Research Article

Output Feedback Adaptive Dynamic Surface Control of Permanent Magnet Synchronous Motor with Uncertain Time Delays via RBFNN

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This paper focuses on an adaptive dynamic surface control based on the Radial Basis Function Neural Network for a fourth-order permanent magnet synchronous motor system wherein the unknown parameters, disturbances, chaos, and uncertain time delays are presented. Neural Network systems are used to approximate the nonlinearities and an adaptive law is employed to estimate accurate parameters. Then, a simple and effective controller has been obtained by introducing dynamic surface control technique on the basis of first-order filters. Asymptotically tracking stability in the sense of uniformly ultimate boundedness is achieved in a short time. Finally, the performance of the proposed control has been illustrated through simulation results.

1. Introduction

Recently, the permanent magnet synchronous motor (PMSM) is the most widely used driven mechanism because of the advantageous merits of cost, reliability, and performances. The PMSM is characterized by complexity, high nonlinearity, time-varying dynamics, inaccessibility of some states, and output for measurements; hence, it can be considered as a challenging engineering problem [1, 2]. It is found that the PMSM is experiencing chaotic behavior at specific parameters and working conditions [3, 4]. Then, the intermittent oscillation of torque and rotational speed, irregular current noise of the system, and unstable control performance appear in the PMSM, which seriously affect the stability and safety. Thus, it is difficult to accomplish the high-performance control of PMSM by using classic PID-type control methods.

A neuron-fuzzy controller (NFC) [5] is suitable for control of systems with uncertainties and nonlinearities. The NFC approach can also achieve self-learning; however, it is unsuitable for online learning real-time control due to the drawback of time consuming [6, 7]. The sliding mode control (SMC) [8] can guarantee the robustness only under the

bounds of the uncertainties and it has a shortage named chattering. The terminal sliding mode control (TSMC) method can assure convergence to the origin in finite time. Hence, the TSMC is successfully applied to PMSM driver system to improve control performance [9]. A position tracking control method via adaptive fuzzy backstepping is presented for the induction motors with unknown parameters [10]. Unfortunately, the traditional backstepping suffers from the “explosion of complexity” caused by the repeated differentiation of virtual control functions [11]. In order to overcome the above shortcomings, a backstepping approach combined with SMC technique is presented to realize synchronization of uncertain fractional-order strict-feedback chaotic system [12].

The dynamic surface control (DSC) developed by Swaroop et al. [13] is a control technique by introducing a first-order filter at each recursive step of the backstepping design procedure, so the differentiation items on the virtual function can be avoided. By incorporating DSC into a neural network-based adaptive control design framework, Wang and Huang [14] proposed a backstepping-based control design for a class of nonlinear systems in strict-feedback form with arbitrary uncertainty. Zhang and Ge [15] further studied the control design for some special nonlinear systems

FIGURE 1: The principle block diagram of PMSM system based on vector control.

with unknown dead-zone using the DSC technique. To control the undesirable chaos in PMSM, an adaptive DSC controller was designed by introducing first-order low-pass filters [16], whereas in [17] DSC method was extended to time-delay uncertain nonlinear systems in parametric strict-feedback form. The new developments referring to DSC for different nonlinear systems and the applications to various engineering fields can be found in [18–20].

In this paper, motivated by the previous studies reported in the literature, the following problems will be addressed. An adaptive DSC based on the Radial Basis Function Neural Network (RBFNN) is discussed for the chaotic PMSM system. The whole process of the system design is performed in a step-by-step manner. For each step, a first-order filter is used to gain the information of derivative of the virtual control function and RBFNN is adopted to approximate the nonlinearity. The uncertain parameters in the system are updated by adaptive laws, which realize accurate parameters estimate. Then, an adaptive virtual control law is designed to stabilize each subsystem in the former steps. Finally, an adaptive DSC is employed via RBFNN in the last step to stabilize the whole system. Further contributions include the design of controller to handle uncertain time delays and disturbances.

2. Dynamics of the PMSM System

The model of a PMSM can be described in the well-known (d - q) frame through the Park transformation as follows. The stator d , q equations in the rotor frame are expressed as follows [21, 22]:

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{3}{2J} n_p [(L_d - L_q) i_d i_q + \Phi i_q] - \frac{B}{J} \omega - \frac{T_L}{J}, \\ \frac{di_d}{dt} &= -\frac{R_s}{L_d} i_d + \frac{n_p L_q}{L_d} \omega i_q + \frac{1}{L_d} u_d, \\ \frac{di_q}{dt} &= -\frac{R_s}{L_q} i_q - \frac{n_p L_d}{L_q} \omega i_d - \frac{n_p \Phi}{L_q} \omega + \frac{1}{L_q} u_q, \\ \frac{d\theta}{dt} &= \omega, \end{aligned} \quad (1)$$

where u_d and u_q are the d - q axis voltages, which denote the system control input. i_d , i_q , θ , and ω are state variables, which stand for the d - q axis currents, the rotor position, and rotor angular velocity, respectively. n_p , J , and B mean the pole pair, moment of inertia, and viscous friction coefficient. L_q and L_r denote the inductance of the stator in the (d - q) frame. T_L , R_s , and Φ stand for the load torque, stator resistance, and magnet flux linkage of inertia.

The general schematic of the PMSM is depicted in Figure 1. The overall system consists of a PMSM with load, space vector pulse width modulation (SVPWM), field-orientation mechanism, voltage-source inverter (VSI), and three controllers. The controllers employ a structure of cascade control loop including a speed loop and two current loops. PI controller, which is used to stabilize the d -axis current errors of the vector controlled drive, is adopted in the d -axis current loop. From Figure 1, the rotor angular velocity ω can be obtained from the position and speed sensor. The currents i_d and i_q can be calculated from i_a and i_b by Clarke and Park transformations. However, in real applications, this method suffers from serious problems, including internal disturbance such as torque ripples, parameters variation, friction forces, and unmodeled dynamics and external disturbance such as load disturbance. Thus, an adaptive dynamic surface controller via RBFNN is developed to solve these problems which can degrade the performance of closed-loop system if the controller does not have enough ability to reject them.

