The passivity for discrete-time stochastic T-S fuzzy systems with time-varying delays is investigated. By constructing appropriate Lyapunov-Krasovskii functionals and employing stochastic analysis method and matrix inequality technique, a delay-dependent criterion to ensure the passivity for the considered T-S fuzzy system is established in terms of linear matrix inequalities (LMIs) that can be easily checked by using the standard numerical software. An example is given to show the effectiveness of the obtained result.

1. Introduction

Fuzzy control offers an alternative control approach for certain nonlinear systems [1, 2]. Among various model-based fuzzy control approaches, the method based on Takagi-Sugeno (T-S) model is thought of as an effective way for the control of complex nonlinear systems, which is presented by a family of fuzzy IF-THEN rules that represent the local linear input-output relations of the system. Over the past decades, there have been significant research efforts on the stability for T-S fuzzy systems; for example, see [3–16] and references therein.

On the other hand, the passivity theory is another effective tool for the stability analysis of system. The reason is mainly twofold: (1) passivity is an expected system behavior, since the storage function induced by passivity is closely related to system energy and therefore serves as a natural candidate for Lyapunov functions and (2) stability and stabilization problems can be solved once the passivity property is assured. The passivity theory was first proposed in the circuit analysis [17] and has then been applied in many areas such as stability, signal processing, complexity, fuzzy control, chaos control, and synchronization [18–21].

Recently, some authors have studied the passivity of some systems and obtained sufficient conditions for checking the passivity of the systems that include linear systems with delays [22–24], delayed neural networks [25, 26], networked control systems [27], nonlinear discrete-time systems with direct input-output link [28], and T-S fuzzy systems [29–33]. In [29], the stability of fuzzy control loops is proven with the unique condition that the controlled plant can be made passive by zero shifting. For linear time-invariant plants, this approach leads to frequency response conditions similar to the previous results in the literature but which are more general and can include robust stability considerations. In [30], the passivity and feedback passification of T-S fuzzy systems with time delays were considered. Both delay-independent and delay-dependent results were presented, and the theoretical results were given in terms of LMIs. In [31], the contiguous-time T-S fuzzy systems with time-varying delays were investigated; several criteria to ensure the passivity and feedback passification were given. In [32], the passivity and feedback passification of T-S fuzzy systems with both discrete and distributed time-varying delays were investigated without assuming the differentiability of the time-varying delays. By employing appropriate Lyapunov-Krasovskii functionals, several delay-dependent criteria for the passivity of the considered T-S fuzzy systems were established in terms of LMIs. In [33], the stochastic T-S fuzzy system with both discrete and distributed time-varying delays was considered; several delay-dependent criteria to ensure the passivity and passification of the considered T-S fuzzy
systems were established. In [34], discrete-time T-S fuzzy systems with delays were considered; some sufficient conditions to verify the passivity of the uncertain discrete-time fuzzy systems were obtained. In [35], the passivity of certain discrete-time T-S fuzzy systems with time delays was investigated; a sufficient condition on the existence of robust passive controller was established based on the Lyapunov stability theory. In [36], stochastic discrete-time T-S fuzzy systems with delay were considered; a sufficient condition in LMIs ensuring the passivity performance of the T-S fuzzy models was presented by utilizing the Lyapunov functional method, the stochastic analysis combined with the matrix inequality techniques. In this paper, we continue to study the passivity for stochastic discrete-time T-S fuzzy systems with time-varying delay. By employing appropriate Lyapunov-Krasovskii functionals and stochastic analysis technique, we obtain a new delay-dependent sufficient condition for checking the passivity of the addressed T-S fuzzy systems. As pointed out in [37–39], the delay-dependent criteria have less conservatism than the delay-independent ones.

