Research Article

Dynamic Behaviors of Holling Type II Predator-Prey System with Mutual Interference and Impulses

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A class of Holling type II predator-prey systems with mutual interference and impulses is presented. Sufficient conditions for the permanence, extinction, and global attractivity of system are obtained. The existence and uniqueness of positive periodic solution are also established. Numerical simulations are carried out to illustrate the theoretical results. Meanwhile, they indicate that dynamics of species are very sensitive with the period matching between species' intrinsic disciplinarians and the perturbations from the variable environment. If the periods between individual growth and impulse perturbations match well, then the dynamics of species periodically change. If they mismatch each other, the dynamics differ from period to period until there is chaos.

1. Introduction

As pointed out by Berryman [1], the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both biology and mathematical biology due its universal existence and importance. Though predator-prey models with periodic or almost periodic coefficients have been studied extensively (see [2–14] and the references cited therein), few papers consider the mutual interference between the predators and prey, which was introduced by Hassell in 1971. During his research of the capturing behavior between hosts and parasites, he found that the hosts or parasites had the tendency to leave each other when they met, which interfered with hosts capture effects. Obviously, the mutual interference will be stronger, while the size of parasite becomes larger. From the observation, Hassell introduced the concept of mutual interference constant m ($0 < m \leq 1$) and established a Lotka-Volterra predator-prey model with mutual interference as follows:

$$\begin{aligned}\dot{x}(t) &= xg(x) - \varphi(x)y^m, \\ \dot{y}(t) &= y(-d + \beta\varphi(x)y^{m-1} - q(y)),\end{aligned}\quad (1)$$

where x and y are the sizes of the prey and predator populations, respectively, and m is mutual interference constant; see

[15, 16] for more details on the biological meaning of mutual interference constant m . Some results have appeared for this type of predator-prey system [17–20].

Recently, much more attention has paid on the functional responses. Based on experiments, Holling [21] investigated three different kinds of functional responses for different kinds of species to model the phenomena of predation, which made the standard Lotka-Volterra system more consistent with the real ecosystem. The Lotka-Volterra system with Holling type II functional response is introduced by Wang and Zhu [22] in the following form:

$$\begin{aligned}\dot{x}(t) &= x(t)(r_1(t) - b_1(t)x(t)) - \frac{c_1(t)x(t)}{k + x(t)}y^m(t), \\ \dot{y}(t) &= y(t)(-r_2(t) - b_2(t)y(t)) + \frac{c_2(t)x(t)}{k + x(t)}y^m(t).\end{aligned}\quad (2)$$

However, ecological system is often deeply perturbed by nature and human exploit activities (e.g., fire, drought, flooding deforestation, hunting, harvesting, breeding, etc.), which are not suitable to be considered continually. To accurately describe the system, impulsive differential equations may be a better candidate than ordinary differential equation. The theory of impulsive differential equations not only is now being recognized to be richer than the corresponding theory

TABLE 1: Parameter values used in the simulations of system (3).

Parameter	Interpretation	Value
$r_1(t)$	Growth rate of the prey	$1 + \sin(2\pi t)$
$r_2(t)$	Death rate of the predator	$0.03 + 0.01 \sin(2\pi t)$
$b_1(t)$	Intracompetition coefficient among the prey	$1 - 0.1 \sin(2\pi t)$
$b_2(t)$	Intracompetition coefficient among the predators	$0.4 - 0.1 \sin(2\pi t)$
$c_1(t)$	Capture rate of predator on prey	$0.7 + 0.1 \cos(2\pi t)$
$c_2(t)$	Conversion rate of nutrition	$0.2 + 0.1 \sin(2\pi t)$
k	The environment carrying capacity	10
m	Mutual interference constant	1/3
h_k	Impulse perturbations on prey	—
g_k	Impulse perturbations on predator	—

of differential equations without impulse, but also represents a more natural framework for mathematical modelling of many real-world phenomena. In recent years, many important and interesting results on the permanence, extinction, global attractivity of systems, the existence and uniqueness of positive periodic solutions, bifurcation and dynamical complexity, and so forth have been proposed in [23–28] and references cited therein. Motivated by above studies, in this paper we will consider the following Holling II predator-prey system with mutual interference and impulses:

$$\begin{aligned}\dot{x}(t) &= x(t)(r_1(t) - b_1(t)x(t)) - \frac{c_1(t)x(t)}{k+x(t)}y^m(t), \\ \dot{y}(t) &= y(t)(-r_2(t) - b_2(t)y(t)) + \frac{c_2(t)x(t)}{k+x(t)}y^m(t), \\ t &\neq \tau_k, \quad (3)\end{aligned}$$

$$\begin{aligned}x(\tau_k^+) &= (1 + h_k)x(\tau_k), \\ y(\tau_k^+) &= (1 + g_k)y(\tau_k), \\ t &= \tau_k, \quad k = 1, 2, \dots,\end{aligned}$$

where x and y are the sizes of the prey and predator populations, respectively. $0 < m < 1$ and k is a positive constant. $r_i(t)$, $b_i(t)$, and $c_i(t)$ ($i = 1, 2$) are continuous periodic functions defined on $[0, +\infty)$ with a common period $T > 0$. $b_i(t)$, $c_i(t)$, ($i = 1, 2$) are strictly positive. Also $\tau_k \rightarrow +\infty$ ($t \rightarrow +\infty$), $0 < \tau_1 < \tau_2 < \tau_3 < \dots < \tau_k < \tau_{k+1} < \dots$. Assume that h_k, g_k ($k = 1, 2, \dots$) are constants. $1 + h_k > 0$ and $1 + g_k > 0$, and there exists an integer $q > 0$ such that $h_{k+q} = h_k$, $g_{k+q} = g_k$, $\tau_{k+q} = \tau_k + T$. The interpretations of parameters can be seen in Table 1.

The organization of this paper is as follows. In Section 2, we give some definitions and state some lemmas which are essential to study the main theorems. In Section 3, sufficient conditions for the permanence, extinction of system (3), and the existence, uniqueness, and global attractivity of positive periodic solution are obtained. In Section 4, numerical examples are presented to illustrate our results. In the last section, we give a brief discussion.

2. Preliminaries

In this section, we introduce some notations and definitions and state some lemmas which will be useful in the subsequent sections.

Let $R_+ = [0, \infty)$, $R_+^2 = \{X \in R^2 \mid X \geq 0\}$. The map $f = (f_1, f_2)^T$ is defined by the right hand of the first and second equations of system (3). Let $V : R_+ \times R_+^2 \rightarrow R_+$; then V is said to belong to class V_0 if

- (1) V is continuous in $(\tau_k, \tau_{k+1}] \times R_+^2$ and for each $X \in R_+^2$, $k = 1, 2, \dots$, there exists $\lim_{(t,Y) \rightarrow (\tau_k^+, X)} V(t, Y) = V(\tau_k^+, X)$;
- (2) V is locally Lipschitzian in X .