Equation (1) are rewritten as follows:

$$\begin{aligned} \frac{d\bar{\omega}}{dt} &= \sigma(\bar{i}_q - \bar{\omega}) - \bar{T}_L, \\ \frac{d\bar{i}_q}{dt} &= -\bar{i}_q - \bar{i}_d \bar{\omega} + \gamma \bar{\omega} + \bar{u}_q, \\ \frac{d\bar{i}_d}{dt} &= -\bar{i}_d + \bar{i}_q \bar{\omega} + \bar{u}_d, \\ \frac{d\bar{\theta}}{dt} &= \bar{\omega}, \end{aligned} \quad (2)$$

where $\gamma = -\Phi/(\kappa L)$, $\sigma = B\tau/J$, $\bar{T}_L = \tau^2 T_L/J$, $\bar{u}_q = u_q/(\kappa R)$, $\bar{u}_d = u_d/(\kappa R)$, $\tau = L/R$, $\kappa = B/(n_p \tau \Phi)$, $\bar{\theta} = \theta$, $\bar{\omega} = \omega\tau$, $\bar{i}_d = i_d/\kappa$, $\bar{i}_q = i_q/\kappa$, $\bar{t} = t/\tau$, $L = L_d = L_q$.

The chaotic behavior is demonstrated in PMSM system when its parameters fall into a certain area. Figures 2 and 3 show the strange attractor and the chaotic time series of the PMSM under the condition of $\sigma = 5.46$, $\gamma = 20$, $\bar{u}_q = \bar{u}_d = 0$, $\bar{T}_L = 0$, $\bar{\theta}(0) = -1$, $\bar{\omega}(0) = -5$, $\bar{i}_q(0) = 0.01$, and $\bar{i}_d(0) = 20$, in which the PMSM appears an aperiodic, random, sudden, or intermittent morbid oscillation.

For simplicity, the following notations are introduced:

$$\begin{aligned} x_1 &= \theta, & x_2 &= \omega, & x_3 &= i_q, & x_4 &= i_d, \\ a_1 &= \frac{3}{2}n_p\Phi, & a_2 &= \frac{3}{2}n_p(L_d - L_q), \\ b_1 &= -\frac{R_s}{L_q}, & b_2 &= -\frac{n_p L_d}{L_q}, \\ b_3 &= -\frac{n_p \Phi}{L_q}, & b_4 &= \frac{1}{L_q}, \\ c_1 &= -\frac{R_s}{L_d}, & c_2 &= \frac{n_p L_q}{L_d}, & c_3 &= \frac{1}{L_d}. \end{aligned} \quad (3)$$

By using these notations, the dynamic model of PMSM with uncertain time delays and disturbance can be described by the following differential equations:

$$\begin{aligned} \dot{x}_1 &= x_2 + \Delta f_1(\bar{x}_1(t - \tau_1)) + d_1, \\ \dot{x}_2 &= \frac{a_1}{J}x_3 + \frac{a_2}{J}x_3x_4 - \frac{B}{J}x_2 - \frac{T_L}{J} + \Delta f_2(\bar{x}_2(t - \tau_2)) + d_2, \\ \dot{x}_3 &= b_1x_3 + b_2x_2x_4 + b_3x_2 + b_4u_q + \Delta f_3(\bar{x}_3(t - \tau_3)) + d_3, \\ \dot{x}_4 &= c_1x_4 + c_2x_2x_3 + c_3u_d + \Delta f_4(x_4(t - \tau_4)) + d_4, \end{aligned} \quad (4)$$

where $\bar{x}_i(\cdot) = [x_1, \dots, x_i]^T$, $i = 1, 2, 3$, $\Delta f_i(\cdot)$, $i = 1, \dots, 4$, denote uncertain nonlinear function owing to the effect of time delay, τ_i , $i = 1, \dots, 4$, represent time-delay constants, and d_i , $i = 1, \dots, 4$, mean the disturbances.

Define the tracking errors

$$\begin{aligned} S_1(t) &= x_1(t) - y_r, \\ S_i(t) &= x_i(t) - \alpha_{if}, \quad i = 2, 3 \\ S_4(t) &= x_4(t), \end{aligned} \quad (5)$$

where α_{if} , $i = 2, 3$, will be given later by the first-order filter and y_r is given continuous tracking signal.

FIGURE 2: The strange attractor of the chaotic PMSM.

Then, for the uncertain nonlinear time-delay functions, the following inequalities can be satisfied:

$$|\Delta f_i(\bar{x}_i(t - \tau_i))| \leq \sum_{j=1}^i |S_j(t - \tau_i)| q_{ij}(\bar{S}_i(t - \tau_i)), \quad i = 1, \dots, 3, \quad (6)$$

$$|\Delta f_4(x_4(t - \tau_4))| \leq |S_4(t - \tau_4)| q_{44}(S_4(t - \tau_4)),$$

where q_{ij} and q_{44} are known continuous functions, $\bar{S}_i(\cdot) = [S_1, \dots, S_i]^T$.

In order to facilitate the design, the following assumption is given.

Assumption 1. $|g_i| \leq g_{Mi}$, $i = 1, \dots, 3$, the desired trajectory y_r is continuous, and its first-order derivative $\dot{y}_r$ and second-order derivative $\ddot{y}_r$ are bounded and available.

In this paper, the control objective is to force the tracking error $S_1(t)$ asymptotical stable in the sense of uniformly ultimate boundedness, which also ensures that the system state $x_1(t)$ can track a reference trajectory y_r .

3. Adaptive Dynamic Surface Controller Based on RBFNN

3.1. RBFNN System. The type of RBFNN shown in Figure 4 is considered as a two-layer network, which contains a hidden layer and an output layer. In this paper, the RBF neural network will be used to approximate the unknown continuous function $f(z) : R^n \rightarrow R$ as

$$\hat{f}(z) = \theta^{*T} \xi(z), \quad (7)$$

where $z \in \Omega \subset R^n$ is the input vector with n being the neural network input dimension, $\theta^* = [\theta_1^*, \theta_2^*, \dots, \theta_l^*]^T \in R^l$ is the weight vector, $l > 1$ is the node number of neuron, and $\xi(z) = [\xi_1(z), \xi_2(z), \dots, \xi_n(z)]^T \in R^l$ is a basic function vector with $\xi_i(z)$ chosen as the commonly used Gaussian function in the following form:

$$\xi_i(z) = \exp \left[-\frac{|z - \mu_i|^2}{2\sigma_i^2} \right] = \exp \left[-\frac{(z - \mu_i)^T (z - \mu_i)}{2\sigma_i^2} \right], \quad i = 1, 2, \dots, l, \quad (8)$$

FIGURE 3: The chaotic time series of PMSM.

where $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{in}]^T$ is the center of the receptive field and σ_i is the width of $\xi_i(z)$.

