Research Article

Distributed Asynchronous Fusion Algorithm for Sensor Networks with Packet Losses

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This paper is concerned with the problem of distributed estimation fusion over peer-to-peer asynchronous sensor networks with random packet dropouts. A distributed asynchronous fusion algorithm is proposed via the covariance intersection method. First, local estimator is developed in an optimal batch fashion by constructing augmented measurement equations. Then the fusion estimator is designed to fuse local estimates in the neighborhood. Both local estimator and fusion estimator are developed by taking into account the random packet losses. The presented estimation method improves local estimates and reduces the estimate disagreement. Simulation results validate the effectiveness of the proposed distributed asynchronous fusion algorithm.

1. Introduction

Distributed fusion and estimation problem over peer-to-peer sensor networks has attracted significant interest in the research community, because of its variety of applications, such as environmental monitoring, surveillance, and target tracking, [1–10]. In these applications, every sensor in the network does not only take measurements in a parallel manner but also acquires information from neighbors and processes it to get an estimate. Compared with the centralized fusion fashion, the main advantage of distributed fusion estimation is the computation burden alleviation and robustness enhancement.

In general, two main approaches are presented in the literature to solve the distributed fusion and estimation problem. The first approach is the consensus approach. The consensus approach was proposed in [11, 12], where local measurements are exchanged among neighbors to get local estimates, and then by using average consensus algorithms among neighbors every sensor in the network gets the same estimate in steady-state. In order to obtain the same estimate at every sensor in the network, the consensus approach may iterate several times for each new measurement. This is highly undesired when estimating the state of dynamic systems where new

measurements need to be processed in a timely manner. The second approach is the diffusion approach. The diffusion approach was presented in [13], in which the estimates of local filters are calculated individually at each sensor by using the data from the neighborhood, and then the local estimates from the neighborhood are fused locally by a convex combination. Therefore, the diffusion approach is well suited for estimating the state of dynamic systems where new measurements are being taken in real time. Though the adaptive weights for the diffusion algorithm were presented in [14], the estimation error covariance information is not taken into account. In [15, 16], the covariance intersection (CI) algorithm was proposed, in which the coefficients of convex combination of the estimates are chosen by considering the error covariance information. Therefore, the CI algorithm was used to design the adaptive weights for the diffusion algorithm in [17]. Though the aforementioned works solved the distributed fusion and estimation problem in their specific perspectives, they did not take into consideration the packet dropouts which are usually unavoidable in networks.

Many results on control and state estimation in networked systems context are already presented for the problem of both unreliable communication links and measurement uncertainties. The interested readers may refer to [18–23]

and the references therein for further information. We will review only those works that are closely related to the state estimation in sensor networks with packet dropouts.

The problem of packet dropouts which are usually unavoidable in sensor networks has gained a lot of attention [24–27]. Schenato addressed a minimum error covariance estimator for sensor networks with random packets losses in [28]. The critical data arrival rate which ensures the stability of Kalman filter is investigated in [29]. An optimal filter is proposed based on the probability of the packet dropouts in [30]. Sun proposed optimal linear estimators in the sense of minimum variance with respect to the packet dropout probability in [31]. Ma and Sun proposed a centralized linear optimal fusion estimator in the minimum variance sense in [32]. Though the aforementioned literatures deal with the random packet losses problem, the sensor asynchronous problem is not taken into account.

Most of the aforementioned works are developed under the synchronous assumption. And the real-world systems may usually be asynchronous. In regard to asynchronous sensor fusion, a series of linear weighted fusion (LWF) algorithms for two and more than two asynchronous sensors with and without feedback had been proposed separately in [33–36]. By establishing state-space models at each sampling rate, a new fusion algorithm for asynchronous sensors had been presented in [37]. A minimum mean-squared error (MMSE) centralized asynchronous fusion algorithm had been presented in [38], which is suitable for arbitrary sensor number, sensor sampling rates, sensor initial sampling time instants, and fusion periods. By reconstructing the optimal centralized asynchronous fusion algorithm in [38], a so-called distributed asynchronous fusion algorithm is proposed in [39], which performs equivalently to the optimal centralized asynchronous fusion algorithm. Though the aforementioned works deal with the asynchronous sensor fusion problem, the random packet dropouts problem is not taken into account. Moreover, these works always need a fusion center to accomplish the estimation task.

