In order to explore carbon market regulation mechanism more effectively, based on carbon accumulation model with jump diffusion, this paper studies the carbon price from two perspectives of quantity instrument and price instrument and quantitatively simulates carbon price regulation mechanisms in the light of actual operation of EU carbon market. The results show that quantity instrument and price instrument both have certain effects on carbon market; according to the comparison of the elasticity change of the expected carbon price, comparative advantages of both instruments rely on the price of carbon finance market. Where the carbon price is excessively high, price instrument is superior to quantity instrument; where carbon price is excessively low, quantity instrument is better than price instrument. Therefore, in the case of carbon market regulation based on expected carbon price, if the carbon price is too high, price instrument should prevail; if the carbon price is excessively low, quantity instrument should prevail.

1. Introduction

Given the market practice in EU ETS and RGGI, the highest risk for carbon market is drastic fluctuation in carbon price. European Union Allowance (EUA) price tumbled from 30 Euros in 2005 to near zero in 2007 at the first stage and from maximum 30 Euros in 2008 to less than 5 Euros in 2013 at the second stage. While, as an artificially designed market, carbon market greatly differs from traditional financial market in terms of price formation mechanism, the price in carbon market is highly vulnerable to the market design and policy mechanism. Therefore, in order to ensure the effective and sustainable development of the carbon market, this paper explores regulation mechanism for guaranteeing carbon price within reasonable fluctuation range and simulates quantitative mechanism of price regulation so as to provide method basis and policy guidance for stable development of carbon market.

The main objective of regulation based on carbon market mechanism is to stabilize carbon market price in the short term and reduce greenhouse gas emission in the long term. Regulation can be divided into two tools, quantity instrument and price instrument, by the mode of economic operation. Price instruments include floor price and safety valve, while quantity instruments may be similar to open market operation of monetary policy, in which allowance is increased or decreased to adjust allowance supply quantity. Debate about comparative advantages between quantity regulation and price regulation always exists. In [1], it is explicitly stated for the first time that the ratio of marginal cost of external control to marginal income of external control was the key determinant factor for the effectiveness of regulation instrument. Pizer [2] proposed composite tool for obtaining price and quantity based on stochastic computable general equilibrium model was more effective than single control. Kelly [3] estimated the effect from quantity regulation and price supervision under the condition that the regulator cannot observe productivity shocks and obtained the result indicating that quantity regulation means always generated more profits, regardless of how to express profit function. Wirl [4] analyzed the issue concerning selection of optimal regulation and control strategy for the government which
served as representative for consumers and enterprises which produced fossil fuels under the differential game framework, which showed that price tool was the optimal strategy choice for both game players.

According to international market practice, for the purpose of preventing drastic fluctuation in market price, regulatory agency generally adopts the following major flexible mechanisms for regulation: safety valve, price collar, allowance reserve, allowance borrowing, carbon credit offsets, European option or American option provided to enterprises by regulator, and so forth. Price safety valve was firstly proposed in market scheme designed by USA for Kyoto Protocol. This scheme pointed out that the objective of Kyoto Protocol was too strict; thus, safety valve plan was put forward (see [5]). Jacoby and Ellerman [6] explored safety valve mechanism and stated that this mechanism designed absolute price ceiling, and when carbon price was higher than such ceiling, allowance can be purchased at such ceiling without limit so as to effectively control cap-and-trade price in carbon market. However, application of price ceiling also exerted negative impact, such as easy reduction in carbon price expectation and subsequent decrease in motive of technical progress in pollution control (see [7, 8]). Price ceiling mechanism cannot make marginal emission reduction cost lower than expected and possibly fails to provide effective incentive for emission reduction. Some scholars proposed to introduce price floor on the basis of price ceiling design, which resulted in price collar mechanism with double-price control. Simulation technique in [9] was separately used to show that price collar mechanism can effectively avoid the defects in price ceiling design. Pizer and Marika [10] further discussed the mode for selling limited quantity of reserve allowance according to safety valve price so as to translate absolute price ceiling into flexible price ceiling and ensure the effect of emission reduction while controlling costs. In addition, the mechanism for providing European option or American option to enterprises can enhance self-regulating subject to information shock, which was firstly proposed by Requate and Unold [11], under which regulator provided enterprises with European option or American option at fixed price at the beginning of compliance period, while they demonstrated that option provision mechanism can drive execution fee to become zero and thus provided the rationality and effectiveness of this mechanism design.