For given scalar $\varepsilon > 0$, by choosing sufficiently large l , the RBFNN can approximate any continuous function over a compact set $\Omega \in R^n$ to arbitrary accuracy as

$$f(z) = \theta^T \xi(z) + \delta(z), \quad \forall z \in \Omega \in R^n, \quad (9)$$

where $\delta(z)$ is the approximation error, satisfying $|\delta(z)| < \varepsilon$, and θ is an unknown ideal constant weight vector, which is an artificial quantity required for analytical purpose. Typically, θ is chosen as the value of θ^* that minimizes $|\delta(z)|$ for all $z \in \Omega$; that is,

$$\theta = \arg \min_{\theta^* \in R^n} \left\{ \sup_{z \in \Omega} |f(z) - \theta^{*T} \xi(z)| \right\}. \quad (10)$$

3.2. Controller Design. In this section, an adaptive dynamic surface control approach of the permanent magnet synchronous motor based on RBF neural network will be developed. The DSC technique based on first-order filters is used to design the controller instead of the traditional backstepping method, and in sequence the explosion of terms problem is avoided.

In order to design dynamic surface controller, the boundary layer errors are defined as follows:

$$y_i = \alpha_{if} - \alpha_i, \quad i = 2, 3, \quad (11)$$

where virtual control input α_i , $i = 2, 3$, will be designed in the subsequent content.

FIGURE 4: The structure of the RBFNN.

The designing process is as follows.

Step 1. Consider the tracking error $S_1 = x_1 - y_r$; the time derivative of S_1 is given by

$$\dot{S}_1 = x_2 + \Delta f_1(\bar{x}_1(t - \tau_1)) + d_1 - \dot{y}_r. \quad (12)$$

Choose Lyapunov-Krasovskii functional as

$$V_1 = \frac{1}{2} S_1^2(t) + \sum_{j=1}^3 \int_{t-\tau_j}^t S_1^2(\tau) q_{j1}^2(\bar{S}_1(\tau)) d\tau. \quad (13)$$

The derivative of V_1 is

$$\begin{aligned} \dot{V}_1 = & [S_2 + y_2 + \alpha_2 + d_1 - \dot{y}_r + \Delta f_1(\bar{x}_1(t - \tau_1))] S_1 \\ & + \sum_{j=1}^3 S_1^2(t) q_{j1}^2(\bar{S}_1(t)) - \sum_{j=1}^3 S_1^2(t - \tau_j) q_{j1}^2(\bar{S}_1(t - \tau_j)). \end{aligned} \quad (14)$$

Using Young's inequality and inequality (6), the following inequalities can be obtained:

$$S_1 y_2 \leq \frac{1}{2} S_1^2 + \frac{1}{2} y_2^2,$$

$$S_1 \Delta f_1(\bar{x}_1(t - \tau_1)) \leq \frac{1}{4} S_1^2 + S_1^2(t - \tau_1) q_{11}^2(\bar{S}_1(t - \tau_1)). \quad (15)$$

Thus, it follows immediately from substituting (15) into (14) that

$$\begin{aligned} \dot{V}_1 \leq & \left[S_2 + \alpha_2 + \frac{3}{4} S_1 - \dot{y}_r + \sum_{j=1}^3 S_1(t) q_{j1}^2(\bar{S}_1(t)) \right] S_1 \\ & + \frac{1}{2} y_2^2 + D_1, \end{aligned} \quad (16)$$

where $D_1 = S_1^2(t - \tau_1) q_{11}^2(\bar{S}_1(t - \tau_1)) - \sum_{j=1}^3 S_1^2(t - \tau_j) q_{j1}^2(\bar{S}_1(t - \tau_j))$.

The virtual control function α_2 is constructed as

$$\alpha_2 = -k_1 S_1 - \frac{3}{4} S_1 + \dot{y}_r - \sum_{j=1}^3 S_1(t) q_{j1}^2(\bar{S}_1(t)), \quad (17)$$

where design parameter $k_1 > 0$.

It is obtained that

$$\dot{V}_1 \leq -k_1 S_1^2 + S_2 S_1 + \frac{1}{2} y_2^2 + D_1. \quad (18)$$

Step 2. Let α_2 be passed through a first-order filter as follows:

$$\tau_2 \dot{\alpha}_{2f} + \alpha_{2f} = \alpha_2, \quad \alpha_{2f}(0) = \alpha_2(0), \quad (19)$$

where τ_2 is a time constant.

With (5) and (19), one has

$$\dot{\alpha}_{2f} = -\frac{y_2}{\tau_2}. \quad (20)$$

The derivative of y_2 is

$$\begin{aligned} \dot{y}_2 = & \dot{\alpha}_{2f} - \dot{\alpha}_2 = -\frac{y_2}{\tau_2} \\ & - \left(-k_1 \dot{S}_1 - \frac{3}{4} \dot{S}_1 + \ddot{y}_r - \sum_{j=1}^3 \dot{S}_1(t) q_{j1}^2(\bar{S}_1(t)) \right. \\ & \left. - \sum_{j=1}^3 S_1(t) \dot{q}_{j1}^2(\bar{S}_1(t)) \right). \end{aligned} \quad (21)$$

It is obtained that

$$\left| \dot{y}_2 + \frac{y_2}{\tau_2} \right| \leq B_2(S_1, S_2, y_2, q_{j1}, y_r, \dot{y}_r, \ddot{y}_r), \quad j = 1, \dots, 3, \quad (22)$$

where $B_2(S_1, S_2, y_2, q_{j1}, y_r, \dot{y}_r, \ddot{y}_r)$, $j = 1, \dots, 3$, is the continuous function.

Using Young's inequality, one has

$$y_2 \dot{y}_2 \leq -\frac{y_2^2}{\tau_2} + y_2^2 + \frac{1}{4} B_2^2. \quad (23)$$

Differentiating S_2 gives

$$\begin{aligned} \dot{S}_2 = & \frac{a_1}{J} x_3 + \frac{a_2}{J} x_3 x_4 - \frac{B}{J} x_2 - \frac{T_L}{J} \\ & + \Delta f_2(\bar{x}_2(t - \tau_2)) + d_2 - \dot{\alpha}_{2f} \end{aligned} \quad (24)$$

$$= g_1 x_3 + f_1 + \Delta f_2(\bar{x}_2(t - \tau_2)) + d_2 - \dot{\alpha}_{2f},$$

where $f_1 = (a_2/J)x_3x_4 - (B/J)x_2 - T_L/J$, $g_1 = a_1/J$.

In the realistic model of PMSM, limited to the working conditions, the parameter g_1 is unknown. So it cannot be used to construct the control signal. Thus, let $\hat{g}_1$ be the estimation of g_1 . Meanwhile, notice that the nonlinear function f_1 contains uncertain parameters, such as a_2 , B , J , and T_L . This will make the traditional adaptive DSC design very troublesome. To avoid this trouble and simplify the control structure, the RBFNN will be employed to approximate f_1 .

According to the description above, for any given $\varepsilon_1 > 0$, there exists a RBFNN system $\theta_1^T \xi_1$ such that

$$f_1 = \theta_1^T \xi_1 + \delta_1, \quad (25)$$

where δ_1 is the approximation error and satisfies $|\delta_1| < \varepsilon_1$.