Definition 1. Let $V \in V_0$; then, for $(t, X) \in (\tau_k, \tau_{k+1}] \times R_+^2$, the upper right derivative of $V(t, X)$ with respect to the impulsive differential system (3) is defined as

$$\begin{aligned}D^+V(t, x) \\ = \lim_{h \rightarrow 0^+} \sup \frac{1}{h} [V(t+h, X+hf(t, X)) - V(t, X)].\end{aligned}\quad (4)$$

Definition 2. System (3) is permanent, if, for any positive solution $(x(t), y(t))^T$ of system (3), there exist positive constants m_1 , m_2 , M_1 , and M_2 such that

$$\begin{aligned}m_1 &\leq \liminf_{t \rightarrow +\infty} x(t) \leq \limsup_{t \rightarrow +\infty} x(t) \leq M_1, \\ m_2 &\leq \liminf_{t \rightarrow +\infty} y(t) \leq \limsup_{t \rightarrow +\infty} y(t) \leq M_2.\end{aligned}\quad (5)$$

Lemma 3 ([29]). Let $V \in V_0$ and $X(t)$ is a solution of (3) with initial value $X(0^+) = X_0$. Assume that

$$\begin{aligned}D^+V(t, X(t)) &\leq g(t, V(t, X(t))), \quad t \neq \tau_k, \quad k = 1, 2, \dots, \\ V(t, X(t^+)) &\leq \psi_k(V(t, X(t))), \quad t = \tau_k, \quad k = 1, 2, \dots,\end{aligned}\quad (6)$$

where $g : R_+ \times R_+ \rightarrow R$ is continuous in $(\tau_k, \tau_{k+1}] \times R_+$ and for $u \in R_+$, $k = 1, 2, \dots$, $\lim_{(t,v) \rightarrow (\tau_k^+, u)} g(t, v) = g(\tau_k^+, u)$ exists

and $\psi_k : R_+ \rightarrow R_+$ is nondecreasing. Let $r(t)$ be the maximal solution of scalar impulsive differential equation

$$\begin{aligned} \dot{u}(t) &= g(t, u(t)), \quad t \neq \tau_k, \quad k = 1, 2, \dots, \\ u(t^+) &= \psi_k(u(t)) \geq 0, \quad t = \tau_k, \quad t_k > 0, \quad k = 1, 2, \dots, \quad (7) \\ u(0^+) &= u_0 \end{aligned}$$

existing on $[0, \infty)$. Then $V(0^+, X_0) \leq u_0$ implies that $V(t, X(t)) \leq r(t)$ for all $t \geq 0$.

Similarly, assume that inequality (6) is reversed. Let $p(t)$ be the minimal solution of (7) existing on $[0, +\infty)$. Then, $V(0^+, X_0) \geq u_0$ implies that $V(t, X(t)) \geq p(t)$ for all $t \geq 0$.

Let $f(t)$ be continuous T -periodic function defined on the R_+ ; we introduce the following notation:

$$m[f(t)] = \frac{1}{T} \int_0^T f(t) dt. \quad (8)$$

Consider the following periodic logistic equation with impulses:

$$\begin{aligned} \dot{x}(t) &= x(t) (r_1(t) - b_1(t) x(t)), \quad t \neq \tau_k, \\ x(\tau_k^+) &= (1 + h_k) x(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (9)$$

We have the following results.

Lemma 4 (see [30]). (1) If

$$\sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t)] > 0, \quad (A_1)$$

then system (9) has a unique T -periodic solution $x^*(t)$, which is globally asymptotically stable.

(2) If

$$\sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t)] \leq 0, \quad (A_2)$$

then $x^*(t) \equiv 0$.

Consider a periodic Logistic equation with impulses:

$$\begin{aligned} \dot{x}(t) &= x(t) (b(t) - a(t) (x(t))^\theta), \quad t \neq \tau_k, \\ x(\tau_k^+) &= (1 + h_k') x(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots, \end{aligned} \quad (10)$$

where θ is a positive constant, $a(t)$ and $b(t)$ are continuous T -periodic functions with $a(t) > 0$ and $m[b(t)] > 0$, and $h'_{k+q} = h'_k$, $\tau_{k+q} = \tau_k + T$. We have the following results.

Lemma 5 (see [31]). (1) If

$$\sum_{k=1}^q \ln(1 + h'_k) + Tm[b] > 0, \quad (11)$$

then system (10) has a unique T -periodic solution $\widetilde{x^*}(t)$, which is globally asymptotically stable.

(2) If

$$\sum_{k=1}^q \ln(1 + h'_k) + Tm[b(t)] < 0, \quad (12)$$

then $\widetilde{x^*}(t) \equiv 0$.

Lemma 6 (see [31]). Let $x^*(t)$ be the unique T -periodic positive solution of system (9) and $\widetilde{x^*}_\varepsilon(t)$ the unique T -periodic positive solution of the following system:

$$\begin{aligned} \dot{x}(t) &= x(t) (b_\varepsilon(t) - a(t) (x(t))^\theta), \quad t \neq \tau_k, \\ x(\tau_k^+) &= (1 + h'_k) x(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots, \end{aligned} \quad (13)$$

where $b_\varepsilon(t)$ is the continuous T -periodic function with $m[b_\varepsilon(t)] > 0$ and $\lim_{\varepsilon \rightarrow 0} b_\varepsilon(t) = b(t)$. Then

$$\lim_{\varepsilon \rightarrow 0} |\widetilde{x^*}_\varepsilon(t) - x^*(t)| = 0. \quad (14)$$

Furthermore, we consider the following periodic logistic equation with impulses:

$$\begin{aligned} \dot{v}(t) &= v(t) \left(r_2(t) - c_2(t) \frac{x^*(t)}{k + x^*(t)} v^{1-m}(t) \right), \quad t \neq \tau_k, \\ v(\tau_k^+) &= \frac{1}{1 + g_k} v(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (15)$$

By Lemma 5, we have the following Corollary.

Corollary 7. If

$$-\sum_{k=1}^q \ln(1 + g_k) + Tm[r_2(t)] > 0, \quad (H_1)$$

then system (15) has a unique T -periodic solution $\widetilde{v^*}(t)$, which is globally asymptotically stable.