For the research on quantity and price regulation mechanisms, Guo et al. [12] analyzed regulation mechanism in Chinese carbon market on the basis of classical risk neutral framework in the light of pilot operation of Shenzhen carbon market. However, given practical experience in EU, external impact was easier to cause jump during carbon accumulation. And Borovkov et al. [13] adopted Poisson process to describe carbon accumulation equation in order to more tally with uncertainties in carbon market. For the sake of more accordance with carbon market price process, based on [13], this paper introduces Poisson process for depicting jump-diffusion process of carbon accumulation and setting upper and lower threshold values in line with actual operation of EU ETS carbon emission market and then separately simulates carbon market price in terms of two aspects including quantity instrument and price instrument (in carbon emission regulation mechanism in this paper, quantity instrument specifically refers to regulation of total allowance and price instrument means regulation of penalty value) and analyzes comparative advantages of two regulation modes. With comparative analysis, we find that comparative advantages of quantity regulation and price regulation rely on price level in carbon market: with elasticity of expected carbon price changes, when carbon price is too high and needs to be downwardly adjusted, price instrument is superior to quantity instrument; conversely, when carbon price is excessively low and needs to be upwardly adjusted, quantity regulation is superior to price regulation. Thus where carbon market is regulated according to expected carbon price, if carbon price is low, price regulation will prevail; if carbon price is excessively high, quantity regulation will prevail.

This paper is arranged as follows. Section 2 provides theoretical framework for carbon price simulation. Section 3 starts with upper and lower threshold values and adopts Monte Carlo method to simulate changes in carbon emission price in the case of quantity instrument and price instrument separately and makes comparison between two regulation modes. Section 4 draws conclusions and offers future research direction.

2. Modeling

EU emissions trading system (EU ETS) is the largest carbon emission trading market in the world and has demonstration effect in the world’s carbon trading market. Thus quantitative simulation in this paper is conducted on the basis of practical data on EU ETS which appears as cap-and-trade scheme. This scheme specifies emission allowance (N) for member states that the sum of emission allowance for all countries will not exceed the emission committed in the protocol. Allocation of emission allowance gives comprehensive considerations to past emission, expected emission and emission standard, and so forth. If actual emission from enterprises in member states (Q_r) exceeds emission allowance within one given compliance period (T), penalty (P) will be paid for each CO_2 unit.

If carbon emission from enterprise exceeds allowance, emission allowance will be purchased from other enterprises or auctions by the government. Thus, allowance can be traded in the market and its trading price is affected by allowance cap, penalty, and supply-demand relationship. In order to explore the impact of allowance cap and penalty price on carbon emission price, this paper excludes the impact of supply-demand relationship on price and only takes into account carbon price under risk-neutral valuation method, given that, in practice, cumulative carbon emission does not necessarily change in a smooth way but presents discontinuous “jump” process. According to literatures [13, 14], cumulative carbon emission Q_r at terminal moment T consists of two parts: continuous part composed of geometrical Brownian motion and “jump” part which is described as Poisson process. Assuming that (Ω, ℋ, P) is one probability space, P is risk-neutral measure; Q_0 is expected carbon accumulation at
initial moment, namely, BAU (Business-As-Usual) emission, \( Q_T \) is total cumulative carbon emission at the moment \( T \), and \( (Q_t)_{0\leq t\leq T} \) is one exogenous given process which is given by the following formula:
\[
\frac{dQ_t}{Q_t} = (\mu - \lambda k) dt + \sigma dW(t) + XdN(t), \quad 0 \leq t \leq T,
\]
where \( W(t) \) is standard Wiener process, \( \mu \) denotes the drift during carbon accumulation, suggesting carbon accumulation speed in the carbon market, and \( \sigma \) is volatility. \( N(t) \) complies with Poisson process with parameter as \( \lambda \). \( W(t) \) and \( N(t) \) are assumed to be independent. \( X \) is the percent of the jump size of cumulative carbon emission.

