loop signals remain bounded and that the tracking error $e(t)$ converges to 0 as $t \rightarrow \infty$.

Proof. In order to analyze the closed-loop stability and tracking performance, choose a Lyapunov function as

$$\begin{aligned} V &= e^T P e + \sum_{i=1}^m \bar{\delta}_{ai}^T \Gamma_a^{-1} \bar{\delta}_{ai} + \text{tr}(\bar{K}_1^T \Gamma_1^{-1} \bar{K}_1) \\ &\quad + \text{tr}(\bar{K}_2^T \Gamma_2^{-1} \bar{K}_2) + \text{tr}(\bar{K}_3^T \Gamma_3^{-1} \bar{K}_3), \end{aligned} \quad (29)$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix and $\bar{\delta}_{ai}$ denotes the i th column of $\bar{\delta}_a$.

Differentiating (29), we can get

$$\begin{aligned} \dot{V} &= 2e^T P (A_M e + B_v \bar{K}_1 x + B_v \bar{K}_2 r - \bar{\delta}_a x - B_m \bar{K}_3 B_f \bar{u}_a) \\ &\quad + \sum_{i=1}^m \bar{\delta}_{ai}^T \Gamma_a^{-1} \dot{\bar{\delta}}_{ai} + \text{tr}(\bar{K}_1^T \Gamma_1^{-1} \dot{\bar{K}}_1) \\ &\quad + \text{tr}(\bar{K}_2^T \Gamma_2^{-1} \dot{\bar{K}}_2) + \text{tr}(\bar{K}_3^T \Gamma_3^{-1} \dot{\bar{K}}_3) \\ &< -e^T Q e - 2 \text{tr}[\bar{\delta}_a^T (P e x^T - \Gamma_a^{-1} \dot{\bar{\delta}}_a)] \\ &\quad + 2 \text{tr}[\bar{K}_1^T (B_v^T P e x^T + \Gamma_1^{-1} \dot{\bar{K}}_1)] \\ &\quad + 2 \text{tr}[\bar{K}_2^T (B_v^T P e r^T + \Gamma_2^{-1} \dot{\bar{K}}_2)] \\ &\quad - 2 \text{tr}[\bar{K}_3^T (B_m^T P e (B_f \bar{u})^T - \Gamma_3^{-1} \dot{\bar{K}}_3)]. \end{aligned} \quad (30)$$

As $\dot{\hat{K}}_1 = \dot{\bar{K}}_1, \dot{\hat{K}}_2 = \dot{\bar{K}}_2, \dot{\hat{K}}_3 = \dot{\bar{K}}_3$, and $\dot{\hat{\delta}}_a = \dot{\bar{\delta}}_a$, substituting (27) and (28) into (30) yields

$$\dot{V} = -e^T Q e \leq 0. \quad (31)$$

Since $V(t)$ is a positive definite function and $\dot{V}(t) \leq 0$, the signals $e(t), \bar{K}_1(t), \bar{K}_2(t), \bar{K}_3(t)$, and $\bar{\delta}_a(t)$ are all bounded. However, due to the modification of the reference model, the stability of the system cannot be directly proven. It is necessary to prove that one of the signals $x(t)$ or $x_m(t)$ is bounded.

Then, we define the following candidate Lyapunov function:

$$W(x) = x^T P_x x. \quad (32)$$

Substituting (4) into (1) yields

$$\begin{aligned} \dot{x} &= (A + \delta_a) x + B_v K_1^T - B_v K_1^T + B_u \Delta u \\ &= (A_m + \delta_a) x - B_v K_1^T + B_u \Delta u. \end{aligned} \quad (33)$$

Based on (25), $A_m + \delta_a$ is assumed to satisfy the following Lyapunov inequality:

$$(A_m + \delta_a)^T P_x + P_x (A_m + \delta_a) < -Q_x, \quad (34)$$

where $P_x = P_x^T, Q_x = Q_x^T \in \mathfrak{R}^{n \times n}$ are both positive definite matrices.

Differentiating (32), we can get

$$\dot{W}(x) < -x^T Q_x x - 2x^T P_x B_v K_1^T + 2x^T P_x B_u \Delta u. \quad (35)$$

Based on the previous results in [14], we know that there always exists a nonempty annulus region such that $\dot{W}(x) < 0$ holds for all x from that region. The boundedness of x can be guaranteed as long as the initial conditions satisfy

Theorem 4.1 in [14]. Since $x(t)$ is bounded and $x_m(t) = x(t) - e(t)$, it is easy to conclude that $x_m(t)$ is bounded.

Consequently, according to the error dynamics in (26) and the boundedness of the closed-loop signals, we can get $\dot{e}(t) \in L_\infty$, and thus $\lim_{t \rightarrow \infty} e(t) = 0$. Due to $e(t) \in L_2 \cap L_\infty$, $\hat{K}_1, \hat{K}_2, \hat{K}_3, \hat{\delta}_a \in L_2 \cap L_\infty$, and $x(t), r(t) \in L_\infty$, all signals and estimated parameters are bounded, realizing $\lim_{t \rightarrow \infty} [x(t) - x_m(t)] = 0$.

The proof is completed. $\square$

Remark 8. Since the reference model is time varying, it is necessary to ensure that the parameter estimate $\hat{\delta}_a$ remains within the stable region. If $\hat{\delta}_a$ exceeds the stable region, a parameter projection method [18] will be utilized to ensure that the adaptive reference model has satisfactory stability and performance.

