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Research Article

New Delay-Dependent Robust Stability Criterion for LPD Discrete-Time Systems with Interval Time-Varying Delays

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This paper investigates the problem of robust stability for linear parameter-dependent (LPD) discrete-time systems with interval time-varying delays. Based on the combination of model transformation, utilization of zero equation, and parameter-dependent Lyapunov-Krasovskii functional, new delay-dependent robust stability conditions are obtained and formulated in terms of linear matrix inequalities (LMIs). Numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed methods.

1. Introduction

Systems with time delay exist in many fields such as electric systems, chemical processes systems, networked control systems, telecommunication systems, and economical systems. Over the past decades, the problem of robust stability analysis for uncertain systems with time delay has been widely investigated by many researchers. Commonly, stability criteria for uncertain systems with time delay are generally divided into two classes: a delay-independent one and a delay-dependent one. The delay-independent stability criteria tends to be more conservative, especially for a small size delay; such criteria do not give any information on the size of delay. On the other hand, delay-dependent stability criteria are concerned with the size of delay and usually provide a maximal delay size.

Discrete-time systems with state delay have strong background in engineering applications, among which network-based control has been well recognized to be a typical example. If the delay is constant in discrete systems, one can transform a delayed system into a delay-free one by using state augmentation techniques. However, when the delay is large, the augmented system will become much complex and thus difficult to analyze and synthesize [1]. In recent years, robust stability analysis of continuous-time and discrete-time systems subject to time-invariant parametric uncertainty has received considerable attention. An important class of linear time-invariant parametric uncertain system is a linear parameter-dependent (LPD) system in which the uncertain state matrices are in the polytope consisting of all convex combination of known matrices. To address this problem, several results have been obtained in terms of sufficient (or necessary and sufficient) conditions, see [1–24] and references cited therein. Most of these conditions have been obtained via the Lyapunov theory approaches in which the parameter-dependent Lyapunov functions have been employed. These conditions are always expressed in terms of LMIs which can be solved numerically by using available tools such as the LMI Toolbox in MATLAB. Recently, delay-dependent robust stability criteria for LPD continuous-time systems with time delay have been taken into consideration. Sufficient conditions for robust stability of time-delay systems have been presented via Lyapunov approaches [8, 16, 21]. However, much attention has been focused on the problem of robust stability analysis for LPD discrete-time systems with time delay [10, 13, 22].

In this paper, we focus on the delay-dependent robust stability criterion for LPD discrete-time systems with interval time-varying delays. Based on the combination of model

transformation, utilization of zero equation, and parameter-dependent Lyapunov functional, new delay-dependent robust stability conditions are obtained and formulated in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to illustrate that the resulting criterion outperforms the existing stability condition.

2. Problem Formulation and Preliminaries

We introduce some notations, definitions, and propositions that will be used throughout the paper. $\mathbb{Z}^+$ denotes the set of nonnegative integer numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in \mathbb{R}^n$, that is, $\|x\|^2 = x^T x$; $M^{n\times r}$ denotes the space of all real matrices of $(n \times r)$-dimensions; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\max}(A(\alpha)) = \max\{\lambda_{\max}(A_i) : i = 1, 2, \ldots, N\}$; $\lambda_{\min}(A(\alpha)) = \min\{\lambda_{\min}(A_i) : i = 1, 2, \ldots, N\}$; matrix A is called a semipositive definite ($A \geq 0$) if $x^T A x \geq 0$, for all $x \in \mathbb{R}^n$; A is a positive definite ($A > 0$) if $x^T A x > 0$ for all $x \neq 0$; matrix B is called a seminegative definite ($B \leq 0$) if $x^T B x \leq 0$, for all $x \in \mathbb{R}^n$; B is a negative definite ($B < 0$) if $x^T B x < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $A \geq B$ means $A - B \geq 0$; $*$ represents the elements below the main diagonal of a symmetric matrix.

Consider the following uncertain LPD discrete-time system with interval time-varying delays of the form

$$x(k+1) = A(\alpha)\, x(k) + B(\alpha)\, x(k - h(k)), \tag{1}$$

$$x(s) = \phi(s), \quad s \in \{-h_2, -h_2 + 1, \ldots, -1, 0,\}, \tag{2}$$

where $k \in \mathbb{Z}^+$, $x(k) \in \mathbb{R}^n$ is the system state and $\phi(s)$ is an initial value at s. $A(\alpha), B(\alpha) \in M^{n\times n}$ are uncertain matrices belonging to the polytope of the form

$$\begin{gathered} A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i, \qquad B(\alpha) = \sum_{i=1}^{N} \alpha_i B_i, \\ \sum_{i=1}^{N} \alpha_i = 1,\ \alpha_i \geq 0,\ A_i, B_i \in M^{n\times n},\ i = 1, \ldots, N. \end{gathered} \tag{3}$$

In addition, we assume that the time-varying delay $h(k)$ is upper and lower bounded. It satisfies the following assumption of the form

$$0 < h_1 \leq h(k) \leq h_2, \tag{4}$$

where h_1 and h_2 are known positive integers.

Definition 1 (see [19]). The system (1) is said to be robustly stable if there exists a positive definite function $V(k, x(k)) : \mathbb{Z}^+ \times \mathbb{R}^n \to \mathbb{R}$ such that

$$\Delta V(k, x(k)) = V(k+1, x(k+1)) - V(k, x(k)) < 0, \tag{5}$$

along any trajectory of the solution of the system (1).

