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Research Article

Options Procurement Policy for Option Contracts with Supply and Spot Market Uncertainty

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Supplier's reliability is a major issue in procurement management. In this paper, we establish a decision making model from the perspective of the firm who will procure from the multiple suppliers and the spot markets. The suppliers are unreliable and provide different types of option-type supply contracts which should be made before demand realization, while the spot market can only be used after demand realization and has both the price and liquidity risks. We establish the optimal portfolio policies for the firm with conditions to find the qualified suppliers. By defining a new function which contains the demand risk, the supplier's risk, and the liquidity risk, we find that the optimal policy is to allocate different curves of this function to different suppliers. We also study some special cases to derive some managerial insights. At last, we numerically study how the various risks affect the choice of suppliers and the value of the option contract.

1. Introduction

Procurement is one of the major elements of a firm's operations management. In most industries, procurement cost usually consists of a large part of the firm's total costs and thus has large influence on the firm's total operation cost and its competitive advantage. To manage the procurement cost, it is a common practice for firms to maintain a list of suppliers with different cost structures to secure their supply. However, the resulting complex procurement structure leads to some complex management problems, for example, how to choose the appropriate suppliers and how to allocate the demand to each of these suppliers. These management problems ask for the practitioners and scholars to develop or provide tools to procurement managers in decision making, to maintain a stable production and a high customer service level under a reasonable procurement cost. In practice, long term contract between a firm and its supplier, with fixed or flexible terms in price or quantity, is still the major way. With such a long term contract, a firm can always secure the quantity and maintains a reasonable procurement cost in the meantime. However, as reported in a global investigation by Mckinsey [1], with the rapid changing of the business environment, the reliability risk of the supplier has also become one of the major issues in the procurement practice and has been paid attention to by the practitioners and scholars. For example, in 2000, Sony's stock price plummets 9% because his supplier cannot supply the LCD and flash memories in time, which leads to Sony's incapability to meet his customer's demand for PlayStation. In 2005, one of the major private-owned coal companies, that is, Peabody Energy, has announced a loss of 3.4$ in his semiannual report because his supplier cannot fulfill the supply contract.

Besides the traditional long term contract, spot market (and B2B market) has played a more and more important role in a firm's procurement strategy as a solution to those mismatches between supply and demand, especially with the development of information technology. It has become a supplement to the traditional long term contract, with its time and quantity flexibility in procurement. Currently, the products traded in the spot market have expanded and include semiconductors, memories, and CPUs besides the

traditional commodities such as oil, metal, and agricultural products. However, spot market has its own deficiency as compared to the long term contract. On the one hand, the price in spot market is volatile. For example, although the price for semiconductors has a decreasing tend in the long term, the short term price increase can arrive to 200% or 300%. On the other hand, the spot market exits liquidity problem, which means that the firm cannot buy the required quantity at the spot price in some times. Although the liquidity issue is not a problem for the liquid market such as the copper and zinc, it is still a problem for iron and paper pulp [2].

To mitigate the procurement risk (i.e., reliability risk, price risk, and liquidity risk) discussed above, HP launches a PRM (proactive risk management) project. The major idea of such project is to segment the demand curve with different risk and then outsource them to the supplier with different cost structures. Our paper is based on the idea proposed by such project. Specifically, we establish a decision making model from the perspective of the firm who will procure from the multiple suppliers and the spot markets. The suppliers are unreliable and provide different types of option-type supply contracts which should be made before demand realization, while the spot market can only be used after demand realization and has both the price and liquidity risks. Under such a decision-making framework, we establish the optimal portfolio policies for the firm with conditions to find the qualified suppliers. Moreover, we show how the various risks affect the choice of suppliers and the value of the option contract. Our contribution has twofold: first, we provide a procurement portfolio policy to a more general environment; second, we provide some insights on how the environment parameters affect the buyer's cost.

The paper is organized as follows. In Section 2, we will review the related literature and position our research among them. In Section 3, we will establish the model and develop the optimal policy. Moreover, we will study two special cases to dig out more insights. In Section 4, the numerical study will be conducted. And last, in Section 5, a conclusion will be made with some discussions for future research.

