Research Article

New Interval Oscillation Criteria for Certain Linear Hamiltonian Systems

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Using a generalized Riccati transformation and the general integral means technique, some new interval oscillation criteria for the linear matrix Hamiltonian system $U' = (A(t) - \lambda(t)I)U + B(t)V$, $V' = C(t)U + (\mu(t)I - A^*(t))V$, $t \geq t_0$ are obtained. These results generalize and improve the oscillation criteria due to Zheng (2008). An example is given to dwell upon the importance of our results.

1. Introduction

Hamiltonian matrix systems arise in many dynamic problems; the qualitative properties of Hamiltonian systems and matrix differential systems have been widely studied by many authors (e.g., see [1–17] and the references quoted there in). In this paper, we consider interval oscillatory properties for the linear Hamiltonian system:

$$\begin{aligned} U' &= (A(t) - \lambda(t)I)U + B(t)V, \\ V' &= C(t)U + (\mu(t)I - A^*(t))V, \end{aligned} \quad (1)$$

where $A(t)$, $B(t)$, and $C(t)$ are $n \times n$ matrices of real valued continuous functions on the interval, $J = [t_0, \infty)$, $B(t) = B^*(t)$, $B(t)$ is positive definite, $C(t) = C^*(t)$, and $\mu(t)$, $\lambda(t)$ are real valued continuous functions on J . Here and in the sequel, the transpose of matrix M is denoted by M^* and its positive definiteness is denoted by $M > 0$. For any $n \times n$ Hermitian matrix M , its eigenvalues are real numbers, and we always denote them by $\lambda_1[M] \geq \lambda_2[M] \geq \dots \geq \lambda_n[M]$. The trace of M is denoted by $\text{tr}(M)$, and recall that $\text{tr}(M) = \sum_{k=1}^n \lambda_k(M)$.

When $\lambda(t) = \mu(t) = 0$, the system (1) reduces to the following linear self-adjoint matrix Hamiltonian system:

$$\begin{aligned} U' &= A_1(t)U + B_1(t)V, \\ V' &= C_1(t)U - A_1^*(t)V, \quad t \in J. \end{aligned} \quad (2)$$

A solution $(U(t), V(t))$ of the system (1) (or (2)) is said to be nontrivial if $\det U(t) \neq 0$ for at least one $t \in J$. A nontrivial solution $(U(t), V(t))$ of the system (1) (or (2)) is said to be prepared or self-conjugate if $U^*(t)V(t) - V^*(t)U(t) = 0$, $t \in J$. System (1) (or (2)) is said to be oscillatory on J if there is a nontrivial prepared solution $(U(t), V(t))$ of (1) such that $\det U(t)$ vanishes at least once on $[T, \infty)$ for each $T \geq t_0$. Otherwise, it is said to be nonoscillatory.

Firstly, we give some of the definitions for the sake of convenience.

Definition 1. Let $\mathcal{M}$ be the linear space of $n \times n$ real matrices, and let $\mathcal{S}$ be the subspace of all symmetric matrices in $\mathcal{M}$. For any $A, B \in \mathcal{S}$, $A \geq B$ means that $A - B \geq 0$ is positive semidefinite and $A > B$ means that $A - B > 0$ is positive definite.

Definition 2. A function $H = H(t, s)$ is called to belong to a function class $\mathcal{H}$, denoted by $H \in \mathcal{H}$, if $H \in C(\overline{\mathbf{D}}, \mathbf{R}_+)$, where $\mathbf{D} = \{(t, s) : t > s \geq t_0\}$ ($\overline{\mathbf{D}}$ is the closure of $\mathbf{D}$) which satisfies $H(t, t) = 0$, $H(t, s) > 0$ for $t > s \geq t_0$. Furthermore, H has continuous derivatives $\partial H / \partial t$ and $\partial H / \partial s$ on $\mathbf{D}$ such that for all $(t, s) \in \mathbf{D}$,

$$\begin{aligned} \frac{\partial H(t, s)}{\partial t} &= h_1(t, s) \sqrt{H(t, s)}, \\ \frac{\partial H(t, s)}{\partial s} &= -h_2(t, s) \sqrt{H(t, s)}, \end{aligned} \quad (3)$$