**Notations.** The notations are quite standard. Throughout this paper, $I$ represents the unitary matrix with appropriate dimensions; $\mathbb{N}$ stands for the set of nonnegative integers; $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript $\dagger$ denotes matrix transposition and the asterisk “*” denotes the elements below the main diagonal of a symmetric block matrix. The notation $X \succeq 0$ (resp., $X \succ 0$) means that $X$ and $Y$ are symmetric matrices and that $X - Y$ is positive semidefinite (resp., positive definite). Also $\| \cdot \|$ is the Euclidean norm in $\mathbb{R}^n$. And $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) denotes the least (resp., largest) eigenvalue of symmetric matrix $A$. For a positive constant $\alpha$, $[a]$ denotes the integer part of $a$. For integers $a, b$ with $a < b$, $\mathbb{N}[a, b]$ denotes the discrete interval given by $\mathbb{N}[a, b] = [a, a + 1, \ldots, b - 1, b]$. Also $C(\mathbb{N}[-\tau, 0], \mathbb{R}^n)$ denotes the set of all functions $\phi : \mathbb{N}[-\tau, 0] \rightarrow \mathbb{R}^n$. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathcal{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets). $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. Denote by $L^2_{\mathcal{F}_0}(\mathbb{N}[-\tau, 0], \mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable $C(\mathbb{N}[-\tau, 0], \mathbb{R}^n)$ valued random variables $\psi = \{\psi(s) : s \in \mathbb{N}[-\tau, 0]\}$ such that $\sup_{s \in \mathbb{N}[-\tau, 0]} \mathbb{E}[|\psi(s)|^2] < \infty$. $\Delta V(k)$ denotes the difference of function $V(k)$ given by $\Delta V(k) = V(k + 1) - V(k)$. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

## 2. Model Description and Preliminaries

In this section, we consider a discrete-time T-S fuzzy system with stochastic disturbances and time-varying delay with the $i$th rule formulated in the following form.

**Plant Rule i.** If $z_{1i}(t)$ is $M_{i1}$ and ... and $z_{pi}(t)$ is $M_{ip}$, then

$$x(k + 1) = A_i x(k) + B_i x(k - \tau(k)) + U_i J(k) + \sigma_i(x(k), x(k - \tau(k))) \omega_i(k),$$

for $k \in \mathbb{N}[-\tau, 0]$.

(1)

Where $\epsilon_i(x, y) \epsilon_i(x, y) \leq \|W_i x\|^2 + \|S_i y\|^2$, (5)

$$\forall x, y \in \mathbb{R}^n, \quad i = 1, 2, \ldots, r.$$
Lemma 3 (see [40]). Suppose that matrices $M_i \in \mathbb{R}^{n \times m} (i = 1, 2, \ldots, s)$ and a positive-semidefinite matrix $P \in \mathbb{R}^{n \times n}$ are given. If $\sum_{i=1}^{s} h_i = 1$ and $0 \leq h \leq 1$, then
\[
\left( \sum_{i=1}^{s} h_i M_i \right)^T P \left( \sum_{i=1}^{s} h_i M_i^T \right) \leq \sum_{i=1}^{s} h_i M_i^T PM_i. \tag{6}
\]

3. Main Results

In this section, we will establish our main criterion based on the LMI approach. For presentation convenience, in the following, we denote that $\delta = [(\tau + \bar{\tau})/2]$.

Theorem 4. Under Assumption 2, model (1) is passive in the sense of Definition 1 if there exist two scalars $\gamma > 0$ and $\lambda > 0$ and eight symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, R_1, R_2, \text{ and } R_3$ such that the following LMIs hold for $i = 1, 2, \ldots, r$:
\[
P + R < \lambda I, \quad \Pi^{(i)} + \Omega_1 < 0, \quad \Pi^{(i)} + \Omega_2 < 0
\]
hold or
\[
P + R < \lambda I, \quad \Pi^{(i)} + \Omega_3 < 0, \quad \Pi^{(i)} + \Omega_4 < 0
\]
hold, where
\[
\Pi^{(i)} = \begin{bmatrix}
\Pi_{i1}^{(i)} & 0 & 0 & 0 & \Pi_{i6}^{(i)} \\
* & \Pi_{i2}^{(i)} & 0 & 0 & \Pi_{i5}^{(i)} \\
* * & -Q_i & 0 & 0 & 0 \\
* * * & -Q_2 & 0 & 0 & 0 \\
* * * * & -Q_3 & 0 & 0 & 0 \\
* * * * * & * & \Pi_{i6}^{(i)} & 0 & 0 \\
\end{bmatrix},
\]
\[
\Omega_1 = \begin{bmatrix}
-R_1 & 0 & 0 & R_1 & 0 & 0 \\
* & -3R_3 & R_3 & 0 & 2R_3 & 0 \\
* & * & -R_2 - R_3 & R_2 & 0 & 0 \\
* & * & * & -R_1 - R_2 & R_1 & 0 \\
* & * & * & * & -R_3 & 0 \\
* & * & * & * & * & 0 \\
\end{bmatrix},
\]
\[
\Omega_2 = \begin{bmatrix}
-R_1 & 0 & 0 & R_1 & 0 & 0 \\
* & -3R_3 & R_3 & 0 & 2R_3 & 0 \\
* & * & -R_2 - R_3 & R_2 & 0 & 0 \\
* & * & * & -R_1 - R_2 & R_1 & 0 \\
* & * & * & * & -R_3 & 0 \\
* & * & * & * & * & 0 \\
\end{bmatrix},
\]
in which $R = \tau^2 R_1 + (\delta - \tau)^2 R_2 + (\tau - \delta)^2 R_3$, $\Pi_{i1}^{(i)} = A_i^T (P + R) A_i - P + Q_1 + Q_2 + Q_3 + (1 + \tau - \tau) Q_4 - R A_i - A_i^T R + R + \lambda W_i W_i^T$, $\Pi_{i2}^{(i)} = A_i^T (P + R) B_i - R B_i$, $\Pi_{i6}^{(i)} = A_i^T (P + R) U_i - R U_i - C_i^T$, $\Pi_{i5}^{(i)} = B_i^T (P + R) B_i - Q_4 + \lambda S_i S_i$, $\Pi_{i4}^{(i)} = B_i^T (P + R) U_i - D_i^T$, and $\Pi_{i3}^{(i)} = U_i^T (P + R) U_i - V_i - V_i^T - \gamma I$.