2. Review of the Literature

One research stream related to our current research is those works on optimal coordination policies with long term contracts, including the quantity flexible contract, price flexible contracts, and time flexible contracts. However, most previous works do not consider the spot market and do not include the supplier's reliability risk. Readers can refer to Cachon [3] for a brief review on these models. Based on these works, Wu et al. [4], Kleindorfer and Wu [5], and Wu and Kleindorfer [6] study the coordination mechanism with the presence of spot market. Specifically, Wu et al. [4] and Kleindorfer and Wu [5] consider the coordination strategy and the options' pricing strategy under two-party tariff contract. Wu and Kleindorfer [6] extend the model to include multiple suppliers and investigate the coordination strategy and the contract's pricing problem. However, different to the model in our paper, in those models, the spot prices are always assumed to be a deterministic function of the demand and, moreover, the suppliers are all reliable.

Another related research stream is those works which consider the optimal procurement policies from the buyer's perspective, to minimize his total procurement cost. Ritchken and Tapiero [7] study the role of the option contracts in the risk management, when the spot market is stochastic. Mattock [8] studies the optimal capacity planning problem when the demand and the spot price are related and proposes an algorithm to solve the corresponding problem. However, these models only consider one supplier. When there are multiple suppliers, Schummer and Vohra [9] consider the optimal procurement policy and find that the problem can be converted to a linear programming problem. They thus designed an incentive compatible auction mechanism for the option-type procurement contract. With a similar framework, Martínez-de-Albéniz and Simichi-Levi [10] study the procurement problems for planning horizon with multiple periods. They derive the optimal procurement policy under the condition that the contract types and numbers in each period are the same. Fu et al. [11] further propose an algorithm for the optimal ordering policy when the stochastic demand and the spot price are correlated. However, all those works neglect the reliability risk and the spot market liquidity risk.

Our current research also contributes to the literature on operations management in the presence of random yield. Yano and Lee [12] and Minner [13] make some conclusions on these works. In general, most of these works only establish the optimal policy under supply risk and the supply chain typically includes only one supplier. With the newsvendor framework, Dada et al. [14] extend previous research to include multiple suppliers with different reliability risks. In their model, the supply risk for each supplier is independent of each other and there are no spot markets. Moreover, the contracts between the suppliers and the buyer are wholesale-price contracts. Our paper considers the random yield problems with multiple suppliers; however, the difference lies in twofold: first, the long term contract we considered is option-type contract; second, the reliability risks for each supplier are totally correlated. Although the second characteristic is strict, it has its own practice background. Moreover, it makes our model more tractable.

3. Model Formulation

We consider a single period procurement model under a supply chain with a buyer and N suppliers (see Figure 1), in which the ith supplier is called supplier i, $i = 1, 2, \ldots, N$. Similar to the settings in Fu et al. [11], the buyer can procure from each of these suppliers before demand realization or can buy from the spot market after demand realization. Thus, we assume the lead time for procurement from suppliers as one and that from the spot market as zero. This is a typical assumption in literature considering both the long term supplier and the spot market. However, different to the models of Fu et al. [11], we consider two more realistic

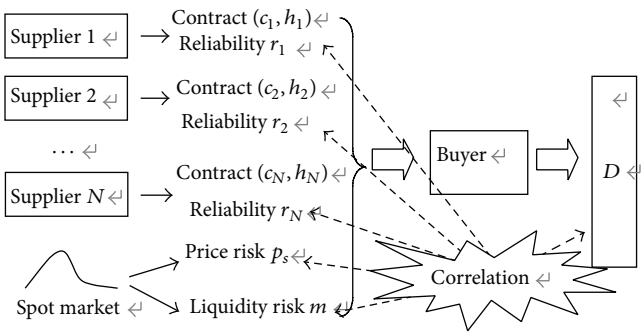

Figure 1: Model description.

settings. First, the procurement from the spot market will face the liquidity risk and thus has uncertainties, even after spot price realization. We denote the probability that the buyer can procure from the spot market at the prevailing spot price by m, $m \in [0, 1]$. The value of m is a stochastic variable. Such probability measures the uncertainty that the buyer can find a proper seller in the spot market. For a discussion on the liquidity risk, readers can refer to Wu and Kleindorfer [6]. Second, the suppliers are unreliable. That is, if the buyer procures Q_i from the ith supplier, he can only get $r_i Q_i$ after demand realization, in which r_i is a random variable realized after demand realization (or equivalently, after products arrive). Such multiplicative type of random yield is also studied in Yano and Lee [12].

