Research Article

Partitioning Technique for Relaxed Stability Criteria of Discrete-Time Systems with Interval Time-Varying Delay

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We demonstrate an improved stability analysis based on a partition oriented technique for discrete-time systems with interval time-varying delay. The partition oriented technique introduces beneficial terms contributing to the negative definiteness of the Lyapunov function difference, meanwhile completely avoiding traditional inequality based approaches. In contrast, nonpartitioning oriented techniques do not put emphasis on further dividing the interval of the summation in the Lyapunov function. Herein, we demonstrate that the advantages of exploiting partitioning techniques manifest the relaxed stability criteria, as well as the flexibility to tune tradeoff between allowable timedelay range performance and computational load. Simulation carried out on a benchmark discrete-time system reveals the significant improvement in terms of maximum allowable upper bound in comparison.

1. Introduction

Stability analysis on continuous and discrete-time systems with interval time-varying delay has been extensively researched in the past decade [1–7]. Traditional techniques focused on introducing inequalities (e.g., Moon et al. [8], Park [9], model transformation [10, 11]) to cope with the double integral problem in the Lyapunov function whereas we term these approaches as nonpartitioning oriented techniques. However, these nonpartitioning oriented techniques increase conservativeness by introducing positive terms to the Lyapunov function difference. This setback led to techniques (e.g., zero equalities [12], averaging technique [13]) which further partition the time-delay interval to introduce beneficial terms contributing to the negative definiteness of the Lyapunov function difference (whereas we term this as partitioning techniques), resulting in relaxed stability criteria. From the above observation, we are motivated to completely avoid nonpartitioning oriented techniques while exploiting partitioning oriented techniques. The advantages are (i) relaxed stability criteria and (ii) flexibility to tune tradeoff between allowable time-delay range performance

and computational load. Simulation results on a discrete-time example reveal the best performance in comparison to previous literature.

2. Main Results

Using the partitioning technique, we have the following main results.

Theorem 1. Given nonnegative integer scalars τ_M and τ_m , the system

$$\begin{aligned} x(k+1) &= Ax(k) + A_h x(k-h(k)), \\ x(k) &= \varphi(k), \quad k \in [-\tau_M, 0] \end{aligned} \quad (1)$$

is asymptotically stable, where $x(k) \in \mathbf{R}^n$ is the state vector; A, A_h are constant matrices with appropriate dimensions; $\varphi(k)$ is the initial condition of $k \in [-\tau_M, 0]$; for any time varying delay $h(k)$ satisfying $0 \leq \tau_m \leq h(k) \leq \tau_M$, there exist real matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$ ($i = 1, 2, 3, 4$), $R = R^T > 0$, $Z = Z^T > 0$, and any appropriately dimensioned

matrices $N = [N_1^T, N_2^T]^T$, $M = [M_1^T, M_2^T]^T$, $S = [S_1^T, S_2^T]^T$, and $T = [T_1^T, T_2^T]^T$ such that the following two sets of LMIs hold.

Case I ($0 \leq \tau_m \leq h(k) \leq \tau_{\text{avg}}$). Consider

$$\begin{aligned}\Theta_1 &= \begin{bmatrix} X_{11} & X_{12} & N_1 \\ X_{12}^T & X_{22} & N_2 \\ N_1^T & N_2^T & Z \end{bmatrix} \geq 0, \\ \Theta_2 &= \begin{bmatrix} X_{11} + Y_{11} & X_{12} + Y_{12} & M_1 \\ X_{12} + Y_{12}^T & X_{22} + Y_{22} & M_2 \\ M_1^T & M_2^T & R + Z \end{bmatrix} \geq 0, \\ \Theta_3 &= \begin{bmatrix} X_{11} + Y_{11} & X_{12} + Y_{12} & S_1 \\ X_{12}^T + Y_{12}^T & X_{22} + Y_{22} & S_2 \\ S_1^T & S_2^T & R + Z \end{bmatrix} \geq 0, \\ \Theta_4 &= \begin{bmatrix} X_{11} & X_{12} & T_1 \\ X_{12}^T & X_{22} & T_2 \\ T_1^T & T_2^T & Z \end{bmatrix} \geq 0,\end{aligned}$$