where $h_1, h_2 \in C(\mathbf{D}, \mathbf{R})$ are functions determined by $H(t, s)$. Note that $(t - s)^\mu$ for $\mu > 1$; $\ln(t/s)$ and $G(t - s)$ with $G \in C^1(0, \infty)$, $G'(t) > 0$ for $t > 0$, and $G(0) = 0$ belong to function class $\mathcal{H}$. For given $a, b \in \mathbf{R}$, $a < b$, a function $z(t)$ is called to belong to a function class $Z[a, b]$, denoted by $z \in Z[a, b]$, if $z(t) \in C[a, b]$, $z'(t) \in C(a, b)$, $z(t) > 0$ for $t \in (a, b)$, and $z(a) = z(b) = 0$. We point out that if $H, \tilde{H} \in \mathcal{H}$, and we define a function with $z(t) = H^\mu(b, t)\tilde{H}^\nu(t, a)$ for $t \in [a, b]$, where $\mu, \nu > 1/2$ are constants, then $z \in Z[a, b]$.

Definition 3. A nonlinear (and possibly discontinuous) function $q : \mathcal{S} \rightarrow \mathbf{R}$ with $q(A) \leq 0$ whenever $A \leq 0$ is called negativity preserving; the class of all such negativity-preserving functionals on $\mathcal{S}$ is denoted by $\mathcal{N}(\mathcal{S})$.

The negativity-preserving functionals are widely used in oscillation criteria for matrix differential systems in paper [2]. Meanwhile, using negativity-preserving functionals, Zheng [11] considered the matrix system (2); oscillation criteria of interval type were obtained with the function $H = H(t, s)$ as follows.

Theorem 4. Suppose that there exists $f(t) \in C(J; \mathbf{R})$ such that fB_1^{-1} is differentiable, and $\alpha(t) = \exp\{-2 \int_t^t f(s)ds\}$. If there exists $q \in \mathcal{N}(\mathcal{S})$, $z \in Z[a, b]$ such that

$$q \left\{ \int_a^b \left[z^2 D_1 - \alpha (zA_1^* - z'I) B_1^{-1} (zA_1 - z'I) \right] (t) dt \right\} > 0, \quad (4)$$

where $D_1(t) = \{\alpha[-C_1 - 2fK_1 + f^2B_1^{-1} - (fB_1^{-1})']\}(t)$ and $K_1(t) = (A_1^*B_1^{-1} + B_1^{-1}A_1)(t)$. Then system (2) is oscillatory.

Theorem 5. Suppose that $0 < B_1^{-1}(t) \leq I$. If, for each $r \geq t_0$ and for some $\mu, \nu > 1$,

$$\begin{aligned} & \limsup_{t \rightarrow \infty} \frac{1}{t^{\mu+\nu-1}} \lambda_1 \\ & \times \left\{ \int_r^t (t-s)^\mu (s-r)^\nu \right. \\ & \times \left(\bar{D}_0(s) + 2 \frac{\nu t - (\mu+\nu)s + \mu r}{(t-s)(s-r)} K_1(s) \right) ds \Big\} \\ & > \mu\nu(\mu+\nu-2) \frac{\Gamma(\mu-1)\Gamma(\nu-1)}{4\Gamma(\mu+\nu)}, \end{aligned} \quad (5)$$

where $\bar{D}_0(s) = (-C_1 - A_1^*B_1^{-1}A_1)(s)$, then system (2) is oscillatory.

In this paper, using negativity-preserving functionals on a suitable matrix space and generalized Riccati transformation, we establish some new oscillation criteria for the system (1), which extend and improve the oscillation criteria mentioned previously, and an example is given which dwells upon the importance of our results.

2. Main Results

Now, we give the main oscillation criteria for system (1) of interval type.