Assuming there is no arbitrage in the market, we use risk neutral method to calculate carbon emission price \( y_t \) at the moment \( t \in [0, T] \) in the case of not using any mechanism:
\[
y_t = E \left[ P \cdot I_{\{Q_T > N\}} \cdot F_{\mathcal{F}_t} \right].
\]

According to formula (2), carbon emission price at time \( t \) is mainly determined by allowance \( N \) and penalty price \( P \); thus, in the following part, the factors affecting carbon emission price are explored mainly by changing allowance \( N \) and penalty price \( P \), and regulation effects of allowance and penalty on carbon emission price are separately simulated and compared.

### 3. Carbon Emission Price Regulation Mechanism

Practical implementation of cap-and-trade scheme is always entrapped in the dilemma in which it is difficult to place equal emphasis on both economic development and emission reduction. On the one hand, strict cap control is beneficial for low-carbon transformation, but high carbon price cost will add burden on economic development at the initial stage of transformation; on the other hand, slack cap constraint is easy to form relatively low carbon price so that the scarcity of carbon emission right disappears, resulting in cap objective losing its constraint. Currently, the EU ETS and RGGI markets in the world are severely questioned because of low carbon price resulting from excessive allowance. Because both excessively high carbon price and excessively low carbon price can affect carbon emission market, in order to ensure steady market operation, we preset upper and lower threshold values of carbon price to the effect that when carbon price reaches upper and lower threshold values, the government will make adjustment.

This section unfolds from regulations of upper threshold and regulations of lower threshold and simulates carbon price separately by changing allowance and penalty. According to practical experience in EU ETS, penalty is set as \( P = 40 \), assuming that acceptable price is 20, the upper limit which we can tolerate is set as 150\% of it, and the lower limit is set as 75\% of it; namely, \( P_{\text{high}} = 30 \), \( P_{\text{low}} = 15 \). Initial expected carbon accumulation is set as \( Q_0 = 200 \) (million tons), cumulative rate and volatility of expected carbon emission are separately set as (setting of this parameter is based on the literature [15]) \( \mu = 0.15, \sigma = 0.15 \), and one compliance period \( T \) is 3 (years) according to the first stage of EU ETS (2005–2007).

#### 3.1 Regulations of Upper Threshold

In this part, we will proceed from the following two aspects: quantity instrument and price instrument. There is relevant practical experience concerning quantity instrument in both RGGI and EU ETS; for example, in 2013, allowance reserve mechanism was introduced in RGGI carbon trading market; namely, limited quantity of reserve allowance was sold at safety valve price. Carbon credit offset is also one kind of quantity instruments and the substitute for allowance; allowance demand decreases with increasing credit offset; carbon credit offset in EU ETS amounted to 555 million tons during 2008–2011, accounting for 7\% of emission.