Motivated by the above discussion, we consider the distributed estimation fusion problem for an asynchronous sensor network with random packet dropouts. The sensor network is considered to be a peer-to-peer network with no fusion center, and the communication channels of the sensor network are unreliable. The main contribution of our paper is the development of the distributed asynchronous fusion algorithm in a peer-to-peer sensor network with random packet dropouts. Our algorithm is developed as follows. Firstly, a Bernoulli random variable is employed to describe the random packet dropouts. Then, by constructing a local augmented measurement with the random packet dropouts under consideration, an asynchronous local estimator is obtained at each sensor to compute local estimate. Further, the fusion estimators at different sensors are presented by utilizing a weighted fusion criterion based on covariance intersection method, and the error covariance information of local estimates is used to compute the fusion weights. The main advantage of the proposed distributed asynchronous fusion algorithm is the practicality when dealing with a

practical asynchronous sensor network with unreliable communication links.

The rest of the paper is organized as follows. In Section 2, the asynchronous distributed estimation problem is formulated. Then, design procedures for the asynchronous distributed fusion algorithm are presented in Section 3. The effectiveness of the proposed algorithm is demonstrated by a simulation example in Section 4. Finally, the conclusion is provided in Section 5.

2. Problem Formulation

Consider a continuous-time dynamic system described by the following linear stochastic differential equation:

$$\dot{x}(t) = A(t)x(t) + G(t)w(t), \quad (1)$$

where $x(t) \in \mathcal{R}^{d_x}$ and $w(t) \in \mathcal{R}^{d_x}$ is zero-mean white Gaussian noise with covariance matrix $q(t)$. Suppose that $\Phi(t_k, t_{k-1})$ is the corresponding state transition matrix of $A(t)$, t_{k-1} is the time when the fusion center executed the last fusion operation or the initial time if $k = 1$, t_k is the oncoming fusion time instant, and $T = t_k - t_{k-1}$ is the fusion interval. Then, by discretizing the continuous-time linear system (1), we have

$$x(t_k) = \Phi(t_k, t_{k-1})x(t_{k-1}) + w(t_k, t_{k-1}), \quad (2)$$

where $w(t_k, t_{k-1}) = \int_{t_{k-1}}^{t_k} \Phi(t_k, v) G(v)w(v)dv$ is zero-mean white Gaussian noise with covariance matrix $Q(t_k, t_{k-1}) = \int_{t_{k-1}}^{t_k} \Phi(t_k, \tau)G(\tau)q(\tau)G^T(\tau)\Phi^T(t_k, \tau)d\tau$.

Observations of system (1) are obtained by a sensor network which deployed spatially. The sensor network which consists of N distributed sensors is considered to be a peer-to-peer network. The interconnection topology of the sensor network is denoted by $\mathcal{A} = \{L^{i,j}\}$. $L^{i,j} = 1$ indicates that sensor i and sensor j are connected, otherwise $L^{i,j} = 0$ implies that sensor i and sensor j are not connected. No fusion center exists in the sensor network, and every sensor acts also as an estimator which collects information only from its neighbors to generate local estimates. The set of sensors connected to the certain sensor i is called the neighborhood of sensor i and it is denoted by $\mathcal{N}_i$. Without loss of generality, assume that $i \in \mathcal{N}_i$ and the number of neighbors of sensor is given by the number of elements of $\mathcal{N}_i$, written as $\mathcal{D}_i$.

Suppose that the number of measurements provided by sensor i during the k th time interval is n_k^i . So observations during the k th time interval of system (1) can be modeled as

$$y_k^{i,j} = h_k^{i,j}x(t_k^{i,j}) + v_k^{i,j}, \quad i = 1, \dots, N; \quad j = 1, \dots, n_k^i, \quad (3)$$

where $h_k^{i,j}$ and $v_k^{i,j}$ are measurement matrix and zero-mean white Gaussian measurement noise with covariance matrix $R_k^{i,j}$, respectively. $y_k^{i,j}$ corresponds to the j th measurement of sensor during time interval k . The set of measurements obtained by sensor i during time interval is denoted by Z_k^i . Assume that $w(t)$, $v_k^{i,j}$, and $x(0)$ are mutually independent.

We also assume that the sensors in the network are time synchronized and time driven; therefore the sensors can easily obtain the time stamp through the digital communication network. The observations are exchanged between neighboring sensors once in every fusion interval. The packet-dropping process at every communication links is modeled as independent identically distributed (i.i.d.) Bernoulli random process. We use Bernoulli random variable $\theta_k^{i,r}$ (for all $L^{i,r} = 1$) to indicate whether a packet is successfully transmitted from sensor i to sensor r at k th time instant. If $\theta_k^{i,r} = 1$, the packet is successfully transmitted, otherwise, $\theta_k^{i,r} = 0$ and the packet is dropped. Without loss of generality, it is assumed that $\theta_k^{i,i} = 1$.