Theorem 6. Suppose that there exists $f(t) \in C(J; \mathbf{R})$ such that fB^{-1} is differentiable, and $\alpha(t) = \exp\{-2 \int_t^t f(s)ds\}$. If there exists $q \in \mathcal{N}(\mathcal{S})$, $z \in Z[a, b]$ such that

$$\begin{aligned} & q \left\{ \int_a^b \left[(z^2 D) (t) \right. \right. \\ & \quad \left. \left. - \alpha(t) e^{-\int_{t_0}^t (\lambda+\mu)(s)ds} \left[(z(A^* - QB) - z'I) B^{-1} \right. \right. \right. \\ & \quad \left. \left. \left. \times (z(A - BQ) - z'I) \right] (t) \right] dt \right\} \\ & > 0, \end{aligned} \quad (6)$$

where

$$\begin{aligned} D(t) &= \left\{ \alpha \left[-e^{-\int (\lambda+\mu)} (QA - QBQ + C - (\lambda+\mu)Q + A^*Q) \right. \right. \\ & \quad \left. \left. - 2fK + e^{-\int (\lambda+\mu)} f^2 B^{-1} - (e^{-\int (\lambda+\mu)} f B^{-1})' \right] \right\} (t), \\ K(t) &= e^{-\int_{t_0}^t (\lambda+\mu)(s)ds} \left[(A^* B^{-1} + B^{-1} A)(t) - 2Q \right], \end{aligned} \quad (7)$$

and Q is any constant nonzero symmetric $n \times n$ matrix, then system (1) is oscillatory.

Proof. Suppose that there exists a prepared solution $(U(t), V(t))$ of (1) which is not oscillatory. Without loss of generality, we may assume that $\det U(t) \neq 0$ for $t \in J$. Define

$$\begin{aligned} W(t) &= \alpha(t) e^{-\int_{t_0}^t (\lambda+\mu)(s)ds} \\ & \times [V(t)U^{-1}(t) + f(t)B^{-1}(t) + Q], \quad t \in J. \end{aligned} \quad (8)$$

Then $W(t)$ is Hermitian and satisfies the Riccati equation

$$\begin{aligned} W'(t) &= -\frac{1}{\alpha(t)} e^{\int_{t_0}^t (\lambda+\mu)(s)ds} \\ & \times [WBW - A^*W - WA + QBW + WBQ - D](t). \end{aligned} \quad (9)$$

Let $E(t) = W(t) + \alpha(t) e^{-\int_{t_0}^t (\lambda+\mu)(s)ds} (B^{-1}A - Q)(t)$. Then we get

$$\begin{aligned} \frac{1}{\alpha} E^* B E &= \frac{1}{\alpha} W^* B W \\ &+ e^{-\int (\lambda+\mu)} (A^* W + WA + \alpha e^{-\int (\lambda+\mu)} A^* B^{-1} A) \\ &- e^{-\int (\lambda+\mu)} (QBW - WBQ) \\ &+ \alpha e^{-2 \int (\lambda+\mu)} (QBQ - A^* Q - QA). \end{aligned} \quad (10)$$

It follows from (9) and (10) that, for $t \in J$,

$$\begin{aligned} W' = & -\frac{1}{\alpha} e^{-\int(\lambda+\mu)} E^* B E - D \\ & + \alpha e^{-\int(\lambda+\mu)} A^* B^{-1} A \\ & - \alpha e^{-\int(\lambda+\mu)} (A Q^* + Q A - Q B Q). \end{aligned} \quad (11)$$

Since $B(t) > 0$, we define $R(t) = [e^{\int(\lambda+\mu)} B]^{1/2}(t)$. Let $G(t) = (z R E R)(t) - \alpha(t) z'(t) I$. Then

$$\begin{aligned} \frac{1}{\alpha} R^{-1} G^* G R^{-1} &= \frac{z^2}{\alpha} E^* \left(e^{\int(\lambda+\mu)} B \right) E - z z' (E^* + E) \\ &+ \alpha z'^2 \left(e^{\int(\lambda+\mu)} B \right)^{-1} \\ &= \frac{z^2}{\alpha} e^{\int(\lambda+\mu)} E^* B E - z z' (E^* + E) \\ &+ \alpha e^{-\int(\lambda+\mu)} z'^2 B^{-1}. \end{aligned} \quad (12)$$

