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Research Article

Codynamics of Four Variables Involved in Dengue Transmission and Its Control by Community Intervention: A System of Four Difference Equations

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In the case of Dengue transmission and control, the interaction of nature and society is captured by a system of difference equations. For the purpose of studying the dynamics of these interactions, four variables involved in a Dengue epidemic, proportion of infected people (P), number of mosquitoes involved in transmission (M), mosquito habitats (H), and population awareness (A), are linked in a system of difference equations: $P_{n+1} = aP_n + (1-e^{-iM_n})(1-P_n)$, $M_{n+1} = lM_n e^{-A_n} + bH_n(1-e^{-M_n})$, $H_{n+1} = cH_n/(1+pA_n) + 1/(1+qA_n)$, and $A_{n+1} = rA_n + fP_n$, $n = 0, 1, \ldots$. The constraints have socioecological meaning. The initial conditions are such that $0 \le P_0 \le 1$, $(M_0, H_0, A_0) \ge (0, 0, 0)$, the parameters $l, a, c, r \in (0, 1)$, and the parameters f, i, b, and p are positive. The paper is concerned with the analysis of solutions of the above system for $p = q$. We studied the global asymptotic stability of the degenerate equilibrium. We also propose extensions of the above model and some open problems. We explored the role of memory in community awareness by numerical simulations. When the memory parameter is large, the proportion of infected people decreases and stabilizes at zero. Below a critical point we observe periodic oscillations.

1. Introduction

The response to an epidemic is triggered by awareness of a coming epidemic or by an existing one. The response is aimed at reducing the incidence of the actual disease. In the case of Dengue fever, the disease is caused by a virus that is transmitted by the bite of the mosquito, usually *Aedes aegypti*. The mosquitoes deposit eggs in small containers of water. These hatch to produce larvae. Some transform into pupae and then adult mosquitoes. The breeding sites may be ephemeral, such as water in an empty beer can or used tire, an animal drinking trough near a human habitation, or in-doors stored water in large containers [1, 2].

The information about a Dengue epidemic can come from the number of reported cases of Dengue, the abundance of mosquitoes, or the numbers of breeding sites for mosquitoes, or some other indicator such as rainfall that predicts breeding sites. The information triggers consciousness, and the response can be either individual and/or one at the community level. In previous work, we studied the dynamics of a discrete time system in which we modeled the awareness as a factor that is triggered by the formation of potential breeding sites and the response was aimed at eliminating them. The system was studied by a pair of two difference equations [3].

By expanding the model to introduce an ongoing educational program, our new model predicted that high consciousness over time kept the number of breeding sites low [4]. In a study with three difference equations, we study a system in which the information is related to the number of adult mosquitoes. The more mosquitoes, the greater the awareness of the population, and this awareness leads to action to reduce the mosquito population by controlling breeding sites [5]. This population awareness is prompted and dissipates at a rate determined by the abundance of

mosquitoes, similar to a birth and death process. The dynamics then is that mosquitoes are produced when adult females locate breeding sites and deposit eggs which develop into adult mosquitoes, and mosquitoes die at a rate depending on their own biology and environmental conditions as a result of control measures implemented as awareness rises. Thus the pair of variables, mosquitoes and awareness, are linked in a negative feedback loop in a system of equations where decay due to control was modeled with a rational fractional term at the environmental level. With another system of three difference equations we have explored an intervention by spraying mosquitoes [6]. The change in the spraying parameter resulted in almost periodic behavior and fluctuations in the populations of mosquitoes. Simulations show that alertness in consciousness, by keeping the memory parameter of previous week high, has an impact on the behavior of solutions and implicitly on the number of mosquitoes. When the memory parameter is high, there will be a steady decrease in the number of mosquitoes. The present study builds upon the previous models. We present a system of four difference equations, with the proportion of infected people as an additional variable that prompts consciousness:

$$\begin{aligned} P_{n+1} &= aP_n + \left(1 - e^{-iM_n}\right)\left(1 - P_n\right), \\ M_{n+1} &= lM_n e^{-gA_n} + bH_n\left(1 - e^{-sM_n}\right), \\ H_{n+1} &= \frac{cH_n}{1 + pA_n} + \frac{d}{1 + pA_n}, \\ A_{n+1} &= rA_n + fP_n, \\ & \qquad n = 0, 1, \ldots. \end{aligned} \tag{1}$$

This discrete system links the proportion of the infected people (P_n), mosquitoes (M_n), habitats (H_n), and awareness A_n. The initial conditions are such that $0 \le P_0 \le 1$, $(M_0, H_0, A_0) \ge (0, 0, 0)$, the parameters $l, a, c, r \in (0, 1)$, and the parameters f, i, b, and p are positive. The current system represents a modification of the system in [5].