Definition 2 (see [19]). The system (1) when $A(\alpha) = A$, $B(\alpha) = B$, and $A, B \in M^{n\times n}$ is said to be asymptotically stable if there exists a positive definite function $V(k, x(k)) : \mathbb{Z}^+ \times \mathbb{R}^n \to \mathbb{R}$ such that

$$\Delta V(k, x(k)) = V(k+1, x(k+1)) - V(k, x(k)) < 0, \tag{6}$$

along any trajectory of the solution of the system (1) when $A(\alpha) = A$, $B(\alpha) = B$.

Proposition 3 ([7, the Schur complement lemma]). *Given constant symmetric matrices X, Y, and Z of appropriate dimensions with $Y > 0$, then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0. \tag{7}$$

Proposition 4 (see [9]). *For any constant matrix $W \in M^{n\times n}, W = W^T > 0$, two integers r_M and r_m satisfying $r_M \geq r_m$, and vector function $x : [r_m, r_M] \to \mathbb{R}^n$, the following inequality holds:*

$$\left(\sum_{i=r_m}^{r_M} x(i)\right)^T W \left(\sum_{i=r_m}^{r_M} x(i)\right) \leq (r_M - r_m + 1) \sum_{i=r_m}^{r_M} x^T(i)\, W x(i). \tag{8}$$

Rewrite the system (1) in the following system:

$$\begin{gathered} x(k+1) = x(k) + y(k), \\ y(k) = [A(\alpha) + B(\alpha) - I]\, x(k) - B(\alpha) \sum_{i=k-h(k)}^{k-1} y(i). \end{gathered} \tag{9}$$

3. Robust Stability Conditions

In this section, we study the robust stability criteria for the system (1) by using the combination of model transformation, the linear matrix inequality (LMI) technique, and the Lyapunov method. We introduce the following notations for later use:

$$\begin{gathered} C_j(\alpha) = \sum_{i=1}^{N} \alpha_i C_i^j, \qquad D_j(\alpha) = \sum_{i=1}^{N} \alpha_i D_i^j, \\ E_j(\alpha) = \sum_{i=1}^{N} \alpha_i E_i^j, \qquad G_j(\alpha) = \sum_{i=1}^{N} \alpha_i G_i^j, \\ L_j(\alpha) = \sum_{i=1}^{N} \alpha_i L_i^j, \qquad P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i, \\ Q(\alpha) = \sum_{i=1}^{N} \alpha_i Q_i, \qquad R(\alpha) = \sum_{i=1}^{N} \alpha_i R_i, \end{gathered}$$

$$S(\alpha)=\sum_{i=1}^{N}\alpha_i S_i,\qquad T(\alpha)=\sum_{i=1}^{N}\alpha_i T_i,$$
$$U(\alpha)=\sum_{i=1}^{N}\alpha_i U_i,\qquad V(\alpha)=\sum_{i=1}^{N}\alpha_i V_i,$$
$$W(\alpha)=\sum_{i=1}^{N}\alpha_i W_i,\qquad X(\alpha)=\sum_{i=1}^{N}\alpha_i X_i,$$
$$Y(\alpha)=\sum_{i=1}^{N}\alpha_i Y_i,\qquad Z(\alpha)=\sum_{i=1}^{N}\alpha_i Z_i,$$
$$\sum_{i=1}^{N}\alpha_i=1,\ \alpha_i\geq 0,$$
$$C_i^j, D_i^j, E_i^j, G_i^j, L_i^j, P_i, Q_i, R_i, S_i, T_i, U_i,$$
$$V_i, W_i, X_i, Y_i, Z_i\in M^{n\times n},$$
$$j=1,2,3,\ i=1,2,\ldots,N,\ \rho=h_2-h_1.$$

Moreover,

$$\prod_{i,j}=\begin{bmatrix}\Sigma_{i,j}^{11} & \Sigma_{i,j}^{12} & \Sigma_{i,j}^{13} & \Sigma_{i,j}^{14} & \Sigma_{i,j}^{15} & \Sigma_{i}^{16} & \Sigma_{i,j}^{17} & \Sigma_{i,j}^{18} & 0\\ * & \Sigma_{i,j}^{22} & 0 & 0 & 0 & 0 & 0 & \Sigma_{i,j}^{28} & 0\\ * & * & \Sigma_{i}^{33} & \Sigma_{i}^{34} & 0 & \Sigma_{i,j}^{36} & 0 & 0 & \Sigma_{i,j}^{18}\\ * & * & * & \Sigma_{i}^{44} & 0 & 0 & \Sigma_{i,j}^{47} & 0 & \Sigma_{i,j}^{18}\\ * & * & * & * & \Sigma_{i}^{55} & 0 & 0 & \Sigma_{i,j}^{18} & 0\\ * & * & * & * & * & \Sigma_{i}^{66} & 0 & 0 & 0\\ * & * & * & * & * & * & \Sigma_{i,j}^{77} & 0 & 0\\ * & * & * & * & * & * & * & \Sigma_{i,j}^{88} & 0\\ * & * & * & * & * & * & * & * & \Sigma_{i,j}^{99}\end{bmatrix},\tag{10}$$

