Research Article

Dynamics of a Market Share Model for Enterprises with Coopetition Strategy

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A deterministic model is used to study the change of the market share with coopetition strategy for enterprises. The model takes into consideration both coopetition enterprises and other enterprises, and the coopetition threshold R_0 is identified and global dynamics are completely determined by R_0 . It shows that R_0 is a global threshold parameter in the sense that if $R_0 < 1$, the coopetition free equilibrium is globally stable and the market share of coopetition enterprises tends to zero, whereas if $R_0 > 1$, there is a unique coopetition equilibrium which is globally attractive with some conditions, and thus the market share of coopetition enterprises tends to a steady state value. By some sensitivity analysis of R_0 on parameters, we conclude that the size of the coopetition threshold R_0 and coopetition equilibrium depended on the cooperation competitiveness of coopetition enterprises.

1. Introduction

Coopetition (cooperation and competition) was put forward by Nalebuff and Brandenburger for the first time in 1996, and they discussed the importance of coopetition in business by using game theory as a theoretical frame [1]. In the same year, Bengtsson and Kock also called the phenomenon coopetition which contains competition and cooperation [2]. Coopetitive relationships are complex as they consist of two diametrically different types of logic interaction. Actors involved in coopetition are involved in a relationship that on the one hand consists of hostility due to conflicting interests and on the other hand consists of friendliness due to common interests. These two types of logic of interaction are in conflict with each other and must be separated in a proper way to make a coopetitive relationship possible. In 2000, Bengtsson and Kock studied the coopetition of corporate network [3]. The coopetition theory of Deutsch divides the target of interest subjects into cooperation, competition and independence; the relationship of the target partners affects their interaction results and then directly affects their action strategies [4]. Hausken introduced competition between groups that may induce cooperation to emerge in defection games despite considerable cost of cooperation [5]. Loebbecke et al. studied

knowledge transfer and distribution which are based on coopetition [6].

With the development of the information, communication technology, networks, and virtual organizations, the coopetition phenomenon has appeared between enterprises. Resource heterogeneity determines that the supply chain of alliance partner enterprises is a symbiotic relationship with coopetition, that is, the coopetition relationships of the enterprise dominant in the supply chain network. Zhuo et al. [7] studied the supply chain alliance partner enterprise coopetition relations based on Volterra model. In their model, they considered the alliance which contains two enterprises. The following system (1) is the main model:

$$\begin{aligned}\frac{dx_1}{dt} &= r_1 x_1 \left(1 - \frac{x_1}{N_1}\right) + \frac{\delta_2 r_1 x_1 x_2}{N_2} - \frac{\lambda_2 r_1 x_1 x_2}{N_2}, \\ \frac{dx_2}{dt} &= r_2 x_2 \left(1 - \frac{x_2}{N_2}\right) + \frac{\delta_1 r_2 x_1 x_2}{N_1} - \frac{\lambda_1 r_2 x_1 x_2}{N_1},\end{aligned}\quad (1)$$

where x_i is the production value of enterprise i ; N_i is the limit production value of enterprise i ; r_i is the natural growth rate of enterprise i . δ_1 represents the contribution rate from enterprise 1 to enterprise 2, and δ_2 describes the contribution rate from enterprise 2 to enterprise 1. λ_1 represents

FIGURE 1: Cooperation diagram of two enterprises.

the competition rate from enterprise 1 to enterprise 2, and λ_2 describes the competition rate from enterprise 2 to enterprise 1. In this model, they only considered the change of production value of enterprise about cooperation and competition and do not include the direction in which the production value has gone. Based on previous work, in this paper, we consider a deterministic model of two cooperation enterprises and other enterprises and study the change direction of the market share with cooperation strategy of these two cooperation enterprises.

The paper is organized as follows. In Section 2, we present and interpret the dynamical model that describes the market share of two cooperation enterprises and other enterprises, give the cooperation threshold of this model and the uniqueness of positive cooperation equilibrium when the cooperation threshold is larger than 1, and prove the stability of the cooperation equilibrium. And in Section 3, some numerical simulations are showed to illustrate the effectiveness of the proposed result. Section 4 gives a brief discussion about main results.