The first equation describes the proportion of infected people (between 0 and 1). They prompt consciousness, while the intervention is against mosquitoes and perhaps habitats. In the relationships among variables, the awareness is prompted by the proportion of sick people. The control of both adult mosquitoes by spraying and habitats is carried out by community intervention.

The parameter i is related to the behavior of infected mosquitoes, and it can be viewed as a transmission rate. An explanation of the term $(1 - e^{-iM_n})$ goes as follows. If Q represents the probability that a mosquito transmits the infection, then $1-Q$ is the probability that it does not transmit the infection. Therefore, $(1 - Q)^{M_n}$ will be the probability that M_n mosquitoes do not transmit the infection. One can rewrite

$$(1 - Q)^{M_n} = e^{\ln(1-Q)^{M_n}} = e^{M_n \ln(1-Q)}. \tag{2}$$

We denote $i = -\ln(1 - Q) > 0$.

One can observe that if $0 \le P_0 \le 1$ then $P_1 \le 1$. This is true because

$$P_1 = aP_0 + \left(1 - e^{-iM_0}\right)\left(1 - P_0\right) \le aP_0 + \left(1 - P_0\right) \le 1. \tag{3}$$

It follows by induction that $0 \le P_n \le 1$. Also, if $(M_0, H_0, A_0) \ge (0, 0, 0)$ then $(M_n, H_n, A_n) \ge (0, 0, 0)$. Thus, we have that $(P_0, M_0, H_0, A_0) \ge (0, 0, 0, 0)$ then $(P_n, M_n, H_n, A_n) \ge (0, 0, 0, 0)$.

By using a series of transformations, one can rescale the parameters g, s. and d in (1). We use the following changes of variables, $M_n = (1/s)m_n$ (in the second and first equation), $A_n = (1/g)a_n$ (in the third and fourth equation), and $H_n = dh_n$. These transformations will not change the nature of parameters a, c, l, and r, as these remain between 0 and 1. Thus, after relabeling the variables and parameters, one can work with a simplified system of equations as below (it is this system that will get analyzed in the next sections):

$$\begin{aligned} P_{n+1} &= aP_n + \left(1 - e^{-iM_n}\right)\left(1 - P_n\right), \\ M_{n+1} &= lM_n e^{-A_n} + bH_n\left(1 - e^{-M_n}\right), \\ H_{n+1} &= \frac{cH_n}{1 + pA_n} + \frac{1}{1 + pA_n}, \\ A_{n+1} &= rA_n + fP_n, \\ & \qquad n = 0, 1, \ldots. \end{aligned} \tag{4}$$

In the sequel, we look at boundedness properties, local and global asymptotic stability of equilibria. Numerical simulations, open problems, and further directions of improvement will be mentioned.

2. Boundedness of Solutions

Lemma 1. *Let $\{P_n, M_n, H_n, A_n\}_{n\ge0}$ be a positive solution of system (4). Parameters are such that $0 < l < 1$, $0 < a < 1$, $0 < c < 1$, and $0 < r < 1$. Then $\limsup_{n\to\infty} P_n \le 1/(1 - a)$, $\limsup_{n\to\infty} M_n \le b/(1-l)(1-c)$, $\limsup_{n\to\infty} H_n \le 1/(1-c)$, and $\limsup_{n\to\infty} A_n \le f/(1 - a)(1 - r)$.*

Proof. First equation of system (4) gives $P_{n+1} \le aP_n + (1-P_n) \le aP_n + 1$. Thus $\limsup_{n\to\infty} P_n \le 1/(1 - a)$ and then for any positive number ϵ_p, there exists N sufficiently large, such that, for all $n \ge N$,

$$P_{n+1} < \frac{1}{1 - a} + \epsilon_p. \tag{5}$$

Making use of (5) in the fourth equation, we get

$$A_{n+1} \le rA_n + \frac{f}{1 - a} + \epsilon_p. \tag{6}$$

Since $0 < r < 1$ we obtain $\limsup_{n\to\infty} A_n \le f/(1 - a)(1 - r)$ and then for any positive number ϵ_a, there exists N sufficiently large, such that, for all $n \ge N$,

$$A_{n+1} < \frac{f}{(1 - a)(1 - r)} + \epsilon_a. \tag{7}$$

The third equation of (4) yields $H_{n+1} \le cH_n + 1$ which combined with $0 < c < 1$ gives $\limsup_{n\to\infty} H_n \le 1/(1-c)$. Thus, for any positive number ϵ_h, there exists N sufficiently large, such that, for all $n \ge N$,