For each $T \geq t_0$, by assumptions, there exist $a, b \in \mathbf{R}$, $T \leq a < b$, and $z \in C^1[a, b]$ satisfying $z(a) = z(b) = 0$. Now, multiplying (11) by $z^2(t)$ and integrating it from a to b , we have

$$\begin{aligned} & \int_a^b z^2(t) D(t) dt \\ & \leq \int_a^b \alpha(t) e^{-\int_{t_0}^t (\lambda+\mu)(s) ds} \\ & \quad \times \left(z^2 \left(A^* B^{-1} A - A^* Q - Q A + Q B Q \right) \right. \\ & \quad \left. - z z' \left(A^* B^{-1} + B^{-1} A - 2Q \right) + z'^2 B^{-1} \right) (t) dt \\ & = \int_a^b \alpha(t) e^{-\int_{t_0}^t (\lambda+\mu)(s) ds} \\ & \quad \times \left[\left(z(A - BQ)^* - z' I \right) B^{-1} \left(z(A - BQ) - z' I \right) \right] (t) dt, \end{aligned} \quad (13)$$

which implies that

$$\begin{aligned} & \int_a^b \left[z^2 D - \alpha e^{-\int(\lambda+\mu)} \left(z(A^* - QB) - z' I \right) \right. \\ & \quad \left. \times B^{-1} \left(z(A - BQ) - z' I \right) \right] (t) dt \leq 0. \end{aligned} \quad (14)$$

Applying the functional q to (14), we obtain a contradiction with assumption (6). This completes the proof. $\square$

Corollary 7. Suppose that there exists $f(t) \in C(J; \mathbf{R})$ such that fB^{-1} is differentiable, and $\alpha(t) = \exp\{-2 \int^t f(s) ds\}$. If, for each $T \geq t_0$, there exist $a, b \in \mathbf{R}$, $T \leq a < b$, and $z \in Z[a, b]$ such that

$$\begin{aligned} & \lambda_i \left\{ \int_a^b \left[z^2 D - \alpha e^{-\int(\lambda+\mu)} \left(z(A^* - QB) - z' I \right) \right. \right. \\ & \quad \left. \left. \times B^{-1} \left(z(A - BQ) - z' I \right) \right] (t) dt \right\} > 0, \end{aligned} \quad (15)$$

for some $i \in \{1, 2, \dots, n\}$, where $D(t)$ is defined as previously mentioned, then system (1) is oscillatory.

Corollary 8. Suppose that there exists $f(t) \in C(J; \mathbf{R})$ such that fB^{-1} is differentiable, and $\alpha(t) = \exp\{-2 \int^t f(s) ds\}$. If there exists $q \in \mathcal{N}(\mathcal{S})$ and, for each $r \geq t_0$, there exist $H, \tilde{H} \in \mathcal{H}$ and $\mu, \nu > 1/2$ such that

$$\begin{aligned} \limsup_{t \rightarrow \infty} q \left\{ \int_r^t \left(H^{2\mu}(t, s) \tilde{H}^{2\nu}(s, r) D(s) - \alpha(s) e^{-\int(\lambda+\mu)} \right. \right. \\ \left. \left. \times \Psi^*(t, s, r) B^{-1}(s) \Psi(t, s, r) \right) ds \right\} > 0, \end{aligned} \quad (16)$$

where $D(s)$ is defined as that in Theorem 6, and

$$\begin{aligned} \Psi(t, s, r) &= H^\mu(t, s) \tilde{H}^\nu(s, r) (A - BQ)(s) \\ &- \nu H^\mu(t, s) \tilde{H}^{\nu-1/2}(s, r) \tilde{h}_1(s, r) \\ &+ \mu h_2(t, s) H^{\mu-1/2}(t, s) \tilde{H}^\nu(s, r), \end{aligned} \quad (17)$$

then system (1) is oscillatory.

Let $H(t, s) = \tilde{H}(t, s) = (t - s)^2$, $\mu = \nu = 1/2$; we have the following useful corollary.

