Research Article

Periodicity and Permanence of a Discrete Impulsive Lotka-Volterra Predator-Prey Model Concerning Integrated Pest Management

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By piecewise Euler method, a discrete Lotka-Volterra predator-prey model with impulsive effect at fixed moment is proposed and investigated. By using Floquets theorem, we show that a globally asymptotically stable pest-eradication periodic solution exists when the impulsive period is less than some critical value. Further, we prove that the discrete system is permanence if the impulsive period is larger than some critical value. Finally, some numerical experiments are given.

1. Introduction

Impulsive equations are found in almost every domain of applied science, such as population dynamics, ecology, biological systems, and optimal control. In recent years, the theory of impulsive differential equations has been an object of active research (see [1–4] and reference therein) since it is much richer than the corresponding theory of differential equations without impulsive effects.

It is well known that continuous-time dynamic systems play an important role in control theory, population dynamics, and so on. But in applications of continuous-time dynamic systems to some practical problems, such as computer simulation, experimental, or computational purposes, it is usual to formulate a discrete-time system which is a version of the continuous-time system. In some sense, the discrete time model inherits the dynamical characteristics of the continuous-time systems. We refer to [4–16] for related discussions of the importance and the need for discrete-time analogs to reflect the dynamics of their continuous-time counterparts. Nevertheless, the discrete-time version can but not always preserve the dynamics of its initial version because the theory of difference equations is a lot richer than the corresponding theory of differential equations as pointed out

in [17, 18]. Therefore, it is important to study the dynamics of its initial version alone.

Due to the above facts, we construct the following discrete impulsive Lotka-Volterra predator-prey model concerning integrated pest management by piecewise Euler method:

$$\begin{aligned}x_{kT+l+1} &= x_{kT+l} \exp \{r - ax_{kT+l} - by_{kT+l}\}, \\y_{kT+l+1} &= y_{kT+l} \exp \{-d + cx_{kT+l}\}, \\x_{kT+0} &= (1 - p_1) x_{kT}, \\y_{kT+0} &= (1 - p_2) y_{kT} + \mu, \quad l = 0, 1, 2, \dots, T-1, \\k &= 0, 1, 2, \dots,\end{aligned}\tag{1}$$

where $r > 0$ is the intrinsic growth rate of pest, $a > 0$ is the coefficient of intraspecific competition, $b > 0$ is the per-capita rate of predation of the predator, $d > 0$ is the death rate of predator, $c > 0$ denotes the product of the per-capita rate of predation and the rate of converting pest into predator, and T is the period of the impulsive effect. $0 \leq p_1 < 1$ ($0 \leq p_2 < 1$) represents the fraction of pest (predator) which dies due to the pesticide, and $\mu > 0$ is the release amount of predator at kT , $k \in \mathbb{Z}_+$. That is, we can use a combination of biological

(periodic releasing natural enemies) and chemical (spraying pesticide) tactics that eradicates the pest to extinction and show the efficiency of integrated pest management strategy.

System (1) can be regarded as a discrete analogy of the following impulsive Lotka-Volterra predator-prey model concerning integrated pest management:

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 \{r - ax_1 - bx_2\}, \\ \frac{dx_2}{dt} &= x_2 \{-d + cx_1\}, \\ \Delta x_1 &= (1 - p_1)x_1, \\ \Delta x_2 &= (1 - p_2)x_2 + \mu,\end{aligned}\quad (2)$$

where $\Delta x_i(t) = x_i(t^+) - x_i(t)$, $i = 1, 2$. Liu et al. [19] discussed the dynamical behavior of model (2).

Recently, the studies of discrete impulsive model have received great attention from more scholar (see [5, 15, 16, 20–22]). The main difficulty for dynamical analysis of such equations comes from impulsive effect on the equations since the corresponding theory for impulsive difference equations have not yet been fully developed. The discrete impulsive model (1) gives a new form of describing the impulsive moment. In some papers, authors use $(kT)^+$ to denote impulsive moment (see [20]). It is obvious that describing the impulsive moment of model (1) is easily realized at computer. In addition, some authors use $kT - 1$ to denote impulsive moment (see [16, 21]). Compared with it, model (1) is a better analogue of the continuous-time dynamic system.

The main aim of this paper is to construct the discrete impulsive model (1) and discuss the dynamical behaviors of the discrete impulsive model (1). We investigate the globally asymptotical stability of pest-eradication periodic solution system (1) and the permanence of system (1).

2. Global Qualitative Analysis for Model (1)

Before our main results, we will give some lemmas which will be useful for our main results. First, we present the Floquet theory for the linear T -periodic difference equation

$$u_{n+1} = A_n u_n, \quad (3)$$

where $u_n \in \mathbb{R}^s$, $n = 0, 1, 2, \dots$. A_n is a real $s \times s$ matrix whose entries are functions of n satisfying $A_n = A_{n+T}$ for a positive integer T and is nonsingular; that is, $\det A_n \neq 0$ for all $n = 0, 1, 2, \dots$. As usual $M = A_{T-1}A_{T-2} \cdots A_0$ denotes the monodromy of (3).

Lemma 1 (see [5, 6]). *Let B be an $s \times s$ nonsingular matrix and let T be any positive integer. Then there exists some $s \times s$ matrix C , such that $C^T = B$.*

Lemma 2 (see [5, 6]). *If we write $M = D^T$, then there exists a change of variable $u_n = S_n y_n$, with $S_{n+T} = S_n$ for all $n = 0, 1, 2, \dots$, such that (3) becomes*

$$y_{n+1} = D y_n. \quad (4)$$

Lemma 3 (see [5]). *Let a_{kT+l} be a solution of the following impulsive inequality:*

$$\begin{aligned}a_{kT+l+1} &\geq a_{kT+l} \exp(-d), \\ a_{kT+0} &= (1 - p_2)a_{kT} + \mu, \quad l = 0, 1, 2, \dots, T-1, \\ a_{0+0} &= a_0, \quad k = 0, 1, 2, \dots\end{aligned}\quad (5)$$

and let b_{kT+l} be a solution of the following impulsive inequality

$$\begin{aligned}b_{kT+l+1} &= b_{kT+l} \exp(-d), \\ b_{kT+0} &= (1 - p_2)b_{kT} + \mu, \quad l = 0, 1, 2, \dots, T-1, \\ b_{0+0} &= a_0, \quad k = 0, 1, 2, \dots\end{aligned}\quad (6)$$

Then $a_{kT+l} \geq b_{kT+l}$, $l = 0, 1, 2, \dots, T-1$.