Φ

$$\begin{aligned}\Phi &= \begin{bmatrix} \Phi_{11} & \Phi_{12} & -N_1 + M_1 & -T_1 & -S_1 + T_1 \\ \Phi_{12}^T & \Phi_{22} & -N_2 + M_2 & -T_2 & -S_2 + T_2 \\ -N_1^T + M_1^T & -N_2^T + M_2^T & -Q_2 & 0 & 0 \\ -T_1^T & -T_2^T & 0 & -Q_3 & 0 \\ -S_1^T + T_1^T & -S_2^T + T_2^T & 0 & 0 & -Q_4 \end{bmatrix} \\ &< 0,\end{aligned}\tag{2}$$

where

$$\begin{aligned}\Phi_{11} &= -P + Q_1 + Q_2 + Q_3 + Q_4 + (\tau_M - \tau_m)Q_1 + N_1 + N_1^T \\ &\quad + \tau_M X_{11} + \delta_1 Y_{11} + A^T P A + (A - I)^T \Theta (A - I), \\ \Phi_{12} &= N_2^T - M_1 + S_1 + \tau_M X_{12} + \delta_1 Y_{12} + A^T P A_h \\ &\quad + (A - I)^T \Theta A_h, \\ \Phi_{22} &= -M_2 - M_2^T + S_2 + S_2^T - Q_1 + \tau_M X_{22} + \delta_1 Y_{22} \\ &\quad + A_h^T P A_h + A_h^T \Theta A_h, \\ \Theta &= \delta_1 R + \tau_M Z.\end{aligned}\tag{3}$$

Case II ($\tau_{\text{avg}} \leq h(k) \leq \tau_M$). Consider

$$\begin{aligned}\bar{\Theta}_1 &= \begin{bmatrix} X_{11} & X_{12} & N_1 \\ X_{12}^T & X_{22} & N_2 \\ N_1^T & N_2^T & Z \end{bmatrix} \geq 0, \\ \bar{\Theta}_2 &= \begin{bmatrix} X_{11} & X_{12} & M_1 \\ X_{12}^T & X_{22} & M_2 \\ M_1^T & M_2^T & Z \end{bmatrix} \geq 0, \\ \bar{\Theta}_3 &= \begin{bmatrix} X_{11} + Y_{11} & X_{12} + Y_{12} & S_1 \\ X_{12} + Y_{12}^T & X_{22} + Y_{22} & S_2 \\ S_1^T & S_2^T & R + Z \end{bmatrix} \geq 0, \\ \bar{\Theta}_4 &= \begin{bmatrix} X_{11} + Y_{11} & X_{12} + Y_{12} & T_1 \\ X_{12}^T + Y_{12}^T & X_{22} + Y_{22} & T_2 \\ T_1^T & T_2^T & R + Z \end{bmatrix} \geq 0,\end{aligned}$$

$\bar{\Phi}$

$$\begin{aligned}\bar{\Phi} &= \begin{bmatrix} \Phi_{11} & \bar{\Phi}_{12} & -N_1 + M_1 & -T_1 & -M_1 + S_1 \\ \bar{\Phi}_{12}^T & \bar{\Phi}_{22} & -N_2 + M_2 & -T_2 & -M_2 + S_2 \\ -N_1^T + M_1^T & -N_2^T + M_2^T & -Q_2 & 0 & 0 \\ -T_1^T & -T_2^T & 0 & -Q_3 & 0 \\ -M_1^T + S_1^T & -M_2^T + S_2^T & 0 & 0 & -Q_4 \end{bmatrix} \\ &< 0,\end{aligned}\tag{4}$$

where

$$\begin{aligned}\bar{\Phi}_{12} &= N_2^T - S_1 + T_1 + \tau_M X_{12} + \delta_2 Y_{12} \\ &\quad + A^T P A_h + (A - I)^T \Theta A_h, \\ \bar{\Phi}_{22} &= -S_2 - S_2^T + T_2 + T_2^T - Q_1 + \tau_M X_{22} \\ &\quad + \delta_2 Y_{22} + A_h^T P A_h + A_h^T \Theta A_h.\end{aligned}\tag{5}$$

Proof. First, denote $\tau_{\text{avg}} = [(1/2)(\tau_M + \tau_m)]$, that is, the floor function of $(1/2)(\tau_M + \tau_m)$, $\delta_1 = \tau_{\text{avg}} - \tau_m$, and $\delta_2 = \tau_M - \tau_{\text{avg}}$. Let

$$y(l) = x(l+1) - x(l).\tag{6}$$

Then, it is clear that

$$y(k) = x(k+1) - x(k) = (A - I)x(k) + A_h(k - h(k)).\tag{7}$$

We divide the proof into two cases.