With (24) and (25), one has

$$\dot{S}_2 = g_1 x_3 + \theta_1^T \xi_1 + \delta_1 + \Delta f_2(\bar{x}_2(t - \tau_2)) + d_2 - \dot{\alpha}_{2f}. \quad (26)$$

Consider the Lyapunov-Krasovskii functional candidate V_2 as

$$\begin{aligned} V_2 = & V_1 + \frac{1}{2} S_2^2 + \frac{1}{2} y_2^2 + \frac{1}{2\gamma_1} (\hat{\theta}_1 - \theta_1)^T (\hat{\theta}_1 - \theta_1) \\ & + \frac{1}{2\Gamma_1} (\hat{g}_1 - g_1)^2 + \sum_{j=2}^3 \int_{t-\tau_j}^t S_2^2(\tau) q_{j2}^2(\bar{S}_2(\tau)) d\tau. \end{aligned} \quad (27)$$

Then, the derivative of V_2 is given by

$$\begin{aligned} \dot{V}_2 \leq & \dot{V}_1 + S_2 [g_1 (S_3 + y_3 + \alpha_3) + \theta_1^T \xi_1 + \delta_1 + d_2 - \dot{\alpha}_{2f}] \\ & + S_2 \Delta f_2(\bar{x}_2(t - \tau_2)) + \left(1 - \frac{1}{\tau_2} \right) y_2^2 + \frac{1}{4} B_2^2 + \frac{1}{\gamma_1} \\ & \times (\hat{\theta}_1 - \theta_1)^T \hat{\theta}_1 + \frac{1}{\Gamma_1} (\hat{g}_1 - g_1) \dot{\hat{g}}_1 \\ & + \sum_{j=2}^3 S_2^2(t) q_{j2}^2(\bar{S}_2(t)) - \sum_{j=2}^3 S_2^2(t - \tau_j) q_{j2}^2 \\ & \times (\bar{S}_2(t - \tau_j)). \end{aligned} \quad (28)$$

Using Young's inequality and inequality Assumption 1, the following inequalities can be obtained:

$$S_2 y_3 \leq \frac{1}{2} S_2^2 + \frac{1}{2} y_3^2,$$

$$S_2 \Delta f_2 (\bar{x}_2(t - \tau_2)) \leq \frac{1}{4} S_2^2 + \sum_{j=1}^2 S_j^2(t - \tau_2) q_{2j}^2(\bar{S}_j(t - \tau_2)). \quad (29)$$

Substituting (29) into (28) gives

$$\begin{aligned} \dot{V}_2 \leq S_2 & \left[g_1(S_3 + \alpha_3) + \frac{1}{2} S_2 g_1 + S_1 + \theta_1^T \xi_1 + \delta_1 \right. \\ & \left. + d_2 - \dot{\alpha}_{2f} + \frac{1}{4} S_2 + \sum_{j=2}^3 S_2(t) q_{j2}^2(\bar{S}_2(t)) \right] \\ & - k_1 S_1^2 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 + \left(1 - \frac{1}{\tau_2}\right) y_2^2 + \frac{1}{4} B_2^2 \\ & + \frac{1}{\gamma_1} (\hat{\theta}_1 - \theta_1)^T \dot{\hat{\theta}}_1 + \frac{1}{\Gamma_1} (\hat{g}_1 - g_1) \dot{\hat{g}}_1 + \sum_{j=1}^2 D_j, \end{aligned} \quad (30)$$

where $D_2 = \sum_{j=1}^2 S_j^2(t - \tau_2) q_{2j}^2(\bar{S}_j(t - \tau_2)) - \sum_{j=2}^3 S_2^2(t - \tau_j) q_{j2}^2(\bar{S}_2(t - \tau_j))$.

Choose the virtual control function α_3 and the corresponding parameters adaptive laws as

$$\begin{aligned} \alpha_3 = & \frac{\hat{g}_1}{\hat{g}_1^2 + \eta_1} \left[- \left(k_2 + \frac{1}{2} \hat{g}_1 + \frac{1}{4} \right) S_2 - S_1 - \hat{\theta}_1^T \xi_1 + \dot{\alpha}_{2f} \right. \\ & \left. - \sum_{j=2}^3 S_2(t) q_{j2}^2(\bar{S}_2(t)) \right], \end{aligned} \quad (31)$$

$$\dot{\hat{\theta}}_1 = \gamma_1 (\xi_1 S_2 - m_1 \hat{\theta}_1), \quad (32)$$

$$\dot{\hat{g}}_1 = \Gamma_1 (S_2 \alpha_3 - c_1 \hat{g}_1), \quad (33)$$

where $k_2 > 0$, $\gamma_1 > 0$, $m_1 > 0$, $\Gamma_1 > 0$, $c_1 > 0$, and $\eta_1 > 0$ are design parameters.

Substituting (31)–(33) into (30), (30) becomes

$$\begin{aligned} \dot{V}_2 \leq & - \frac{\eta_1}{\hat{g}_1^2 + \eta_1} \left[- \left(k_2 + \frac{1}{2} \hat{g}_1 + \frac{1}{4} \right) S_2 - S_1 - \hat{\theta}_1^T \xi_1 + \dot{\alpha}_{2f} \right. \\ & \left. - \sum_{j=2}^3 S_2(t) q_{j2}^2(\bar{S}_2(t)) \right] S_2 (\delta_1 + d_2) S_2 \end{aligned}$$

$$\begin{aligned} & + g_1 S_2 S_3 - \hat{\theta}_1^T \xi_1 S_2 - \hat{g}_1 S_2 \alpha_3 - \sum_{j=1}^2 k_j S_j^2 - \frac{1}{2} S_2^2 \hat{g}_1 + \frac{1}{2} y_2^2 \\ & + \frac{1}{2} g_1 y_3^2 + \left(1 - \frac{1}{\tau_2}\right) y_2^2 + \frac{1}{4} B_2^2 + (\hat{\theta}_1 - \theta_1)^T \\ & \times (\xi_1 S_2 - m_1 \hat{\theta}_1) + (\hat{g}_1 - g_1) (S_2 \alpha_3 - c_1 \hat{g}_1) + \sum_{j=1}^2 D_j \\ & \leq \frac{1}{4} \vartheta_1^2 + S_2^2 + g_1 S_2 S_3 - \sum_{j=1}^2 k_j S_j^2 - \frac{1}{2} S_2^2 \hat{g}_1 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 \\ & + \left(1 - \frac{1}{\tau_2}\right) y_2^2 + \frac{1}{4} B_2^2 - m_1 \hat{\theta}_1^T \dot{\hat{\theta}}_1 - c_1 \hat{g}_1 \dot{\hat{g}}_1 + \sum_{j=1}^2 D_j, \end{aligned} \quad (34)$$

where $\vartheta_1 \geq |\varepsilon_1 + d_2 - (\eta_1 / (\hat{g}_1^2 + \eta_1))[-(k_2 + (1/2)\hat{g}_1 + 1/4)S_2 - S_1 - \hat{\theta}_1^T \xi_1 + \dot{\alpha}_{2f} - \sum_{j=2}^3 S_2(t) q_{j2}^2(\bar{S}_2(t))]|$ is a continuous function.