Proof. First, we know that $a_{0+0} = a_0 = b_{0+0}$. By induction, we assume that $a_{kT+l} \geq b_{kT+l}$, $k = 0, 1, 2, \dots$, $l = 1, 2, \dots, T-1$.

There are two cases.

Case 1. Considering $l \neq 0$, $k = 0, 1, 2, \dots$ or $k = 0, l = 0, 1, 2, \dots, T-1$, we have

$$\begin{aligned}a_{kT+l+1} &\geq a_{kT+l} \exp(-d) \\ &\geq b_{kT+l} \exp(-d) = b_{kT+l+1}.\end{aligned}\quad (7)$$

Case 2. Considering $l = 0$, $k = 1, 2, \dots$, we have

$$\begin{aligned}a_{kT+1} &\geq a_{kT+0} \exp(-d) \\ &= ((1 - p_2)a_{kT} + \mu) \exp(-d) \\ &\geq ((1 - p_2)b_{kT} + \mu) \exp(-d) = b_{kT+1}.\end{aligned}\quad (8)$$

Therefore, for both cases we have $a_{kT+l} \geq b_{kT+l}$. The proof is complete. $\square$

Lemma 4. *Suppose (x_{kT+l}, y_{kT+l}) is a solution of (1) subject to $x_0 > 0$, $y_0 > 0$; then $x_{kT+l} > 0$, $y_{kT+l} > 0$ for all $kT + l \geq 0$, $k = 0, 1, 2, \dots$, $l = 0, 1, 2, \dots, T-1$.*

2.1. Global Stability of the Pest-Eradication Periodic Solution. Consider the following system:

$$\begin{aligned}y_{kT+l+1} &= y_{kT+l} \exp(-d), \\ y_{kT+0} &= (1 - p_2)y_{kT} + \mu, \quad l = 0, 1, 2, \dots, T-1, \\ y_{0+0} &= y_0 > 0, \quad k = 0, 1, 2, \dots\end{aligned}\quad (9)$$

Clearly, $\widehat{y_{kT+l}} = (\mu / (1 - (1 - p_2) \exp(-dT))) \exp(-dl)$, $l = 0, 1, 2, \dots, T-1$, with $y_{0+0} = (\mu / (1 - (1 - p_2) \exp(-dT)))$, is a positive periodic solution of (9). Since the solution of (9) is $y_{kT+l} = (1 - p_2)^k (y_{0+0} - (\mu / (1 - (1 - p_2) \exp(-dT)))) \exp(-(kT + l)d) + \widehat{y_{kT+l}}$, $l = 0, 1, 2, \dots, T-1$, $k = 0, 1, 2, 3, \dots$, we have the following lemma.

Lemma 5. *Equation (9) has a positive periodic solution y_{kT+l} , $k = 0, 1, 2, \dots$, $l = 0, 1, 2, \dots, T-1$, and, for every solution y_{kT+l} of (9), we have $y_{kT+l} \rightarrow \widehat{y_{kT+l}}$ as $k \rightarrow \infty$.*

Therefore, system (1) has a pest-eradication periodic solution:

$$(0, \widetilde{y_{kT+l}}) = \left(0, \frac{\mu}{1 - (1 - p_2) \exp(-dT)} \exp(-dl) \right) \quad (10)$$

and $y_{0+0} = \mu / (1 - (1 - p_2) \exp(-dT))$.

Now we give the conditions which assure the globally stability of the pest-eradication periodic solution $(0, \widetilde{y_{kT+l}})$.

Theorem 6. Let (x_{kT+l}, y_{kT+l}) be any solution of (1); then $(0, \widetilde{y_{kT+l}})$ is globally asymptotically stable provided

$$\begin{aligned} rT - \frac{b\mu(1 - \exp(-dT))}{(1 - (1 - p_2) \exp(-dT))(1 - \exp(-d))} \\ < \ln\left(\frac{1}{1 - p_1}\right). \end{aligned} \quad (11)$$

Proof. Firstly, we proved the local stability. The local stability of periodic solution $(0, \widetilde{y_{kT+l}})$ may be determined by considering the behavior of small amplitude perturbations of the

solution. Defining $x_{kT+l} = u_{kT+l}$, $y_{kT+l} = v_{kT+l} + \widetilde{y_{kT+l}}$, there may be written

$$\begin{pmatrix} u_{kT+l+1} \\ v_{kT+l+1} \end{pmatrix} = \begin{pmatrix} u_{kT+l} \exp(r - au_{kT+l} - b(v_{kT+l} + \widetilde{y_{kT+l}})) \\ (v_{kT+l} + \widetilde{y_{kT+l}}) \exp(-d + cu_{kT+l}) \end{pmatrix}. \quad (12)$$

Equation (12) can be expanded in a Taylor series: after neglecting higher-order terms, the linearized equations read as

$$\begin{aligned} u_{kT+l+1} &= \exp\{r - b\widetilde{y_{kT+l}}\} u_{kT+l}, \\ v_{kT+l+1} &= \widetilde{y_{kT+l}} c \exp(-d) u_{kT+l} + \exp(-d) v_{kT+l}, \\ u_{kT+0} &= (1 - p_1) u_{kT}, \\ v_{kT+0} &= (1 - p_2) v_{kT}, \quad l = 0, 1, 2, \dots, T-1, \\ k &= 0, 1, 2, \dots \end{aligned} \quad (13)$$