Next, one considers the following periodic logistic equation with impulses:

$$\begin{aligned} \dot{x}(t) &= x(t) \left(r_1(t) - \frac{c_1(t)}{k} \widetilde{y^*}(t)^m - b_1(t) x(t) \right), \quad t \neq \tau_k, \\ x(\tau_k^+) &= (1 + h_k) x(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (16)$$

By Lemma 5, we have the following Corollary.

Corollary 8. If

$$\sum_{k=1}^q \ln(1 + h_k) + Tm \left[r_1(t) - \frac{c_1(t)}{k} \widetilde{y^*}(t)^m \right] > 0, \quad (H_2)$$

then system (16) has a unique T -periodic solution $x_*(t)$, which is globally asymptotically stable.

3. Main Results

In this section, we will study the permanence, extinction, and global attractivity of system (3) and the existence and uniqueness of positive periodic solution.

Lemma 9. *For any solution $(x(t), y(t))^T$ of system (3), if $x(0^+) > 0$, $y(0^+) > 0$, then $x(t) > 0$ and $y(t) > 0$ for all $t \geq 0$.*

Proof. From the first equation of (3), one has

$$\dot{x}(t) = p(t)x(t), \quad t \neq \tau_k, \quad (17)$$

where

$$p(t) = r_1(t) - b_1(t)x(t) - \frac{c_1(t)}{k+x(t)}y^m(t); \quad (18)$$

thus, from $x(0^+) > 0$, we have

$$x(t) = \prod_{0 < \tau_k < t} (1 + h_k) x(0^+) \exp\left(\int_0^t p(s) ds\right) > 0. \quad (19)$$

Similarly, it follows from the second equation of system (3) that $y(t) > 0$. This completes the proof of Lemma 9. $\square$

Lemma 10. *Assuming that (A_1) and (H_1) hold, for any positive solution $(x(t), y(t))^T$ of system (3), there exist constants M_i ($i = 1, 2$) such that*

$$\limsup_{t \rightarrow +\infty} x(t) \leq M_1, \quad \limsup_{t \rightarrow +\infty} y(t) \leq M_2. \quad (20)$$

Proof. First, we show that $x(t)$ is ultimate upper bounded. It follows from (3) that

$$\begin{aligned} \dot{x}(t) &\leq x(t)(r_1(t) - b_1(t)x(t)), \quad t \neq \tau_k, \\ x(\tau_k^+) &= (1 + h_k)x(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (21)$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{z}(t) &= z(t)(r_1(t) - b_1(t)z(t)), \quad t \neq \tau_k, \\ z(\tau_k^+) &= (1 + h_k)z(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (22)$$

By Lemma 3, we have $x(t) \leq z(t)$, where $z(t)$ is the solution of (22) with $z(0^+) = x(0^+)$. By (A_1) , from Lemma 4 we get that system (22) has a unique T -periodic solution $x^*(t)$, which is globally stable. Therefore, for any $\varepsilon > 0$ being small enough, there exists a $T_1 > 0$ such that

$$|z(t) - x^*(t)| < \varepsilon, \quad (23)$$

for any $t > T_1$. Let $M_1 = \sup\{x^*(t) : t \in [0, T]\}$; we have

$$x(t) \leq z(t) < x^*(t) + \varepsilon \leq M_1 + \varepsilon \quad (24)$$

for any $t > T_1$. Let $\varepsilon \rightarrow 0$; we obtain

$$\limsup_{t \rightarrow +\infty} x(t) \leq M_1. \quad (25)$$

Next, we prove that $y(t)$ is ultimately upper bounded. Define $y(t) = 1/u(t)$. From system (3), we have

$$\begin{aligned} \dot{u}(t) &= u(t) \left(r_2(t) + b_2(t) \frac{1}{u(t)} \right. \\ &\quad \left. - c_2(t) \frac{x(t)}{k+x(t)} u^{1-m}(t) \right), \quad t \neq \tau_k, \end{aligned} \quad (26)$$

$$u(\tau_k^+) = \frac{1}{1+g_k} u(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots$$

For any $t > T_1$ and $t \neq \tau_k$ ($k = 1, 2, \dots$), we have

$$\begin{aligned} \dot{u}(t) &\geq u(t) \left(r_2(t) - c_2(t) \frac{x(t)}{k+x(t)} u^{1-m}(t) \right) \\ &\geq u(t) \left(r_2(t) - c_2(t) \frac{x^*(t) + \varepsilon}{k+x^*(t) + \varepsilon} u^{1-m}(t) \right). \end{aligned} \quad (27)$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{\psi}(t) &= \psi(t) \left(r_2(t) - c_2(t) \frac{x^*(t) + \varepsilon}{k+x^*(t) + \varepsilon} \psi^{1-m}(t) \right), \\ t &\neq \tau_k, \end{aligned} \quad (28)$$

$$\psi(\tau_k^+) = \frac{1}{1+g_k} \psi(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots$$

By Lemma 3, for any $t > T_1$, we have $u(t) \geq \psi(t)$, where $\psi(t)$ is the solution of (28) with $\psi(T^+) = u(T^+)$. By (H_1) , from Lemma 5, system (28) has a unique T -periodic solution $\widetilde{v}_\varepsilon^*(t)$, which is globally stable. Therefore, letting $\varepsilon \rightarrow 0$, from Lemma 6, we have $\widetilde{v}_\varepsilon^*(t) \rightarrow \widetilde{v}^*(t)$. Letting $M_2^* = \inf\{v^*(t) : t \in [0, T]\}$, for any $\varepsilon_0 > 0$ being small enough, there exists $T_2 > T_1$, and we have

$$u(t) = \frac{1}{y(t)} \geq \psi(t) > \widetilde{v}^*(t) - \varepsilon_0 \geq M_2^* - \varepsilon_0 \quad (29)$$

for any $t > T_2$. Let $\varepsilon_0 \rightarrow 0$; we have

$$\limsup_{t \rightarrow +\infty} y(t) \leq \frac{1}{M_2^*} \triangleq M_2. \quad (30)$$

This completes the proof of Lemma 10. $\square$

Theorem 11. *Assume that (H_1) and (H_2) hold, and*

$$\sum_{k=1}^q \ln(1 + g_k) + Tm \left[-r_2(t) + \frac{c_2(t)x_*(t)}{k+x_*(t)} \widetilde{y}^{m-1}(t) \right] > 0, \quad (H_3)$$

$$m \left[r_1(t) - \frac{c_1(t)}{k} (\widetilde{y}^*(t))^m \right] > 0, \quad (H_4)$$

$$m \left[-r_2(t) + \frac{c_2(t)x_*(t)}{k+x_*(t)} (\widetilde{y}^*(t))^{m-1} \right] > 0, \quad (H_5)$$

where $x_*(t)$ is the unique T -periodic solution of system (16) and $\widetilde{y}^*(t) \triangleq 1/\widetilde{v}^*(t)$. Then species x and y are permanent.