2. Mathematical Modeling and Analysis

2.1. Model Formulation. In our model, we classify the market quantity of all enterprises into three compartments: $S_1(t)$ describes the market share of the cooperation enterprise 1, $S_2(t)$ denotes the market share of the cooperation enterprise 2, and $S_o(t)$ represents the market share of other enterprises at time t . Let $C(t)$ denote the density of cooperation awareness programs between two enterprises at time t . There are some assumptions about this dynamical model. (i) It is assumed that the market quantity of all enterprises is kept constant. (ii) The increase and decrease of the market share of these two cooperation enterprises only depended on the cooperation and competition between these two enterprises. (iii) The growth rate of density of cooperation awareness programs is assumed to be proportional to the market share of the cooperation enterprise 1 and enterprise 2. There are also some other assumptions about this dynamical model in the flowchart. More details can be found in Figure 1.

For the market share of cooperation enterprise 1 compartment $S_1(t)$, the input of compartment $S_1(t)$ includes $k_1\beta S_o C$ and $pd_{21}S_2$. $k_1\beta S_o C$ denotes the market share from compartment $S_o(t)$ to compartment $S_1(t)$ through the cooperation between enterprise 1 and enterprise 2, and $pd_{21}S_2$ represents the market share from compartment $S_2(t)$ to compartment

$S_1(t)$ through the competition between enterprise 1 and enterprise 2. Due to the competition between enterprise 1 and enterprise 2, the output of $S_1(t)$ is $d_{12}S_1$. So the change of $S_1(t)$ is as follows:

$$\frac{dS_1}{dt} = k_1\beta S_o C - d_{12}S_1 + pd_{21}S_2. \quad (2)$$

In the same way, we can obtain the change over time in the size of the compartment $S_2(t)$, which is as follows:

$$\frac{dS_2}{dt} = k_2\beta S_o C - d_{21}S_2 + pd_{12}S_1. \quad (3)$$

For the market share of other enterprises compartment $S_o(t)$, the output of compartment $S_o(t)$ includes $k_1\beta S_o C$ and $k_2\beta S_o C$, and the input of compartment $S_o(t)$ includes $(1-p)d_{12}S_1$ and $(1-p)d_{21}S_2$. So the change over time in the size of the compartment $S_o(t)$ is as follows:

$$\frac{dS_o}{dt} = (1-p)(d_{12}S_1 + d_{21}S_2) - (k_1 + k_2)\beta S_o C. \quad (4)$$

For the density of cooperation awareness programs compartment $C(t)$, the input of the compartment $C(t)$ contains $k_1S_1 + k_2S_2$, which is the density of cooperation awareness programs discharged by the cooperation enterprises 1 and 2. The output of the compartment $C(t)$ is the depletion of cooperation awareness programs, which is δC . So the change over time in the size of the compartment $C(t)$ is as follows:

$$\frac{dC}{dt} = k_1S_1 + k_2S_2 - \delta C. \quad (5)$$

So the mathematical model is described as the following ordinary differential equations:

$$\begin{aligned} \frac{dS_o}{dt} &= (1-p)(d_{12}S_1 + d_{21}S_2) - (k_1 + k_2)\beta S_o C, \\ \frac{dS_1}{dt} &= k_1\beta S_o C - d_{12}S_1 + pd_{21}S_2, \\ \frac{dS_2}{dt} &= k_2\beta S_o C - d_{21}S_2 + pd_{12}S_1, \\ \frac{dC}{dt} &= k_1S_1 + k_2S_2 - \delta C, \end{aligned} \quad (6)$$

where

$$S_o(t) + S_1(t) + S_2(t) = 1. \quad (7)$$

All parameters are assumed to be nonnegative in system (6). k_1 and k_2 represent the rate with which cooperation awareness programs are being implemented from enterprises 1 and 2, respectively. δ denotes the depletion rate of cooperation awareness programs. d_{21} represents the competition awareness programs rate from enterprise 1 to enterprise 2, and d_{12} represents the competition awareness programs rate from enterprise 2 to enterprise 1. p ($0 < p < 1$) is the proportion of the market share from enterprise 1 to enterprise 2 or from enterprise 2 to enterprise 1 due to the competition between

enterprise 1 and enterprise 2, and β is the transmission rate of enterprise cooperation.