$$H_{n+1} < \frac{1}{(1-c)} + \epsilon_h. \tag{8}$$

Finally, (8) and $M_{n+1} \le lM_n + bH_n \le lM_n + b/(1-c) + \epsilon_h$ produce $\limsup_{n\to\infty} M_n \le b/(1-l)(1-c)$. Thus for any positive number ϵ_m, there exists N sufficiently large, such that, for all $n \ge N$,

$$M_{n+1} < \frac{b}{(1-l)(1-c)} + \epsilon_m. \tag{9}$$

□

Some notations that will be used throughout the paper are, in order,

$$\begin{aligned} &\limsup_{n\to\infty} P_n = S_P, && \liminf_{n\to\infty} P_n = I_P, \\ &\limsup_{n\to\infty} H_n = S_H, && \liminf_{n\to\infty} H_n = I_H, \\ &\limsup_{n\to\infty} M_n = S_M, && \liminf_{n\to\infty} M_n = I_M, \\ &\limsup_{n\to\infty} A_n = S_A, && \liminf_{n\to\infty} A_n = I_A. \end{aligned} \tag{10}$$

3. Equilibria

Clearly,

$$\left(0, 0, \frac{1}{1-c}, 0\right) \tag{11}$$

is an equilibrium point of system (4) for all the values of the parameters.

Lemma 2. *(1) Assume that $b \le (1-c)(1-l)$. Then the degenerate equilibrium $(0, 0, 1/(1-c), 0)$ is the only equilibrium point.*

(2) Assume that $b > (1-c)(1-l)$; then there are two equilibrium points, namely, the degenerate one and a positive one denoted by $(\overline{P}, \overline{M}, \overline{H}, \overline{A})$. The positive equilibrium can take the form

$$\left(\frac{1-e^{-i\overline{M}}}{2-a-e^{-i\overline{M}}}, \overline{M}, \frac{1}{1-c+\left(pf\left(1-e^{-i\overline{M}}\right)/(1-r)\left(2-a-e^{-i\overline{M}}\right)\right)}, \frac{f\left(1-e^{-i\overline{M}}\right)}{(1-r)\left(2-a-e^{-i\overline{M}}\right)}\right). \tag{12}$$

Proof. The equilibrium solutions verify the system

$$\begin{aligned} \overline{P} &= a\overline{P} + \left(1-e^{-i\overline{M}}\right)\left(1-\overline{P}\right), \\ \overline{M} &= l\overline{M}e^{-\overline{A}} + b\overline{H}\left(1-e^{-\overline{M}}\right), \\ \overline{H} &= \frac{c\overline{H}}{1+p\overline{A}} + \frac{1}{1+p\overline{A}}, \\ \overline{A} &= r\overline{A} + f\overline{P}. \end{aligned} \tag{13}$$

The fourth equation in the above system gives

$$\overline{P} = \frac{(1-r)\overline{A}}{f}. \tag{14}$$

Solving for $\overline{H}$ in the third equation yields

$$\overline{H} = \frac{1}{1-c+p\overline{A}}. \tag{15}$$

Combining (15) with the second equation of system (13) produces

$$\left(1-le^{-\overline{A}}\right)\overline{M} = \frac{b\left(1-e^{-\overline{M}}\right)}{1-c+p\overline{A}}. \tag{16}$$

Replacing (14) in first system equation and multiplying by f to both sides,

$$(1-a)(1-r)\overline{A} = \left(1-e^{-i\overline{M}}\right)\left(f-(1-r)\overline{A}\right). \tag{17}$$

Since $(1-c+p\overline{A}) \ne 0$ and $(1-le^{-\overline{A}}) \ne 0$, (16) can be written in the form

$$\frac{\overline{M}}{1-e^{-\overline{M}}} = \frac{b}{\left(1-c+p\overline{A}\right)\left(1-le^{-\overline{A}}\right)}. \tag{18}$$

Equation (17) gives

$$\overline{A} = \frac{f\left(1-e^{-i\overline{M}}\right)}{(1-r)\left(2-a-e^{-i\overline{M}}\right)}. \tag{19}$$

Notice that $(2-a-e^{-i\overline{M}}) > 0$. Set

$$w\left(\overline{M}\right) = \frac{f\left(1-e^{-i\overline{M}}\right)}{(1-r)\left(2-a-e^{-i\overline{M}}\right)}. \tag{20}$$

Notice that

$$\overline{A} = w\left(\overline{M}\right), \tag{21}$$

where function $w(M)$ has the property that it is an increasing function, first order derivative

$$w'(M) = \frac{f(1-a)ie^{-iM}}{(1-r)\left(2-a-e^{-iM}\right)^2} > 0 \tag{22}$$

for $M \in (0, \infty)$. Set the real valued functions

$$\Phi_1(M) = \frac{M}{1 - e^{-M}}, \qquad g(A) = \frac{b}{(1 - c + pA)(1 - le^{-A})}. \tag{23}$$

We have that $\Phi_1(M)$ is an increasing function in M, $\Phi_1(0^+) = 1$ and $\Phi_1(\overline{M}) = g(\overline{A})$.