Proof. Defining $\eta(k) = x(k + 1) - x(k)$, we consider the following Lyapunov-Krasovskii functional candidate for model (1) as
\[
V(k) = \sum_{i=1}^{4} V_i(k), \tag{10}
\]
where
\[
V_1(k) = x^T(k) P x(k), \tag{11}
\]
\[
V_2(k) = \sum_{i=k-\delta}^{k-1} x^T(i) Q_1 x(i) + \sum_{i=k}^{k+1} x^T(i) Q_2 x(i) + \sum_{i=k-\tau}^{k-1} x^T(i) Q_3 x(i), \tag{12}
\]
\[
V_3(k) = \sum_{i=k-\tau}^{k-1} x^T(i) Q_4 x(i) + \sum_{i=k-\tau-1}^{k-1} x^T(i) Q_5 x(i), \tag{13}
\]
\[
V_4(k) = \tau \sum_{i=k-\tau}^{k-1} \sum_{i=k-\tau+1}^{k-1} \sum_{i=k-\tau}^{k-1} \sum_{i=k-\tau-1}^{k-1} \eta^T(i) R_1 \eta(i) + (\delta - \tau) \sum_{i=k-\tau}^{k-1} \sum_{i=k-\tau-1}^{k-1} \eta^T(i) R_2 \eta(i) + (\tau - \delta) \sum_{i=k-\tau}^{k-1} \sum_{i=k-\tau-1}^{k-1} \eta^T(i) R_3 \eta(i). \tag{14}
\]

Calculating the difference of $V_i(k) (i = 1, 2, 3, 4)$ along the trajectories of model (1) and taking the mathematical expectation, we obtain that
\[
E \{ \Delta V_1(k) \} = E \left\{ \sum_{i=1}^{r} \mu_i(t) \left[ A_i x(k) + B_i x(k - \tau(k)) + U_i J(k) \right] \right\}
\]
\[
\begin{align*}
\mathbb{E}\{\Delta V_4(k)\} &= \mathbb{E}\left\{\eta^T(k) R\eta(k) - \tau \sum_{i=k-\tau}^{k-1} \eta^T(i) R_1 \eta(i)\right. \\
&\quad - (\delta - \tau) \sum_{i=k-\delta}^{k-\tau-1} \eta^T(i) R_2 \eta(i) \left.\right\}.
\end{align*}
\]

In deriving inequalities (15) and (17), Lemma 3 and the condition \(\tau \leq \tau(k) \leq \bar{\tau}\) have been used, respectively.

From the definition of \(\eta(k)\) and (3) and applying Lemma 3, we have

\[
\begin{align*}
\mathbb{E}\{\Delta V_3(k)\} &= \mathbb{E}\left\{x^T(k) \left(A^T \eta(k) - RA_1 - A_1^T R + R\right) x(k) \right. \\
&\quad + 2x^T(k) (A_1^T R B_1 - R B) x(k - \tau(k)) \\
&\quad + 2x^T(k) (A_1^T R U_1 - R U) x(k - \tau(k)) \\
&\quad + x^T(k - \tau(k)) B_1^T R B x(k - \tau(k)) \\
&\quad + 2x^T(k - \tau(k)) B_1^T R U_1 x(k - \tau(k)) \\
&\quad + J^T(k) U_1^T R U_1 J(k) \\
&\quad + \sigma^T_i(x(k), x(k - \tau(k))) \\
&\quad \left.\right\} \right\}.
\end{align*}
\]