For system (6), if there is no cooperation, system (6) will become the following system:

$$\frac{dS_o}{dt} = 0, \quad \frac{dS_1}{dt} = 0, \quad \frac{dS_2}{dt} = 0, \quad \frac{dC}{dt} = 0. \quad (8)$$

It has an equilibrium $P_0 = (S_o(0), S_1(0), S_2(0), C(0))$.

The positive cooperation equilibrium $P^* = (S_o^*, S_1^*, S_2^*, C^*)$ and cooperation free equilibrium $P^0 = (S_o^0, 0, 0, 0)$ of system (6) are determined by equations

$$\begin{aligned} (1-p)(d_{12}S_1 + d_{21}S_2) - (k_1 + k_2)\beta S_o C &= 0, \\ k_1\beta S_o C - d_{12}S_1 + pd_{21}S_2 &= 0, \\ k_2\beta S_o C - d_{21}S_2 + pd_{12}S_1 &= 0, \\ k_1S_1 + k_2S_2 - \delta C &= 0. \end{aligned} \quad (9)$$

Calculating system (9), if $S_1 = S_2 = 0$, we can obtain that $C = 0$ and $S_o = S_o^0 = S$, and system (6) has a cooperation free equilibrium $P^0 = (S_o^0, 0, 0, 0)$; if $S_1 \neq 0, S_2 \neq 0$, we can obtain that

$$\begin{aligned} S_o^* &= \frac{d_{12}d_{21}\delta(1-p^2)}{\beta(d_{12}k_2(k_1p + k_2) + d_{21}k_1(k_1 + k_2p))}, \\ C^* &= \frac{d_{12}k_2(k_2 + k_1p) + d_{21}k_1(k_1 + k_2p)}{d_{12}(k_2 + k_1p)\delta} S_2^*, \\ S_1^* &= \frac{d_{21}(k_1 + k_2p)}{d_{12}(k_1p + k_2)} S_2^*, \\ S_2^* &= \frac{d_{12}(k_1p + k_2)}{d_{12}(k_1p + k_2) + d_{21}(k_1 + k_2p)} \\ &\quad \times \left(1 - \frac{d_{12}d_{21}\delta(1-p^2)}{\beta(d_{12}k_2(k_1p + k_2) + d_{21}k_1(k_1 + k_2p))} \right). \end{aligned} \quad (10)$$

For the positive cooperation equilibrium P^* of system (6), we must make $S_2^* > 0$, and it means that $1 - d_{12}d_{21}\delta(1-p^2)/\beta(d_{12}k_2(k_1p + k_2) + d_{21}k_1(k_1 + k_2p)) > 0$ and $\beta(d_{12}k_2(k_1p + k_2) + d_{21}k_1(k_1 + k_2p))/d_{12}d_{21}\delta(1-p^2) > 1$. So we can define the cooperation threshold by the following equation:

$$R_0 = \frac{\beta(d_{12}k_2(k_1p + k_2) + d_{21}k_1(k_1 + k_2p))}{d_{12}d_{21}\delta(1-p^2)}. \quad (11)$$

From the above analysis, we can conclude that when $R_0 < 1$, system (6) only has a cooperation free equilibrium $P^0 = (S_o^0, 0, 0, 0)$; when $R_0 > 1$, system (6) has a unique positive cooperation equilibrium $P^* = (S_o^*, S_1^*, S_2^*, C^*)$. Define the set

$$\begin{aligned} X &= \left\{ (S_o, S_1, S_2, C) \mid S_o, S_1, S_2 \geq 0, \right. \\ &\quad \left. 0 \leq C \leq \frac{d_{12}k_2(k_2 + k_1p) + d_{21}k_1(k_1 + k_2p)}{d_{12}(k_2 + k_1p)\delta} \right\}, \end{aligned} \quad (12)$$

which is invariant with respect to system (6).

2.2. Stability of the Cooperation Equilibrium. In this section, we will prove that the cooperation free equilibrium $P^0 = (S_o^0, 0, 0, 0)$ is globally asymptotically stable. In order to prove global stability of the cooperation free equilibrium, the Lyapunov function will be used. The Lyapunov function is a powerful tool for the stability analysis of autonomous differential system, and it has been used for some epidemiological models with constant inflow and bilinear incidences or nonlinear incidences [8–16]. In the following, we will prove global stability of the cooperation free equilibrium by using a Lyapunov function.