From the above,

$$g(\overline{A}) = g(w(\overline{M})) = (g \circ w)(\overline{M}), \tag{24}$$

where we denote Φ_2 as

$$\Phi_2(M) = (g \circ w)(M) = g(w(M)). \tag{25}$$

Function Φ_2 is decreasing. Let $M_1 < M_2$. Since function w is increasing, one has $w(M_1) < w(M_2)$. But g is a decreasing function and

$$\begin{aligned}\Phi_2(M_1) &= (g \circ w)(M_1) \\ &= g(w(M_1)) > g(w(M_2)) = \Phi_2(M_2).\end{aligned} \tag{26}$$

Using that $w(0^+) = 0$, we have that

$$\Phi_2(0^+) = \frac{b}{(1 - c)(1 - l)}. \tag{27}$$

For (18) to have a unique solution (and thus system to have a unique solution), one must have $\Phi_1(0^+) < \Phi_2(0^+)$ or equivalently $1 < b/(1 - c)(1 - l)$ and the proof ends. □

4. Stability of Equilibrium Points

Next we are concerned with the local and global asymptotic stability of equilibrium points. Notations for our map are as follows:

$$\begin{aligned}
&P_{n+1} = \Theta(P_n, M_n, H_n, A_n) \\
&\text{with } \Theta(P, M, H, A) = aP + \left(1 - e^{-iM}\right)(1 - P), \\
&M_{n+1} = g(P_n, M_n, H_n, A_n) \\
&\text{with } g(P, M, H, A) = lMe^{-A} + bH\left(1 - e^{-M}\right), \\
&H_{n+1} = h(P_n, M_n, H_n, A_n) \\
&\text{with } h(P, M, H, A) = \frac{cH}{1 + pA} + \frac{1}{1 + pA}, \\
&A_{n+1} = \Phi(P_n, M_n, H_n, A_n) \\
&\text{with } \Phi(P, M, H, A) = rA + fP.
\end{aligned} \tag{28}$$

The Jacobian evaluated at the equilibrium point $(\overline{P}, \overline{M}, \overline{H}, \overline{A})$ has the form

$$J\left(\overline{P}, \overline{M}, \overline{H}, \overline{A}\right) = \begin{pmatrix} a - \left(1 - e^{-i\overline{M}}\right) & \left(1 - \overline{P}\right) ie^{-i\overline{M}} & 0 & 0 \\ 0 & le^{-\overline{A}} + b\overline{H}e^{-\overline{M}} & b - be^{-\overline{M}} & -l\overline{M}e^{-\overline{A}} \\ 0 & 0 & \dfrac{c}{1 + p\overline{A}} & \dfrac{-p\left(1 + c\overline{H}\right)}{\left(1 + p\overline{A}\right)^2} \\ f & 0 & 0 & r \end{pmatrix}. \tag{29}$$

Using the third equilibrium equation, $1 + c\overline{H} = \overline{H}(1 + p\overline{A})$. Thus, $-p(1 + c\overline{H})/(1 + p\overline{A})^2 = -p\overline{H}/(1 + p\overline{A})$. The characteristic equation associated with $(\overline{P}, \overline{M}, \overline{H}, \overline{A})$ is given by the fourth order polynomial:

$$\begin{aligned}
&\left[a - \left(1 - e^{-i\overline{M}}\right) - \lambda\right]\left[le^{-\overline{A}} + b\overline{H}e^{-\overline{M}} - \lambda\right] \\
&\quad \times \left[\frac{c}{\left(1 + p\overline{A}\right)} - \lambda\right][r - \lambda] \\
&\quad - \left(1 - \overline{P}\right) ie^{-i\overline{M}} f \left[\frac{-p\overline{H}b\left(1 - e^{-\overline{M}}\right)}{1 + p\overline{A}}\right. \\
&\qquad \left. + l\overline{M}e^{-\overline{A}}\left(\frac{c}{1 + p\overline{A}} - \lambda\right)\right] = 0.
\end{aligned} \tag{30}$$