Corollary 9. Suppose that there exists $f(t) \in C(J; \mathbf{R})$ such that, $e^{-\int(\lambda+\mu)} fB^{-1}$ is differentiable, and $\alpha(t) = \exp\{-2 \int^t f(s) ds\}$. If there exists $q \in \mathcal{N}(\mathcal{S})$ such that for each $r \geq t_0$,

$$\begin{aligned} \limsup_{t \rightarrow \infty} q \left\{ \int_r^t \left((t - s)^2 (s - r)^2 D(s) - \alpha(s) e^{-\int(\lambda+\mu)} \right. \right. \\ \left. \left. \times \Psi_1^*(t, s, r) B^{-1}(s) \Psi_1(t, s, r) \right) ds \right\} > 0, \end{aligned} \quad (18)$$

where $D(s)$ is defined as in Theorem 6, and

$$\Psi_1(t, s, r) = (t - s)(s - r)(A - BQ)(s) - (t + r - 2s)I, \quad (19)$$

then system (1) is oscillatory.

Theorem 10. Suppose that $0 < B^{-1}(t) \leq e^{\int(\lambda+\mu)} I$. If, for each $r \geq t_0$ and for some $\mu, \nu > 1$,

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t^{\mu+\nu-1}} \lambda_1 \left\{ \int_r^t (t - s)^\mu (s - r)^\nu \right. \\ \left. \times \left(D_0(s) + \frac{\nu t - (\mu + \nu)s + \mu r}{2(t - s)(s - r)} K(s) \right) ds \right\} \\ > \mu \nu (\mu + \nu - 2) \frac{\Gamma(\mu - 1) \Gamma(\nu - 1)}{4\Gamma(\mu + \nu)}, \end{aligned} \quad (20)$$

where

$$D_0(s) = \left[-e^{-\int(\lambda+\mu)} (C + A^* B^{-1} A - (\lambda + \mu) Q) \right] (s), \quad (21)$$

and $K(s)$ are defined as in Theorem 6, then system (1) is oscillatory.

Proof. We choose $f(t) \equiv 0$, so $a(t) \equiv 1$, and

$$D(t) = \left[\alpha \left(-e^{-\int(\lambda+\mu)} \right. \right. \\ \left. \left. \times (QA - QBQ + C - (\lambda + \mu) Q + A^* Q) \right) \right] (t). \quad (22)$$

As in the proof of Theorem 6, we obtain (14), with a being replaced by r , b replaced by t , and the integral variable being replaced by s , that is,

$$\int_r^t z^2(s) D_0(s) ds \\ \leq - \int_r^t (zz'K)(s) ds + \int_r^t z'^2(s) e^{-\int(\lambda+\mu)} B^{-1}(s) ds. \quad (23)$$

Suppose that $z(s) = (t-s)^{\mu/2}(s-r)^{\nu/2}$, we get

$$u'(s) = z(s) \frac{\nu t - (\mu + \nu)s + \mu r}{2(t-s)(s-r)}. \quad (24)$$

So

$$\int_r^t (t-s)^\mu (s-r)^\nu \left(D_0(s) + \frac{\nu t - (\mu + \nu)s + \mu r}{2(t-s)(s-r)} K(s) \right) ds \\ \leq \frac{1}{4} \int_r^t (t-s)^{\mu-2} (s-r)^{\nu-2} \\ \times [\nu(t-s) - \mu(s-r)]^2 e^{-\int(\lambda+\mu)} B^{-1}(s) ds \\ \leq \frac{1}{4} \left[\int_r^t \nu^2 (t-s)^\mu (s-r)^{\nu-2} ds \right. \\ \left. - 2\mu\nu \int_r^t (t-s)^{\mu-1} (s-r)^{\nu-1} ds \right. \\ \left. + \mu^2 \int_r^t (t-s)^{\mu-2} (s-r)^\nu ds \right] I. \quad (25)$$