Case I. We choose the Lyapunov function candidate

$$V(k) = \sum_{i=1}^5 V_i(k), \quad (8)$$

where

$$V_1(k) = x^T(k) P x(k),$$

$$V_2(k) = \sum_{i=k-h(k)}^{k-1} x^T(i) Q_1 x(i),$$

$$V_3(k) = \sum_{i=k-\tau_m}^{k-1} x^T(i) Q_2 x(i) + \sum_{i=k-\tau_M}^{k-1} x^T(i) Q_3 x(i) + \sum_{i=k-\tau_{\text{avg}}}^{k-1} x^T(i) Q_4 x(i),$$

$$V_4(k) = \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{i=k+j}^{k-1} x^T(i) Q_1 x(i),$$

$$V_5(k) = \sum_{j=-\tau_{\text{avg}}}^{-\tau_m-1} \sum_{i=k+j}^{k-1} y^T(i) R y(i) + \sum_{j=-\tau_M}^{-1} \sum_{i=k+j}^{k-1} y^T(i) Z y(i), \quad (9)$$

and P, Q_1, Q_2, Q_3, Q_4, R , and Z are positive definite matrices to be determined. Define $\Delta V_i(k) = V_i(k+1) - V_i(k)$ which leads to

$$\begin{aligned} \Delta V_1(k) &= x^T(k) A^T P A x(k) + x^T(k) A^T P A_h x(k-h(k)) \\ &\quad + x^T(k-h(k)) A_h^T P A x(k) + x^T(k-h(k)) \\ &\quad \times A_h^T P A_h x(k-h(k)) - x^T(k) P x(k), \end{aligned}$$

$$\begin{aligned} \Delta V_2(k) &= \sum_{i=k+1-h(k+1)}^{k-1} x^T(i) Q_1 x(i) + x^T(k) Q_1 x(k) \\ &\quad - \sum_{i=k+1-h(k)}^{k-1} x^T(i) Q_1 x(i) \\ &\quad - x^T(k-h(k)) Q_1 x(k-h(k)) \\ &\leq \sum_{i=k+1-\tau_m}^{k-1} x^T(i) Q_1 x(i) + \sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i) Q_1 x(i) \\ &\quad + x^T(k) Q_1 x(k) - \sum_{i=k+1-\tau_m}^{k-1} x^T(i) Q_1 x(i) \\ &\quad - x^T(k-h(k)) Q_1 x(k-h(k)) \\ &= \sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i) Q_1 x(i) + x^T(k) Q_1 x(k) \\ &\quad - x^T(k-h(k)) Q_1 x(k-h(k)), \end{aligned}$$

$$\begin{aligned} \Delta V_3(k) &= x^T(k) (Q_2 + Q_3 + Q_4) x(k) - x^T(k-\tau_m) \\ &\quad \times Q_2 x(k-\tau_m) - x^T(k-\tau_M) Q_3 x(k-\tau_M) \\ &\quad - x^T(k-\tau_{\text{avg}}) Q_4 x(k-\tau_{\text{avg}}), \end{aligned}$$

$$\Delta V_4(k) = (\tau_M - \tau_m) x^T(k) Q_1 x(k) - \sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i) Q_1 x(i),$$

$$\begin{aligned} \Delta V_5(k) &= (\tau_{\text{avg}} - \tau_m) y^T(k) R y(k) - \sum_{i=k-\tau_{\text{avg}}}^{k-\tau_m-1} y^T(i) R y(i) \\ &\quad + \tau_M y^T(k) Z y(k) - \sum_{i=k-\tau_M}^{k-1} y^T(i) Z y(i). \end{aligned} \quad (10)$$

From (6), the following zero equations

$$2\zeta_1^T(k) N \left(x(k) - x(k-\tau_m) - \sum_{i=k-\tau_m}^{k-1} y(i) \right) = 0, \quad (11)$$

$$2\zeta_1^T(k) M \left(x(k-\tau_m) - x(k-h(k)) - \sum_{i=k-h(k)}^{k-\tau_m-1} y(i) \right) = 0, \quad (12)$$

$$2\zeta_1^T(k) S \left(x(k-h(k)) - x(k-\tau_{\text{avg}}) - \sum_{i=k-\tau_{\text{avg}}}^{k-h(k)-1} y(i) \right) = 0, \quad (13)$$

$$2\zeta_1^T(k) T \left(x(k-\tau_{\text{avg}}) - x(k-\tau_M) - \sum_{i=k-\tau_M}^{k-\tau_{\text{avg}}-1} y(i) \right) = 0, \quad (14)$$