where

$$\begin{aligned}
\Sigma_{i,j}^{11} &= Q_i+R_i+T_i+h_1M_i+N_i+N_i^T+h_2K_i+L_i+L_i^T\\
&\quad+C_i^1+C_i^{1^T}+D_i^1+D_i^{1^T}+E_i^1+E_i^{1^T}+L_i^{2^T}A_j\\
&\quad+A_i^TL_j^2+L_i^{2^T}B_j+B_i^TL_j^2-L_i^2-L_i^{2^T},\\
\Sigma_{i,j}^{12} &= A_i^TL_j^1+B_i^TL_j^1-L_i^1-L_i^{2^T}+P_i,\\
\Sigma_i^{13} &= C_i^2-N_i-C_i^{1^T},\qquad \Sigma_i^{14}=D_i^2-L_i-D_i^{1^T},\\
\Sigma_i^{15} &= -E_i^{1^T}+E_i^2,\qquad \Sigma_i^{16}=-C_i^{1^T}+C_i^3,\\
\Sigma_i^{17} &= -D_i^{1^T}+D_i^3,\\
\Sigma_{i,j}^{18} &= -E_i^{1^T}+E_i^3-L_i^{2^T}B_j+A_i^TL_j^3+B_i^TL_j^3-L_i^3,\\
\Sigma_i^{22} &= P_i+h_1^2U_i+h_2^2V_i+h_2^2W_i+\rho^2X_i+h_1Y_i\\
&\quad+h_2Z_i-L_i^1-L_i^{1^T},\qquad \Sigma_{i,j}^{28}=-L_i^{1^T}B_j-L_i^3,\\
\Sigma_i^{33} &= -Q_i+S_i-C_i^2-C_i^{2^T}+G_i^1+G_i^{1^T},\\
\Sigma_i^{34} &= G_i^2-G_i^{1^T},\qquad \Sigma_i^{36}=-C_i^3-C_i^{2^T},\\
\Sigma_i^{39} &= G_i^3-G_i^{1^T},\qquad \Sigma_i^{44}=-R_i-S_i-D_i^2-D_i^{2^T}-G_i^2-G_i^{2^T},\\
\Sigma_i^{47} &= -D_i^3-D_i^{2^T},\qquad \Sigma_i^{49}=-G_i^3-G_i^{2^T},\\
\Sigma_i^{55} &= -T_i-E_i^2-E_i^{2^T},\qquad \Sigma_i^{58}=-E_i^3-E_i^{2^T},\\
\Sigma_i^{66} &= -U_i-C_i^3-C_i^{3^T},\qquad \Sigma_i^{77}=-V_i-D_i^3-D_i^{3^T},\\
\Sigma_{i,j}^{88} &= -W_i-E_i^3-E_i^{3^T}-L_i^{3^T}B_j-B_i^TL_j^3,\\
\Sigma_{i,j}^{99} &= -X_i-G_i^3-G_i^{3^T}.
\end{aligned}\tag{11}$$

Theorem 5. *The system* (1)-(2) *is robustly stable if there exist positive definite symmetric matrices* $P_i, Q_i, R_i, S_i, T_i, U_i, V_i, W_i, X_i, Y_i$, *and* Z_i, *any appropriate dimensional matrices* $M_i, N_i, K_i, L_i, C_i^j, D_i^j, E_i^j, G_i^j$, *and* L_i^j, $j=1,2,3$, $i=1,2,\ldots,N$ *satisfying the following LMIs:*

$$\prod_{i,i}<-I,\quad i=1,2,\ldots,N,\tag{12}$$

$$\prod_{i,j}+\prod_{j,i}<\frac{2}{N-1}I,\quad i=1,\ldots,N-1,\ j=i+1,\ldots,N,\tag{13}$$

$$\begin{bmatrix}M_i & N_i\\ N_i^T & Y_i\end{bmatrix}\geq 0,\quad i=1,2,\ldots,N,\tag{14}$$

$$\begin{bmatrix}K_i & L_i\\ L_i^T & Z_i\end{bmatrix}\geq 0,\quad i=1,2,\ldots,N.\tag{15}$$

Proof. Consider the following parameter-dependent Lyapunov-Krasovskii function for the system (9) of the form

$$V(k)=\sum_{i=1}^{5}V_i(k),\tag{16}$$

where

$$\begin{aligned}
V_1(k) &= x^T(k)P(\alpha)x(k),\\
V_2(k) &= \sum_{i=k-h_1}^{k-1}x^T(i)Q(\alpha)x(i)+\sum_{i=k-h_2}^{k-1}x^T(i)R(\alpha)x(i)\\
&\quad+\sum_{i=k-h_2}^{k-h_1-1}x^T(i)S(\alpha)x(i)+\sum_{i=k-h(k)}^{k-1}x^T(i)T(\alpha)x(i),
\end{aligned}$$

$$\begin{aligned}
V_3(k) &= h_1 \sum_{j=-h_1}^{-1} \sum_{i=k+j}^{k-1} y^T(i) U(\alpha) y(i) \\
&\quad + h_2 \sum_{j=-h_2}^{-1} \sum_{i=k+j}^{k-1} y^T(i) V(\alpha) y(i) \\
&\quad + h_2 \sum_{j=-h_2}^{-1} \sum_{i=k+j}^{k-1} y^T(i) W(\alpha) y(i), \\
V_4(k) &= \rho \sum_{j=-h_2}^{-h_1-1} \sum_{i=k+j}^{k-1} y^T(i) X(\alpha) y(i), \\
V_5(k) &= \sum_{j=-h_1+1}^{0} \sum_{i=k-1+j}^{k-1} y^T(i) Y(\alpha) y(i) \\
&\quad + \sum_{j=-h_2+1}^{0} \sum_{i=k-1+j}^{k-1} y^T(i) Z(\alpha) y(i).
\end{aligned} \tag{17}$$