Proof. From Lemma 9, there exist constants M_i ($i = 1, 2$) such that

$$\limsup_{t \rightarrow +\infty} x(t) \leq M_1, \quad \limsup_{t \rightarrow +\infty} y(t) \leq M_2, \quad i = 1, 2. \quad (31)$$

From the proof of Lemma 10, for above $\varepsilon > 0$ being small enough, there exists $T_2 > 0$ such that

$$y(t) \leq \frac{1}{\nu^*(t) - \varepsilon} \triangleq \widetilde{y}_\varepsilon^*(t) \quad (32)$$

for any $t > T_2$. From the first equation of system (3), for $t > T_2$ and $t \neq \tau_k$, we have

$$\begin{aligned} \dot{x}(t) &\geq x(t) \left(r_1(t) - \frac{c_1(t)}{k + x(t)} \widetilde{y}_\varepsilon^*(t)^m - b_1(t) x(t) \right), \\ &\geq x(t) \left(r_1(t) - \frac{c_1(t)}{k} \widetilde{y}_\varepsilon^*(t)^m - b_1(t) x(t) \right). \end{aligned} \quad (33)$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{\phi}(t) &= \phi(t) \left(r_1(t) - \frac{c_1(t)}{k} \widetilde{y}_\varepsilon^*(t)^m - b_1(t) \phi(t) \right), \quad t \neq \tau_k, \\ \phi(\tau_k^+) &= (1 + h_k) \phi(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots, \end{aligned} \quad (34)$$

where $\phi(t)$ is the solution of (34) with $\phi(T_2^+) = x(T_2^+)$. By Lemma 3, for any $t > T_2$, we have $x(t) \geq \phi(t)$. From condition (H_2) , we have

$$\sum_{k=1}^q \ln(1 + h_k) + Tm \left[r_1(t) - \frac{c_1(t)}{k} (\widetilde{y}_\varepsilon^*(t))^m \right] > 0. \quad (35)$$

From inequality (35) and Lemma 4, system (34) has a unique T -periodic solution $x_{*\varepsilon}(t)$, which is globally stable. Since condition (H_4) holds, for the above $\varepsilon > 0$ being small enough, we have $m[r_1(t) - (c_1(t)/k)(\widetilde{y}_\varepsilon^*(t))^m] > 0$; letting $\varepsilon \rightarrow 0$, from Lemma 6, we have $x_{*\varepsilon}(t) \rightarrow x_*(t)$. Letting $m_1 = \inf\{x_*(t) : t \in [0, T]\}$, for the above ε being small enough, there exists $T_3 > T_2$ such that $x(t) \geq \phi(t) \geq x_*(t) - \varepsilon > m_1 - \varepsilon$, for any $t > T_3$. Let $\varepsilon \rightarrow 0$; we have $\lim_{t \rightarrow +\infty} \inf x(t) \geq m_1$. From system (3), for $t > T_3$ and $t \neq \tau_k$, we have

$$\begin{aligned} \dot{y}(t) &\geq y(t) \left(-r_2(t) - b_2(t) y(t) \right. \\ &\quad \left. + \frac{c_2(t)(x_*(t) - \varepsilon)}{k + (x_*(t) - \varepsilon)} (\widetilde{y}_\varepsilon^*(t))^{m-1} \right). \end{aligned} \quad (36)$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{w}(t) &= w(t) \left(-r_2(t) - b_2(t) w(t) \right. \\ &\quad \left. + \frac{c_2(t)(x_*(t) - \varepsilon)}{k + (x_*(t) - \varepsilon)} (\widetilde{y}_\varepsilon^*(t))^{m-1} \right), \quad t \neq \tau_k, \\ w(\tau_k^+) &= (1 + g_k) w(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots, \end{aligned} \quad (37)$$

where $w(t)$ is the solution of (37) with $w(T_3^+) = y(T_3^+)$. By Lemma 3, for all $t > T_3$, we have $y(t) \geq w(t)$, for the above $\varepsilon > 0$ being small enough. By condition (H_3) , we obtain

$$\begin{aligned} \sum_{k=1}^q \ln(1 + g_k) + Tm \left[-r_2 + \frac{c_2(t)(x_*(t) - \varepsilon)}{k + (x_*(t) - \varepsilon)} (\widetilde{y}_\varepsilon^*(t))^{m-1} \right] \\ > 0. \end{aligned} \quad (38)$$

From inequality (38), Lemma 4, system (37) has a unique T -periodic solution $y_{*\varepsilon}(t)$, which is globally asymptotically stable. Since condition (H_5) holds, for the above $\varepsilon > 0$ being small enough, such that $m[-r_2(t) + ((c_2(t)(x_*(t) - \varepsilon))/(k + (x_*(t) - \varepsilon)))(\widetilde{y}_\varepsilon^*(t))^{m-1}] > 0$, let $\varepsilon \rightarrow 0$, and from Lemma 6, we have $y_{*\varepsilon}(t) \rightarrow y_*(t)$, where $y_*(t)$ is the unique T -periodic globally asymptotically stable solution of the following system:

$$\begin{aligned} \dot{p}(t) &= p(t) \left(-r_2(t) - b_2(t) p(t) \right. \\ &\quad \left. + \frac{c_2(t)x_*(t)}{k + x_*(t)} (\widetilde{y}_\varepsilon^*(t))^{m-1} \right), \quad t \neq \tau_k, \\ p(\tau_k^+) &= (1 + g_k) p(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots. \end{aligned} \quad (39)$$

Letting $m_2 = \inf\{y_*(t) : t \in [0, T]\}$, for the above $\varepsilon > 0$ being small enough, there exists $T_4 > T_3$ such that $y(t) \geq w(t) > y_*(t) - \varepsilon$, for any $t > T_4$. Let $\varepsilon \rightarrow 0$; we have $\lim_{t \rightarrow +\infty} y(t) \geq m_2$. $\square$

Theorem 12. Assume that

$$\sum_{k=1}^q \ln(1 + g_k) + Tm \left[-r_2(t) - \frac{c_2(t)x^*(t)}{k + x^*(t)} (y_*(t))^{m-1} \right] < 0, \quad (H_6)$$

$$\sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t)] > 0. \quad (H_7)$$

Then, for any positive solution $(x(t), y(t))^T$ of system (3), one has

$$\lim_{t \rightarrow +\infty} y(t) = 0, \quad \lim_{t \rightarrow +\infty} |x(t) - x^*(t)| = 0, \quad (40)$$

where $x^*(t)$ and $y_*(t)$ are unique T -periodic solutions of systems (9) and (39), respectively.