Step 3. Filter α_3 through the following first-order filter

$$\tau_3 \dot{\alpha}_{3f} + \alpha_{3f} = \alpha_3, \quad \alpha_{3f}(0) = \alpha_3(0), \quad (35)$$

where τ_3 is a time constant.

With (5) and (35), one has

$$\dot{\alpha}_{3f} = -\frac{y_3}{\tau_3}. \quad (36)$$

The derivative of y_3 is

$$\begin{aligned} \dot{y}_3 = & \dot{\alpha}_{3f} - \dot{\alpha}_3 \\ = & -\frac{y_3}{\tau_3} - \left\{ \frac{\hat{g}_1}{\hat{g}_1^2 + \eta_1} \left[- \left(k_2 + \frac{1}{2} \hat{g}_1 + \frac{1}{4} \right) \dot{S}_2 - \dot{S}_1 \right. \right. \\ & - \dot{\hat{\theta}}_1^T \xi_1 - \hat{\theta}_1^T \frac{\partial \xi_1}{\partial x_3} \dot{x}_3 + \dot{\alpha}_{2f} \\ & - \sum_{j=2}^3 \dot{S}_2(t) q_{j2}^2(\bar{S}_2(t)) \\ & \left. \left. - \sum_{j=2}^3 S_2(t) q_{j2}^2(\bar{S}_2(t)) \right] \right. \\ & - \frac{(\eta_1 - \hat{g}_1^2) \dot{\hat{g}}_1}{(\hat{g}_1^2 + \eta_1)^2} \\ & \left. \times \left[- \left(k_2 + \frac{1}{2} \hat{g}_1 + \frac{1}{4} \right) S_2 - S_1 - \hat{\theta}_1^T \xi_1 + \dot{\alpha}_{2f} \right. \right. \\ & \left. \left. - \sum_{j=2}^3 S_2(t) q_{j2}^2(\bar{S}_2(t)) \right] \right\}. \end{aligned} \quad (37)$$

It is obtained that

$$\left| \dot{y}_3 + \frac{y_3}{\tau_3} \right| \leq B_3 (S_1, \dots, S_3, y_2, y_2, q_{11}, \dots, q_{31}, q_{22}, q_{32}, \hat{\theta}_1, \hat{g}_1, y_r, \dot{y}_r, \ddot{y}_r), \quad (38)$$

where $B_3(S_1, \dots, S_3, y_2, y_2, q_{11}, \dots, q_{31}, q_{22}, q_{32}, \hat{\theta}_1, \hat{g}_1, y_r, \dot{y}_r, \ddot{y}_r)$ is the continuous function.

Using Young's inequality, one has

$$y_3 \dot{y}_3 \leq -\frac{y_3^2}{\tau_3} + y_3^2 + \frac{1}{4} B_3^2. \quad (39)$$

Differentiating S_3 results in the following equation:

$$\dot{S}_3 = f_2 + b_4 u_q + \Delta f_3 (\bar{x}_3 (t - \tau_3)) + d_3 - \dot{\alpha}_{3f}, \quad (40)$$

where $f_2 = b_1 x_3 + b_2 x_2 x_4 + b_3 x_2$.

Notice that the unknown parameters, such as b_1 , b_2 , and b_3 , appear in the expression of f_2 . To facilitate the design of controller, the RBFNN will be employed to approximate the nonlinear function f_2 .

According to (9), for any given $\varepsilon_2 > 0$, there exists a neural network $\theta_2^T \xi_2$ such that

$$f_2 = \theta_2^T \xi_2 + \delta_2, \quad (41)$$

where δ_2 is the approximation error and satisfies $|\delta_2| \leq \varepsilon_2$.

Thus, (40) is rewritten as follows:

$$\dot{S}_3 = \theta_2^T \xi_2 + \delta_2 + b_4 u_q + \Delta f_3 (\bar{x}_3 (t - \tau_3)) + d_3 - \dot{\alpha}_{3f}. \quad (42)$$

Define Lyapunov-Krasovskii functional

$$V_3 = V_2 + \frac{1}{2} S_3^2 + \frac{1}{2} y_3^2 + \frac{1}{2\gamma_2} (\hat{\theta}_2 - \theta_2)^T (\hat{\theta}_2 - \theta_2) + \frac{1}{2\Gamma_2} (\hat{g}_2 - g_2)^2 + \int_{t-\tau_3}^t S_3^2(\tau) q_{33}^2(\bar{S}_3(\tau)) d\tau. \quad (43)$$

Then, the derivative of V_3 is computed by

$$\begin{aligned} \dot{V}_3 \leq & \dot{V}_2 + S_3 [g_2 u_q + \theta_2^T \xi_2 + \delta_2 + d_2 - \dot{\alpha}_{3f}] \\ & + S_3 \Delta f_3 (\bar{x}_3 (t - \tau_3)) + \left(1 - \frac{1}{\tau_3}\right) y_3^2 + \frac{1}{4} B_3^2 \\ & + \frac{1}{\gamma_2} (\hat{\theta}_2 - \theta_2)^T \dot{\hat{\theta}}_2 + \frac{1}{\Gamma_2} (\hat{g}_2 - g_2) \dot{\hat{g}}_2 \\ & + S_3^2(t) q_{33}^2(\bar{S}_3(t)) - S_3^2(t - \tau_3) q_{33}^2(\bar{S}_3(t - \tau_3)), \end{aligned} \quad (44)$$

where $g_2 = b_4$.