(19)

It is easy to get

\[
-\tau \sum_{i=k-\tau}^{k-1} \eta^T(i) R_1 \eta(i)
\]

\[
\leq - \sum_{i=k-\tau}^{k-1} \eta^T(i) R_1 \sum_{i=k-\tau}^{k-1} \eta(i)
\]

(20)

When \(\tau \leq \tau(k) \leq \delta\), let \(a(k) = (\tau(k) - \tau)/(\delta - \tau)\); then \(0 \leq a(k) \leq 1\). It is easy to get that

\[
- (\delta - \tau) \sum_{i=k-\delta}^{k-\tau-1} \eta^T(i) R_2 \eta(i)
\]

(17)
\[ - (\delta - \tau) \sum_{i=k-r(k)}^{k-r(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ = - (\delta - \tau(k)) \sum_{i=k-\delta}^{k-\tau(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ - (\tau(k) - \tau) \sum_{i=k-\tau(k)}^{k-\tau(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ - (\delta - \tau(k)) \sum_{i=k-\delta}^{k-\tau(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ \leq - (\delta - \tau(k)) \sum_{i=k-\tau(k)}^{k-\tau(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ - a(k) (\delta - \tau(k)) \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ - (1 - a(k)) (\tau(k) - \tau) \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ - (\tau(k) - \tau) \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \eta(i) \]

\[ \leq - \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \sum_{i=k-\delta}^{k-\tau(k)-1} \eta(i) \]

\[ - a(k) \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \sum_{i=k-\delta}^{k-\tau(k)-1} \eta(i) \]

\[ - (1 - a(k)) \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta(i) \]

\[ - \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta^T(i) R_2 \sum_{i=k-\tau(k)}^{k-r(k)-1} \eta(i) \]

\[ = \left[ x(k - \tau(k)) \right]^T \begin{bmatrix} -2R_2 & R_2 & 0 \\ * & -R_2 & 0 \\ * & * & -R_2 \end{bmatrix} \left[ x(k - \tau(k)) \right] \]

\[ + a(k) \left[ x(k - \tau(k)) \right]^T \begin{bmatrix} -R_2 & R_2 \\ * & -R_2 \end{bmatrix} \left[ x(k - \tau(k)) \right] \]

\[ + (1 - a(k)) \left[ x(k - \tau(k)) \right]^T \begin{bmatrix} -R_2 & R_2 \\ * & -R_2 \end{bmatrix} \left[ x(k - \tau(k)) \right] \]

\[ \leq \left[ x(k - \delta) \right]^T \begin{bmatrix} -R_3 & R_1 \\ x(k - \tau) \end{bmatrix} \left[ x(k - k - \tau) \right]. \]

From the first inequality of condition (7) and Assumption 2, we get

\[ \sigma_i^T(x(k), x(k - \tau(k))) (P + R) \sigma_i(x(k), x(k - \tau(k))) \]

\[ \leq \lambda \left[ \|W_i x(k)\|^2 + \|S_i x(k - \tau(k))\|^2 \right]. \]

Denote that \( a(k) = (x^T(k), x^T(k - \tau(k)), x^T(k - \delta), x^T(k - \tau), x^T(k - \tau), J^T(k))^T \). It follows from (15) to (23) that

\[ E \left\{ \Delta V(k) - 2 y^T(k) J(k) - \gamma J^T(k) J(k) \right\} \]

\[ \leq E \left\{ \sum_{i=1}^{T} \mu_i(t) a(k) \left( a(k) \left( P(i) + \Omega_1 \right) + (1 - a(k)) \right) \times \left( P(i) + \Omega_2 \right) \right\} \]

\[ \times a(k) \right\}. \]

(24)

From the second inequality and the third inequality of condition (7), we get

\[ E \left\{ \Delta V(k) - 2 y^T(k) J(k) - \gamma J^T(k) J(k) \right\} \leq 0 \]

(25)

and therefore we have

\[ 2 \sum_{k=0}^{T} E \left\{ y^T(k) J(k) \right\} \]

\[ \geq \sum_{k=0}^{T} E \left\{ \Delta V(k) \right\} - \gamma \sum_{k=0}^{T} E \left\{ J^T(k) J(k) \right\} \]

\[ = \sum_{k=0}^{T} E \left\{ V(T) - V(0) \right\} - \gamma \sum_{k=0}^{T} E \left\{ J^T(k) J(k) \right\} \]