Theorem 1. *The cooperation free equilibrium P^0 of system (6) is globally asymptotically stable when $R_0 < 1$.*

Proof. In order to investigate the asymptotic behavior of equilibrium $P^0 = (S_o^0, 0, 0, 0)$ for system (6), we can reduce (6) to the following equations:

$$\begin{aligned} \frac{dS_1}{dt} &= k_1\beta S_o C - d_{12}S_1 + pd_{21}S_2, \\ \frac{dS_2}{dt} &= k_2\beta S_o C - d_{21}S_2 + pd_{12}S_1, \\ \frac{dC}{dt} &= k_1S_1 + k_2S_2 - \delta C. \end{aligned} \quad (13)$$

For the cooperation free equilibrium P^0 , we define the following Lyapunov function:

$$L_1 = mS_1 + nS_2 + C. \quad (14)$$

Then the derivative of L_1 along solutions of system (13) is

$$\begin{aligned} \frac{dL_1}{dt} &= mS_1' + nS_2' + C' \\ &= m(k_1\beta S_o C - d_{12}S_1 + pd_{21}S_2) \\ &\quad + n(k_2\beta S_o C - d_{21}S_2 + pd_{12}S_1) \\ &\quad + k_1S_1 + k_2S_2 - \delta C = A + B, \end{aligned} \quad (15)$$

where

$$\begin{aligned} A &= mk_1\beta S_o C + nk_2\beta S_o C - \delta C, \\ B &= m(-d_{12}S_1 + pd_{21}S_2) \\ &\quad + n(-d_{21}S_2 + pd_{12}S_1) + k_1S_1 + k_2S_2. \end{aligned} \quad (16)$$

Considering the following equations

$$\begin{aligned} -md_{12}S_1 + npd_{12}S_1 + k_1S_1 &= 0, \\ mpd_{21}S_2 - nd_{21}S_2k_2S_2 &= 0, \end{aligned} \quad (17)$$

we have

$$\begin{aligned} m &= \frac{d_{12}k_2p + d_{21}k_1}{d_{12}d_{21}(1-p^2)}, \\ n &= \frac{d_{12}k_2 + d_{21}k_1p}{d_{12}d_{21}(1-p^2)}. \end{aligned} \quad (18)$$

Letting (18) generate into (15), we can obtain that

$$\begin{aligned} \frac{dL_1}{dt} &= A + B \\ &= \left(\frac{d_{12}k_2p + d_{21}k_1}{d_{12}d_{21}(1-p^2)}k_1 + \frac{d_{12}k_2 + d_{21}k_1p}{d_{12}d_{21}(1-p^2)}k_2 \right) \\ &\quad \times \beta S_o C - \delta C \\ &\leq (R_0 - 1)\delta C. \end{aligned} \quad (19)$$

Therefore, when $R_0 < 1$, $dL_1/dt < 0$, and the equality $dL_1/dt = 0$ holds if and only if $R_0 = 1$. Thus the coopetition free equilibrium P^0 is globally asymptotically stable in X by LaSalle's invariance principle [17]. This completes the proof. $\square$

Next, we will show that when $R_0 > 1$, the unique positive coopetition equilibrium $P^* = (S_o^*, S_1^*, S_2^*, C^*)$ of system (6) is globally attractive.

For system (6), the first equation is independent of the last three equations. And as for $S_o(t) + S_1(t) + S_2(t) = 1$, system (6) has the following limiting system:

$$\begin{aligned} \frac{dS_1}{dt} &= (1 - S_1 - S_2)k_1\beta C - d_{12}S_1 + pd_{21}S_2, \\ \frac{dS_2}{dt} &= (1 - S_1 - S_2)k_2\beta C - d_{21}S_2 + pd_{12}S_1, \\ \frac{dC}{dt} &= k_1S_1 + k_2S_2 - \delta C. \end{aligned} \quad (20)$$

For system (20), we define the vector $\bar{x} = (S_1, S_2, C)$.

Theorem 2. When $R_0 > 1$, $pd_{21} \geq \max\{C(t)\}k_1\beta$, and $pd_{12} \geq \max\{C(t)\}k_2\beta$, the positive coopetition equilibrium $E^* = (S_1^*, S_2^*, C^*)$ of system (20) is globally asymptotically stable with respect to $\bar{x}(0) \in X$.