Hence

$$A_{kT+l} = \begin{pmatrix} \exp(r - b\widetilde{y_{kT+l}}) & 0 \\ \widetilde{y_{kT+l}} c \exp(-d) & \exp(-dT) \end{pmatrix}; \quad (14)$$

the fundamental solution matrix is

$$\begin{aligned} M &= A_{kT+0} A_{kT+1} A_{kT+2} \cdots A_{kT+T-1} \begin{pmatrix} 1 - p_1 & 0 \\ 0 & 1 - p_2 \end{pmatrix} \\ &= \begin{pmatrix} \exp\left(rT - \frac{b\mu}{1 - (1 - p_2) \exp(-dT)} \frac{1 - \exp(-dT)}{1 - \exp(-d)}\right) & 0 \\ * & \exp(-dT) \end{pmatrix} \begin{pmatrix} 1 - p_1 & 0 \\ 0 & 1 - p_2 \end{pmatrix} \\ &= \begin{pmatrix} (1 - p_1) \exp\left(rT - \frac{b\mu}{1 - (1 - p_2) \exp(-dT)} \frac{1 - \exp(-dT)}{1 - \exp(-d)}\right) & 0 \\ * & (1 - p_2) \exp(-dT) \end{pmatrix}. \end{aligned} \quad (15)$$

There is no need to calculate the exact form of (*) as it is not required in the analysis that follows.

The stability of the periodic solution $(0, \widetilde{y_{kT+l}})$ is determined by the eigenvalues of M .

Let λ_1, λ_2 be eigenvalues of matrix M . Then

$$\begin{aligned} \lambda_1 &= (1 - p_1) \\ &\times \exp\left(rT - \frac{b\mu}{1 - (1 - p_2) \exp(-dT)} \frac{1 - \exp(-dT)}{1 - \exp(-d)}\right), \\ \lambda_2 &= (1 - p_2) \exp(-dT), \end{aligned} \quad (16)$$

according to Lemmas 1 and 2, $(0, \widetilde{y_{kT+l}})$ is locally stable if $|\lambda_1| < 1$ and $|\lambda_2| < 1$. Obviously, $\lambda_2 < 1$. So, $(0, \widetilde{y_{kT+l}})$ is

locally stable if $|\lambda_1| < 1$. That is,

$$\begin{aligned} rT - \frac{b\mu(1 - \exp(-dT))}{(1 - (1 - p_2) \exp(-dT))(1 - \exp(-d))} \\ < \ln\left(\frac{1}{1 - p_1}\right). \end{aligned} \quad (17)$$

In the following we prove the global attractivity. Choose $\varepsilon > 0$ such that

$$\begin{aligned} \delta &\triangleq (1 - p_1) \\ &\times \exp\left(rT + b\varepsilon T \right. \\ &\quad \left. - \frac{b\mu(1 - \exp(-dT))}{(1 - (1 - p_2) \exp(-dT))(1 - \exp(-d))}\right) \\ &< 1. \end{aligned} \quad (18)$$

Noting that $y_{kT+l+1} \geq y_{kT+l} \exp(-d)$, consider the following impulsive equation:

$$\begin{aligned} z_{kT+l+1} &= z_{kT+l} \exp\{-d\}, \\ z_{kT+0} &= (1 - p_2) z_{kT} + \mu, \quad l = 0, 1, 2, \dots, T-1, \\ k &= 0, 1, 2, \dots, \end{aligned} \quad (19)$$

$$z_{0+0} = y_0 \geq 0.$$

By Lemmas 3 and 5, we have

$$y_{kT+l} \geq z_{kT+l} > \widetilde{y_{kT+l}} - \varepsilon, \quad (20)$$

for k large enough, so

$$\begin{aligned} x_{kT+l+1} &\leq x_{kT+l} \exp(r - by_{kT+l}) \\ &\leq x_{kT+l} \exp(r - b(\widetilde{y_{kT+l}} - \varepsilon)), \end{aligned} \quad (21)$$

which leads to

$$\begin{aligned} x_{kT+0} &= (1 - p_1) x_{kT} \\ &\leq (1 - p_1) x_{kT-1} \exp(r - b(\widetilde{y_{kT-1}} - \varepsilon)) \\ &\leq (1 - p_1) x_{kT-2} \exp(r - b(\widetilde{y_{kT-2}} - \varepsilon)) \\ &\quad \times \exp(r - b(\widetilde{y_{kT-1}} - \varepsilon)) \\ &\vdots \\ &\leq (1 - p_1) x_{(k-1)T+0} \\ &\quad \times \exp(Tr + bT\varepsilon \\ &\quad - b(\widetilde{y_{(k-1)T+0}} + \widetilde{y_{(k-1)T-1}} + \dots + \widetilde{y_{kT-1}})) \\ &\leq (1 - p_1) x_{(k-1)T+0} \\ &\quad \times \exp\left(Tr + bT\varepsilon - \frac{b\mu(1 - \exp(-dT))}{(1 - (1 - p_2)\exp(-dT))(1 - \exp(-d))}\right). \end{aligned} \quad (22)$$

Therefore, $x_{kT+0} \leq x_{0+0} \delta^k$ and $x_{kT+0} \rightarrow 0$ as $k \rightarrow +\infty$. Moreover, $0 < x_{kT+l} \leq x_{kT+0} \exp(rT + bT\varepsilon)$, so we have $x_{kT+l} \rightarrow 0$ as $k \rightarrow \infty$.