Proof. From system (3), for $t > T_4$ and $t \neq \tau_k$, we have

$$\dot{y}(t) \leq y(t) \left(-r_2(t) - b_2(t) y(t) + \frac{c_2(t)x^*(t)}{k + x^*(t)} (y_*(t))^{m-1} \right). \quad (41)$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{\rho}(t) &= \dot{y}(t) \\ &\leq \rho(t) \left(-r_2(t) - b_2(t) \rho(t) \right. \\ &\quad \left. + \frac{c_2(t) x^*(t)}{k + x^*(t)} (\rho_*(t))^{m-1} \right), \quad t \neq \tau_k, \\ \rho(\tau_k^+) &= (1 + g_k) \rho(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (42)$$

From (H₆) and Lemma 4, we have

$$\lim_{t \rightarrow +\infty} y(t) = 0. \quad (43)$$

Therefore, for $\varepsilon > 0$ being small enough, there exists $T_5 > 0$ such that $y(t) < \varepsilon$ for any $t > T_5$. Since (H₇) holds, we obtain

$$\sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t) - c_1(t)\varepsilon^m] > 0. \quad (44)$$

Thus, from the first equation of system (3), we have

$$\begin{aligned} x(t)(r_1(t) - b_1(t)x(t) - c_1(t)\varepsilon^m) \\ \leq \dot{x}(t) \leq x(t)(r_1(t) - b_1(t)x(t)) \end{aligned} \quad (45)$$

for any $t > T_5$. Consider the following auxiliary system

$$\begin{aligned} \dot{\varphi}(t) &= \varphi(t)(r_1(t) - b_1(t)\varphi(t) - c_1(t)\varepsilon^m), \quad t \neq t_k, \\ \varphi(\tau_k^+) &= (1 + h_k)\varphi(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots, \\ \dot{\sigma}(t) &= \sigma(t)(r_1(t) - b_1(t)\sigma(t)), \quad t \neq t_k, \\ \sigma(\tau_k^+) &= (1 + h_k)\sigma(\tau_k), \quad t = \tau_k, \quad k = 1, 2, \dots \end{aligned} \quad (46)$$

From (H₇) and Lemma 4, system (46) has unique T -periodic solutions $\varphi_\varepsilon^*(t)$ and $\sigma^*(t)$, respectively, which are globally asymptotically stable. By Lemma 3, for any $\varepsilon_1 > 0$ being small enough, there exists $T_6 > 0$ such that $\varphi_\varepsilon^*(t) - \varepsilon_1 \leq x(t) \leq \sigma^*(t) + \varepsilon_1$ for any $t > T_6$; letting $\varepsilon \rightarrow 0$, we have $x^*(t) - \varepsilon_1 \leq x(t) \leq x^*(t) + \varepsilon_1$; since ε_1 is arbitrarily small, we have $\lim_{t \rightarrow +\infty} |x(t) - x^*(t)| = 0$. $\square$

Theorem 13. Suppose that (H₆) holds and

$$\sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t)] < 0. \quad (H_8)$$

Then species x and y are extinct; that is,

$$\lim_{t \rightarrow \infty} x(t) = 0, \quad \lim_{t \rightarrow \infty} y(t) = 0. \quad (47)$$

Proof. From (H₈) and Lemma 4, system (9) has a unique T -periodic solution $x^*(t)$, and $x^*(t) = 0$. By Lemma 3, we have $\lim_{t \rightarrow \infty} x(t) \equiv 0$; therefore, $\lim_{t \rightarrow \infty} y(t) = 0$. $\square$

Theorem 14. Suppose that all the conditions of Theorem 11 hold and there exist positive constants α and δ such that

$$\min_{t \in [0, T]} \left\{ \alpha b_1(t) - \frac{\alpha c_1(t)(M_2)^m}{(k + m_1)^2} - \frac{\delta c_2(t)k(m_2)^{m-1}}{(k + m_1)^2} \right\} > 0, \quad (H_9)$$

$$\begin{aligned} \min_{t \in [0, T]} \left\{ \delta b_2(t) - \frac{\alpha c_1(t)m(m_2)^{m-1}}{k + m_1} \right. \\ \left. - \frac{\delta c_2(t)M_1(m-1)(m_2)^{m-2}}{k + m_1} \right\} > 0. \end{aligned} \quad (H_{10})$$

Then system (3) has a unique positive T -periodic solution which is globally asymptotically stable.

Proof. Let $(x(t), y(t))^T$ and $(\bar{x}(t), \bar{y}(t))^T$ be any two positive solutions of system (3); for any sufficiently small $\varepsilon > 0$, from Theorem 11, there is a large enough $T_4 > 0$ such that for any $t > T_4$, we have

$$\begin{aligned} m_1 - \varepsilon &\leq x(t), & \bar{x}(t) &\leq M_1 + \varepsilon, \\ m_2 - \varepsilon &\leq y(t), & \bar{y}(t) &\leq M_2 + \varepsilon. \end{aligned} \quad (48)$$

Define the Lyapunov function as follows:

$$V(t) = \alpha |\ln x(t) - \ln \bar{x}(t)| + \delta |\ln y(t) - \ln \bar{y}(t)|. \quad (49)$$

By the mean value theorem and (48), for any $t \in (\tau_k, \tau_{k+1}]$, $k = p, p+1, \dots$, and $\tau_p > T_4$, we have

$$\begin{aligned} \frac{1}{M_1 + \varepsilon} |x(t) - \bar{x}(t)| &\leq |\ln x(t) - \ln \bar{x}(t)| \leq |x(t) - \bar{x}(t)|, \\ \frac{1}{M_2 + \varepsilon} |y(t) - \bar{y}(t)| &\leq |\ln y(t) - \ln \bar{y}(t)| \leq |y(t) - \bar{y}(t)|, \\ \frac{1}{k + \bar{x}(t)} - \frac{1}{k + x(t)} &= \frac{1}{(k + \xi_{11}(t))^2} (\bar{x}(t) - x(t)), \\ \frac{x(t)}{k + x(t)} - \frac{\bar{x}(t)}{k + \bar{x}(t)} &= \frac{k}{(k + \xi_{12}(t))^2} (x(t) - \bar{x}(t)), \\ (\bar{y}(t))^m - (y(t))^m &= m(\xi_{21}(t))^{m-1} (\bar{y}(t) - y(t)), \\ (y(t))^{m-1} - (\bar{y}(t))^{m-1} &= (m-1)(\xi_{22}(t))^{m-2} (y(t) - \bar{y}(t)), \end{aligned} \quad (50)$$