Evaluating the forward deference of $V(k)$, it is defined as

$$\Delta V(k) = V(k+1) - V(k). \tag{18}$$

Let us define, for $i = 1, 2, \ldots, 5$,

$$\Delta V_i(k) = V_i(k+1) - V_i(k). \tag{19}$$

Then along the solution of the system (9), we obtain

$$\begin{aligned}
\Delta V_1(k) &= V_1(k+1) - V_1(k) \\
&= x^T(k+1) P(\alpha) x(k+1) - x^T(k) P(\alpha) x(k) \\
&= 2x^T(k) P(\alpha) y(k) + y^T(k) P(\alpha) y(k) \\
&\quad + 2x^T(k) L_1^T(\alpha) \Bigg[-y(k) + [A(\alpha) + B(\alpha) - I] \\
&\qquad \times x(k) - B(\alpha) \sum_{i=k-h(k)}^{k-1} y(i) \Bigg] \\
&\quad + 2y^T(k) L_2^T(\alpha) \Bigg[-y(k) + [A(\alpha) + B(\alpha) - I] \\
&\qquad \times x(k) - B(\alpha) \sum_{i=k-h(k)}^{k-1} y(i) \Bigg] \\
&\quad + 2 \sum_{i=k-h(k)}^{k-1} y(i)^T L_3^T(\alpha) \Bigg[-y(k) + [A(\alpha) + B(\alpha) - I] \\
&\qquad \times x(k) - B(\alpha) \sum_{i=k-h(k)}^{k-1} y(i) \Bigg], \\
\Delta V_2(k) &= V_2(k+1) - V_2(k) \\
&= x^T(k) Q(\alpha) x(k) - x^T(k-h_1) Q(\alpha) x(k-h_1) \\
&\quad + x^T(k) R(\alpha) x(k) - x^T(k-h_2) R(\alpha) x(k-h_2) \\
&\quad + x^T(k-h_1) S(\alpha) x(k-h_1) - x^T(k-h_2) \\
&\quad \times S(\alpha) x(k-h_2) + x^T(k) T(\alpha) x(k) \\
&\quad - x^T(k-h(k)) T(\alpha) x(k-h(k)).
\end{aligned} \tag{20}$$

From Proposition 4, we have

$$\begin{aligned}
\Delta V_3(k) &= V_3(k+1) - V_3(k) \\
&= h_1^2 y^T(k) U(\alpha) y(k) - h_1 \sum_{i=k-h_1}^{k-1} y^T(i) U(\alpha) y(i) \\
&\quad + h_2^2 y^T(k) V(\alpha) y(k) - h_2 \sum_{i=k-h_2}^{k-1} y^T(i) V(\alpha) y(i) \\
&\quad + h_2^2 y^T(k) W(\alpha) y(k) - h_2 \sum_{i=k-h_2}^{k-1} y^T(i) W(\alpha) y(i) \\
&\le h_1^2 y^T(k) U(\alpha) y(k) - \left(\sum_{i=k-h_1}^{k-1} y(i) \right)^T \\
&\quad \times U(\alpha) \left(\sum_{i=k-h_1}^{k-1} y(i) \right) + h_2^2 y^T(k) V(\alpha) y(k) \\
&\quad - \left(\sum_{i=k-h_2}^{k-1} y(i) \right)^T V(\alpha) \left(\sum_{i=k-h_2}^{k-1} y(i) \right) \\
&\quad + h_2^2 y^T(k) W(\alpha) y(k) - \left(\sum_{i=k-h(k)}^{k-1} y(i) \right)^T \\
&\quad \times W(\alpha) \left(\sum_{i=k-h(k)}^{k-1} y(i) \right), \\
\Delta V_4(k) &= V_4(k+1) - V_4(k) \\
&= \rho^2 y^T(k) X(\alpha) y(k) - \rho \sum_{i=k-h_2}^{k-h_1-1} y^T(i) X(\alpha) y(i)
\end{aligned}$$

$$\leq \rho^2 y^T(k) X(\alpha) y(k) - \left(\sum_{i=k-h_2}^{k-h_1-1} y(i)\right)^T X(\alpha) \times \left(\sum_{i=k-h_2}^{k-h_1-1} y(i)\right),$$

$$\begin{aligned}\Delta V_5(k) &= V_5(k+1) - V_5(k) \\ &= h_1 y^T(k) Y(\alpha) y(k) - \sum_{i=-h_1+1}^{0} y^T(k-1+i) Y(\alpha) \times y(k-1+i) \\ &\quad + h_2 y^T(k) Z(\alpha) y(k) - \sum_{i=-h_2+1}^{0} y^T(k-1+i) Z(\alpha) \times y(k-1+i).\end{aligned} \tag{21}$$

We can show that

$$\begin{aligned}&2x^T(k) N(\alpha) \sum_{i=-h_1+1}^{0} y(k-1+i) \\ &\quad + \sum_{i=-h_1+1}^{0} y^T(k-1+i) Y(\alpha) y(k-1+i) + h_1 x^T(k) M(\alpha) x(k) \\ &= \sum_{i=-h_1+1}^{0} \begin{bmatrix} x(k) \\ y(k-1+i) \end{bmatrix}^T \begin{bmatrix} M(\alpha) & N(\alpha) \\ N^T(\alpha) & Y(\alpha) \end{bmatrix} \times \begin{bmatrix} x(k) \\ y(k-1+i) \end{bmatrix} \geq 0.\end{aligned} \tag{22}$$

It is easy to see that

$$\begin{aligned}&2x^T(k) L(\alpha) \sum_{i=-h_1+1}^{0} y(k-1+i) \\ &\quad + \sum_{i=-h_2+1}^{0} y^T(k-1+i) Z(\alpha) y(k-1+i) + h_2 x^T(k) K(\alpha) x(k) \\ &= \sum_{i=-h_2+1}^{0} \begin{bmatrix} x(k) \\ y(k-1+i) \end{bmatrix}^T \begin{bmatrix} K(\alpha) & L(\alpha) \\ L^T(\alpha) & Z(\alpha) \end{bmatrix} \times \begin{bmatrix} x(k) \\ y(k-1+i) \end{bmatrix} \geq 0.\end{aligned} \tag{23}$$