One can look at the characteristic equation in the form

$$\begin{aligned}
&\lambda^4 - (A_1 + A_2 + A_3 + A_4)\lambda^3 \\
&\quad + (A_1A_2 + A_1A_3 + A_1A_4 + A_2A_3 + A_2A_4 + A_3A_4)\lambda^2 \\
&\quad - (A_1A_2A_3 + A_1A_2A_4 \\
&\qquad + A_1A_3A_4 + A_2A_3A_4 + A_5A_7)\lambda \\
&\quad + A_1A_2A_3A_4 + A_5A_6 + A_3A_5A_7 = 0,
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
A_1 &= a - \left(1 - e^{-i\overline{M}}\right), \\
A_2 &= le^{-\overline{A}} + b\overline{H}e^{-\overline{M}}, \\
A_3 &= \frac{c}{1 + p\overline{A}}, \\
A_4 &= r, \\
A_5 &= -\left(1 - \overline{P}\right) ife^{-i\overline{M}}, \\
A_6 &= \frac{-p\overline{H}b\left(1 - e^{-\overline{M}}\right)}{1 + p\overline{A}}, \\
A_7 &= l\overline{M}e^{-\overline{A}}.
\end{aligned} \tag{32}$$

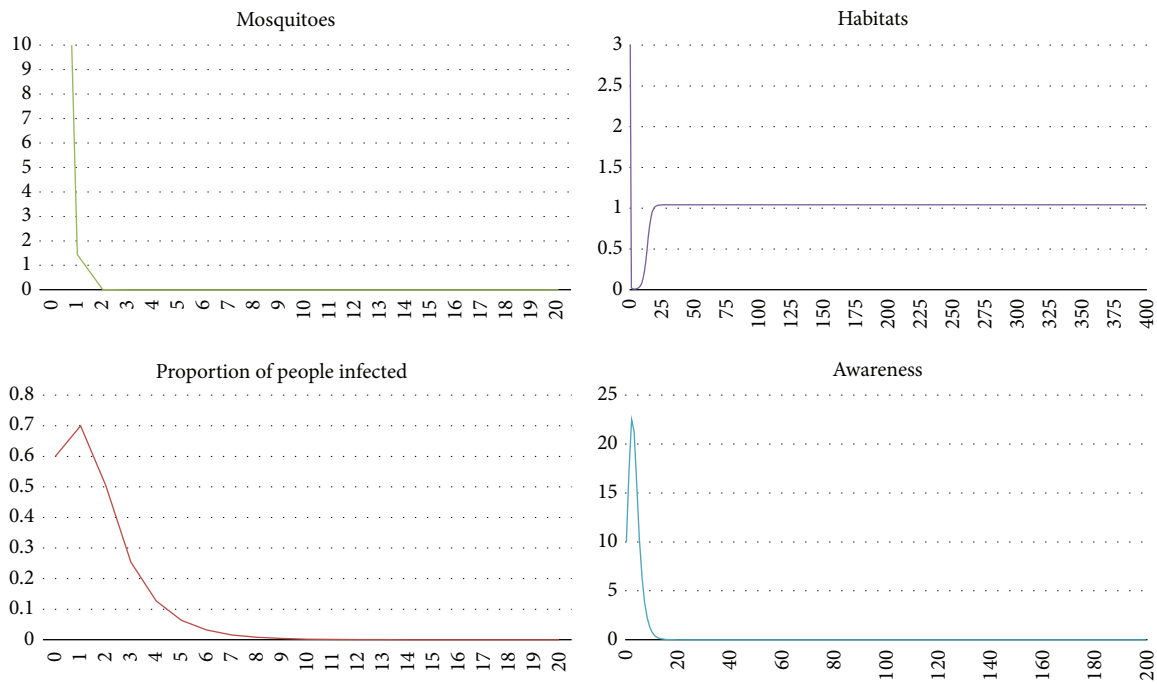

FIGURE 1: The above graph is generated with parameter values $a = 0.5$, $p = 5$, $b = 0.48$, $c = 0.04$, $l = 0.5$, $i = 0.5$, $r = 0.5$, and $f = 20$ (the parameters are placed in the region where $b = (1-c)(1-l)$). One can see that the solutions converge to the degenerate equilibrium point.

In the region of existence of positive equilibrium point, $b > (1-c)(1-l)$, the values of parameters for which the roots of the fourth order polynomial are inside unit disc generate a locally asymptotically stable equilibrium point. The positive equilibrium point is not always locally asymptotically stable in the region $b > (1-c)(1-l)$ (see Figure 3).

The following theorem about the degenerate equilibrium point (Figure 1) $(0, 0, 1/(1-c), 0)$ holds.