where ξ_{1i} is suited between $x(t)$ and $\bar{x}(t)$, ξ_{2i} is suited between $y(t)$ and $\bar{y}(t)$, and $i = 1, 2$. For $t > T_4$, $t \neq \tau_k$ ($k = 1, 2, \dots$)

and the above small enough $\varepsilon > 0$; from conditions (H_9) and (H_{10}) , there exist positive constants η and θ such that

$$\begin{aligned} \min_{t \in [0, T]} \left\{ \alpha b_1(t) - \frac{\alpha c_1(t) (M_2 + \varepsilon)^m}{(k + m_1 - \varepsilon)^2} \right. \\ \left. - \frac{\delta c_2(t) k (m_2 - \varepsilon)^{m-1}}{(k + m_1 - \varepsilon)^2} \right\} > \eta, \\ \min_{t \in [0, T]} \left\{ \delta b_2(t) - \frac{\alpha c_1(t) (m) (m_2 - \varepsilon)^{m-1}}{k + m_1 - \varepsilon} \right. \\ \left. - \frac{\delta c_2(t) (M_1 + \varepsilon) (m-1) (m_2 - \varepsilon)^{m-2}}{k + m_1 + \varepsilon} \right\} > \eta, \\ \eta \min \left\{ \frac{m_1 - \varepsilon}{\alpha}, \frac{m_2 - \varepsilon}{\delta} \right\} > \theta. \end{aligned} \quad (51)$$

Calculating the upper right derivative of $V(t)$, we obtain

$$\begin{aligned} D^+ V(t) &= \alpha \operatorname{sgn}(x(t) - \bar{x}(t)) \left(\frac{\dot{x}(t)}{x(t)} - \frac{\dot{\bar{x}}(t)}{\bar{x}(t)} \right) \\ &\quad + \delta \operatorname{sgn}(y(t) - \bar{y}(t)) \left(\frac{\dot{y}(t)}{y(t)} - \frac{\dot{\bar{y}}(t)}{\bar{y}(t)} \right) \\ &= \alpha \operatorname{sgn}(x(t) - \bar{x}(t)) \\ &\quad \times \left[-b_1(t) (x(t) - \bar{x}(t)) \right. \\ &\quad \left. - c_1(t) \left(\frac{(y(t))^m}{k + x(t)} - \frac{(\bar{y}(t))^m}{k + \bar{x}(t)} \right) \right] \\ &\quad + \delta \operatorname{sgn}(y(t) - \bar{y}(t)) \\ &\quad \times \left[-b_2(t) (y(t) - \bar{y}(t)) \right. \\ &\quad \left. - c_2(t) \left(\frac{x(t) (y(t))^{m-1}}{k + x(t)} \right. \right. \\ &\quad \left. \left. - \frac{\bar{x}(t) (\bar{y}(t))^{m-1}}{k + \bar{x}(t)} \right) \right] \\ &= -\alpha b_1(t) |x(t) - \bar{x}(t)| - \delta b_2(t) |y(t) - \bar{y}(t)| \\ &\quad + \alpha c_1(t) \operatorname{sgn}(x(t) - \bar{x}(t)) \\ &\quad \times \left(\frac{(\bar{y}(t))^m}{k + \bar{x}(t)} - \frac{(y(t))^m}{k + x(t)} \right) \\ &\quad + \delta c_2(t) \operatorname{sgn}(y(t) - \bar{y}(t)) \\ &\quad \times \left(\frac{x(t) (y(t))^{m-1}}{k + x(t)} - \frac{\bar{x}(t) (\bar{y}(t))^{m-1}}{k + \bar{x}(t)} \right). \end{aligned} \quad (52)$$

In view of

$$\begin{aligned} c_1(t) \operatorname{sgn}(x(t) - \bar{x}(t)) &\left(\frac{(\bar{y}(t))^m}{k + \bar{x}(t)} - \frac{(y(t))^m}{k + x(t)} \right) \\ &= c_1(t) \operatorname{sgn}(x(t) - \bar{x}(t)) \\ &\quad \times \left[\frac{(\bar{y}(t))^m}{k + \bar{x}(t)} - \frac{(\bar{y}(t))^m}{k + x(t)} + \frac{(\bar{y}(t))^m}{k + x(t)} + \frac{(y(t))^m}{k + x(t)} \right] \\ &= c_1(t) \operatorname{sgn}(x(t) - \bar{x}(t)) (\bar{y}(t))^m \\ &\quad \times \left[\frac{1}{(k + \xi_{11})^2} (\bar{x}(t) - x(t)) \right. \\ &\quad \left. + \frac{m(\xi_{21}(t))^{m-1}}{k + x(t)} (\bar{y}(t) - y(t)) \right], \end{aligned} \quad (53)$$

$$\begin{aligned} c_2(t) \operatorname{sgn}(y(t) - \bar{y}(t)) &\left(\frac{x(t) (y(t))^{m-1}}{k + x(t)} - \frac{\bar{x}(t) (\bar{y}(t))^{m-1}}{k + \bar{x}(t)} \right) \\ &= c_2(t) \operatorname{sgn}(y(t) - \bar{y}(t)) \\ &\quad \times \left[\frac{x(t) (y(t))^{m-1}}{k + x_1(t)} - \frac{x(t) (\bar{y}(t))^{m-1}}{k + x(t)} \right. \\ &\quad \left. + \frac{x(t) (\bar{y}(t))^{m-1}}{k + x(t)} - \frac{\bar{x}(t) (\bar{y}(t))^{m-1}}{k + \bar{x}(t)} \right] \\ &= c_2(t) \operatorname{sgn}(y(t) - \bar{y}(t)) \\ &\quad \times \left[\frac{x(t) (m-1) (\xi_{22}(t))^{m-2}}{k + x(t)} (y(t) - \bar{y}(t)) \right. \\ &\quad \left. + \frac{k}{(k + \xi_{12}(t))^2} (\bar{y}(t))^{m-1} (x(t) - \bar{x}(t)) \right], \end{aligned} \quad (54)$$

substituting (53) and (54) into (52), we have

$$\begin{aligned} D^+ V(t) &\leq - \left(\alpha b_1(t) - \alpha c_1(t) \frac{(\bar{y}(t))^m}{(k + \xi_{11})^2(t)} \right. \\ &\quad \left. - \frac{\delta c_2(t) k (\bar{y}(t))^{m-1}}{(k + \xi_{12}^2(t))} \right) \\ &\quad \times |x(t) - \bar{x}(t)| \\ &\quad - \left[\delta b_2(t) - \frac{\alpha c_1(t) (\bar{y}(t))^m m (\xi_{21}^{m-1}(t))}{k + x} \right. \\ &\quad \left. - \frac{\delta c_2(t) x(t) (m-1) (\xi_{22}^{m-2}(t))}{k + x} \right] \end{aligned}$$

TABLE 2: The effects of h_k and g_k on the dynamical behavior of system (3).