By (22) and (23), we can obtain

$$\begin{aligned}&- \sum_{i=-h_1+1}^{0} y^T(k-1+i) Y(\alpha) y(k-1+i) \\ &\quad \leq h_1 x^T(k) M(\alpha) x(k) + 2x^T(k) N(\alpha) \times \sum_{i=-h_1+1}^{0} y(k-1+i) \\ &\quad = h_1 x^T(k) M(\alpha) x(k) + 2x^T(k) N(\alpha) x(k) - 2x^T(k) N(\alpha) x(k-h_1),\end{aligned} \tag{24}$$

and we conclude that

$$\begin{aligned}&- \sum_{i=-h_2+1}^{0} y^T(k-1+i) Z(\alpha) y(k-1+i) \\ &\quad \leq h_2 x^T(k) K(\alpha) x(k) + 2x^T(k) L(\alpha) \times \sum_{i=-h_2+1}^{0} y(k-1+i) \\ &\quad = h_2 x^T(k) K(\alpha) x(k) + 2x^T(k) L(\alpha) x(k) - 2x^T(k) L(\alpha) x(k-h_2).\end{aligned} \tag{25}$$

It is obvious that

$$\begin{aligned}\Upsilon &\equiv x(k) - x(k-h_1) - \sum_{i=k-h_1}^{k-1} y(i) = 0, \\ \Phi &\equiv x(k) - x(k-h_2) - \sum_{i=k-h_2}^{k-1} y(i) = 0, \\ \Psi &\equiv x(k) - x(k-h(k)) - \sum_{i=k-h(k)}^{k-1} y(i) = 0, \\ \Omega &\equiv x(k-h_1) - x(k-h_2) - \sum_{i=k-h_2}^{k-h_1-1} y(i) = 0.\end{aligned} \tag{26}$$

The following equations are true for any polytopic matrices with appropriate dimensions:

$$\left[2x^T(k) C_1^T(\alpha) + 2x^T(k-h_1) C_2^T(\alpha) + 2\left(\sum_{i=k-h_1}^{k-1} y(i)\right)^T C_3^T(\alpha)\right] \times \Upsilon = 0, \tag{27}$$

$$\left[2x^T(k) D_1^T(\alpha) + 2x^T(k-h_2) D_2^T(\alpha) + 2\left(\sum_{i=k-h_2}^{k-1} y(i)\right)^T D_3^T(\alpha)\right] \times \Phi = 0, \tag{28}$$

$$\left[2x^T(k)E_1^T(\alpha)+2x^T(k-h(k))E_2^T(\alpha)\right.$$
$$\left.+2\left(\sum_{i=k-h(k)}^{k-1}y(i)\right)^T E_3^T(\alpha)\right]\times\Psi=0, \tag{29}$$

$$\left[2x^T(k-h_1)G_1^T(\alpha)+2x^T(k-h_2)G_2^T(\alpha)\right.$$
$$\left.+2\left(\sum_{i=k-h_2}^{k-h_1-1}y(i)\right)^T G_3^T(\alpha)\right]\times\Omega=0. \tag{30}$$

It follows from (18)–(30) that

$$\Delta V(k)\le\xi^T(k)\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\prod_{i,j}\xi(k), \tag{31}$$

where $\xi^T(k)=[x(k)^T\ y(k)^T\ x(k-h_1)^T\ x(k-h_2)^T\ x(k-h(k))^T\ \sum_{i=k-h_1}^{k-1}y^T(i)\ \sum_{i=k-h_2}^{k-1}y^T(i)\ \sum_{i=k-h(k)}^{k-1}y(i)^T\ \sum_{i=k-h_2}^{k-h_1-1}y^T(i)]$ and $\prod_{i,j}$ is defined in (10). Due to the fact that $\sum_{i=1}^{N}\alpha_i=1$, we obtain the following identities:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\prod_{i,j}=\sum_{i=1}^{N}\alpha^2\prod_{i,i}+\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_i\left[\prod_{i,j}+\prod_{j,i}\right], \tag{32}$$

$$(N-1)\sum_{i=1}^{N}\alpha_i^2-2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_j=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left[\alpha_i-\alpha_j\right]^2\ge 0. \tag{33}$$

By (31)–(33), if the conditions (12)–(15) are true, then

$$\Delta V(k)<-\omega\|x\|^2, \tag{34}$$

where $\omega>0$. This means that the system (1)-(2) is robustly stable. The proof of the theorem is complete. □

If $A(\alpha)=A$ and $B(\alpha)=B$ when $A,B\in M^{n\times n}$ then the system (1)-(2) reduces to the following system:

$$x(k+1)=Ax(k)+Bx(k-h(k)), \tag{35}$$

$$x(s)=\phi(s),\quad s\in\{-h_2,\dots,-1,0\}. \tag{36}$$

Take the Lyapunov-Krasovskii functional as (16), where $P(\alpha)=P, Q(\alpha)=Q, R(\alpha)=R, S(\alpha)=S, T(\alpha)=T, U(\alpha)=U, V(\alpha)=V, W(\alpha)=W, X(\alpha)=X, Y(\alpha)=Y$, and $Z(\alpha)=Z$ when $P,Q,R,S,T,U,V,W,X,Y,Z\in M^{n\times n}$. Moreover, let us set polytopic matrices with appropriate dimensions of the forms $C_j(\alpha)=C^j$, $D_j(\alpha)=D^j$, $E_j(\alpha)=E^j$, $G_j(\alpha)=G^j$, and $L_j(\alpha)=L^j$ when $C^j,D^j,E^j,G^j,L^j\in M^{n\times n}$, $j=1,2,3$.