Using Young's inequality and inequality (6), one has

$$S_3 \Delta f_3 (\bar{x}_3 (t - \tau_3)) \leq \frac{1}{4} S_3^2 + \sum_{j=1}^3 S_j^2 (t - \tau_3) q_{3j}^2 (\bar{S}_j (t - \tau_3)). \quad (45)$$

Thus, it follows immediately from substituting (45) into (44) that

$$\begin{aligned} \dot{V}_3 \leq & S_3 \left[g_2 u_q + \theta_2^T \xi_2 + \delta_2 + d_2 - \dot{\alpha}_{3f} + \frac{1}{4} S_3 + S_3(t) q_{33}^2 \right. \\ & \times (\bar{S}_3(t)) + g_1 S_2 \left. \right] + \frac{1}{4} \vartheta_1^2 + \frac{1}{\gamma_2} (\hat{\theta}_2 - \theta_2)^T \dot{\hat{\theta}}_2 \\ & + \frac{1}{\Gamma_2} (\hat{g}_2 - g_2) \dot{\hat{g}}_2 + S_2^2 - \frac{1}{2} S_2^2 \tilde{g}_1 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 \\ & - m_1 \tilde{\theta}_1^T \hat{\theta}_1 - c_1 \tilde{g}_j \hat{g}_j - \sum_{j=1}^2 k_j S_j^2 + \sum_{j=2}^3 \left(1 - \frac{1}{\tau_j}\right) y_j^2 \\ & + \sum_{j=2}^3 \frac{1}{4} B_j^2 + \sum_{j=1}^3 D_j, \end{aligned} \quad (46)$$

where $D_3 = \sum_{j=1}^3 S_j^2(t - \tau_3) q_{3j}^2(\bar{S}_j(t - \tau_3)) - S_3^2(t - \tau_3) q_{33}^2(\bar{S}_3(t - \tau_3))$.

At the present stage, the control law u_q and the related parameters adaptive laws are designed as

$$u_q = \frac{\hat{g}_2}{\hat{g}_2^2 + \eta_2} \left[-\left(k_3 + \frac{1}{4}\right) S_3 - \tilde{\theta}_2^T \xi_2 - g_1 S_2 + \dot{\alpha}_{3f} - S_3(t) q_{33}^2(\bar{S}_3(t)) \right], \quad (47)$$

$$\dot{\hat{\theta}}_2 = \gamma_2 (\xi_2 S_3 - m_2 \hat{\theta}_2), \quad (48)$$

$$\dot{\hat{g}}_2 = \Gamma_2 (S_3 u_q - c_2 \hat{g}_2), \quad (49)$$

where $k_3 > 0$, $\gamma_2 > 0$, $m_2 > 0$, $\Gamma_2 > 0$, $c_2 > 0$, and $\eta_2 > 0$ are design parameters.

By similar manipulations previously done in Step 2, one has

$$\begin{aligned}
\dot{V}_3 \leq & (\delta_2 + d_3) S_3 - \frac{\eta_2}{\tilde{g}_2^2 + \eta_2} \left[- \left(k_3 + \frac{1}{4} \right) S_3 - \tilde{\theta}_2^T \xi_2 - g_1 S_2 \right. \\
& \left. + \dot{\alpha}_{3f} - S_3(t) q_{33}^2(\tilde{S}_3(t)) \right] S_3 \\
& + \frac{1}{4} \vartheta_1^2 - k_3 S_3^2 - \tilde{\theta}_2^T \xi_2 S_3 - \tilde{g}_2 u_q S_3 + \tilde{\theta}_2^T (\xi_2 S_3 - m_2 \hat{\theta}_2) \\
& + \tilde{g}_2 (S_3 u_q - c_2 \hat{g}_2) + S_2^2 - \frac{1}{2} S_2^2 \tilde{g}_1 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 \\
& - m_1 \tilde{\theta}_1^T \hat{\theta}_1 - c_1 \hat{g}_1 \tilde{g}_1 - \sum_{j=1}^2 k_j S_j^2 + \sum_{j=2}^3 \left(1 - \frac{1}{\tau_j} \right) y_j^2 \\
& + \sum_{j=2}^3 \frac{1}{4} B_j^2 + \sum_{j=1}^3 D_j \leq \sum_{j=1}^2 \frac{1}{4} \vartheta_j^2 + \sum_{j=1}^2 S_j^2 - \frac{1}{2} S_2^2 \tilde{g}_1 + \frac{1}{2} y_2^2 \\
& + \frac{1}{2} g_1 y_3^2 - \sum_{j=1}^2 m_j \tilde{\theta}_j^T \hat{\theta}_j - \sum_{j=1}^2 c_j \tilde{g}_j \hat{g}_j - \sum_{j=1}^3 k_j S_j^2 \\
& + \sum_{j=2}^3 \left(1 - \frac{1}{\tau_j} \right) y_j^2 + \sum_{j=2}^3 \frac{1}{4} B_j^2 + \sum_{j=1}^3 D_j,
\end{aligned} \tag{50}$$

where $\vartheta_2 \geq |\varepsilon_2 + d_3 - (\eta_2/(\tilde{g}_2^2 + \eta_2))[-(k_3 + 1/4)S_3 - \tilde{\theta}_2^T \xi_2 - g_1 S_2 + \dot{\alpha}_{3f} - S_3(t)q_{33}^2(\tilde{S}_3(t))]|$ is a continuous function.

Step 4. At this step, the control law u_d will be constructed. In this end, choose Lyapunov-Krasovskii functional as $V_4 = V_3 + (1/2)S_4^2 + (1/2\gamma_3)(\hat{\theta}_3 - \theta_3)^T(\hat{\theta}_3 - \theta_3) + (1/2\Gamma_3)(\hat{g}_3 - g_3)^2 + \int_{t-\tau_4}^t S_4^2(\tau)q_{44}^2(S_4(\tau))d\tau$. Then, the derivative of V_4 is given by

$$\begin{aligned}
\dot{V}_4 = & \dot{V}_3 + S_4 [f_3 + g_3 u_d + d_4 + \Delta f_4(x_4(t - \tau_4))] \\
& + \frac{1}{\gamma_3} (\hat{\theta}_3 - \theta_3)^T \dot{\hat{\theta}}_3 + \frac{1}{\Gamma_3} (\hat{g}_3 - g_3) \dot{\hat{g}}_3 + S_4^2(t) q_{44}^2 \\
& \times (S_4(t)) - S_4^2(t - \tau_4) q_{44}^2(S_4(t - \tau_4)),
\end{aligned} \tag{51}$$

where $f_3 = c_1 x_4 + c_2 x_2 x_3$, $g_3 = c_3$.

Similarly, the RBFNN system $\theta_3^T \xi_3$ is utilized to approximate the nonlinear function f_3 with unknown parameters c_1 and c_2 . For any given $\varepsilon_3 > 0$, one has

$$f_3 = \theta_3^T \xi_3 + \delta_3, \tag{52}$$

where δ_3 is the approximation error and satisfies $|\delta_3| < \varepsilon_3$.