Then, the decision sequences of the model are as follows.

(1) At the beginning of the planning horizon, denoted by time $t = 0$, supplier i, $i = 1, 2, \ldots, N$, will provide an option-type supply contract (c_i, h_i) to the buyer. Here, c_i is the unit reservation price which the buyer should pay when making the contract for reserving a fixed quantity, h_i is the unit exercise price which will be paid by the buyer on the actual quantity of the products received. The different pairs of (c_i, h_i) represent the different cost structure and risk attitude of the suppliers. From the perspective of the buyer, these pairs provide him different flexibilities. Thus, the buyer should decide the quantity Q_i to reserve from each of the suppliers.

(2) Then, the selling season starts and the products from the supplier are delivered at time $t = 1$. Depending on the realization of the customer demand D, the spot price p_s, the market liquidity m, and the suppliers' supply risk r_i's, the buyer should decide the exercise quantity from each of these suppliers and the spot market to satisfy the demand. We denote the order quantities from the supplier i after demand realization by x_i when the spot market has enough liquidity and denote that from the supplier i by $\overline{x}_i$ when the spot market has no liquidity to satisfy the buyer's demand.

(3) At last, all unsatisfied demands will incur a shortage cost s to the buyer, and the buyer calculates his total procurement cost. We assume $s \geq \max_i\{h_i, i = 1, 2, \ldots, N\}$ without loss of generality.

To make our model tractable, we propose the following assumption on the random yield r_i for each supplier.

Assumption 1. $r_i = k_i \eta$, in which η is a random variable with mean $\overline{\eta}$.

Although this assumption is required mainly by technical reason, it has its own practical support. This assumption implies that the supply risks of these suppliers are all associated with a common environment variable η. For example, if those suppliers all have the same upstream supplier with disruption risk, or if all these suppliers' risks associated with some common variable (e.g., location, macroeconomic environment, spot price, etc.), we can regard that the supply risks of those suppliers have the form which this assumption asks for. Here, k_i measures the magnitude that the supplier i's supply risk affected by the environment variable. The higher the value of k_i is, the higher the mean supply quantity and the associated variance are. However, the coefficient of variant for each supplier's random yield r_i remains the same.

Without loss of generality, we assume the suppliers are ordered by the values of c_i. That is, $c_1 > c_2 > \cdots > c_N$. Then, we should have $h_1 < h_2 < \cdots < h_N$, as other contract pairs unsatisfying such condition will be ruled out. Moreover, we assume $h_1 = 0$. Then, the contract provided by supplier i is a wholesale price contract with wholesale price c_1, while other contracts are option contracts. We assume that even when the realized quantity from any supplier is less than the quantity reserved by the buyer, the supplier will not refund those reservation prices to the buyer. The other notation is as follows. Denote the cumulative distribution function and the probability density function of m, p_s, D, η by $F(m, p_s, D, \eta)$ and $f(m, p_s, D, \eta)$, respectively. Denote the marginal cumulative distribution function and the marginal probability density function of random variable X by $F_X(X)$ and $f_X(X)$, respectively. Denote the joint cumulative distribution function and the joint probability density function of random variables X and Y by $F_{XY}(X, Y)$ and $f_{XY}(X, Y)$, respectively. Denote the marginal cumulative distribution function and the marginal probability density function of random variable X, when $Z = z$, by $F_X(X, z)$ and $f_X(X, z)$, respectively.

With the model description (see Notation Section), we can establish the optimization model at time $t = 0$ for the buyer as follows:

$$\min_{Q_i, i=1,2,\ldots,N} \sum_{i=1}^{N} c_i Q_i + E_{D,m,p,\eta}\left[(1-m)\,\Phi_1(\mathbf{Q}, D, p, \eta) + m\Phi_2(\mathbf{Q}, D, p, \eta)\right], \tag{1}$$

in which $\mathbf{Q} = [Q_1, \ldots, Q_N]$ and $\Phi_1(\mathbf{Q}, D, p, \eta)$ and $\Phi_2(\mathbf{Q}, D, p, \eta)$ are the decision models at time $t = 1$, after realization of