According to Theorem 5, we have Corollary 6 for the delay-dependent asymptotically stability criterion of the system (35)-(36). We introduce the following notations for later use:

$$\prod=\begin{bmatrix}\Sigma^{11}&\Sigma^{12}&\Sigma^{13}&\Sigma^{14}&\Sigma^{15}&\Sigma^{16}&\Sigma^{17}&\Sigma^{18}&0\\ *&\Sigma^{22}&0&0&0&0&0&\Sigma^{28}&0\\ *&*&\Sigma^{33}&\Sigma^{34}&0&\Sigma^{36}&0&0&\Sigma^{18}\\ *&*&*&\Sigma^{44}&0&0&\Sigma^{47}&0&\Sigma^{18}\\ *&*&*&*&\Sigma^{55}&0&0&\Sigma^{18}&0\\ *&*&*&*&*&\Sigma^{66}&0&0&0\\ *&*&*&*&*&*&\Sigma^{77}&0&0\\ *&*&*&*&*&*&*&\Sigma^{88}&0\\ *&*&*&*&*&*&*&*&\Sigma^{99}\end{bmatrix}, \tag{37}$$

where

$$\begin{aligned}
\Sigma^{11}&=Q+R+T+h_1M+N+N^T+h_2K+L+L^T\\
&\quad+C^1+C^{1^T}+D^1+D^{1^T}+E^1+E^{1^T}+L^{2^T}A\\
&\quad+A^TL^2+L^{2^T}B+B^TL^2-L^2-L^{2^T},\\
\Sigma^{12}&=A^TL^1+B^TL^1-L^1-L^{2^T}+P,\\
\Sigma^{13}&=C^2-N-C^{1^T},\qquad \Sigma^{14}=D^2-L-D^{1^T},\\
\Sigma^{15}&=-E^{1^T}+E^2,\qquad \Sigma^{16}=-C^{1^T}+C^3,\\
\Sigma^{17}&=-D^{1^T}+D^3,\\
\Sigma^{18}&=-E^{1^T}+E^3-L^{2^T}B+A^TL^3+B^TL^3-L^3,\\
\Sigma^{22}&=P+h_1^2U+h_2^2V+h_2^2W+\rho^2X\\
&\quad+h_1Y+h_2Z-L^1-L^{1^T},\\
\Sigma^{28}&=-L^{1^T}B-L^3,\\
\Sigma^{33}&=-Q+S-C^2-C^{2^T}+G^1+G^{1^T},\qquad \Sigma^{34}=G^2-G^{1^T},\\
\Sigma^{36}&=-C^3-C^{2^T},\qquad \Sigma^{39}=G^3-G^{1^T},\\
\Sigma^{44}&=-R-S-D^2-D^{2^T}-G^2-G^{2^T},\\
\Sigma^{47}&=-D^3-D^{2^T},\\
\Sigma^{49}&=-G^3-G^{2^T},\qquad \Sigma^{55}=-T-E^2-E^{2^T},\\
\Sigma^{58}&=-E^3-E^{2^T},\\
\Sigma^{66}&=-U-C^3-C^{3^T},\qquad \Sigma^{77}=-V-D^3-D^{3^T},\\
\Sigma^{88}&=-W-E^3-E^{3^T}-L^{3^T}B-B^TL^3,\\
\Sigma^{99}&=-X-G^3-G^{3^T}.
\end{aligned} \tag{38}$$

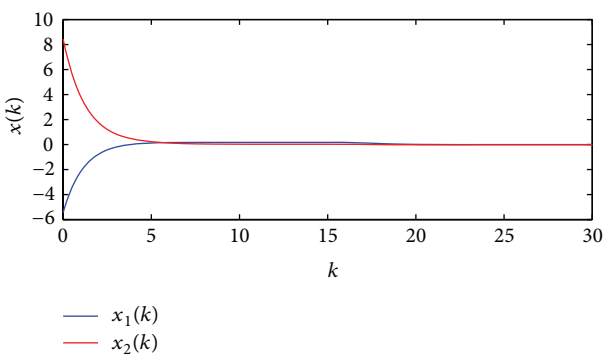

FIGURE 1: The Simulation solution of the states $x_1(k)$ and $x_2(k)$ in Example 7 for LPD discrete-time delayed system with initial conditions $x_1(k) = -5.5$ and $x_2(k) = 8.5$, $k = -16, \ldots, -2, -1, 0$ and $\alpha_1 = 1/3$, $\alpha_2 = 2/3$ by using method of Runge-Kutta order 4 ($h = 0.01$) with Matlab.

Corollary 6. *The system* (35)-(36) *is asymptotically stable if there exist positive definite symmetric matrices* $P, Q, R, S, T, U, V, W, X, Y$, *and* Z, *any appropriate dimensional matrices* $M, N, K, L, C^j, D^j, E^j, G^j$, *and* L^j, $j = 1, 2, 3$, *satisfying the following LMIs:*

$$\prod < 0,$$
$$\begin{bmatrix} M & N \\ N^T & Y \end{bmatrix} \geq 0, \tag{39}$$
$$\begin{bmatrix} K & L \\ L^T & Z \end{bmatrix} \geq 0.$$

4. Numerical Examples

Example 7. Consider the following LPD discrete-time system with interval time-varying delays (1)-(2) with

$$A_1 = \begin{bmatrix} 0.80 & 0 \\ 0.01 & 0.60 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.90 & 0 \\ 0.05 & 0.90 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.10 & 0 \\ 0.20 & 0.10 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.10 & 0 \\ -0.20 & -0.10 \end{bmatrix}, \tag{40}$$

$h(k) = 2 + 14\cos^2(k\pi/2)$ with initial condition $\phi(k) = \left[\begin{smallmatrix} -5.5 \\ 8.5 \end{smallmatrix}\right]$, $k \in [-16, 0]$. The numerical solutions $x_1(k)$ and $x_2(k)$ of (1)-(2) with (40) are plotted in Figure 1.