Note the fact that

$$S_4 \Delta f_4(x_4(t - \tau_4)) \leq \frac{1}{4} S_4^2 + S_4^2(t - \tau_4) q_{44}^2(S_4(t - \tau_4)). \tag{53}$$

Substituting (52)-(53) into (51) gives

$$\begin{aligned}
\dot{V}_4 \leq & S_4 \left[g_3 u_d + \theta_3^T \xi_3 + \delta_3 + d_4 + \frac{1}{4} S_4 + S_4(t) q_{44}^2(S_4(t)) \right] \\
& + \frac{1}{\gamma_3} (\hat{\theta}_3 - \theta_3)^T \dot{\hat{\theta}}_3 + \frac{1}{\Gamma_3} (\hat{g}_3 - g_3) \dot{\hat{g}}_3 + \sum_{j=1}^2 \frac{1}{4} \vartheta_j^2 + \sum_{j=1}^2 S_j^2 \\
& - \frac{1}{2} S_2^2 \tilde{g}_1 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 - \sum_{j=1}^2 m_j \tilde{\theta}_j^T \hat{\theta}_j - \sum_{j=1}^2 c_j \tilde{g}_j \hat{g}_j \\
& - \sum_{j=1}^3 k_j S_j^2 + \sum_{j=2}^3 \left(1 - \frac{1}{\tau_j} \right) y_j^2 + \sum_{j=2}^3 \frac{1}{4} B_j^2 + \sum_{j=1}^4 D_j,
\end{aligned} \tag{54}$$

where $D_4 = S_4^2(t - \tau_4)q_{44}^2(S_4(t - \tau_4)) - S_4^2(t - \tau_4)q_{44}^2(S_4(t - \tau_4))$.

Now design u_d , $\dot{\hat{\theta}}_3$, and $\dot{\hat{g}}_3$ as

$$u_d = \frac{\hat{g}_3}{\tilde{g}_3 + \eta_3} \left[- \left(k_4 + \frac{1}{4} \right) S_4 - \tilde{\theta}_3^T \xi_3 - S_4(t) q_{44}^2(S_4(t)) \right], \tag{55}$$

$$\dot{\hat{\theta}}_3 = \gamma_3 (\xi_3 S_4 - m_3 \hat{\theta}_3), \tag{56}$$

$$\dot{\hat{g}}_3 = \Gamma_3 (S_4 u_d - c_3 \hat{g}_3), \tag{57}$$

where $k_4 > 0$, $\gamma_3 > 0$, $m_3 > 0$, $\Gamma_3 > 0$, $c_3 > 0$, and $\eta_3 > 0$ are design parameters.

By similar manipulations previously done in Step 2, one has

$$\begin{aligned}
\dot{V}_4 \leq & \sum_{j=1}^3 \frac{1}{4} \vartheta_j^2 + \sum_{j=1}^2 S_j^2 - \frac{1}{2} S_2^2 \tilde{g}_1 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 \\
& - \sum_{j=1}^4 k_j S_j^2 - \sum_{j=1}^3 m_j \tilde{\theta}_j^T \hat{\theta}_j - \sum_{j=1}^3 c_j \tilde{g}_j \hat{g}_j \\
& + \sum_{j=2}^3 \left(1 - \frac{1}{\tau_j} \right) y_j^2 + \sum_{j=2}^3 \frac{1}{4} B_j^2 + \sum_{j=1}^4 D_j,
\end{aligned} \tag{58}$$

where $\vartheta_3 \geq |\delta_3 + d_4 - (\eta_3/(\tilde{g}_3^2 + \eta_3))[-(k_4 + 1/4)S_4 - \tilde{\theta}_3^T \xi_3 - S_4(t)q_{44}^2(S_4(t))]|$ is a continuous function.

Up to now, the whole design process of the controller of PMSM is completed, and the configuration of the proposed control system is depicted in Figure 5.

4. Stability Analysis

For any given $p > 0$, the closed sets can be defined as follows:

$$\begin{aligned}\Omega_1 &= \{(S_1, q_{11}, \dots, q_{31}) : V_1 \leq p\}, \\ \Omega_2 &= \{(S_1, S_2, \hat{\theta}_1, y_2, \hat{g}_1, q_{11}, \dots, q_{31}, q_{22}, q_{32}) : V_2 \leq p\}, \\ \Omega_3 &= \{(S_1, \dots, S_3, \hat{\theta}_1, \hat{\theta}_2, y_2, y_3, \hat{g}_1, \hat{g}_2, q_{11}, \dots, \\ &\quad q_{31}, q_{22}, q_{32}, q_{33}) : V_3 \leq p\}, \\ \Omega_4 &= \{(S_1, \dots, S_4, \hat{\theta}_1, \dots, \hat{\theta}_3, y_2, y_3, \hat{g}_1, \dots, \hat{g}_3, \\ &\quad q_{11}, \dots, q_{31}, q_{22}, q_{32}, q_{33}, q_{44}) : V_4 \leq p\}.\end{aligned}\quad (59)$$

Theorem 2. Suppose that the control law in (47) and (55) with adaptive law in (32), (33), (48), (49), (56), and (57) is applied to the PMSM (4), Assumption 1 stands, and if there exists a positive constant p that the initial condition satisfies $V_4 \leq p$ and the design constants k_i, τ_i , and a_0 satisfy the following conditions:

$$\begin{aligned}k_1 &\geq 1, & k_2 &\geq 2g_{M1} + 1 + a_0, \\ k_3 &\geq \frac{1}{4}g_{M2} + 1 + a_0, & k_4 &\geq \frac{1}{4}g_{M3} + 1 + a_0, \\ \frac{1}{\tau_i} &\geq \frac{1}{4}g_{Mi} + 1 + a_0, & a_0 &\leq \frac{1}{2} \min_{1 \leq i \leq 3} \{\gamma_i, \Gamma_i, m_i, c_i\},\end{aligned}\quad (60)$$

then the closed-loop control system is semiglobal uniformly ultimately bounded, and the output tracking error converges to a neighborhood of zero.