Theorem 3. *Assume that* $b < (1-c)(1-l)$. *Then* $(0, 0, 1/(1-c), 0)$ *is globally asymptotically stable.*

Proof. When $\overline{P} = 0$, $\overline{M} = 0$, $\overline{H} = 1/(1-c)$, and $\overline{A} = 0$, the Jacobian becomes

$$J\left(0, 0, \frac{1}{1-c}, 0\right) = \begin{pmatrix} a & i & 0 & 0 \\ 0 & l + \dfrac{b}{1-c} & 0 & 0 \\ 0 & 0 & c & \dfrac{-cp}{1-c} - p \\ f & 0 & 0 & r \end{pmatrix} \tag{33}$$

with the characteristic equation a polynomial that factors into

$$(a-\lambda)\left(l + \frac{b}{1-c} - \lambda\right)(c-\lambda)(r-\lambda) = 0. \tag{34}$$

Three of the roots, namely, $\lambda_1 = a$, $\lambda_2 = c$, and $\lambda_4 = r$ are less than 1 and if $l + b/(1-c) < 1$ (or $b < (1-c)(1-l)$) then the degenerate equilibrium is a sink and thus locally asymptotically stable. It remains to be shown that this equilibrium is a global attractor. We offer a proof by contradiction as in [5]. Let us suppose $S_P > 0$ and $S_M > 0$. Then using the last equation in the system, we conclude

$$S_A \le rS_A + \frac{f}{1-a}. \tag{35}$$

Using that $1 - e^{-M_n} < M_n$ in the second equation of the reduced system yields

$$M_n \le lM_n + bH_nM_n. \tag{36}$$

Thus

$$S_M \le lS_M + bS_HS_M. \tag{37}$$

Dividing by $S_M > 0$ to both sides one obtains

$$\frac{1-l}{b} \le S_H \le \frac{1}{1-c} \tag{38}$$

which implies that $(1-l)(1-c) \le b$ (hence the contradiction). Thus $S_M = 0$.

First equation in the reduced system yields the inequality

$$P_{n+1} \le aP_n + iM_n(1-P_n) \tag{39}$$

or further $P_{n+1} \le aP_n + iM_n$. Passing to the limit one has

$$S_P \le aS_P + iS_M = aS_P. \tag{40}$$

Dividing by $(1-a) > 0$ the above yields $S_P \le 0$ which in combination with $S_P \ge 0$ gives $S_P = 0$.