D, m, p, η, depends on whether the buyer can or cannot buy from the spot market. Specifically,

$$\begin{aligned}\Phi_1(\mathbf{Q}, D, p, \eta) = \min_{\overline{x}_i, i=1,2,\ldots,N} & \left(\sum_{i=1}^{N} h_i \overline{x}_i + s\overline{y}\right) \\ \text{s.t.} \quad & \overline{x}_i \le r_i Q_i, \quad i = 1, 2, \ldots, N \\ & \sum_{i=1}^{N} \overline{x}_i + \overline{y} = D \\ & \overline{x}_i \ge 0, \quad i = 1, 2, \ldots, N\end{aligned} \tag{2}$$

is the decision on the procurement quantity from each supplier when the spot market has liquidity problem, and

$$\begin{aligned}\Phi_2(\mathbf{Q}, D, p, \eta) = \min_{x_i, i=1,2,\ldots,N} & \left(\sum_{i=1}^{N} h_i x_i + \min\{p_s, s\}\, y\right) \\ \text{s.t.} \quad & x_i \le r_i Q_i, \quad i = 1, 2, \ldots, N \\ & \sum_{i=1}^{N} x_i + y = D \\ & x_i \ge 0, \quad i = 1, 2, \ldots, N\end{aligned} \tag{3}$$

is the decision on the procurement quantities from each supplier and the spot market when the spot market has ample supply.

We now first consider the decision problems in the time $t = 1$. The policy in this stage is rather simple as all market uncertainties have realized. Specifically, the buyer satisfies his demand from the supplier with the lowest exercise price to the one with the largest exercise price and bears the shortage cost at last when the spot market is unavailable (or procures from the spot market if the spot market is available and the spot price is lower than the shortage cost). In the following part, we will only focus on the decision problems on the optimal reservation quantity Q_i from supplier i at time $t = 0$. Specifically, we first analyze the case, the general case, when all stochastic factors are considered. Then, we consider two special cases. One case only considers the demand risk and assuming $r_i = 1$, for all $i = 1, 2, \ldots, N$, while the other case assumes that the spot price p_s and the market liquidity probability m are deterministic. The purpose of these two special cases is to derive some managerial insights.

Before going further to the analysis, we first introduce the definition of active contract as defined in Fu et al. [11]. We use Q_i^* to denote the optimal reservation quantity to supplier i.

Definition 2. Contract i is called active, if $Q_i^* > 0$. Contracts i and j are called consecutive active contracts, if

$$Q_i^* > 0, \quad Q_j^* > 0, \quad Q_r^* = 0 \quad \text{for } i < r < j. \tag{4}$$

Contract i is called the last active contract, if

$$Q_i^* > 0, \quad Q_r^* = 0, \quad r > i. \tag{5}$$

Moreover, we define the effective exercise price h_i', $i = 1, 2, \ldots, N$, and the effective shortage cost s' as

$$\begin{aligned} h_i' &= \begin{cases} h_i, & \text{if } h_i \le p_s; \\ (1-m)\, h_i + m p_s, & \text{if } h_i > p_s, \end{cases} \\ s' &= \begin{cases} s, & \text{if } s \le p_s; \\ (1-m)\, s + m p_s, & \text{if } s > p_s. \end{cases}\end{aligned} \tag{6}$$

3.1. Analysis for the General Setting. For the general setting, we can define a series of functions, $\{g_n\}_{n=1}^{N}$, as follows:

$$g_n(Q_1, Q_2, \ldots, Q_n) = \eta F_D\left(m, p_s, \eta\left(\sum_{i=1}^{n} k_i Q_i\right), \eta\right). \tag{7}$$

Then, we have the following property associated with the optimal solutions to the procurement problem.

Theorem 3. *There exists a series of active contracts $\{Q_i^*\}$ for the optimization problem* (1). *For any consecutive active contracts i and j, $(j > i)$, one has the following.*

(1) *If i is not the last active contract, then one has*

$$\begin{aligned}\iiint_{\eta, p_s, m} & \left(\eta - g_i(Q_1^*, Q_2^*, \ldots, Q_i^*)\right)\left(h_j' - h_i'\right) \\ & \times f_{m,p_s,\eta}(m, p_s, \eta)\, d\eta\, dp_s\, dm = \frac{c_i}{k_i} - \frac{c_j}{k_j}.\end{aligned} \tag{8}$$

(2) *If i is the last active contract, then one has*

$$\begin{aligned}\iiint_{\eta, p_s, m} & \left(\eta - g_i(Q_1^*, Q_2^*, \ldots, Q_i^*)\right)\left(s' - h_i'\right) \\ & \times f_{m,p_s,\eta}(m, p_s, \eta)\, d\eta\, dp_s\, dm = \frac{c_i}{k_i}.\end{aligned} \tag{9}$$