Solution. By using the LMI Toolbox in MATLAB (with accuracy 0.01) and conditions (12)–(15) of Theorem 5, this system is robustly stable for discrete delay time satisfying $h_1 = 2$, $h_2 = 16$, and

$$P_1 = 10^6 \times \begin{bmatrix} 4.3970 & -0.4104 \\ -0.4104 & 0.3531 \end{bmatrix},$$
$$P_2 = 10^5 \times \begin{bmatrix} 5.2274 & -0.0613 \\ -0.0613 & 0.3488 \end{bmatrix},$$
$$Q_1 = 10^4 \times \begin{bmatrix} 4.9438 & -1.3988 \\ -1.3988 & 1.3372 \end{bmatrix},$$
$$Q_2 = 10^4 \times \begin{bmatrix} 1.6042 & -0.3173 \\ -0.3173 & 0.1431 \end{bmatrix},$$
$$R_1 = 10^4 \times \begin{bmatrix} 1.8144 & -0.5737 \\ -0.5737 & 0.4286 \end{bmatrix},$$
$$R_2 = 10^3 \times \begin{bmatrix} 4.2669 & -0.8605 \\ -0.8605 & 0.3167 \end{bmatrix},$$
$$S_1 = 10^4 \times \begin{bmatrix} 2.3161 & -0.7409 \\ -0.7409 & 0.5638 \end{bmatrix},$$
$$S_2 = 10^3 \times \begin{bmatrix} 5.5557 & -1.1238 \\ -1.1238 & 0.4007 \end{bmatrix},$$
$$T_1 = 10^5 \times \begin{bmatrix} 2.2862 & -0.3474 \\ -0.3474 & 0.2764 \end{bmatrix},$$
$$T_2 = 10^4 \times \begin{bmatrix} 5.0867 & 0.3198 \\ 0.3198 & 0.2709 \end{bmatrix},$$
$$U_1 = 10^4 \times \begin{bmatrix} 9.6266 & -0.3736 \\ -0.3736 & 2.6793 \end{bmatrix},$$
$$U_2 = 10^4 \times \begin{bmatrix} 6.7104 & -1.2896 \\ -1.2896 & 0.6493 \end{bmatrix},$$
$$V_1 = 10^3 \times \begin{bmatrix} 2.6354 & -0.1810 \\ -0.1810 & 0.2286 \end{bmatrix},$$
$$V_2 = 10^3 \times \begin{bmatrix} 1.1098 & -0.2116 \\ -0.2116 & 0.0795 \end{bmatrix},$$
$$W_1 = 10^4 \times \begin{bmatrix} 1.2581 & 0.0031 \\ 0.0031 & 0.0279 \end{bmatrix},$$
$$W_2 = 10^3 \times \begin{bmatrix} 9.8860 & 0.9947 \\ 0.9947 & 0.4654 \end{bmatrix},$$
$$X_1 = 10^3 \times \begin{bmatrix} 3.6535 & -0.2316 \\ -0.2316 & 0.3050 \end{bmatrix},$$
$$X_2 = 10^3 \times \begin{bmatrix} 1.5049 & -0.2858 \\ -0.2858 & 0.1068 \end{bmatrix},$$
$$Y_1 = 10^5 \times \begin{bmatrix} 1.0677 & -0.0668 \\ -0.0668 & 0.3783 \end{bmatrix},$$
$$Y_2 = 10^4 \times \begin{bmatrix} 8.4989 & -1.5451 \\ -1.5451 & 1.0863 \end{bmatrix},$$

$$Z_1 = 10^4 \times \begin{bmatrix} 4.8154 & -0.3746 \\ -0.3746 & 0.6475 \end{bmatrix},$$

$$Z_2 = 10^4 \times \begin{bmatrix} 2.5697 & -0.4901 \\ -0.4901 & 0.2043 \end{bmatrix},$$

$$C_1^1 = 10^4 \times \begin{bmatrix} -1.5983 & -0.1755 \\ -0.1755 & -1.6231 \end{bmatrix},$$

$$C_2^1 = 10^4 \times \begin{bmatrix} -2.1448 & 0.1127 \\ 0.1127 & -1.6904 \end{bmatrix},$$

$$C_1^2 = 10^4 \times \begin{bmatrix} 1.2837 & 0.1710 \\ 0.1710 & 1.5002 \end{bmatrix},$$

$$C_2^2 = 10^4 \times \begin{bmatrix} 1.6947 & -0.0260 \\ -0.0260 & 1.6512 \end{bmatrix},$$

$$C_1^3 = 10^3 \times \begin{bmatrix} -7.3049 & 0.2491 \\ 0.2491 & 9.7027 \end{bmatrix},$$

$$C_2^3 = 10^4 \times \begin{bmatrix} -0.1374 & 0.3389 \\ 0.3389 & 1.4742 \end{bmatrix},$$

$$D_1^1 = 10^4 \times \begin{bmatrix} -0.9350 & -0.1962 \\ -0.1962 & -1.4853 \end{bmatrix},$$

$$D_2^1 = 10^4 \times \begin{bmatrix} -1.7099 & 0.0229 \\ 0.0229 & -1.6489 \end{bmatrix},$$

$$D_1^2 = 10^4 \times \begin{bmatrix} 0.5714 & 0.2950 \\ 0.2950 & 1.3754 \end{bmatrix},$$

$$D_2^2 = 10^4 \times \begin{bmatrix} 1.3928 & 0.0393 \\ 0.0393 & 1.6269 \end{bmatrix},$$

$$D_1^3 = 10^4 \times \begin{bmatrix} 1.9066 & -0.0891 \\ -0.0891 & 1.7055 \end{bmatrix},$$