Next, we prove that $y_{kT+l} \rightarrow \widetilde{y_{kT+l}}$ as $k \rightarrow \infty$ if $\lim_{k \rightarrow \infty} x_{kT+l} = 0$. For $0 < \varepsilon_1 < d/c$, there exists an $K_0 > 0$ such that $0 < x_{kT+l} < \varepsilon_1$ for all $l = 0, 1, 2, \dots, T-1$, and $k > K_0$; then from (1) we have

$$y_{kT+l} \exp(-d) < y_{kT+l+1} < y_{kT+l} \exp(-d + c\varepsilon_1). \quad (23)$$

By Lemmas 3 and 5, we obtain

$$\begin{aligned} y_{kT+l}^1 &\leq y_{kT+l} \leq y_{kT+l}^2, \\ y_{kT+l}^1 &\rightarrow \widetilde{y_{kT+l}}, \quad y_{kT+l}^2 \rightarrow \widetilde{y_{kT+l}} \quad \text{as } k \rightarrow \infty, \end{aligned} \quad (24)$$

where y_{kT+l}^1 and y_{kT+l}^2 are solutions of

$$\begin{aligned} y_{kT+l+1}^1 &= y_{kT+l}^1 \exp\{-d\}, \\ y_{kT+0}^1 &= (1 - p_2) y_{kT}^1 + \mu, \quad l = 0, 1, 2, \dots, T-1, \\ k &= 0, 1, 2, \dots, \end{aligned} \quad (25)$$

$$\begin{aligned} y_{0+0}^1 &= y_0 \geq 0, \\ y_{kT+l+1}^2 &= y_{kT+l}^2 \exp\{-d + c\varepsilon_1\}, \\ y_{kT+0}^2 &= (1 - p_2) y_{kT}^2 + \mu, \quad l = 0, 1, 2, \dots, T-1, \\ k &= 0, 1, 2, \dots, \end{aligned} \quad (26)$$

respectively, and $\widetilde{y_{kT+l}^2} = (\mu/(1 - (1 - p_2)\exp((-d + c\varepsilon_1)T))) \exp((-d + c\varepsilon_1)l)$, $l = 0, 1, 2, \dots, T-1$.

Therefore, for any $\varepsilon_2 > 0$ there exists a $\widehat{K} > 0$ such that

$$\widetilde{y_{kT+l}} - \varepsilon_2 < y_{kT+l} < \widetilde{y_{kT+l}^2} + \varepsilon_2. \quad (27)$$

Let $\varepsilon_1 \rightarrow 0$; we have

$$\widetilde{y_{kT+l}^2} \rightarrow \widetilde{y_{kT+l}} \quad \text{as } k \rightarrow \infty, \quad (28)$$

which implies $y_{kT+l} \rightarrow \widetilde{y_{kT+l}}$ as $k \rightarrow \infty$. This completes the proof. $\square$

2.2. Permanence. Now we investigate the permanence of system (1). For convenience, let $n = kT + l$. Firstly, we give the following definition.

Definition 7. System (1) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions (x_n, y_n) with all initial values $(x_{0+0} > 0, y_{0+0} > 0)$,

$$m < x_n < M, \quad m < y_n < M \quad (29)$$

hold for all $n > T_0$. Here T_0 may depend on the initial values (x_{0+0}, y_{0+0}) .

Lemma 8. There exist two positive constants M and N , such that for every solution (x_n, y_n) of system (1), we have

$$x_n < M \quad \text{for } n > N. \quad (30)$$

Proof. To prove (30), we have two cases.

Case I. $n \neq kT + 0$, $k = 0, 1, 2, 3, \dots$. For any $x_0 > 0$, we have

$$\begin{aligned} x_{n+1} &= x_n \exp\{r - ax_n - by_n\} \\ &\leq x_n \exp\{r - ax_n\} \\ &\leq \frac{\exp\{r - 1\}}{a} \triangleq M; \end{aligned} \quad (31)$$

here we used

$$\max_{x \geq 0} \{x \exp \{b - ax\}\} = \frac{\exp \{b - 1\}}{a}. \quad (32)$$

Case II. $n = kT + 0$, $k = 0, 1, 2, 3, \dots$, we have

$$\begin{aligned} x_{kT+0} &= (1 - p_1) x_{kT} \\ &= (1 - p_1) x_{kT-1} \exp \{r - ax_{kT-1} - by_{kT-1}\} \\ &\leq (1 - p_1) x_{kT-1} \exp \{r - ax_{kT-1}\} \\ &\leq (1 - p_1) M < M. \end{aligned} \quad (33)$$

This completes the proof. $\square$

In the following, without loss of generality, we assume $M > d/c$ is large enough.

Lemma 9. *There exists a constant M_1 such that, for every solution (x_n, y_n) of (1), we have*

$$\limsup_{n \rightarrow \infty} y_n < M_1. \quad (34)$$

Proof. We first prove that there exists a $K > 0$ such that

$$\liminf_{n \rightarrow \infty} y_n < K. \quad (35)$$

There exist $K, \varepsilon_0, M_2 > \mu/p_2 > 0$ such that

$$\begin{aligned} \varepsilon_0 < \frac{d}{c}; \quad M \exp(r - bK) < \varepsilon_0; \quad K > \frac{\mu}{e^B - 1 + p_2}, \\ \frac{\mu}{(1 - p_2) \exp(AT) - 1} < K + 1, \end{aligned} \quad (36)$$

$$\frac{1}{A} \ln \frac{M_2}{2(K + 1)} > 3, \quad (37)$$

where $A = -d + cM$, $B = d - c\varepsilon_0$. Now we claim that (35) holds. Otherwise, there would exist $N_1 \in \mathbb{N}$ and $\varepsilon > 0$, such that $y_n > K + \varepsilon$ for $n > N_1$. By Lemma 8, there exists an $N_2 > N_1$ such that

$$x_n < M, \quad \text{for } n > N_2. \quad (38)$$

Then if $n > N_2$

$$\begin{aligned} x_{n+1} &\leq x_n \exp(r - b(K + \varepsilon)) \\ &\leq M \exp(r - bK) < \varepsilon_0. \end{aligned} \quad (39)$$

There are two cases as follows.

(1) If $n = kT + 0 > N_2 + 1$,

$$\begin{aligned} y_{n+1} &= y_{kT+1} \\ &\leq y_{kT+0} \exp(-d + cx_{kT+0}) \\ &\leq ((1 - p_2) y_{kT} + \mu) \exp(-d + c\varepsilon_0) \\ &\leq ((1 - p_2) y_n + \mu) \exp(-B). \end{aligned} \quad (40)$$

We claim that $y_{n+1} < y_n$. Otherwise, $y_{n+1} \geq y_n$, so

$$((1 - p_2) y_n + \mu) \exp(-B) \geq y_n; \quad (41)$$

hence

$$y_n \leq \frac{\mu}{e^B - (1 - p_2)} < K, \quad (42)$$

which is a contradiction.