#### 3.1.1. Quantity Instrument

According to formula (2), carbon emission price is jointly determined by allowance cap, actual carbon emission, and penalty price. If the government anticipates that emission reduction is excessive, too little issuance of free-of-charge allowance will cause relatively high carbon emission price and short supply in carbon emission market, which can lead to relatively large economic burden on market bubble. Based on practical experience in EU ETS and RGGI, we set specific quantity regulation as follows: if carbon price is higher than upper threshold value for \( n \) consecutive months in one compliance period \( T \), allowance would increase by \( \alpha \) times so as to lower the carbon price. Assuming that \( Q_T \) is actual total carbon emission of all enterprises, its process satisfies jump-diffusion process; such mechanism can be expressed as follows:
\[
y_t = E \left[ P \cdot I_{\{Q_T > N\}} \cdot I_{\{\tau > T\}} + P \cdot I_{\{Q_T > (1 + \alpha)N\}} \cdot I_{\{\tau < T\}} \mathcal{F}_t \right],
\]
where \( \tau = \inf \{n \leq s < t, \int_0^s I_{\{y_u > P_{\text{high}}\}} du = n\} \).

We use Monte Carlo method to validate the effectiveness of such mechanism; if emission reduction is expected to be excessively 50\%, namely, \( N = 100 \) (million tons), generating \( M = 1,000,000 \) different routes, simulation (parameters are set as \( Q_0 = 200 \), \( N = 100 \), \( P = 40 \), \( P_{\text{high}} = 25 \), \( \mu = 0.15, \sigma = 0.15, \alpha = 0.15, \lambda = 0.5, T = 3, n = 6 \)) is conducted to obtain \( y_t \) frequency distribution diagram of carbon emission prices in \( M \) routes at each time point \( t \).

As shown in Figure 1, axis \( x \) and axis \( y \) represent time \( t \) and the carbon emission price distribution, respectively, and most prices are distributed at about 30, axis \( z \); namely, height of curved surface in diagram represents the frequency of carbon emission price. According to simulated price, carbon emission price at \( t = 0 \), namely, the initial stage of market, is 33.13 higher than the upper limit 30. If it is higher than 30 for six consecutive months, allowance cap will increase to be \( \alpha \) times the previous one in the 7th month. As indicated in Figure 1, when \( t > 6 \), compared with diagram in the case of not using quantity instrument, diagram in the case of using this instrument shows decreased probability of distribution in relatively high price range [30, 40] and increased probability of distribution in relatively low price range [0, 10].

Furthermore, in order to show the effect from using quantity instrument more markedly, we can also obtain
expected values of carbon prices at all routes and compare them with those in the case of not using this instrument, and we can find that, after this instrument is used, carbon emission price is 29.92 at $t = 7$ (when carbon price has been higher than price ceiling for six consecutive months), down 5.62% compared with price 31.2 in the case of not using this mechanism.

3.1.2. Price Instrument. If carbon emission price is extremely high, we can reduce the expectation of market participants about carbon price by decreasing penalty so as to lessen the demand for carbon emission right for cutting down carbon price. Thus, we design the following price regulation: if carbon price is higher than the upper threshold value for $n$ consecutive months in one compliance period $T$, penalty $P$ is decreased by $\alpha$ percentage points so as to lower the carbon price; this mechanism can be expressed as follows:

$$y_t = E \left[ P \cdot I_{\{Q_t \geq N\}} \cdot I_{\{\tau \geq T\}} + (1 - \alpha) \cdot P \cdot I_{\{Q_t \geq N\}} \cdot I_{\{\tau < T\}} \right],$$

(4)

where $\tau = \inf \{ n \leq s < t, \int_{n}^{s} I_{\{y_u > P_{\text{high}}\}} du = n \}$.

Parameters are the same as those in Figure 1; we simulate this mechanism to obtain frequency distribution diagram in the case of using price instrument. According to Figure 2, initial price of carbon emission is still about 33, namely, 33.2 (Monte Carlo method simulation can cause slight difference in each result under the same parameters). After price instrument is used, frequency distribution of carbon emission price at $t = 7$ generally presents relatively obvious shift towards low price range [20, 30] from high price range [30, 40] and compared with the case where this mechanism is not used. And the frequency distribution at low price range [0, 10] also obviously increases. As shown in Figure 2, after price instrument is used, decreasing in carbon price distribution is more obvious than that in the case of using quantity instrument, which suggests that price instrument has significant effect on controlling carbon price.