Proof. We will use the theory of cooperate system to prove the global stability; therefore, we only need to verify the assumption in Corollary 3.2 in [18] for system (20).

Let $\bar{f} : X \rightarrow X$ be defined by the right-hand side of system (20), $\bar{f} = (f_1, f_2, f_3)$. Clearly $\bar{f}$ is continuously differentiable, $\bar{f}(0) = 0$, $\bar{f}(\bar{x}) \geq 0$, and $D\bar{f}(\bar{x})$ is Irreducible for all $\bar{x} \in X$. Calculating the jacobian matrix of system (20) at the unique positive coopetition equilibrium $E^* = (S_1^*, S_2^*, C^*)$,

$$\begin{aligned} D\bar{f}(E^*) &= \begin{pmatrix} -d_{12} - k_1\beta C^* & pd_{21} - k_1\beta C^* & k_1\beta(1 - S_1^* - S_2^*) \\ pd_{12} - k_2\beta C^* & -d_{21} - k_2\beta C^* & k_1\beta(1 - S_1^* - S_2^*) \\ k_1 & k_2 & -\delta \end{pmatrix}. \end{aligned} \quad (21)$$

Because $pd_{21} \geq \max\{C(t)\}k_1\beta$ and $pd_{12} \geq \max\{C(t)\}k_2\beta$, $\bar{f}$ is cooperative.

Note that, for $\forall \alpha \in (0, 1)$ and $\bar{x} > 0$,

$$\begin{aligned} f_1(\alpha\bar{x}) &= \alpha[(1 - \alpha S_1 - \alpha S_2)k_1\beta C - d_{12}S_1 + pd_{21}S_2] \\ &\geq \alpha[(1 - S_1 - S_2)k_1\beta C - d_{12}S_1 + pd_{21}S_2] \\ &= \alpha f_1(\bar{x}). \end{aligned} \quad (22)$$

In a similar way, we can obtain that

$$f_2(\alpha\bar{x}) \geq \alpha f_2(\bar{x}), \quad f_3(\alpha\bar{x}) = \alpha f_3(\bar{x}). \quad (23)$$

Thus $\bar{f}(\alpha\bar{x}) \geq \alpha\bar{f}(\bar{x})$, which is sublinear on X . By Lemma 2 and Corollary 3.2 in [18], we can conclude that the positive coopetition equilibrium $E^* = (S_1^*, S_2^*, C^*)$ of system (20) is globally asymptotically stable with respect to $\bar{x}(0) \in X$. $\square$

Next, by a similar proof to that of Theorem 3.1 in [19], we will prove the following theorem.

Theorem 3. When $R_0 > 1$, the unique positive coopetition equilibrium $P^* = (S_o^*, S_1^*, S_2^*, C^*)$ of system (6) is globally attractive with respect to $(S_o(0), \bar{x}(0)) \in X$.

Proof. Let $\Phi(t) : R_+^4 \rightarrow R_+^4$ be the solution semiflow of system (6), and let ω be omega limit set of $\Phi(S_o(0), \bar{x}(0))$, $(S_o(0), \bar{x}(0)) \in X$. By Lemmas 1 and 1.2.1 in [20], ω is an internal chain transitive set for $\Phi(t)$. Obviously, for system (6), there are only two equilibria P^0 and P^* when $R_0 > 1$. By Theorems 1 and 3, it is not difficult to verify that $\Phi(t)$ satisfies the condition of Theorem 1.2.2 in [20]; thus, ω should be either P^0 or P^* .

Next, we prove that $\omega = \{P^*\}$. If this were not true; then, $\omega = \{P^0\}$, then we should prove $\lim_{t \rightarrow \infty} \sup S_o = 1$, $\lim_{t \rightarrow \infty} \sup S_1 = 0$, $\lim_{t \rightarrow \infty} \sup S_2 = 0$, and $\lim_{t \rightarrow \infty} \sup C = 0$. Let

$$M = \begin{pmatrix} -d_{12} & pd_{21} & k_1\beta \\ pd_{12} & -d_{21} & k_1\beta \\ k_1 & k_2 & -\delta \end{pmatrix}. \quad (24)$$