(2) If $n \neq kT + 0 > N_2 + 1$,

$$\begin{aligned} y_{n+1} &\leq y_n \exp(-d + cx_{kT+0}) \\ &\leq y_n \exp(-d + c\varepsilon_0) \\ &\leq y_n \exp(-B) < y_n. \end{aligned} \quad (43)$$

Therefore, for both cases, we have $y_{n+1} < y_n$, so $\lim_{n \rightarrow \infty} y_n = 0$, which is a contradiction.

For M_2 given by (37), in the following we will prove that (34) holds.

Otherwise,

$$\limsup_{n \rightarrow \infty} y_n > M_2. \quad (44)$$

By (35) and (44), there exist

$$n_2 > n_1 > N_2, \quad (45)$$

such that

$$\begin{aligned} y_{n_1} &\leq K + 1, \quad y_{n_2} \geq M_2, \\ K + 1 &< y_n < M_2 \quad \text{for } n \in (n_1, n_2). \end{aligned} \quad (46)$$

(i) If n_1, n_2 are at the same region $\in [kT, (k + 1)T)$, then

$$\begin{aligned} y_{n_2} &\leq y_{n_2-1} \exp(A) \\ &\leq y_{n_2-2} \exp(2A) \\ &\vdots \\ &\leq y_{n_1} \exp((n_2 - n_1)A) \end{aligned} \quad (47)$$

so

$$n_2 - n_1 \geq \frac{1}{A} \ln \frac{M_2}{K + 1} > 3. \quad (48)$$

(ii) If n_1, n_2 are not at the same region, we assume $n_2 \in [kT, (k + 1)T)$ and $n_1 \in [\bar{k}T, (\bar{k} + 1)T)$. Then

$$\begin{aligned} y_{n_2} &\leq y_{n_2-1} \exp(A) \leq y_{n_2-2} \exp(2A) \\ &\vdots \\ &\leq (1 - p_2) y_{kT} \exp((n_2 - kT)A) \\ &\quad + \mu \exp((n_2 - kT)A) \\ &\vdots \end{aligned}$$

$$\begin{aligned}
&\leq (1-p_2)^2 y_{(k-1)T} \exp[(n_2 - (k-1)T)A] \\
&\quad + \mu(1-p_2) \exp[(n_2 - (k-1)T)A] \\
&\quad + \mu \exp((n_2 - kT)A) \\
&\vdots \\
&\leq (1-p_2)^{k-\tilde{k}} y_{(\tilde{k}+1)T} \exp[(n_2 - (\tilde{k}+1)T)A] \\
&\quad + \mu(1-p_2)^{k-\tilde{k}-1} \exp[(n_2 - (\tilde{k}+1)T)A] \\
&\quad + \mu(1-p_2)^{k-\tilde{k}-2} \exp[(n_2 - (\tilde{k}+2)T)A] \\
&\quad + \cdots + \mu(1-p_2)^2 \exp[(n_2 - (k-2)T)A] \\
&\quad + \mu(1-p_2) \exp[(n_2 - (k-1)T)A] \\
&\quad + \mu \exp((n_2 - kT)A) \\
&\leq (1-p_2)^{k-\tilde{k}} y_{n_1} \exp((n_2 - n_1)A) \\
&\quad + \frac{\mu \exp((n_2 - kT)A) [1 - (1-p_2) \exp((k-\tilde{k})AT)]}{1 - (1-p_2) \exp(AT)} \\
&\leq y_{n_1} \exp((n_2 - n_1)A) \\
&\quad + \frac{\mu(1-p_2) \exp[(n_2 - \tilde{k}T)A - \exp((n_2 - kT)A)]}{(1-p_2) \exp(AT) - 1} \\
&\leq 2(K+1) \exp[(n_2 - n_1)A].
\end{aligned} \tag{49}$$

So

$$n_2 - n_1 \geq \frac{1}{A} \ln \frac{M_2}{2(K+1)} > 3. \tag{50}$$

Therefore, the set $\{n \mid n_1 < n < n_2\}$ is nonempty. So if $n_2 - 1 \neq kT + 0 > N_2 + 1$, then

$$\begin{aligned}
x_{n_2-1} &= x_{n_2-2} \exp(r - ax_{n_2-2} - by_{n_2-2}) \\
&\leq x_{n_2-2} \exp(r - by_{n_2-2}) \\
&\leq M \exp(r - b(K+1)) \\
&\leq \varepsilon_0.
\end{aligned} \tag{51}$$

If $n_2 - 1 = kT + 0 > N_2 + 1$, then

$$\begin{aligned}
x_{n_2-1} &= x_{kT+0} = (1-p_1) x_{kT} \\
&\leq (1-p_1) x_{kT-1} \exp(r - b(K+1)) \\
&\leq M \exp(r - b(K+1)) \\
&\leq \varepsilon_0.
\end{aligned} \tag{52}$$

There are three case as follows.