The j th measurement sensor r got from sensor i during time interval k can be written as

$$\begin{aligned} z_k^{i,j} &= \theta_k^{i,r} y_k^{i,j} + (1 - \theta_k^{i,r}) y_{k-1}^{i,j}, \\ i &= 1, \dots, N, \quad j = 1, \dots, n_k^{i,j}, \quad \forall L^{i,r} = 1, \end{aligned} \quad (4)$$

where $\lambda \triangleq E\{\theta_k^{i,r}\} = \text{Prob}\{\theta_k^{i,r} = 1\}$ is the packet arriving probability (PAP) and $1 - \lambda \triangleq 1 - E\{\theta_k^{i,r}\} = \text{Prob}\{\theta_k^{i,r} = 0\}$ is the packet loss probability (PLP) [17].

3. Distributed Asynchronous Fusion Algorithm

3.1. Local Asynchronous Estimator. Without loss of generality, it is assumed that sensors in $\mathcal{N}_i$ totally obtain a set of N_k^i asynchronous measurements during the k th time interval. Measurement $z_k^{l,m}$ is provided by sensor l ($l \in \mathcal{N}_i$) at time instant $\bar{t}_k^{l,m}$ ($\bar{t}_k^{l,m} \in (t_{k-1}, t_k]$). After sensor i collects all N_k^i asynchronous measurements which are provided by sensors in $\mathcal{N}_i$, it reorders these measurements in time sequence. Suppose the j th measurement at sensor during the k th time interval which is denoted by $Y_k^{i,j}$ corresponds to $z_k^{l,m}$, then we have

$$Y_k^{i,j} = z_k^{l,m} = \theta_k^{l,i} y_k^{l,m} + (1 - \theta_k^{l,i}) y_{k-1}^{l,m}. \quad (5)$$

As mentioned before, we can obtain the time stamp of current measurement easily. The packet is successfully transmitted, if the time stamp of the latest measurement is in the present fusion interval, that is, $\theta_k^{l,i} = 1$, otherwise $\theta_k^{l,i} = 0$. We can simplify the measurement $Y_k^{i,j}$ as

$$\begin{aligned} Y_k^{i,j} &= \theta_k^{l,i} y_k^{l,m} \\ &= \theta_k^{l,i} h_k^{l,m} x(t_k^{l,m}) + \theta_k^{l,i} v_k^{l,m} \\ &\triangleq \bar{\theta}_k^{i,j} \bar{H}_k^{i,j} x(t_k^{i,j}) + \bar{\theta}_k^{i,j} \bar{v}_k^{i,j}, \end{aligned} \quad (6)$$

where $\bar{H}_k^{i,j} = h_k^{l,m}$, $t_k^{i,j} = t_k^{l,m}$, $\bar{v}_k^{i,j} = v_k^{l,m}$, and $\bar{\theta}_k^{i,j} = \theta_k^{l,i}$.

The state $x(t_k^{i,j})$ can be written by using (2) as

$$x(t_k^{i,j}) = \Phi^{-1}(t_k, t_k^{i,j}) x(t_k) - \Phi^{-1}(t_k, t_k^{i,j}) w(t_k, t_k^{i,j}), \quad (7)$$

where $\Phi^{-1}(t_k, t_k^{i,j}) = \Phi(t_k^{i,j}, t_k)$ is the backward state transition matrix.

By using (6) and (7), $Y_k^{i,j}$ can be rewritten as

$$\begin{aligned} Y_k^{i,j} &= \bar{H}_k^{i,j} x(t_k^{i,j}) + \bar{v}_k^{i,j} \\ &= \bar{\theta}_k^{i,j} \bar{H}_k^{i,j} \Phi^{-1}(t_k, t_k^{i,j}) x(t_k) \\ &\quad - \bar{\theta}_k^{i,j} \bar{H}_k^{i,j} \Phi^{-1}(t_k, t_k^{i,j}) w(t_k, t_k^{i,j}) + \bar{\theta}_k^{i,j} \bar{v}_k^{i,j} \\ &= \xi_k^{i,j} H_k^{i,j} x(t_k) + \xi_k^{i,j} \Lambda_k^{i,j}, \end{aligned} \quad (8)$$

where $H_k^{i,j} = \bar{H}_k^{i,j} \Phi^{-1}(t_k, t_k^{i,j})$, $\Lambda_k^{i,j} = \bar{v}_k^{i,j} - H_k^{i,j} w(t_k, t_{k-1})$, and $\xi_k^{i,j} = \bar{\theta}_k^{i,j}$.