(26)

\[ \geq - \gamma \sum_{k=0}^{T} E \left\{ J^T(k) J(k) \right\} \]

\[ = - \gamma \sum_{k=0}^{T} E \left\{ J^T(k) J(k) \right\} \]

for all integers \( T \geq 0 \). From Definition 1, we know that (26) implies that the stochastic T-S fuzzy system (1) is globally passive in the sense of expectation.
When $\delta \leq \tau(t) \leq \tau$, let $b(k) = (\tau - \tau(k)) / (t - \delta)$; then $0 \leq b(k) \leq 1$. In the similitude of the proof of inequality (21), we have

$$- (\tau - \delta) \sum_{i=\tau}^{k-\delta-1} \eta^T(i) R_3 \eta(i) \leq \begin{bmatrix} x(k - \tau(k)) \n x(k - \tau) \n x(k - \delta) \end{bmatrix}^T \begin{bmatrix} -2R_3 & R_3 & R_3 \\
 -R_3 & 0 & 0 \\
 -R_3 & -R_3 & 0 \end{bmatrix} \begin{bmatrix} x(k - \tau(k)) \\
 x(k - \tau) \\
 x(k - \delta) \end{bmatrix}$$

$$+ b(k) \begin{bmatrix} x(k - \tau(k)) \\
 x(k - \tau) \\
 x(k - \delta) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 & R_3 \\
 * & 0 & 0 \\
 * & -R_3 & 0 \end{bmatrix} \begin{bmatrix} x(k - \tau(k)) \\
 x(k - \tau) \\
 x(k - \delta) \end{bmatrix}$$

$$+ (1 - b(k)) \begin{bmatrix} x(k - \tau(k)) \\
 x(k - \tau) \\
 x(k - \delta) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 & R_3 \\
 * & 0 & 0 \\
 * & -R_3 & 0 \end{bmatrix} \begin{bmatrix} x(k - \tau(k)) \\
 x(k - \tau) \\
 x(k - \delta) \end{bmatrix},$$

(27)

It follows from (15)–(20), (23), (27), and (28) that

$$E \left\{ \nabla V(k) - 2 y^T(k) J(k) - \gamma y^T(k) J(k) \right\} \leq E \left\{ \sum_{i=1}^{\tau} \mu_i(t) \alpha^T(k) \begin{bmatrix} b(k) (\Pi^{(0)} + \Omega_3) + (1 - b(k)) \end{bmatrix} \right\}.$$

(29)

From the second inequality and the third inequality of condition (8), we get

$$E \left\{ \nabla V(k) - 2 y^T(k) J(k) - \gamma y^T(k) J(k) \right\} \leq 0.$$  

(30)

By using the Matlab LMI Control Toolbox, we can find a solution to the LMIs in (7) as follows:

$$P = 10^4 \begin{bmatrix} 1.0868 & -0.0007 \\
 -0.0007 & 1.0878 \end{bmatrix},$$

$$Q_1 = 10^3 \begin{bmatrix} 1.3480 & -0.0014 \\
 -0.0014 & 1.3499 \end{bmatrix},$$

$$Q_2 = 10^3 \begin{bmatrix} 1.3488 & -0.0011 \\
 -0.0011 & 1.3501 \end{bmatrix},$$

$$Q_3 = 10^3 \begin{bmatrix} 1.3489 & -0.0009 \\
 -0.0009 & 1.3502 \end{bmatrix},$$

$$Q_4 = 10^3 \begin{bmatrix} 1.3603 & -0.0037 \\
 -0.0037 & 1.3639 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 1.7200 & 0.8232 \\
 0.8232 & 0.5407 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.2118 & 0.1525 \\
 0.1525 & 0.1121 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 1.8563 & 0.8703 \\
 0.8703 & 0.5674 \end{bmatrix},$$

$$\gamma = 1.3546 \times 10^3, \quad \lambda = 1.2234 \times 10^4.$$

According to Theorem 4, the considered model (1) is passive in the sense of Definition 1.
5. Conclusions

In this paper, the passivity for discrete-time stochastic T-S fuzzy systems with time-varying delays has been investigated. By constructing appropriate Lyapunov-Krasovskii functionals and employing stochastic analysis method and matrix inequality technique, a delay-dependent criterion to ensure the passivity for the considered T-S fuzzy systems has been established in terms of linear matrix inequalities (LMIs) that can be easily checked by using the standard numerical software. An example is also given to show the effectiveness of the obtained result.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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