(1) If $n_2 = kT + 1, k = 1, 2, 3, \dots$ and $M_2 > \mu/p_2$,

$$\begin{aligned}
y_{n_2} &= y_{kT+1} \\
&= y_{kT+0} \exp(-d + cx_{kT+0}) \\
&= ((1-p_2) y_{kT} + \mu) \exp(-d + cx_{kT+0}) \\
&\leq ((1-p_2) y_{kT} + \mu) \exp(-d + c\varepsilon_0) \\
&\leq ((1-p_2) M_2 + \mu) \exp(-d + c\varepsilon_0) \\
&\leq (1-p_2) M_2 + \mu \leq M_2.
\end{aligned} \tag{53}$$

(2) If $n_2 = kT + 0, k = 1, 2, 3, \dots$ and $M_2 > \mu/p_2$,

$$\begin{aligned}
y_{n_2} &= y_{kT+0} \\
&= ((1-p_2) y_{kT} + \mu) \\
&= ((1-p_2) y_{kT-1} \exp(-d + cx_{kT-1}) + \mu) \\
&\leq ((1-p_2) y_{kT-1} \exp(-d + c\varepsilon_0) + \mu) \\
&\leq ((1-p_2) M_2 \exp(-d + c\varepsilon_0) + \mu) \leq M_2.
\end{aligned} \tag{54}$$

(3) Other cases are as follows:

$$\begin{aligned}
y_{n_2} &= y_{n_2-1} \exp(-d + cx_{n_2-1}) \\
&= y_{n_2-1} \exp(-d + cx_{n_2-1}) \\
&\leq M_2 \exp(-d + c\varepsilon_0) \leq M_2,
\end{aligned} \tag{55}$$

which is a contradiction. This completes the proof. $\square$

Theorem 10. Equation (1) is permanence provided that

$$\begin{aligned}
rT - \frac{b\mu(1 - \exp(-dT))}{(1 - (1-p_2) \exp(-dT))(1 - \exp(-d))} \\
> \ln\left(\frac{1}{1-p_1}\right)
\end{aligned} \tag{56}$$

holds true.

Proof. Suppose (x_n, y_n) is a solution of (1) with $(x_0 > 0, y_0 > 0)$. By Lemmas 8 and 9, we have proved there exists a constant $M > 0$ such that $(x_n < M, y_n < M)$ for n large enough.

From (20), we know $y_n > \tilde{y}_n - \varepsilon$ for all n large enough and some $\varepsilon > 0$, so $y_n \geq (\mu/(1-(1-p_2) \exp(-dT))) \exp(-dT) - \varepsilon \triangleq m_2$ for n large enough.

Thus, we only need to find $m_1 > 0$ such that $x_n \geq m_1$ for n large enough.

We will do it in the following two steps.

Step I. From condition (56), let $0 < m_3 < d/c$, $\varepsilon_1 > 0$ be small enough such that $\eta \triangleq (1 - p_1) \exp(T(r - am_3 - b\varepsilon_1) - (\mu/(1 - (1 - p_2) \exp((-d + cm_3)T))) (b(1 - \exp((-d + cm_3)T))/(1 - \exp(-d + cm_3)))) > 1$; we will prove that $x_n < m_3$ cannot hold for all $n > 0$. Otherwise,

$$\begin{aligned} y_{n+1} &\leq y_n \exp(-d + cm_3), \\ y_{kT+0} &= (1 - p_2) y_{kT} + \mu. \end{aligned} \quad (57)$$

So we have $y_n \leq z_n$ and $z_n \rightarrow \tilde{z}_n$, $n \rightarrow \infty$, where z_n is the solution of

$$\begin{aligned} z_{n+1} &= z_n \exp(-d + cm_3), \\ z_{kT+0} &= (1 - p_2) z_{kT} + \mu, \\ z_{0+0} &= y_{0+0}, \end{aligned} \quad (58)$$

and $\tilde{z}_n = (\mu/(1 - (1 - p_2) \exp((-d + cm_3)T))) \exp((-d + cm_3)l)$, $l = 0, 1, 2, \dots, T - 1$, with $y_{0+0} = \mu/(1 - (1 - p_2) \exp((-d + cm_3)T))$, is a positive periodic solution of (58).

Therefore, there exists a $N_1 > 0$ such that

$$y_n \leq z_n < \tilde{z}_n + \varepsilon_1, \quad (59)$$

$$x_{n+1} \geq x_n \exp(r - am_3 - b(\tilde{z}_n + \varepsilon_1)), \quad (60)$$

$$x_{kT+0} = (1 - p_1) x_{kT}$$

for $n > N_1$. Therefore there exists a K_1 , such that if $k > K_1$

$$\begin{aligned} x_{kT+T} &\geq x_{kT+T-1} \exp(r - am_3 - b(\widetilde{z_{kT+T-1}} + \varepsilon_1)) \\ &\geq x_{kT+T-1} \exp(r - am_3 - b\varepsilon_1) \exp(-b\widetilde{z_{kT+T-1}}) \\ &\geq x_{kT+T-2} \exp(2(r - am_3 - b\varepsilon_1)) \\ &\quad \times \exp(-b(\widetilde{z_{kT+T-1}} + \widetilde{z_{kT+T-2}})) \\ &\geq \vdots \\ &\geq x_{kT+0} \exp(T(r - am_3 - b\varepsilon_1)) \\ &\quad \times \exp(-b(\widetilde{z_{kT+T-1}} + \widetilde{z_{kT+T-2}} + \cdots + \widetilde{z_{kT+0}})) \quad (61) \\ &\geq (1 - p_1) x_{kT} \\ &\quad \times \exp\left(T(r - am_3 - b\varepsilon_1) - \frac{\mu}{1 - (1 - p_2) \exp((-d + cm_3)T)} \times \frac{b(1 - \exp((-d + cm_3)T))}{1 - \exp(-d + cm_3)}\right) \\ &= x_{kT} \eta, \end{aligned}$$

then $x_{(k+K_1)T} \geq x_{kT} \eta^k \rightarrow \infty$ as $k \rightarrow \infty$, which is a contradiction to the boundedness of x_{kT+l} . Hence there exists a $n_1 > 0$ such that $x_{n_1} \geq m_3$.

Step II. If $x_n \geq m_3$ for all $n \geq n_1$, then our aim is achieved. Otherwise, $x_n < m_3$ for some $n > n_1$. Set $n^* = \min_{n > n_1} \{n \mid x_n < m_3\}$. We have $x_n \geq m_3$ for $n \in [n_1, n^*)$.

Assume that $n^* \in [kT, (k+1)T)$. It is easy to see that $x_{n^*} < m_3$, $x_{n^*-1} \geq m_3$.