$$D_2^3 = 10^4 \times \begin{bmatrix} 1.7454 & -0.0290 \\ -0.0290 & 1.6519 \end{bmatrix},$$

$$E_1^1 = 10^4 \times \begin{bmatrix} -4.2910 & -1.6701 \\ -1.6701 & -1.5104 \end{bmatrix},$$

$$E_2^1 = 10^4 \times \begin{bmatrix} -3.7600 & -0.2382 \\ -0.2382 & -1.7532 \end{bmatrix},$$

$$E_1^2 = 10^4 \times \begin{bmatrix} -7.0237 & 0.5763 \\ 0.5763 & 0.8408 \end{bmatrix},$$

$$E_2^2 = 10^4 \times \begin{bmatrix} -0.7534 & -0.1269 \\ -0.1269 & 1.4957 \end{bmatrix},$$

$$E_1^3 = 10^5 \times \begin{bmatrix} 1.0553 & -0.0488 \\ -0.0488 & 0.2403 \end{bmatrix},$$

$$E_2^3 = 10^4 \times \begin{bmatrix} 3.8446 & 0.2721 \\ 0.2721 & 1.7426 \end{bmatrix},$$

$$G_1^1 = 10^4 \times \begin{bmatrix} -1.1174 & -0.1492 \\ -0.1492 & -1.4929 \end{bmatrix},$$

$$G_2^1 = 10^4 \times \begin{bmatrix} -1.6145 & -0.0032 \\ -0.0032 & -1.6397 \end{bmatrix},$$

$$G_1^2 = 10^4 \times \begin{bmatrix} 0.6849 & 0.2738 \\ 0.2738 & 1.3873 \end{bmatrix},$$

$$G_2^2 = 10^4 \times \begin{bmatrix} 1.3546 & 0.0545 \\ 0.0545 & 1.6203 \end{bmatrix},$$

$$G_1^3 = 10^4 \times \begin{bmatrix} 1.8742 & -0.0945 \\ -0.0945 & 1.7071 \end{bmatrix},$$

$$G_2^3 = 10^4 \times \begin{bmatrix} 1.7014 & -0.0146 \\ -0.0146 & 1.6454 \end{bmatrix},$$

$$L_1^1 = 10^6 \times \begin{bmatrix} 5.3779 & -0.2824 \\ -0.2824 & 0.4796 \end{bmatrix},$$

$$L_2^1 = 10^6 \times \begin{bmatrix} 2.3117 & 0.0294 \\ 0.0294 & 0.3636 \end{bmatrix},$$

$$L_1^2 = 10^6 \times \begin{bmatrix} 4.0552 & -0.3696 \\ -0.3696 & 0.2311 \end{bmatrix},$$

$$L_2^2 = 10^4 \times \begin{bmatrix} 9.4267 & 0.9110 \\ 0.9110 & 7.4877 \end{bmatrix},$$

$$L_1^3 = 10^5 \times \begin{bmatrix} -2.9313 & -0.2209 \\ -0.2209 & -0.4362 \end{bmatrix},$$

$$L_2^3 = 10^5 \times \begin{bmatrix} 2.1784 & -0.0837 \\ -0.0837 & -0.1597 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 897.9190 & -266.0787 \\ -266.0787 & 187.1557 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 236.7231 & -47.9750 \\ -47.9750 & 19.2914 \end{bmatrix},$$

$$M_1 = 10^3 \times \begin{bmatrix} 6.1473 & -1.9404 \\ -1.9404 & 1.4158 \end{bmatrix},$$

$$M_2 = 10^3 \times \begin{bmatrix} 1.6184 & -0.3310 \\ -0.3310 & 0.1305 \end{bmatrix},$$

$$N_1 = 10^3 \times \begin{bmatrix} -5.4598 & 0.9628 \\ 0.9628 & -2.2399 \end{bmatrix},$$

$$N_2 = 10^3 \times \begin{bmatrix} -3.9963 & 0.7762 \\ 0.7762 & -0.4562 \end{bmatrix},$$

$$L_1 = 10^3 \times \begin{bmatrix} -2.4982 & 0.3102 \\ 0.3102 & -0.3738 \end{bmatrix},$$

$$L_2 = 10^3 \times \begin{bmatrix} -1.1912 & 0.2324 \\ 0.2324 & -0.0945 \end{bmatrix}. \tag{41}$$

For the given h_1, Table 1 lists the comparison of the upper bounds delay h_2 for robust stability of the system (1)-(2) with (40) by the different method. By a conditions in Theorem 5, we can see from Table 1 that our result is superior to those in [22, Theorem 1].

Table 1: Comparison of the maximum allowed time delay h_2 for different conditions.

h_1	2	4	5	6
Liu et al. [10] (2006)	2	4	5	6
Zhang et al. [22] (2010)	12	13	14	15
Theorem 5	>15	>16	>17	>18

Table 2: Comparison of the maximum allowed time delay h_2 for different conditions.

h_1	7	10	15	20
Fridman and Shaked [6] (2005)	13	15	19	23
Gao and Chen [1](2007)	14	15	18	22
Zhang et al. [22] (2010)	15	17	21	25
Zhang et al. [23] (2011)	15	17	21	25
Corollary 6	>18	>19	>24	>27

Example 8. Consider the system (35) with

$$A = \begin{bmatrix} 0.80 & 0 \\ 0.05 & 0.90 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.10 & 0 \\ -0.20 & -0.10 \end{bmatrix}. \tag{42}$$

For the given h_1, we calculate the allowable maximum value of h_2 that guarantees the asymptotic stability of the system (35) with (42). By using different methods, the calculated results are presented in Table 2. From the table, we can see that Corollary 6 in this paper provides the less conservative results.

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