$$\begin{aligned} &\tau_M \zeta_1^T(k) X \zeta_1(k) - \sum_{i=k-\tau_m}^{k-1} \zeta_1^T(k) X \zeta_1(k) \\ &- \sum_{i=k-h(k)}^{k-\tau_m-1} \zeta_1^T(k) X \zeta_1(k) - \sum_{i=k-\tau_{\text{avg}}}^{k-h(k)-1} \zeta_1^T(k) X \zeta_1(k) \end{aligned} \quad (15)$$

$$\begin{aligned} &- \sum_{i=k-\tau_M}^{k-\tau_{\text{avg}}-1} \zeta_1^T(k) X \zeta_1(k) = 0, \\ &\delta_1 \zeta_1^T(k) Y \zeta_1(k) - \sum_{i=k-h(k)}^{k-\tau_m-1} \zeta_1^T(k) Y \zeta_1(k) \\ &- \sum_{i=k-\tau_{\text{avg}}}^{k-h(k)-1} \zeta_1^T(k) Y \zeta_1(k) = 0 \end{aligned} \quad (16)$$

hold for any appropriate dimensions matrices N , M , S , and T , where $\zeta_1^T(k) = [x^T(k) \ x^T(k-h(k))]$, and semi-positive-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}. \quad (17)$$

Taking the forward difference of $V(k)$ and adding all the terms of (11)–(16) to $\Delta V(k)$ leads to

$$\begin{aligned} \Delta V(k) \leq & \zeta_2^T(k) \Phi \zeta_2(k) - \sum_{i=k-\tau_m}^{k-1} \zeta_3^T(k, i) \Theta_1 \zeta_3(k, i) \\ & - \sum_{i=k-h(k)}^{k-\tau_m-1} \zeta_3^T(k, i) \Theta_2 \zeta_3(k, i) - \sum_{i=k-\tau_{\text{avg}}}^{k-h(k)-1} \zeta_3^T(k, i) \Theta_3 \zeta_3(k, i) \\ & - \sum_{i=k-\tau_M}^{k-\tau_{\text{avg}}-1} \zeta_3^T(k, i) \Theta_4 \zeta_3(k, i), \end{aligned} \quad (18)$$

where $\zeta_2^T(k) = [\zeta_1^T(k) \ x^T(k-\tau_m) \ x^T(k-\tau_M) \ x^T(k-\tau_{\text{avg}})]$ and $\zeta_3(k, i)^T = [\zeta_2^T(k) \ y(i)^T]$. Thus, LMIs (2) assure that $\Delta V(k)$ is negative definite.

Case II. We choose the Lyapunov function candidate

$$V(k) = \sum_{i=1}^5 \bar{V}_i(k), \quad (19)$$

where the $\bar{V}_i(k) = V_i(k)$ for $i = 1, 2, 3, 4$ (are same as Case I) except for $i = 5$ where

$$\bar{V}_5(k) = \sum_{j=-\tau_M}^{-\tau_{\text{avg}}-1} \sum_{i=k+j}^{k-1} y^T(i) R y(i) + \sum_{j=-\tau_M}^{-1} \sum_{i=k+j}^{k-1} y^T(i) Z y(i), \quad (20)$$

which yields

$$\begin{aligned} \Delta \bar{V}_5(k) = & (\tau_M - \tau_{\text{avg}}) y^T(k) R y(k) - \sum_{i=k-\tau_M}^{k-\tau_{\text{avg}}-1} y^T(i) R y(i) \\ & + \tau_M y^T(k) Z y(k) - \sum_{i=k-\tau_M}^{k-1} y^T(i) Z y(i). \end{aligned} \quad (21)$$