Proof. Define the following Lyapunov function candidate for the fourth step:

$$\begin{aligned}V &= \frac{1}{2} \left(\sum_{j=1}^4 S_j^2 + \sum_{j=2}^3 y_j^2 + \sum_{j=1}^3 \frac{1}{\gamma_j} \tilde{\theta}_j^T \tilde{\theta}_j + \sum_{j=1}^3 \frac{1}{\Gamma_j} \tilde{g}_j^2 \right) \\ &\quad + \sum_{j=1}^3 \int_{t-\tau_j}^t S_1^2(\tau) q_{j1}^2(\bar{S}_1(\tau)) d\tau \\ &\quad + \sum_{j=2}^3 \int_{t-\tau_j}^t S_2^2(\tau) q_{j2}^2(\bar{S}_2(\tau)) d\tau \\ &\quad + \int_{t-\tau_3}^t S_3^2(\tau) q_{33}^2(\bar{S}_3(\tau)) d\tau \\ &\quad + \int_{t-\tau_4}^t S_4^2(\tau) q_{44}^2(S_4(\tau)) d\tau.\end{aligned}\quad (61)$$

For the term $-m_j \tilde{\theta}_j^T \hat{\theta}_j$, one has $-m_j \tilde{\theta}_j^T \hat{\theta}_j = -m_j \tilde{\theta}_j^T (\tilde{\theta}_j + \theta_j) \leq -(1/2)m_j \|\tilde{\theta}_j\|^2 + (1/2)m_j \|\theta_j\|^2$. Similarly, $-c_j \tilde{g}_j \hat{g}_j \leq$

$-(1/2)c_j \tilde{g}_j^2 + (1/2)c_j g_j^2$ holds. Consequently, the derivative of V is

$$\begin{aligned}\dot{V} &= -\sum_{j=1}^4 k_j S_j^2 + \sum_{j=1}^2 S_j^2 - \sum_{j=1}^3 \frac{1}{2} m_j \|\tilde{\theta}_j\|^2 - \sum_{j=1}^3 \frac{1}{2} c_j \tilde{g}_j^2 \\ &\quad + \sum_{j=2}^3 \left(1 - \frac{1}{\tau_j}\right) y_j^2 + \frac{1}{2} y_2^2 + \frac{1}{2} g_1 y_3^2 - \frac{1}{2} S_2^2 \tilde{g}_1 \\ &\quad + \sum_{j=1}^4 D_j + \sum_{j=1}^3 \frac{1}{4} \vartheta_j^2 + \sum_{j=2}^3 \frac{1}{4} B_j^2 + \sum_{j=1}^3 \frac{1}{2} m_j \|\theta_j\|^2 \\ &\quad + \sum_{j=1}^3 \frac{1}{2} c_j g_j^2 \leq -a_0 V + b_0,\end{aligned}\quad (62)$$

where $b_0 = \sum_{j=1}^3 (1/4) \vartheta_j^2 + \sum_{j=2}^3 (1/4) B_j^2 + \sum_{j=1}^3 (1/2) m_j \|\theta_j\|^2 + \sum_{j=1}^3 (1/2) c_j g_j^2$.

Furthermore, (62) implies that

$$0 \leq V(t) \leq \frac{b_0}{a_0} + \left(V(0) - \frac{b_0}{a_0} \right) e^{-a_0 t} \leq \frac{b_0}{a_0} + V(0). \quad (63)$$

By Lyapunov function, it is proved that all the signals $S_j, y_j, \tilde{\theta}_j$, and $\tilde{g}_j$ in the closed-loop system are uniformly ultimately bounded. Consequently, x_i, α_i , and α_{if} are also uniformly ultimately bounded. $\square$

5. Simulation

To illustrate the effectiveness of the proposed controller, the simulation will be done for the PMSM with the parameters: $J = 0.00379 \text{ Kg m}^2$, $R_s = 0.68 \Omega$, $L_d = 0.00315 \text{ H}$, $L_q = 0.00285 \text{ H}$, $B = 0.001158 \text{ Nm/(rad/s)}$, $\Phi = 0.1245 \text{ Wb}$, $n_p = 3$,

$T_L = 1.5$, $q_{11} = 1$, $q_{21} = 1 - \sqrt{2 - S_1^2}$, $q_{31} = |S_1|$, $q_{22} = |S_2|$, $q_{32} = \sqrt{2 - S_2^2}$, $q_{33} = 1$, $q_{44} = 1$, $d_1 = d_2 = d_3 = d_4 = 0.2 \times x_1^3 \times \sin^5(t)$, $\Delta f_1(x_1(t - \tau_1)) = 0.05 \sin(x_1(t - \tau_1))$, $\Delta f_2(\bar{x}_2(t - \tau_2)) = x_1(t - \tau_2)x_2(t - \tau_2)$, $\Delta f_3(\bar{x}_3(t - \tau_3)) = x_1(t - \tau_3)x_2(t - \tau_3)x_3(t - \tau_3)$, $\Delta f_4(x_4(t - \tau_4)) = 0.05 \sin(x_4(t - \tau_4))$, $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.4 \text{ s}$.

The simulation is running under the assumption that the system parameters and nonlinear functions are unknown. The initial conditions of system $x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0$. Then, the reference signal is taken as $y_r = \sin 3t$, the control parameters are chosen as follows: $k_1 = k_2 = 25$, $k_3 = k_4 = 20$, $\Gamma_1 = \Gamma_2 = \Gamma_3 = 0.3$, $c_1 = c_2 = c_3 = 0.9$, $\tau_2 = 0.01$, $\tau_3 = 0.02$, $\gamma_1 = \gamma_2 = \gamma_3 = 25$, $m_1 = m_2 = m_3 = 0.1$, $\eta_1 = \eta_2 = \eta_3 = 0.001$, and RBFNN is chosen in the following way. Neural network $\theta_j^T \xi_j, j = 1, 2, 3$, contains nine nodes with centers spaced evenly in the interval $[-5, 5]$ and width being equal to two.

With the proposed RBFNN-based adaptive dynamic surface control, from Figures 6 and 7, it is observed that the tracking performance is satisfactory and the tracking error quickly converges to zero in a short time for PMSM system in the presence of unknown parameters, disturbances, and uncertain time delays. The final controllers u_q and q_d are

FIGURE 5: Block diagram of the adaptive DSC system based on RBFNN for PMSM.

FIGURE 6: Output x_1 (solid line) follows desired trajectory y_r (dashed line).

FIGURE 7: The rotor position tracking error.

FIGURE 8: q -axis voltage of control input. FIGURE 9: d -axis voltage of control input.

shown in Figures 8 and 9. Moreover, the robustness of the proposed controllers against uncertainty in system parameters and disturbance is analyzed by simulation. According to

Figure 10, it is seen clearly that all state variables are bounded and convergent. Meanwhile, by contrast with Figure 3, the proposed controller can suppress the chaos of PMSM. Therefore, these simulation results demonstrate the effectiveness of the proposed control in achieving its objective.

FIGURE 10: The curves of system state variables when utilizing the proposed controller.

6. Conclusion

A novel adaptive DSC based on the RBFNN is presented for the fourth-order chaotic PMSM system with uncertain time delays so that the first state can track a given trajectory with little time. The unknown PMSM parameters, disturbances, and uncertain time delays are all considered. Compared with the traditional backstepping approach, the main advantage of the proposed method is its simplicity without repetitive differentiation of the nonlinear functions under the condition. It is noted that the system has excellent tracking performance and antidisturbing by choosing appropriate design parameters. Finally, the simulation results are provided to demonstrate the effectiveness and robustness against the unknown parameters, disturbances, chaos, and uncertain time delays.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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