There are two possible cases for $n \in [n^*, (k+1)T)$. Here, for M large enough, we have $\eta_1 = r - am_3 - bM < 0$ and $\eta_2 = r - aM - bM < 0$.

Case (a). There exists a $n'' \in [n^*, (k+1)T)$, such that $x_{n''} \geq m_3$. Let $\hat{n} = \min_{n > n^*} \{n \mid x_n \geq m_3\}$; then $x_n < m_3$ for $n \in [n^*, \hat{n})$ and $x_{\hat{n}} \geq m_3$.

For $n \in [n^*, \hat{n} - 1)$,

$$\begin{aligned} x_{n+1} &\geq x_n \exp(\eta_1) \\ &\vdots \\ &\geq x_{n^*} \exp((n + 1 - n^*) \eta_1). \end{aligned} \quad (62)$$

If $n^* \neq kT + 0$, then

$$\begin{aligned} x_{n^*} &= x_{n^*-1} \exp(r - ax_{n^*-1} - by_{n^*-1}) \\ &\geq m_3 \exp(\eta_2), \\ x_{n+1} &\geq x_{n^*} \exp((n + 1 - n^*) \eta_1) \\ &\geq m_3 \exp(\eta_2) \exp(\eta_1) \\ &\geq (1 - p_1) m_3 \exp(\eta_2) \exp(\eta_1). \end{aligned} \quad (63)$$

If $n^* = kT + 0$, then $kT + 0 > n_1$.

Case (1). Consider $kT = n_1$,

$$x_{n^*} = (1 - p_1) x_{kT} \geq (1 - p_1) m_3 \geq (1 - p_1) m_3 \exp(\eta_2). \quad (64)$$

Case (2). Consider $kT > n_1$,

$$\begin{aligned} x_{n^*} &= (1 - p_1) x_{kT} \\ &\geq (1 - p_1) x_{kT-1} \exp(r - ax_{kT-1} - by_{kT-1}) \\ &\geq (1 - p_1) m_3 \exp(\eta_2), \\ x_{n+1} &\geq x_{n^*} \exp((n + 1 - n^*) \eta_1) \\ &\geq (1 - p_1) m_3 \exp(\eta_2) \exp(\eta_1) \\ &\geq (1 - p_1) m_3 \exp(\eta_2) \exp(\eta_1). \end{aligned} \quad (65)$$

Let $m_1 = (1 - p_1) m_3 \exp(\eta_2) \exp(\eta_1)$. So we have $x_n \geq m_1$ for $n \in [n^*, \hat{n} - 1)$. For $n \geq \hat{n}$ the same arguments can be continued since $x_{\hat{n}} \geq m_3$.

Case (b). There does not exist a $n'' \in [n^*, (k+1)T)$, such that $x_{n''} \geq m_3$; namely, $x_n < m_3$ for $n \in [n^*, (k+1)T)$.

Choose $n_2, n_3 \in \mathbb{Z}^+$ such that

$$n_2 T > \frac{1}{-d + cm_3} \ln \frac{\varepsilon'_1}{M + \mu}, \quad (66)$$

$$\exp(\eta_2) (1 - p_1)^{n_2+1} \exp(\eta_1 (n_2 + 1) T) \eta_1^{n_3} > 1.$$

Let $T' = (n_2 + n_3)T$. We claim that there must be a $\bar{n} \in [(k+1)T, (k+1)T + T']$, such that $x_{\bar{n}} \geq m_3$. Otherwise, $x_n < m_3, n \in [(k+1)T, (k+1)T + T']$. Consider (58) with $z_{(k+1)T+0} = y_{(k+1)T+0}$; we have

$$z_{mT+l} = \left(z_{(k+1)T+0} - \frac{\mu}{1 - (1 - p_1) \exp((-d + cm_3)T)} \right) \times \exp((-d + cm_3)(mT + l - (k+1)T)) + \tilde{z}_{mT+l} \quad (67)$$

for $mT + l \in (nT, (n+1)T)$ and $k+1 \leq n \leq k+1+n_2+n_3$. Then for $m \in (k+1+n_2, k+1+n_2+n_3)$,

$$\begin{aligned} & |z_{mT+l} - \tilde{z}_{mT+l}| \\ & < (M + \mu) \exp((-d + cm_3)(mT + l - (k+1)T)) \\ & \leq (M + \mu) \exp((-d + cm_3)(mT - (k+1)T)) \quad (68) \\ & \leq (M + \mu) \exp((-d + cm_3)(n_2 T)) < \varepsilon'_1, \end{aligned}$$

$$y_{mT+l} \leq z_{mT+l} \leq \tilde{z}_{mT+l} + \varepsilon'_1,$$

which implies that (60) holds for $m \in (k+1+n_2, k+1+n_2+n_3)$.

So as in Step I, we have

$$x_{(k+1+n_2+n_3)T} \geq x_{(k+1+n_2)T} \eta_1^{n_3}. \quad (69)$$

Since $x_n < m_3, n \in [n^*, (k+1)T]$, then $x_n < m_3$ for all $n \in [n^*, ((k+1) + n_2)T]$.

So,

$$x_{n+1} \geq x_n \exp(\eta_1), \quad (70)$$

$$\begin{aligned} x_{((k+1)+n_2)T} & \geq x_{((k+1)+n_2)T-1} \exp(\eta_1) \\ & \geq \vdots \\ & \geq x_{n^*} (1 - p_1)^{n_2} \exp(\eta_1 ((k+1+n_2)T) - n^*). \end{aligned} \quad (71)$$

If $n^* \neq kT + 0$, then

$$\begin{aligned} x_{((k+1)+n_2)T} & = x_{n^*-1} \exp[r - ax_{n^*-1} - by_{n^*-1}] \\ & \quad \times (1 - p_1)^{n_2} \exp(\eta_1 ((k+1+n_2)T) - n^*) \\ & \geq m_3 \exp(\eta_2) (1 - p_1)^{n_2} \exp(\eta_1 ((k+1+n_2)T) - n^*) \\ & \geq m_3 \exp(\eta_2) (1 - p_1)^{n_2} \exp(\eta_1 (n_2 + 1)T). \end{aligned} \quad (72)$$