Proof. By expanding the optimization problem, given the reservation quantity Q_i form supplier i, we get that the objective function has the form of

$$\begin{aligned}\sum_{i=1}^{N} c_i Q_i + \int_{\eta} \Bigg\{ & \sum_{i=1}^{N} \int_{T_{i-1}}^{T_i} \left(\sum_{t=1}^{i-1} h_t' r_t Q_t + h_i'\left(D - T_{i-1}\right)\right) f_{D,\eta}\, dD \\ & + \int_{T_N}^{\infty} \left(\sum_{t=1}^{N} h_t' r_t Q_t + s'\left(D - T_k\right)\right) f_{D,\eta}\, dD \Bigg\}\, d\eta\end{aligned} \tag{10}$$

in which $T_i = \sum_{t=1}^{i} r_t Q_t$, $T_0 = 0$, and k is the last contract whose exercise price is less than the spot price.

Taking the first derivative with respect to Q_i and equaling it to zero, we get

$$\begin{aligned} c_i + \int_{\eta} \Bigg\{ & \sum_{j=i}^{N-1} \int_{T_j}^{T_{j+1}} r_i\left(h_i' - h_j'\right) f_{D,\eta}\, dD \\ & + \int_{T_N}^{\infty} r_i\left(h_i' - s'\right) f_{D,\eta}\, dD \Bigg\}\, d\eta = 0.\end{aligned} \tag{11}$$

It is obvious that the Hessian Matrix of the objective function is positive definite and thus there exist unique solutions for Q_i, for $i = 1, 2, \ldots, N$, within the restriction that $Q_i \geq 0$.

For any consecutive active contracts i, j, and the last active contract l, $(l > j > i)$, we have

$$\begin{aligned} c_i + \int_\eta \left\{ \sum_{t=i}^{l-1} \int_{T_t}^{T_{t+1}} r_i \left(h_i' - h_t'\right) f_{D,\eta} dD \right. \\ \left. + \int_{T_l}^{\infty} r_i \left(h_i' - s'\right) f_{D,\eta} dD \right\} d\eta = 0, \\ c_j + \int_\eta \left\{ \sum_{t=j}^{l-1} \int_{T_t}^{T_{t+1}} r_j \left(h_j' - h_t'\right) f_{D,\eta} dD \right. \\ \left. + \int_{T_l}^{\infty} r_j \left(h_j' - s'\right) f_{D,\eta} dD \right\} d\eta = 0, \\ c_l + \int_\eta \left\{ \int_{T_l}^{\infty} r_l \left(h_l' - s'\right) f_{D,\eta} dD \right\} d\eta = 0. \end{aligned} \tag{12}$$

Rearranging these terms yields the result. □

Thus, this result shows how to allocate the various risks to each supplier by the new function $g_n(Q_1, Q_2, ..., Q_n)$. As $g_n(Q_1, Q_2, ..., Q_n) > 0$ and $h_j' > h_i'$, we can get the following relationship between two consecutive active contracts i, j, $(j > i)$:

$$\frac{c_j}{k_j} + E\left[\eta h_j'\right] > \frac{c_i}{k_i} + E\left[\eta h_i'\right]. \tag{13}$$

This means that the adjusted cost of the active option contract, that is, $c_j/k_j + E[\eta h_j']$, has an increasing trend.

3.2. Special Case with Only Demand Uncertainty. In this case, there is only demand risk, and there are no reliable risks from the supplier and without loss of generality, we assume that $r_i = 1$, for all i and m and p_s are constants. Then, we have the following result.

Proposition 4. *For any consecutive active contracts i and j, $(j > i)$, one has the following.*

(1) *If i is not the last active contract, then one has*

$$1 - F_D\left(\sum_{t=1}^{i} Q_i^*\right) = -\frac{c_j - c_i}{h_j' - h_i'}. \tag{14}$$

(2) *If i is the last active contract, then one has*

$$1 - F_D\left(\sum_{t=1}^{i} Q_i^*\right) = -\frac{c_i}{h_i' - s'}. \tag{15}$$

Note that the active contract with exercise price less than the spot market is independent of the market liquidity m, while other active contracts depend on m; we have the following corollary concerning the difference between the optimal policies when the spot market is available or not.