Case	h_k	g_k	x	y	Figure
1	0.2	0.03	Permanence	Permanence	Figure 1
2	-0.2	-0.6	Permanence	Extinction	Figure 2

FIGURE 1: Time series of $x(t)$ and $y(t)$. FIGURE 2: Time series of $x(t)$ and $y(t)$.

$$\begin{aligned}
& \times |y(t) - \bar{y}(t)| \\
& \leq - \left[\alpha b_1(t) - \alpha c_1(t) \frac{(M_2 + \varepsilon)^m}{(k + m_1 - \varepsilon)^2} \right. \\
& \quad \left. - \frac{\delta c_2(t) k (m_2 - \varepsilon)^{m-1}}{(k + m_1 - \varepsilon)^2} \right] |x(t) - \bar{x}(t)| \\
& \quad - \left[\delta b_2(t) - \frac{\alpha c_1(t) (M_2 + \varepsilon)^m m (m_2 - \varepsilon)^{m-1}}{k + m_1 - \varepsilon} \right. \\
& \quad \left. - \frac{\delta c_2(t) (M_1 + \varepsilon) (m-1) (m_2 - \varepsilon)^{m-2}}{k + m_1 - \varepsilon} \right] \\
& \quad \times |y(t) - \bar{y}(t)| \\
& \leq -\eta \left(\frac{m_1 - \varepsilon}{\alpha} \alpha |\ln x(t) - \ln \bar{x}(t)| \right. \\
& \quad \left. + \frac{m_2 - \varepsilon}{\delta} \delta |\ln y(t) - \ln \bar{y}(t)| \right). \tag{55}
\end{aligned}$$

For all $t \in R_+$ and $t = \tau_k^+$ ($k = 1, 2, \dots$), we have

$$\begin{aligned}
\dot{V}(\tau_k^+) &= \alpha |\ln x(\tau_k^+) - \ln \bar{x}(\tau_k^+)| + \delta |\ln y(\tau_k^+) - \ln \bar{y}(\tau_k^+)| \\
&= \alpha |\ln(1 + h_k)x(\tau_k) - \ln(1 + h_k)\bar{x}(\tau_k)|
\end{aligned}$$

TABLE 3: The effects of h_k on the dynamical behavior of system (3).

Case	h_k	g_k	Figure
—	Bifurcation parameter	0.3	Figure 3(a)
1	-0.4	0.3	Figure 3(b)
2	0	0.3	Figure 3(c)
3	0.6	0.3	Figure 3(d)

$$\begin{aligned}
& + \delta |\ln(1 + g_k)y(\tau_k) - \ln(1 + g_k)\bar{y}(\tau_k)| \\
& = V(\tau_k). \tag{56}
\end{aligned}$$

Hence, $V(t)$ is continuous for all $t > \tau_p > T_4$. The above analysis shows that, for all $t > \tau_p > T_4$,

$$D^+V(t) \leq -\theta V(t), \tag{57}$$

and we further have

$$V(t) \leq V(\tau_p) \exp(-\theta(t - \tau_p)). \tag{58}$$

It is obvious that $V(t) \rightarrow 0$ as $t \rightarrow +\infty$; that is,

$$\begin{aligned}
\lim_{t \rightarrow +\infty} |\ln x(t) - \ln \bar{x}(t)| &= 0, \\
\lim_{t \rightarrow +\infty} |\ln y(t) - \ln \bar{y}(t)| &= 0. \tag{59}
\end{aligned}$$

We further have

$$\lim_{t \rightarrow +\infty} |x(t) - \bar{x}(t)| = 0, \quad \lim_{t \rightarrow +\infty} |y(t) - \bar{y}(t)| = 0. \tag{60}$$

FIGURE 3: (a) Bifurcation diagram of h_k when g_k is fixed at 0.3; (b)–(d) phase portraits.

Choose constants $A = \min\{m_1, m_2\}$ and $B = \max\{M_1, M_2\}$. By Theorem 11, we have

$$A \leq x(t), y(t), \bar{x}(t), \bar{y}(t) \leq B, \quad \forall t \geq T_4. \quad (61)$$

Now let us consider the $(\bar{x}(mT, z_0), \bar{y}(mT, z_0)) = z(mT, z_0)$, $m = 1, 2, \dots$, and $z_0 = (\bar{x}(0), \bar{y}(0))$. It is compact in the domain $[A, B]^2$, since $A \leq \bar{x}(t)$ and $\bar{y}(t) \leq B$ for all $t \geq T_4$. Let $\bar{z}$ be a limit point of this sequence $\bar{z} = \lim_{n \rightarrow \infty} z(m_n T, z_0)$; then we have $z(T, \bar{z}) = \bar{z}$. In fact, since $z(T, z(m_n T, z_0)) = z(m_n T, z(T, z_0))$ and $z(m_n T, z(T, z_0)) - z(m_n T, z_0) \rightarrow 0$ as $m_n \rightarrow \infty$, we have

$$\begin{aligned} & \|z(T, \bar{z}) - \bar{z}\|_{[A, B]^2} \\ & \leq \|z(T, \bar{z}) - z(T, z(m_n T, z_0))\|_{[A, B]^2} \\ & \quad + \|z(T, z(m_n T, z_0)) - z(m_n T, z_0)\|_{[A, B]^2} \\ & \quad + \|z(m_n T, z_0) - \bar{z}\|_{[A, B]^2} \rightarrow 0. \end{aligned} \quad (62)$$

The sequence $z(mT, z_0)$ ($m = 1, 2, \dots$) has a unique limit point; otherwise, let the sequence have two limit points $\bar{z} =$

$\lim_{n \rightarrow \infty} z(m_n T, z_0)$ and $\tilde{z} = \lim_{n \rightarrow \infty} z(m_n T, z_0)$. Then, taking into account (60) and $\tilde{z} = z(m_n T, \tilde{z})$, we have

$$\begin{aligned} \|\bar{z} - \tilde{z}\|_{[A, B]^2} & \leq \|\bar{z} - z(m_n T, z_0)\|_{[A, B]^2} \\ & \quad + \|z(m_n T, z_0) - \tilde{z}\|_{[A, B]^2} \\ & \rightarrow 0, \quad n \rightarrow \infty, \end{aligned} \quad (63)$$

and hence $\bar{z} = \tilde{z}$; the solution $(\bar{x}(t, x_0), \bar{y}(t, y_0))$ is the unique periodic solution of system (3). By (60), it is globally asymptotically stable. This completes the proof of Theorem 14. $\square$

4. Numerical Simulations

In system (3), we take $T = 1, q = 1$, which means that system (3) is a one-period system with one impulse perturbation during one period.