Obviously, M is irreducible and has nonnegative off-diagonal elements. Define $s(M) = \max\{\text{Re } \lambda : \lambda \text{ as an eigenvalue of } M\}$, so $s(M)$ is a simple eigenvalue of M with a positive eigenvector [21]. By Theorem 2 of van den Driessche and Watmough [22], there hold two equivalences:

$$\begin{aligned} \mathcal{R}_0 > 1 &\iff s(M) > 0, \\ \mathcal{R}_0 < 1 &\iff s(M) < 0. \end{aligned} \quad (25)$$

Since $s(M) > 0$, we can choose a small $\varepsilon > 0$ such that $s(M_2) > 0$, $(M_2 = M + \varepsilon M_1)$, where

$$M_1 = \begin{pmatrix} 0 & 0 & k_1\beta \\ 0 & 0 & k_1\beta \\ 0 & 0 & 0 \end{pmatrix}. \quad (26)$$

It follows that there exists a t_0 such that $1 - S_1 - S_2 > 1 - \varepsilon$, for $t > t_0$. Thus, we have

$$\begin{aligned}\frac{dS_1}{dt} &> (1 - \varepsilon)k_1\beta C - d_{12}S_1 + pd_{21}S_2, \\ \frac{dS_2}{dt} &> (1 - \varepsilon)k_2\beta C - d_{21}S_2 + pd_{12}S_1, \\ \frac{dC}{dt} &= k_1S_1 + k_2S_2 - \delta C.\end{aligned}\quad (27)$$

Consider the following system:

$$\begin{aligned}\frac{dS'_1}{dt} &= (1 - \varepsilon)k_1\beta C' - d_{12}S'_1 + pd_{21}S'_2, \\ \frac{dS'_2}{dt} &= (1 - \varepsilon)k_2\beta C' - d_{21}S'_2 + pd_{12}S'_1, \\ \frac{dC'}{dt} &= k_1S'_1 + k_2S'_2 - \delta C'.\end{aligned}\quad (28)$$

Since the matrix M_2 has positive eigenvalue $s(M_2)$ with a positive eigenvector, it is easy to see that $(S'_1(t), S'_2(t), C'(t)) \rightarrow (\infty, \infty, \infty)$, $t \rightarrow \infty$. Using the comparison principle of Smith and Waltman [21], we also know that $(S_1(t), S_2(t), C(t)) \rightarrow (\infty, \infty, \infty)$, $t \rightarrow \infty$, which leads to a contradiction. Consequently the unique positive coopetition equilibrium $P^* = (S_o^*, S_1^*, S_2^*, C^*)$ is globally attractive. $\square$

So far all our analyses are focused on the mathematical models and their dynamic behavior, such as the coopetition threshold, the uniqueness of positive coopetition equilibrium, the global stability of the coopetition free equilibrium, and the global attraction of the positive coopetition equilibrium. We want to seek out how the coopetition strategy influences the market share of the coopetition enterprises, so in the next section, we will present some numerical simulations about the global stability of the coopetition equilibrium and give some sensitivity analysis of the coopetition threshold R_0 on parameters.

3. Numerical Simulations

In this coopetition model, the coopetition threshold R_0 is calculated and shown to be a threshold for the dynamics of the coopetition model. The main purpose is to let the market share of the two enterprises coexist by making the threshold R_0 to be more than 1, so we must know how the coopetition threshold depends on the model parameter values. In the following result, we will show that the coopetition threshold R_0 is a global threshold parameter for the extinction and persistence of the coopetition.

Taking $\beta = 0.025$, $p = 0.4$, $d_{12} = 0.7$, $d_{21} = 0.6$, and $\delta = 3$ and using MATLAB ODE solver, we run numerical simulations for the case with $R_0 < 1$ (see Figure 2) and the case with $R_0 > 1$ (see Figure 3) to demonstrate the conclusions in Theorems 1 and 3.

From Figure 2, we can see that the trajectory of system (6) will tend to the coopetition free equilibrium with different

FIGURE 2: The phase diagram of $S_1(t)$ and $S_2(t)$ with $k_1 = 4$ and $k_2 = 5$. This gives $R_0 \approx 0.8858 < 1$ and the coopetition makes the market share of the enterprise not coexist.