Corollary 5. *Given the spot price p_s, denote the first active contract with exercise price higher than p_s by j^* when the spot market exists and j^o when there is no spot market. Then, $j^* \geq j^o$. Moreover, if $j^* = j^o$, then all the active contracts are the same whenever there is a spot market.*

This corollary implies that it is possible for the buyer to buy from those suppliers with exercise price higher than the spot price when there is market liquidity risk. Furthermore, as compared to the optimal policy without the spot market, the existence of the spot market will induce the buyer to choose those suppliers with higher exercise price and lower reservation price (i.e., $j^* \geq j^o$).

When $j^* = j^o$, if we use $T_i^o = \sum_{t=1}^{i} Q_t$ and $T_i^* = \sum_{t=1}^{i} Q_t^*$ to represent the optimal order up to level from the active supplier i. Denote the last active contract with exercise price lower than p_s as k. Then, when $i \leq k$, $T_i^* = T_i^o$ and when $i > k$, we have

$$\frac{1 - F_D\left(T_i^*\right)}{1 - F_D\left(T_i^o\right)} = \frac{1}{1 - m}. \tag{16}$$

We have the following result concerning how the order up to level from supplier i changes as the market liquidity varies.

Corollary 6. *Consider $T_i^* \leq T_i$ and $\partial T_i^*/\partial m = -(1 - F_D(T_i^o))/f_D(T_i^*)(1-m)^2 < 0$.*

This result implies that the order up to level from supplier i when there is a spot market will be lower than that when there is no spot market. Moreover, such order up to level is increasing in the market liquidity risk, that is, $1 - m$.

3.3. Special Case with Only Supply Uncertainty and Demand Uncertainty. In this case, the spot price p_s and the market liquidity probability m are deterministic; we can define a series of functions, $\{G_n\}_{n=1}^N$, as follows:

$$G_n\left(Q_1, Q_2, \ldots, Q_n\right) = E\left[\eta F_D\left(\eta\left(\sum_{i=1}^{n} k_i Q_i\right), \eta\right)\right]. \tag{17}$$

Then, we have the following property associated with these functions. This property is an application of the stochastic small property and we neglect the proof of this result here.

Lemma 7. *Consider $G_n(\mathbf{Q}) \leq G_{n+1}(\mathbf{Q})$.*

With the assistant of this property, one can establish the following result on the optimal reservation policies at time $t = 0$.

Proposition 8. *For any consecutive active contracts i and j, $(j > i)$ one has the following.*

(1) *If i is not the last active contract, then one has*

$$\overline{\eta} - G_i\left(Q_1^*, Q_2^*, \ldots, Q_i^*\right) = -\frac{c_j/k_j - c_i/k_i}{h_j' - h_i'}. \tag{18}$$

Table 1: Parameters for numerical studies.

Demand D	Spot price p_s	Market liquidity m	Shortage cost s
$N(100, 30)$	$N(20, 6)$	$N(0.5, 0.15)$	30

(2) *If i is the last active contract, then one has*

$$\overline{\eta} - G_i\left(Q_1^*, Q_2^*, \ldots, Q_i^*\right) = \frac{c_i/k_i}{h_i' - s'}. \tag{19}$$

This result, combined with the analysis in Fu et al. [11], also implies that the algorithm to choose the active contracts is as follows: (1) find the contract with the lowest adjusted cost $c_i/\widehat{\eta}k_i$ as the first active contract; (2) given the previous active contract i, find the next active contract as the one that has the largest value of $(c_i/k_i\overline{\eta} - c_j/k_j\overline{\eta})/(h_j' - h_i')$. Moreover, we have the following result.

Corollary 9. *If $k_1 = k_2 = \cdots = k_N$, then, the active contracts are independent of η.*

This result is interesting and shows that the active contracts are independent of the supply uncertainty when the suppliers all have the same yield risk. However, the order up to level from each active contract will depend on the distribution of η.

4. Numerical Analysis

In this section, we numerically study how different risks affect the buyer's optimal reservation quantity. We only focus on the case when there is no supply risk, that is, $r_i = 1$ for $i = 1, 2, \ldots, N$.

We assume that the demand, the spot price, and the market liquidity have joint normal distributions with parameters on their marginal distribution as showed in Table 1.