FIGURE 4: (a) Bifurcation of g_k when h_k is fixed at 0.4; (b)–(d) phase portraits.TABLE 4: The effects of g_k on the dynamical behavior of system (3).

Case	h_k	g_k	Figure
—	0.4	Bifurcation parameter	Figure 4(a)
1	0.4	0.1	Figure 4(b)
2	0.4	0.3	Figure 4(c)
3	0.4	0.4	Figure 4(d)

In case 1, (H_1) – (H_5) are, respectively, equivalent to the following inequalities:

$$-\sum_{k=1}^q \ln(1 + g_k) + Tm[r_2(t)] = 0.0004 > 0,$$

$$\sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t) - c_1(t)(\widetilde{y}^*(t))^m] > 1.143 > 0,$$

$$\begin{aligned} & \sum_{k=1}^q \ln(1 + g_k) + Tm \left[-r_2(t) + \frac{c_2(t)x_*(t)}{k + x_*(t)} \widetilde{y}^*(t)^{m-1} \right] \\ & > 0.0536 > 0. \\ & m[r_1(t) - c_1(t)(\widetilde{y}^*(t))^m] > 0.961 > 0, \\ & m \left[-r_2(t) + \frac{c_2(t)x_*(t)}{k + x_*(t)} \widetilde{y}^*(t)^{m-1} \right] > 0.024 > 0. \end{aligned} \tag{64}$$

Taking $\alpha = 1.5$, $\delta = 1$, conditions (H_9) and (H_{10}) are, respectively, equivalent to the following inequalities:

$$\begin{aligned} & \min_{t \in [0, T]} \left\{ \alpha b_1(t) - \frac{\alpha c_1(t)(M_2)^m}{(k + m_1)^2} - \frac{\delta c_2(t)k(m_2)^{m-1}}{(k + m_1)^2} \right\} \\ & \approx 1.221 > 0, \\ & \min_{t \in [0, T]} \left\{ \delta b_2(t) - \frac{\alpha c_1(t)(m)(m_2)^{m-1}}{k + m_1} \right\} \end{aligned}$$

TABLE 5: The effects of impulse period T on the dynamical behavior of system (3).

Case	T	x	y	Figure
1	1	Periodic	Periodic	Figures 5(a) and 5(b)
2	1.002	Almost periodic	Almost periodic	Figure 5(c)
3	1.014	Chaotic	Chaotic	Figure 5(d)

FIGURE 5: Dynamical behavior of system (3); here, we take initial value $(x(0), y(0)) = (0.721, 0.156)$.

$$\begin{aligned}
 & - \frac{\delta c_2(t) M_1 (m-1) (m_2)^{m-2}}{k + m_1} \Big\} \\
 & \simeq 0.5498 > 0.
 \end{aligned}$$

(65)

By calculation, conditions (H_1) – (H_5) , (H_9) , and (H_{10}) hold; from Theorem 11 and Theorem 14, system (3) is globally asymptotically stable. Numerical simulations of these results can be seen in Figure 1.

Figure 1 shows the dynamical behavior of the solutions $(x(t), y(t))$ with seven group initial values $(x(0), y(0)) =$

(0.5, 0.1), (0.6, 0.2), (0.7, 0.3), (0.8, 0.4), (0.9, 0.5), (1.0, 0.6), and (1.1, 0.7), respectively. From Figure 1, we can obtain that there exists a unique positive periodic solution of system (3) such that any solution of system (3) with initial value $(x(0), y(0))$ tends to the positive periodic solution as $t \rightarrow \infty$.

In case 2, conditions (H_6) and (H_7) are, respectively, equivalent to the following inequalities:

$$\begin{aligned} \sum_{k=1}^q \ln(1 + g_k) + Tm \left[-r_2(t) - \frac{c_2(t)x^*(t)}{k + x^*(t)} (y_*(t))^{m-1} \right] \\ = -0.9463 < 0, \\ \sum_{k=1}^q \ln(1 + h_k) + Tm[r_1(t)] = 0.7769 > 0. \end{aligned} \quad (66)$$

By calculation, conditions (H_6) and (H_7) hold. From Theorem 12, we have that species y will be extinct and species x will be globally asymptotically stable. Numerical simulations of these results are plotted in Figure 2.

Figure 2 shows the dynamical behavior of the solutions $(x(t), y(t))$ with seven group initial values $(x(0), y(0)) = (0.2, 0.2), (0.3, 0.5), (0.4, 0.8), (0.5, 1.1), (0.6, 1.4), (0.7, 1.7),$ and $(0.8, 2.0)$, respectively. From Figure 2, we can see that species y will be extinct and species x will be globally asymptotically stable (Table 2).

Furthermore, we investigate the effects of impulse coefficients h_k and g_k on the dynamical behavior of system (3); we keep the parameter g_k fixed at $g_k = 0.3$. From Table 3, it is found that the impulse coefficient h_k has an important influence on the sizes of prey and predator populations. Therefore, we plot the bifurcation diagram of system (3) when $h_k \in [-0.4, 0.8]$; see Figure 3(a); it is very clear that the number of prey populations increases with increasing parameter h_k .

On the other hand, we keep the parameter h_k fixed at $h_k = 0.4$. For $0.26 \leq g_k \leq 0.36$, the effects of g_k on the dynamical behavior of system (3) have been also investigated. From Table 4, it is found that the impulse coefficient g_k has an important influence on the sizes of prey and predator populations.

Moreover, on increasing the period of impulse perturbations, some interesting phenomena happen; the details are given in Table 5.

5. Conclusion

In this paper, a class of Holling type II predator-prey systems with mutual interference and impulses is studied. Some sufficient conditions for the permanence and extinction of the system and existence and uniqueness of positive periodic solution are obtained. These obtained results are new and the complement to previously known results. From the view point of biology, the mathematical results are full of biological meanings and can be used to provide reliable foundation for making control strategy. The conditions of Theorem 13 show that human activities may save the extinct species living in a weak environment.

From the numerical simulations, we find that the behavior of impulse perturbations of species has an important influence on the system. Besides, the dynamics of species are very sensitive with the period matching between species' intrinsic disciplinarians and the perturbations from the variable environment. If the periods between individual growth and impulse perturbations match well, the dynamics behavior of species periodically changes. If they mismatch each other, the dynamic behaviors would change from period to period until there is chaos. It is very consistent with real ecosystems, which helps us interpret the diversity and many phenomena of ecosystems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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