Overall, in the upper threshold regulation, there is relatively vigorous control over carbon emission, less allowance cap, and relatively high carbon emission price, in which case the effect of price instrument is obviously better than that of quantity instrument (see Table 1). This result is attributable to the following two aspects: because this original price and allowance trading volume are relatively high, regulating the price has relatively large impact on expected income for enterprise; thus, price instrument produces better effect than quantity instrument; another reason is that, at this time, the allowance cap is relatively low and base number is relatively small, and only $\alpha$ time increase in allowance causes less obvious change in allowance; thus, the effect is not distinct.

3.2. Regulations of Lower Threshold. In practice, compared with excessively high carbon price, low carbon price emerges more frequently. Currently, excessively low carbon price resulting from the loose allowance cap occurs in both EU ETS and RGGI; thus, there is no corresponding restraint effect on carbon emission.

Allowance cap (6.47 billion tons) issued in EU ETS at the first stage (2005–2007) was 4.3% higher than emission (6.2 billion tons). At the second stage (2008–2012), allowance cap was lower than emission in the first year, while allowance in subsequent three years was 10.9%, 7.7%, and 9.9% higher than emission, respectively. The problem that RGGI allowance cap was too loose became more severe, annual allowance cap set for the first stage (2009–2011) was 188 million short tons, emission in the first year was 34% lower than allowance cap, and emission in the first three years was only 124, 136, and 119
### Table 1: Upper and lower threshold regulations comparative.

<table>
<thead>
<tr>
<th></th>
<th>Using quantity instrument (t = 7)</th>
<th>Using price instrument (t = 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_t) (no instrument)</td>
<td>(y_t) (using instrument)</td>
<td>Variation</td>
</tr>
<tr>
<td>Upper threshold</td>
<td>31.7</td>
<td>29.92</td>
</tr>
<tr>
<td>Lower threshold</td>
<td>14.88</td>
<td>16.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(y_t) (no instrument)</th>
<th>(y_t) (using instrument)</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper threshold</td>
<td>31.59</td>
<td>26.77</td>
<td>15.26%</td>
</tr>
<tr>
<td>Lower threshold</td>
<td>14.91</td>
<td>15.59</td>
<td>4.56%</td>
</tr>
</tbody>
</table>

3.2.1. **Quantity Instrument.** Excessively high allowance cap is the major cause for low carbon price. In response to excessively low carbon price, we design the following mechanism: if carbon emission price is lower than lower threshold for \(n\) consecutive months within one compliance period \(T\), allowance is decreased by \(\alpha\) times so as to increase carbon price. Its specific expression is shown below:

\[
y_t = E \left[ P \cdot I_{Q_T > N} \cdot I_{(\tau \geq T)} + P \cdot I_{(Q_T \geq (1-\alpha)N)} \cdot I_{(\tau < T)} \right],
\]

where \(\tau = \inf \{n \leq s < t, \int_s^{s-n} I_{y_s < P_{\text{Low}}} du = n\}.\)

If allowance is excessively issued by 50%, namely, \(N = 300\), low carbon price is predictable, and cap control mechanism does not have its due effect. Similarly, we simulate expected carbon price changes and obtain frequency distribution diagram (parameters are set as \(Q_0 = 200, N = 300, P = 40, P_{\text{Low}} = 15, \mu = 0.15, \sigma = 0.15, \alpha = 0.15, \lambda = 0.1, T = 3, n = 6\) of carbon emission prices at different time \(t\). As shown in Figure 3, initial carbon price at \(t = 0\) is 13.18, below the lower threshold value 15. According to the diagram, after quantity instrument is used, where \(t > 6\), with increasing time \(t\), the frequency of carbon emission price distribution in relatively high price range [20, 30] is higher than that in the case of not using this mechanism, and the probability at low price range [0, 20] is lower than that in the case of not using this mechanism.