Using the inequality in the third equation

$$I_H \ge \frac{cI_H}{1+pS_A} + \frac{1}{1+pS_A} \ge cI_H + 1. \tag{41}$$

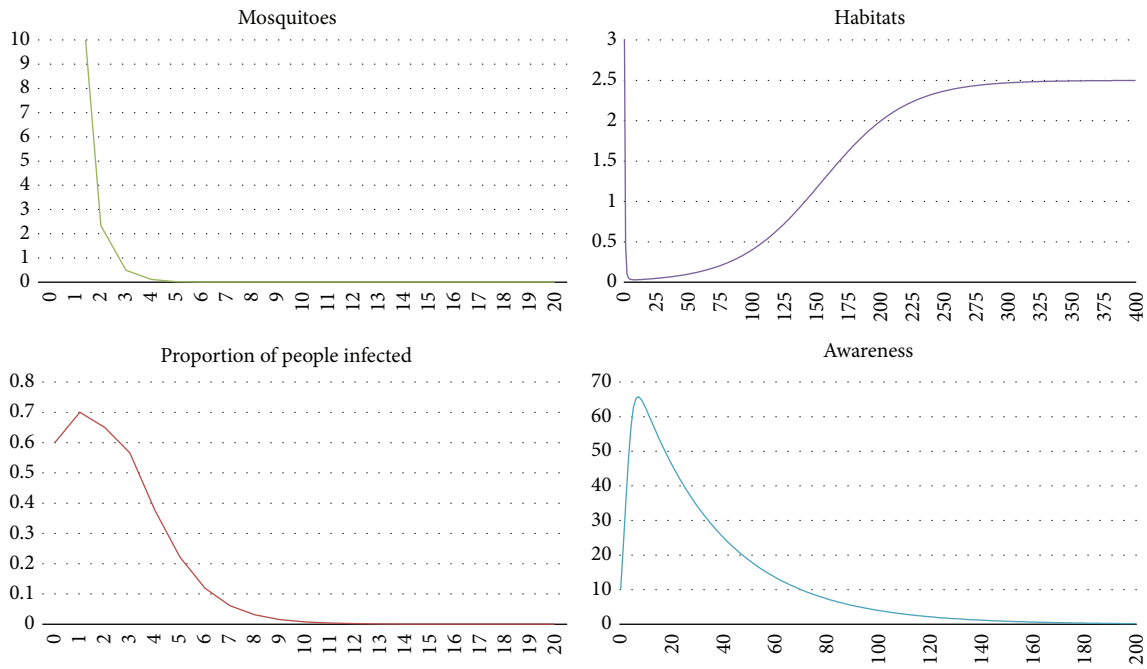

Figure 2: The above graph is generated with parameter values $a = 0.5$, $p = 0.5$, $b = 5$, $c = 0.6$, $l = 0.5$, $i = 0.5$, $r = 0.97$, and $f = 20$ (the parameters are placed in the region where $b > (1-c)(1-l)$). One can see that the solutions display convergence to the equilibrium for big values of r.

It follows $I_H \geq 1/(1-c)$. But $S_H \leq 1/(1-c) \leq I_H$ and thus $S_H = I_H = 1/(1-c)$.

$S_A = 0$ follows easily. □

5. Conclusions and Open Problems

The global asymptotic stability of the degenerate equilibrium was investigated (but the global asymptotic stability of the positive equilibrium remains an open problem that is worth investigating mathematically). An interesting result pertains to the role that the memory plays in controlling the epidemic. We observed oscillatory behavior for marginally low memory parameter values ($r = 0.5$, Figure 3), meaning that the population might recover only for a short period of time and then get periodically infected. Computer simulations indicate that high awareness ($r = 0.97$, Figure 2) leads to a complete decrease in the proportion of infected people and the solutions stabilize.

Simulations done with various parameter values seem to suggest that the memory parameter has a threshold below which there are oscillations and above which it exhibits the equilibria, leading to the extinction of the infection. This is consistent with other findings from studies specifically designed to discover thresholds (see [7]). In [7] the authors considered the rate of contact between susceptible people and infectious vectors, a component captured in our system in the first equation by the term $(1 - e^{-iM_n})(1 - P_n)$. Their study reports that they were surprised to discover that the size of the viral introduction *"was not seen to significantly influence the magnitude of the threshold."* In our future study, we shall focus on finding the memory parameter threshold value that leads to the extinction of the infection and to testing whether changing the initial conditions of the proportion of infected people P_0 has an impact on the threshold value or not.

The average number of mosquitoes per breeding site (parameter b) was estimated to be 9.5, ranging from 3 to 30, in field studies (see [2]). We used $b = 5$ (see Figures 2 and 3), a value within the range suggested by field studies in the aforementioned reference. Computer simulations on system (1) indicate that it is possible that for large values of parameter d (high pollution level such as new empty cans and tires that collect water), the memory parameter r alone may not be sufficiently strong enough to eliminate the infection from the population, and the infection might equilibrate at levels higher than zero. In future work we shall explore the relationship between environmental pollution and the memory that creates awareness in the community.

In this section we also want to bring attention to some extensions and open problems related to system (1). An interesting question to be analytically investigated in a further study is the global asymptotic stability of nondegenerate equilibrium of system (1) especially in the case when the system incorporates different parameters that measure the sensitivity of surviving habitats to communal awareness and individual awareness (hence $p \neq q$). Thus, in this case the third equation reads

$$H_{n+1} = cH_n h_1(pA_n) + dh_2(qA_n). \tag{42}$$

Based on biological considerations, one can take $h_1(\cdot)$ and $h_2(\cdot)$ as decreasing functions, $h_1, h_2 \in C^1((0,\infty) \to (0,1])$ with properties (i) $h_1(0) = 1$ and $h_2(0) = 1$ and (ii) $\lim_{y\to\infty} h_1(y) = 0$ and $\lim_{y\to\infty} h_2(y) = 0$. Two most used