If $n^* = kT + 0$, then $kT + 0 > n_1$

$$\begin{aligned} x_{((k+1)+n_2)T} & = x_{n^*} (1 - p_1)^{n_2+1} \exp(\eta_1 ((k+1+n_2)T) - n^*) \\ & = x_{n^*-1} \exp[r - ax_{n^*-1} - by_{n^*-1}] \\ & \quad \times (1 - p_1)^{n_2+1} \exp(\eta_1 ((k+1+n_2)T) - n^*) \\ & \geq m_3 \exp(\eta_2) (1 - p_1)^{n_2+1} \exp(\eta_1 ((k+1+n_2)T) - n^*) \\ & \geq m_3 \exp(\eta_2) (1 - p_1)^{n_2+1} \exp(\eta_1 (n_2 + 1)T). \end{aligned} \quad (73)$$

So,

$$\begin{aligned} x_{(k+1)+n_2+n_3} & \geq m_3 \exp(\eta_2) (1 - p_1)^{n_2+1} \\ & \quad \times \exp(\eta_1 (n_2 + 1)T) \eta_1^{n_3} > m_3, \end{aligned} \quad (74)$$

which is a contradiction.

Let $\bar{n} = \min_{n > n^*} \{n \mid x_n \geq m_3\}$. Then $x_{\bar{n}} \geq m_3, x_{\bar{n}-1} < m_3$. For $n \in [n^*, \bar{n} - 1]$, (70) holds. Suppose $\bar{n} \in [(k+1)T + (k' - 1)T, (k+1)T + k'T]$, $k' \leq n_2 + n_3$.

We have

$$\begin{aligned} x_n & \geq x_{n_1} \exp(\eta_1) \\ & \vdots \\ & \geq x_{n^*} (1 - p_1)^{k'} \exp((n - n^*)\eta_1). \end{aligned} \quad (75)$$

If $n^* \neq kT + 0$, then

$$\begin{aligned} x_n & \geq m_3 \exp(\eta_2) (1 - p_1)^{k'} \exp((n - n^*)\eta_1) \\ & \geq m_3 \exp(\eta_2) (1 - p_1)^{k'} \exp((n_2 + n_3 + 1)T\eta_1). \end{aligned} \quad (76)$$

If $n^* = kT + 0$, then $kT + 0 > n_1$

$$\begin{aligned} x_n & \geq m_3 \exp(\eta_2) (1 - p_1)^{k'+1} \exp((n - n^*)\eta_1) \\ & \geq m_3 \exp(\eta_2) (1 - p_1)^{k'+1} \exp((n_2 + n_3 + 1)T\eta_1). \end{aligned} \quad (77)$$

Let $m'_1 = m_3 \exp(\eta_2) (1 - p_1)^{n_2+n_3+1} \exp((n_2 + n_3 + 1)T\eta_1) < m_1$. So $x_n > m'_1$ for $n \in [n^*, \bar{n} - 1]$. For $n > \bar{n} - 1$, the same arguments can be continued since $x_{\bar{n}} \geq m_3$.

The proof is completed. $\square$

Remark II. Let

$$\begin{aligned} f(T) & = rT - \frac{b\mu(1 - \exp(-dT))}{(1 - (1 - p_2) \exp(-dT))(1 - \exp(-d))} \\ & \quad - \ln\left(\frac{1}{1 - p_1}\right). \end{aligned} \quad (78)$$

FIGURE 1: Dynamical behavior of the pest-predator system of (1).

FIGURE 2: Dynamical behavior of the pest-predator system of (1).

Since $f(0) = -\ln(1/(1-p_1)) < 0$, $f(T) \rightarrow \infty$ as $T \rightarrow \infty$ and

$$f''(T) = \frac{b\mu p_2 d^2}{1 - \exp(-d)} \frac{\exp(-dT)(1 + (1-p_2)\exp(-dT))}{(1 - (1-p_2)\exp(-dT))^3} > 0, \quad (79)$$

so $f(T) = 0$ has a unique positive root, denoted by $T_{\max}$. From Theorem 6 we know that the pest-eradication periodic solution is globally stable when $T < T_{\max}$. From Theorem 10, system (1) is permanent if $T > T_{\max}$.

3. Numerical Experiments

In Figure 1, we choose parameters of system (1) as $d = 0.4$, $a = 0.4$, $b = 0.9$, $c = 0.6$, $r = 0.2$, $p_1 = 0.8$, $p_2 = 0.4$, $\mu = 0.2$, $T = 10$, and initial value $x_0 = 1$, $y_0 = 1$. It is easy to verify that condition (11) holds, and, by Theorem 6, the pest-eradication periodic solution of (1) is the global stability.

In Figure 2, we choose parameters of system (1) as $d = 0.4$, $a = 0.4$, $b = 1$, $c = 0.6$, $r = 0.3$, $p_1 = 0.2$, $p_2 = 0.01$, $\mu = 0.2$, $T = 10$, and initial value $x_0 = 1$, $y_0 = 1$. It is easy to verify that condition (56) holds, and, by Theorem 10, (1) is permanence.

4. Conclusion

In this paper, by piecewise Euler method, we construct a discrete impulsive Lotka-Volterra predator-prey model

concerning integrated pest management. The discrete impulsive model gives a new form of describing the impulsive moment. On the other hand, model (1) is a better analogue of the continuous-time impulsive dynamic system. By using Floquets theorem, we show that a globally asymptotically stable pest-eradication periodic solution exists when the impulsive period is less than some critical value and the discrete system is permanence if the impulsive period is larger than some critical value. By (78), the impulsive period critical value $T_{\max}$ can be obtained. Since $T_{\max}$ is a direct function with respect to p_1 , p_2 , and μ , in order to obtain the object of integrated pest management, we can determine the impulsive period T according to effect of the chemical pesticides on the populations and cost of the releasing natural enemies.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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