According to calculation at \(t = 7\), when allowance is lowered, expected carbon emission price is 16.98, and the price in the case of not using this mechanism is 14.88. If allowance cap decreases by 15%, expected carbon emission price increases by 14.11%, and the effect is very obvious.

3.2.2. **Price Instrument.** When carbon emission price is too low, increasing penalty is also one effective way for increasing carbon emission price and improving carbon trading market downturn. We design specific mechanism as follows: if carbon price is lower than lower threshold for \(n\) consecutive months within one compliance period \(T\), penalty \(P\) is increased by \(\alpha\) percentage points so as to increase carbon price:

\[
y_t = E \left[ P \cdot I_{Q_T > N} \cdot I_{(\tau \geq T)} + (1 + \alpha) \cdot P \cdot I_{(Q_T \geq (1-\alpha)N)} \cdot I_{(\tau < T)} \right],
\]

where \(\tau = \inf \{n \leq s < t, \int_s^{s-n} I_{y_s < P_{\text{Low}}} du = n\}.

We simulate this mechanism with parameters the same as above; specific frequency distribution of carbon emission...
prices is shown in Figure 4. Compared with quantity instrument, price regulation mechanism produces effect which is not particularly obvious in the case of excessively low carbon price; according to this diagram, where \( t > 6 \), with increasing time, the frequency of carbon prices distributed in low price range \([0, 20]\) in the case of using price instrument is lower than that in the case of not using this mechanism. Thus, when price is low, price regulation has less effect than quantity regulation. According to calculation of expected values of carbon emission prices at various routes, where penalty is upwardly adjusted by 15% at time \( t = 7 \), expected carbon emission price is 15.59, up 4.56% compared with price 14.91 in the case of not using this mechanism.

Therefore, with respect to lower threshold regulation, quantity instrument is superior to price instrument since original price is relatively low; namely, allowance cap is excessively issued, the demand for allowance is relatively low, trading volume is small, and certain change in penalty price has little impact on expected return for enterprises.

Overall, based on carbon accumulation model with jump diffusion, we explore carbon price change routes from two perspectives of quantity instrument and price instrument in the light of actual operation of EU carbon market. We conclude that where carbon price is too high, price instrument should be adopted; if the carbon price is excessively low, quantity instrument should be mainly considered.

### 4. Conclusions and Suggestions

Neither EU ETS nor RGGI has perfect regulatory operation in international carbon market, for which one of the major causes lies in the case where allowance is oversupplied, leading to low carbon emission price; thus, it is difficult for cap-and-trade scheme to play the guiding role in emission reduction and low-carbon investment. Based on analysis
of influencing factors, it is believed that rigid cap setting mechanism will result in lack of flexibility in allowance supply and excessive credit offset may further aggravate excessive allowance. Therefore, this paper is designed to provide effective regulation method for currently unstable carbon market and attempt to carry out basic work on this aspect. We introduce Poisson process for depicting jump diffusion process in carbon accumulation and setting upper and lower threshold in line with actual operation of EU ETS carbon emission market, separately simulate carbon market price in terms of two aspects including quantity instrument and price instrument using Monte Carlo simulation, and analyze comparative advantages of two different instruments. With comparative analysis, we find that comparative advantages of quantity regulation and price regulation depend on price level in carbon market: with regard to elasticity of expected carbon price changes, when carbon price is excessively high and needs to be downwardly adjusted, price instrument is superior to quantity instrument; conversely, when carbon price is excessively low and needs to be upwardly adjusted, quantity instrument is superior to price instrument. In subsequent research, in order to further tally with carbon market, we will further switch pricing framework to hybrid model and, based on detailed description about endogenous mechanism for carbon price formation, explore rational regulation mechanism to guide development of carbon market and other relevant markets. Furthermore, we estimate parameters of the diffusion process more accurately based on the method in [16, 17].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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