FIGURE 3: The phase diagram of $S_1(t)$ and $S_2(t)$ with $k_1 = 5$ and $k_2 = 6$. This gives $R_0 \approx 1.3180 > 1$ and the coopetition makes the market share of the enterprise coexist.

initial conditions when the coopetition threshold R_0 is less than 1, so it is concluded that the coopetition free equilibrium is stable when the coopetition threshold $R_0 < 1$. From Figure 3, we can also conclude that the coopetition equilibrium is stable when the coopetition threshold R_0 is more than 1. For the coopetition enterprises, the main purpose is

FIGURE 4: (a) The cooperation threshold R_0 in terms of k_1, k_2 . (b) The cooperation threshold R_0 in terms of d_{12}, d_{21} .

keeping mutual benefit equilibrium in the cooperation and competition. In this situation, we must make the cooperation threshold R_0 more than 1.

From the expression of the cooperation threshold R_0 of system (6), it is easy to see that all parameter values are directly or indirectly contained in R_0 . So some sensitivity analyses of the cooperation threshold R_0 about parameters are given in the following section.

Figure 4(a) shows that increasing k_1 or k_2 can increase the cooperation threshold R_0 . However, in Figure 4(b), an increase in d_{12} or d_{21} will decrease the cooperation threshold R_0 . When the cooperation awareness programs rate (k_1, k_2) or the competition awareness programs rate (d_{12}, d_{21}) reaches a certain level, it will make the cooperation threshold R_0 more than 1, and then the cooperation enterprises will keep mutual benefit equilibrium in the cooperation and competition.

When the cooperation threshold R_0 is more than 1, we want to know the market share of these two cooperation enterprises at the equilibrium state. From the expression of the cooperation equilibrium P^* , we also know that S_1^* and S_2^* are increasing with respect to k_1 or k_2 , and an increase in d_{12} or d_{21} will decrease S_1^* and S_2^* . In the same way, if the cooperation enterprises want to have enough market share, the cooperation awareness programs rate (k_1, k_2) and the competition awareness programs rate (d_{12}, d_{21}) must also reach a certain level.

4. Conclusion and Discussion

Cooperation is different competition and cooperation, which contains two components of competition and cooperation by using game theory. In network organizations, enterprises often cooperate with other enterprises in order to compete more effectively. In order to seize the opportunity from cooperation, enterprises will also seek attractive partners with a competitive attitude. In fact, cooperation and competition are a relative force, which gives the positive and negative effect in the market [7]. So in this paper, we propose a deterministic model of two cooperation enterprises and other

enterprises to describe the change of market share of these two cooperation enterprises. It is found that the model has the cooperation threshold R_0 and two nonnegative equilibria, the cooperation free equilibrium and the cooperation equilibrium. The cooperation free equilibrium exists without any condition whereas the cooperation equilibrium exists provided $R_0 > 1$. Through the analysis of the model, it has been found that the global asymptotic behavior of system (6) is completely determined by the size of the cooperation threshold R_0 ; that is, the cooperation free equilibrium is globally asymptotically stable if $R_0 < 1$ while a cooperation equilibrium exists uniquely and is globally attractive if $R_0 > 1$.

By some sensitivity analysis of the cooperation threshold R_0 on parameters, we find that increasing the cooperation awareness programs rate (k_1, k_2) or decreasing the competition awareness programs rate (d_{12}, d_{21}) can increase the cooperation threshold R_0 . From the expression of the cooperation equilibrium P^* , we also know that S_1^* and S_2^* are increasing with respect to k_1 or k_2 , and an increase in d_{12} or d_{21} will decrease S_1^* and S_2^* . So we can conclude that the size of the cooperation equilibrium depended on the cooperation competitiveness of these two cooperation enterprises. This conclusion is well explained that there is a symbiotic relationship between enterprises (which contains both cooperation and competition, and both are paid and rewarding) and the cooperation enterprise can achieve mutual benefit equilibrium in the cooperation and competition.

The cooperative competition relationship between enterprises is involved in many aspects. In this paper, we only study the basic rule of cooperation between enterprises, and there also exist some assumptions on this dynamical model. We do not consider the situation without these assumptions, and the information sharing and blockade of cooperation enterprise can not be also taken into account in this model. So we need to continue research in the future. Furthermore, many researches have been performed to study spatial dynamics [23] which can be used to model the relationships between enterprises.

Conflict of Interests

The author declares that she has no conflict of interests.

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