Similar to (11)–(16), (11) and the following zero equations

$$\begin{aligned} & 2\zeta_1^T(k) M \left(x(k-\tau_m) - x(k-\tau_{\text{avg}}) - \sum_{i=k-\tau_{\text{avg}}}^{k-\tau_m-1} y(i) \right) = 0, \\ & 2\zeta_1^T(k) S \left(x(k-\tau_{\text{avg}}) - x(k-h(k)) - \sum_{i=k-h(k)}^{k-\tau_{\text{avg}}-1} y(i) \right) = 0, \\ & 2\zeta_1^T(k) T \left(x(k-h(k)) - x(k-\tau_M) - \sum_{i=k-\tau_M}^{k-h(k)-1} y(i) \right) = 0, \\ & \tau_M \zeta_1^T(k) X \zeta_1(k) - \sum_{i=k-\tau_m}^{k-1} \zeta_1^T(k) X \zeta_1(k) \\ & - \sum_{i=k-\tau_{\text{avg}}}^{k-\tau_m-1} \zeta_1^T(k) X \zeta_1(k) - \sum_{i=k-h(k)}^{k-\tau_{\text{avg}}-1} \zeta_1^T(k) X \zeta_1(k) \\ & - \sum_{i=k-\tau_M}^{k-h(k)-1} \zeta_1^T(k) X \zeta_1(k) = 0, \\ & \delta_2 \zeta_1^T(k) Y \zeta_1(k) - \sum_{i=k-h(k)}^{k-\tau_{\text{avg}}-1} \zeta_1^T(k) Y \zeta_1(k) \\ & - \sum_{i=k-\tau_M}^{k-h(k)-1} \zeta_1^T(k) Y \zeta_1(k) = 0. \end{aligned} \quad (22)$$

Taking the forward difference of $V(k)$ and adding all the terms of (11) and (22) to $\Delta V(k)$ allow us to write $\Delta V(k)$ as

$$\begin{aligned} \Delta V(k) \leq & \zeta_2^T(k) \bar{\Phi} \zeta_2(k) - \sum_{i=k-\tau_m}^{k-1} \zeta_3^T(k, i) \bar{\Theta}_1 \zeta_3(k, i) \\ & - \sum_{i=k-\tau_{\text{avg}}}^{k-\tau_m-1} \zeta_3^T(k, i) \bar{\Theta}_2 \zeta_3(k, i) \\ & - \sum_{i=k-h(k)}^{k-\tau_{\text{avg}}-1} \zeta_3^T(k, i) \bar{\Theta}_3 \zeta_3(k, i) \\ & - \sum_{i=k-\tau_M}^{k-h(k)-1} \zeta_3^T(k, i) \bar{\Theta}_4 \zeta_3(k, i). \end{aligned} \quad (23)$$

Thus, LMIs (4) assure that $\Delta V(k)$ is negative definite. $\square$

Remark 2. The main goal of the partitioning technique is to divide the summation interval $[0, \tau_M]$ of the Lyapunov function difference to as many subintervals (e.g., $[0, \tau_m]$, $[\tau_m, \tau_{\text{avg}}]$, $[\tau_{\text{avg}}, h(k)]$, $[h(k), \tau_M]$, $[0, \tau_m]$, $[\tau_m, h(k)]$, $[h(k), \tau_{\text{avg}}]$, and $[\tau_{\text{avg}}, \tau_M]$ in the analysis) as possible. Using this technique along with the zero equations (11)–(16) and (22), we introduce free weighting matrices which increase feasibility of the resulting LMIs.

TABLE 1: Maximum allowable upper bound τ_M for various τ_m .

		(a)			
Methods		$\tau_m = 6$	$\tau_m = 7$	$\tau_m = 10$	$\tau_m = 15$
Shao and Han [14] Proposition 2	τ_M	16	16	18	21
Shao and Han [14] Proposition 1	τ_M	18	18	20	23
Theorem 1	τ_M	22	23	24	27
		(b)			
Methods		$\tau_m = 20$	$\tau_m = 25$	$\tau_m = 30$	
Shao and Han [14] Proposition 2	τ_M	25	30	34	
Shao and Han [14] Proposition 1	τ_M	27	31	35	
Theorem 1	τ_M	31	35	38	

3. Numerical Simulations

In this section, two examples are used to demonstrate the effectiveness of theorem.

Example 1. Consider the time-delay system with the following matrices:

$$A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_h = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}. \quad (24)$$

Comparing the results of this work to [14], the maximum allowable upper bound (MAUB) of τ_M with various τ_m is presented in Table 1, which shows that our conditions give significantly better results.

Example 2. Consider the time-delay system with the following matrices:

$$A = \begin{bmatrix} 0.6 & 0 \\ 0.35 & 0.7 \end{bmatrix}, \quad A_h = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}. \quad (25)$$

This example is discussed in [15], which assumes that $\tau_m = 2$ and $\tau_M = 10$. For this example and by Theorem 1 in this paper, the MAUB $\tau_M = 34$.

4. Conclusion

In this work, we have presented an exploitation of the partitioning oriented technique to deal with the stability problem of discrete-time systems with an interval time-varying delay. In the analysis, a new Lyapunov function was defined and some conventional inequalities were avoided resulting in relaxed stability criteria formulated into LMIs. A benchmark numerical example was given and results reflect the criteria which show significant improvement over existing results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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