We consider three contracts with parameters as $(10, 0), (5.3237, 6.7810), (1.1580, 16.7230)$. Thus, the first contract is the wholesale price contract and the other two are option-type contracts. The purpose of this study is twofold. First, we will analyze how the correlation between the spot market price and spot market liquidity affects the optimal policy. Second, we will study the effect of the correlation between the market liquidity and the customer demand. In both cases, we will vary the correlation tested while keeping other correlations as zero.

4.1. Effect of the Correlation between Market Liquidity and Spot Price. Keeping other parameters unchanged, we vary the correlation between the market liquidity and spot price from −0.8 to 0.8. The high correlation implies that the event that the spot price is high and the market is available has a large probability. The corresponding results are listed in Table 2.

From Table 2, we find that the higher the correlation, the larger the quantity reserved by the option-type contract. This implies that when the correlation is high, the buyer will rely more on the long-term contract, and when the correlation is negative, he will rely more on the spot market. This is reasonable as the large correlation implies that the buyer can only buy from spot market at high spot price with a high probability. Moreover, the quantity procured from the wholesale contract is insensitive to such correlation, while the option-type contract with the lowest reservation price and the highest exercise price is the most sensitive to the correlation. This is because those option-type contracts play a role mainly as a risk hedging tool and thus are more sensitive to the risk.

Table 2: Effect of correlation between spot market price and liquidity.

The correlation	Optimal solution			
	Wholesale contract	Option-type contract		
	Q_1^*	Q_2^*	Q_3^*	Total
0.8	85.0244	19.7656	29.4336	49.1992
0.4	85.0244	19.7656	29.4336	49.1992
0	85.0244	19.2285	29.3262	48.5574
−0.4	85.0244	19.2285	28.7891	48.0176
−0.8	85.0244	18.5840	28.7891	47.3731

Table 3: Effect of correlation between market liquidity and demand.

The correlation	Optimal solution			
	Wholesale contract	Option-type contract		
	Q_1^*	Q_2^*	Q_3^*	Total
0.8	85.0244	18.5840	25.7812	44.3652
0.4	85.0244	19.2285	27.5000	46.7285
0	85.0244	19.2285	29.3262	48.5547
−0.4	85.0244	19.2285	31.1534	50.3809
−0.8	85.0244	19.7656	32.4414	52.2070

4.2. Effect of the Correlation between Market Liquidity and Demand. Keeping other parameters unchanged, we vary the correlation between the market liquidity and customer demand from −0.8 to 0.8. The high correlation implies that the event that the customer demand is high and the spot market is unavailable has a high probability. The corresponding results are listed in Table 3.

From Table 3, we find similar effect of the correlation on the sensitivity of the contracts, with the possible same reasons. However, the effect of the correlation on the optimal reservation quantity by the option-type contract is reversed. This is also intuitive as the higher the correlation, the higher the probability that the buyer can use the spot market to satisfy his demand. Thus, this results in the low reservation quantity by the option-type contract, and the buyer will rely more on the spot market procurement.

5. Conclusion

In this paper, we establish a decision making model from the perspective of the firm who will procure from the multiple suppliers and the spot markets. The suppliers are unreliable

and provide different types of option-type supply contracts which should be made before demand realization, while the spot market can only be used after demand realization and has both the price and liquidity risks. Thus, compared to existing literature, one major feature of our model is that we take the supply uncertainty and spot market liquidity risk into consideration. From our analysis, we develop the optimal portfolio policies for the firm with conditions to find the qualified suppliers. Specifically, we find that we can define a new series of functions that consist of all the associated risks and then allocate different curves of these functions to the suppliers with different contract structures. We also analyze two special cases of the model to dig out some managerial insights. Moreover, we numerically study how the various risks affect the choice of suppliers and the value of the option contract.

Notations

N:	The number of suppliers
m:	The probability the spot market is illiquid
$\mathbf{Q} = [Q_1, \ldots, Q_N]$:	Q_i is the order quantity from supplier i
$r_i = k_i\eta$:	The random yield for supplier i
(c_i, h_i):	The reservation price and exercise price provided by supplier i
s:	The shortage penalty cost
p:	The spot market price
D:	The stochastic demand
x_i:	The exercise quantity from supplier i when spot market is liquid
$\overline{x}_i$:	The exercise quantity from supplier i when spot market is illiquid.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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