4. Application Example

To investigate the effectiveness of the proposed adaptive fault-tolerant control scheme, we utilize a flight control example based on the ADMIRE model [19]. A low-speed flight case is considered, where the control surface efficiency is poor.

4.1. System Description. The state space of the model is given by (1), where the nominal values of the system matrices are in the appendix and

$$\begin{aligned} x &= [\alpha \ \beta \ p \ q \ r]^T, \\ y &= [\alpha \ \beta \ p]^T, \\ u &= [\delta_c \ \delta_{re} \ \delta_{le} \ \delta_r]^T. \end{aligned} \quad (36)$$

System states α , β , p , q , and r represent the angle of attack, the sideslip angle, the roll rate, the pitch rate, and the yaw rate, respectively. Control inputs δ_c , δ_{re} , δ_{le} , and δ_r represent the positions of the canard wings, the right elevons, the left elevons, and the rudder, respectively.

The virtual control input v contains the angular accelerations in roll, pitch, and yaw produced by the control surface.

The actuator position constraints are given by

$$\begin{aligned} \delta_{\min} &= [-25^\circ \ -30^\circ \ -30^\circ \ -30^\circ]^T, \\ \delta_{\max} &= [25^\circ \ 30^\circ \ 30^\circ \ 30^\circ]^T. \end{aligned} \quad (37)$$

It is assumed that the actuator δ_c loses 99% of effectiveness at 1.01 s while the remaining actuators are functional throughout and the aircraft experiences damage at 0 s; therefore, a model mismatch exists in the system matrix from the outset. The model mismatch δ_a in the system is

$$\delta_a = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (38)$$

The reference inputs are given in Figure 2.

4.2. Controller Design. The matrices K_1 , K_2 are designed using the linear quadratic regulator (LQR) method. The reference model is designed based on (4).

The quadratic stable region is obtained as $\delta_{a_{14}}$ and $\delta_{a_{25}} \in [-1.1564, 1.1564]$. The security threshold is arbitrarily set at 80% of the maximum stable region. The corresponding Lyapunov matrix P in (25) is obtained as

$$P = \begin{bmatrix} 0.0892 & 0.0568 & -0.007 & 0.0807 & 0.127 \\ 0.0568 & 657.846 & 15.8863 & -0.1136 & -44.8585 \\ -0.007 & 15.8863 & 0.6474 & -0.0084 & -2.3445 \\ 0.0807 & -0.1136 & -0.0084 & 0.1653 & 0.1152 \\ 0.127 & -44.8585 & -2.3445 & 0.1152 & 47.7764 \end{bmatrix}. \quad (39)$$

Using (18), the matrix K_3 can be decided. The gains Γ_1 , Γ_2 , Γ_3 , and Γ_a are selected by trial and error.

The initial value of the weighting matrix $\widehat{W}$ is chosen as $\text{diag}\{0.01, 1, 1, 1\}$. Once actuator failures occur, the weighting matrix will be updated.

4.3. Simulation Results

Case 1. In the first case, all adaption is turned off; that is, the weighting matrix, the gains, and the parameter estimates are not updated. The plant states are shown in Figure 3. Obviously, the stability of the system cannot be guaranteed due to the presence of actuator failures, model mismatch, and input saturation.

Case 2. We utilize the adaptive fault-tolerant scheme in this case. The controller gain and parameter estimates are generated by the adaptive laws (27)-(28). At $t = 1.01$ s, the first actuator fails while the remaining actuators continue to be functional, adaptively compensating for the failed first actuator. Unfortunately, actuators δ_2 and δ_4 have exceeded the input constraints as shown in Figure 4. Using the anti-saturation algorithm, all of the actuators can be rigorously kept within the bounds, as shown in Figure 5. During the simulation, the parameter estimates $\hat{\delta}_{a_{14}}$ and $\hat{\delta}_{a_{25}}$ lie in the quadratic stable region throughout and approach the true value of model mismatch adaptively as shown in Figure 7. The plant with the adaptive reference model states and the tracking errors are illustrated in Figures 7 and 8, respectively. Simulation results show that the tracking errors remain small and converge to zero asymptotically.

5. Conclusion

This paper proposed a novel fault-tolerant control scheme for flight systems in the presence of simultaneous actuator failures, model mismatch, and input saturation. The scheme combined CA and MRAC to guarantee the system's stability and tracking performance. Practical conditions such as model mismatch and input saturation were taken into consideration in this paper. Due to model mismatch and input saturation, the original reference model might no longer be appropriate; therefore, we designed an adaptive reference model that also provides satisfactory performance.

FIGURE 2: The reference inputs.

FIGURE 3: The system states of Case 1.

FIGURE 4: The adaptive control input u_a of Case 2.

FIGURE 5: The commanded control input u_c of Case 2.

FIGURE 6: The model mismatch estimate of Case 2.

FIGURE 7: The system states of Case 2.

FIGURE 8: The tracking errors of Case 2.

Simulation results indicated that mismatch estimation, input compensation, and satisfactory tracking performance could be obtained in the presence of actuator failures, model mismatch, and input saturation. Future investigation will be necessary to achieve adaptive fault estimation and test more complex and realistic models. The FTC of multiple flight systems will be explored in our future study.

Appendix

Consider the following:

$$\begin{aligned}
 A &= \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{bmatrix}, \\
 C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{bmatrix}, \\
 B_v &= \begin{bmatrix} 0_{2 \times 3} \\ I_{3 \times 3} \end{bmatrix}, \quad B_u = B_v B.
 \end{aligned} \tag{A.1}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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