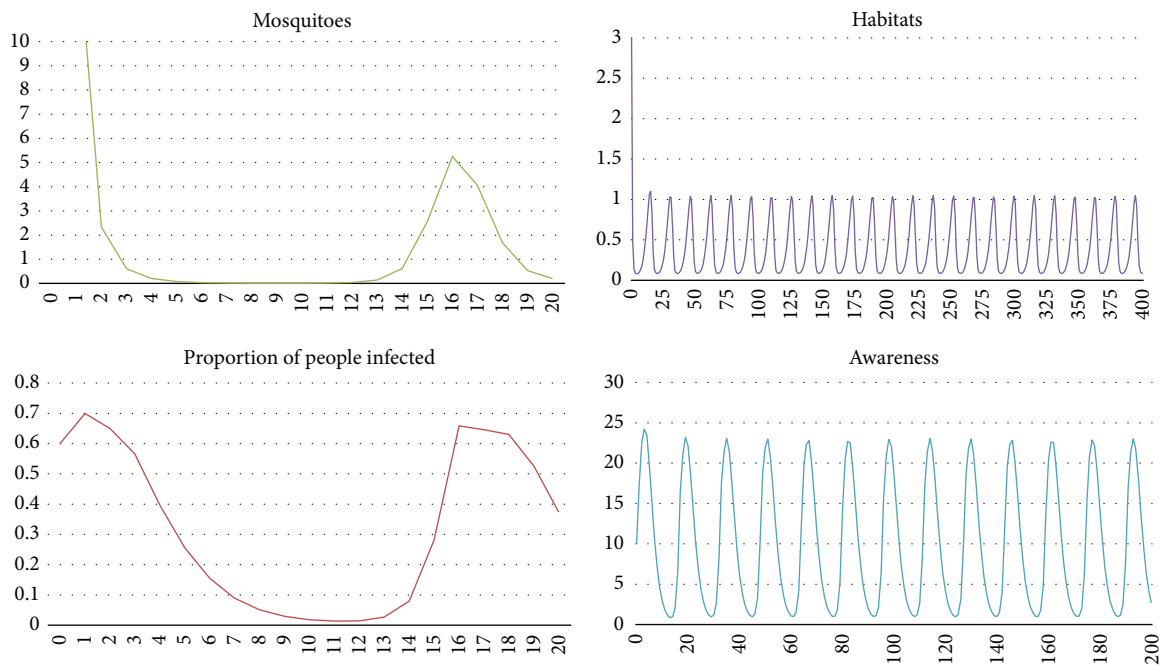

Figure 3: The above graph is generated with parameter values $a = 0.5$, $p = 0.5$, $b = 5$, $c = 0.6$, $l = 0.5$, $i = 0.5$, $r = 0.5$, and $f = 20$ (the parameters are placed in the region where $b > (1-c)(1-l)$). One can see that the solutions display oscillatory behavior for smaller values of r.

examples of such functions (used in the previous work, [3]) are for instance $h_1(y) = 1/(1+py)$ and $h_2(y) = 1/(1+qy)$. Thus, an open problem that we want to pose here refers to the study of the existence and global asymptotic stability of the positive equilibrium of the general system:

$$\begin{aligned} P_{n+1} &= aP_n + \left(1 - e^{-iM_nP_n}\right)(1-P_n), \\ M_{n+1} &= lM_ne^{-gA_n} + bH_n\left(1 - e^{-sM_n}\right), \\ H_{n+1} &= cH_nh_1(pA_n) + dh_2(qA_n), \\ A_{n+1} &= rA_n + fP_n, \\ & \qquad n = 0, 1, \ldots. \end{aligned} \tag{43}$$

Mathematical models may serve at designing policy interventions and provide a better understanding of phenomena at study [3]. Because at times interventions are implemented when consciousness is prompted by an increase in the incidence of sick people, one can work with the original system in a form as such:

$$\begin{aligned} P_{n+1} &= aP_n + \left(1 - e^{-iM_nP_n}\right)(1-P_n), \\ M_{n+1} &= lM_ne^{-gA_n} + bH_n\left(1 - e^{-sM_n}\right), \\ H_{n+1} &= \frac{cH_n}{1+pA_n} + \frac{d}{1+qA_n}, \\ A_{n+1} &= rA_n + fP_n, \\ & \qquad n = 0, 1, \ldots. \end{aligned} \tag{44}$$

The first equation describes the proportion of infected people in the population (between 0 and 1). Proportion of sick people is assumed to prompt consciousness, while the intervention is against mosquitoes and perhaps habitats. The control of both, adult mosquitoes (M) and habitats (H) where mosquitoes lay their eggs, is carried out by spraying and community intervention by reducing breeding sites. One may use this system (system (44)) to compare a few control strategies, where increase in the proportion of infected people is linked to consciousness. Insecticide spraying is a common method in mosquito control despite its many disadvantages; and new ones are continuously being developed and tested [8–10]. In the long run the mosquitoes become resistant and the insecticide ineffective [11]; it poses serious risks to humans and the environment [10, 12–14]. In order to assess the effect of insecticide spraying without habitat management, the equations are modified so that we eliminate the rational control on H_n, and keep the population control on M_n. To assess the effect of habitat control only, through citizens intervention, the equation will keep its intervention parameters as such, $H_{n+1} = cH_n/(1+pA_n) + d/(1+qA_n)$, for example. We believe that system (44) is not only useful biologically but also interesting mathematically. Both systems (43) and (44) possess bounded solutions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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