As in the proof of [11, Theorem 2.7], we obtain

$$\int_r^t (t-s)^\mu (s-r)^\nu \left(D_0(s) + \frac{\nu t - (\mu + \nu)s + \mu r}{2(t-s)(s-r)} K(s) \right) ds \\ \leq \frac{1}{4} (t-r)^{\mu+\nu-1} \left[\frac{\mu\nu^2 \Gamma(\nu-1) \Gamma(\mu)}{\Gamma(\mu+\nu)} \right. \\ \left. - \frac{2\mu\nu \Gamma(\nu) \Gamma(\mu)}{\Gamma(\mu+\nu)} \right]$$

$$+ \frac{\mu^2 \nu \Gamma(\nu) \Gamma(\mu-1)}{\Gamma(\mu+\nu)} \Big] I \\ = (t-r) \mu^{\mu+\nu-1} \mu\nu (\mu+\nu-2) \frac{\Gamma(\mu-1) \Gamma(\nu-1)}{4\Gamma(\mu+\nu)} I. \quad (26)$$

Dividing both sides by $t^{\mu+\nu-1}$ and taking the largest eigenvalue, we have

$$\limsup_{t \rightarrow \infty} \frac{1}{t^{\mu+\nu-1}} \lambda_1 \\ \times \left\{ \int_r^t (t-s)^\mu (s-r)^\nu \right. \\ \left. \times \left(D_0(s) + \frac{\nu t - (\mu + \nu)s + \mu r}{2(t-s)(s-r)} K(s) \right) ds \right\} \\ \leq \mu\nu (\mu+\nu-2) \frac{\Gamma(\mu-1) \Gamma(\nu-1)}{4\Gamma(\mu+\nu)}. \quad (27)$$

This contradicts with condition (20). The proof of Theorem 10 is completed. $\square$

3. Examples

We give the following example to illustrate the applicability of our theorems.

Example 1. Consider the 4-dimensional system (1), where

$$A(t) = \begin{bmatrix} 0 & \frac{1}{t} - t \\ -\frac{1}{t} - t & 0 \end{bmatrix}, \\ C(t) = \begin{bmatrix} \left[\frac{\gamma}{t} \cos t + \frac{3}{4t^3} + \frac{2}{t} + t \right] & 0 \\ 0 & \left[\frac{\gamma}{t} \cos t + \frac{3}{4t^3} - \frac{2}{t} + t \right] \end{bmatrix}, \\ B(t) = tI_2, \quad t \geq 1, \quad \lambda = 1, \quad \mu = -1, \\ Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (28)$$

We get that $K(t) = 0$. Let $f(t) = -1/2t$; let $\alpha(t) = t$, and we have $D(t) = \gamma \cos t I_2$. For each $T \geq 1$, select k to be large enough such that $T < (-\pi/2 + 2k\pi) = a < (\pi/2 + 2k\pi) = b$. Let $u(t) = \sin t$. Using Theorem 6, we have

$$\int_a^b \left[z^2 D - \alpha e^{-\int(\lambda+\mu)} (z(A - BQ)^* - z'I) \right. \\ \left. \times B^{-1} (z(A - BQ) - z'I) \right] (t) dt \\ = \int_{-\pi/2+2k\pi}^{\pi/2+2k\pi} \gamma \sin^2 t \cos t I_2$$

$$\begin{aligned}
& - \begin{pmatrix} -\cos t & -\frac{\sin t}{t} \\ \frac{\sin t}{t} & -\cos t \end{pmatrix} \\
& \times \begin{pmatrix} -\cos t & \frac{\sin t}{t} \\ -\frac{\sin t}{t} & -\cos t \end{pmatrix} \Bigg] dt \\
& = \left[\int_{-\pi/2+2k\pi}^{\pi/2+2k\pi} \left(\gamma \sin^2 t \cos t - \cos^2 t - \frac{\sin^2 t}{t^2} \right) dt \right] I_2 \\
& \geq \left[\int_{-\pi/2+2k\pi}^{\pi/2+2k\pi} \left(\gamma \sin^2 t \cos t - \cos^2 t - \sin^2 t \right) dt \right] I_2 \\
& = \left[\int_{-\pi/2}^{\pi/2} \sin^2 t (\gamma \cos t - 1) dt \right] I_2 \\
& > 0 \quad \left(\text{for } \gamma > \frac{1}{2} \right).
\end{aligned} \tag{29}$$

So we obtain that the Hamiltonian system is oscillatory for $\gamma > 1/2$.

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