By denoting

$$\mathcal{Y}_k^i = \left[(Y_k^{i,1})^T, (Y_k^{i,2})^T, \dots, (Y_k^{i,N_k^i})^T \right]^T, \quad (9)$$

$$\mathcal{H}_k^i = \left[(H_k^{i,1})^T, (H_k^{i,2})^T, \dots, (H_k^{i,N_k^i})^T \right]^T, \quad (10)$$

$$\mathcal{L}_k^i = \left[(\Lambda_k^{i,1})^T, (\Lambda_k^{i,2})^T, \dots, (\Lambda_k^{i,N_k^i})^T \right]^T, \quad (11)$$

$$\Xi_k^i = \text{diag} \left\{ \xi_k^{i,1} I_k^{i,1}, \xi_k^{i,2} I_k^{i,2}, \dots, \xi_k^{i,N_k^i} I_k^{i,N_k^i} \right\}, \quad (12)$$

where $I_k^{i,j}$, $j = 1, 2, \dots, N_k^i$, are identity matrices with proper dimensions.

We can get an augmented measurement equation at sensor i as

$$\mathcal{Y}_k^i = \Xi_k^i \mathcal{H}_k^i x(t_k) + \Xi_k^i \mathcal{L}_k^i. \quad (13)$$

From (2), (3), (6), and (8) we have

$$\begin{aligned} E[\xi_k^{i,j} \Lambda_k^{i,j} | \xi_k^{i,j}] &= E[\bar{v}_k^{i,j} - H_k^{i,j} w(t_k, t_{k-1}) | \xi_k^{i,j}] \\ &= E[v_k^{l,m} | \xi_k^{i,j}] - H_k^{i,j} E[w(t_k, t_{k-1}) | \xi_k^{i,j}] \\ &= 0, \\ E[\xi_k^{i,j} \Lambda_k^{i,j} (\xi_k^{i,j} \Lambda_k^{i,j})^T | \xi_k^{i,j}] &= \xi_k^{i,j} E[(\bar{v}_k^{i,j} - H_k^{i,j} w(t_k, t_{k-1})) \\ &\quad \times (\bar{v}_k^{i,j} - H_k^{i,j} w(t_k, t_{k-1}))^T] \end{aligned}$$

$$\begin{aligned}
&= \xi_k^{i,j} E \left[(\bar{v}_k^{i,j}) (\bar{v}_k^{i,j})^T \right] \\
&\quad + \xi_k^{i,j} E \left[(H_k^{i,j} w(t_k, t_k^{i,j})) \right. \\
&\quad \quad \left. \times (H_k^{i,j} w(t_k, t_k^{i,j}))^T \right] \\
&= \xi_k^{i,j} R_k^{i,j} + \xi_k^{i,j} H_k^{i,j} Q(t_k, t_k^{i,j}) (H_k^{i,j})^T.
\end{aligned} \tag{14}$$

Suppose $t_k^{i,j} > t_k^{i,l}$; then

$$\begin{aligned}
&E \left[\xi_k^{i,j} \Lambda_k^{i,j} (\xi_k^{i,l} \Lambda_k^{i,l})^T \mid \xi_k^{i,j}, \xi_k^{i,l} \right] \\
&= \xi_k^{i,j} E \left[(\bar{v}_k^{i,j} - H_k^{i,j} w(t_k, t_k^{i,j})) (\bar{v}_k^{i,l} - H_k^{i,l} w(t_k, t_k^{i,l}))^T \right] \\
&= \xi_k^{i,j} E \left[(H_k^{i,j} w(t_k, t_k^{i,j})) (H_k^{i,l} w(t_k, t_k^{i,l}))^T \right] \\
&= \xi_k^{i,j} \xi_k^{i,l} H_k^{i,j} Q(t_k, t_k^{i,j}) (H_k^{i,l})^T.
\end{aligned} \tag{15}$$

Thus, the augmented measurement noise $\mathcal{Z}_k^i$ is zero-mean white Gaussian noise with covariance:

$$\begin{aligned}
R_k^i &= \begin{bmatrix} \xi_k^{i,1} (R_k^{i,1} + H_k^{i,1} Q(t_k, t_k^{i,1}) H_k^{i,1}) & \dots & \xi_k^{i,1} \xi_k^{i,N_k^i} H_k^{i,1} Q(t_k, t_k^{i,N_k^i}) H_k^{i,N_k^i} \\ \vdots & \ddots & \vdots \\ \xi_k^{i,N_k^i} \xi_k^{i,1} H_k^{i,N_k^i} Q(t_k, t_k^{i,N_k^i}) H_k^{i,1} & \dots & \xi_k^{i,N_k^i} (R_k^{i,N_k^i} + H_k^{i,N_k^i} Q(t_k, t_k^{i,N_k^i}) H_k^{i,N_k^i}) \end{bmatrix} \\
&= \Xi_k^i \mathcal{R}_k^i \Xi_k^i,
\end{aligned} \tag{16}$$

where

$$\mathcal{R}_k^i = \begin{bmatrix} R_k^{i,1} + H_k^{i,1} Q(t_k, t_k^{i,1}) H_k^{i,1} & \dots & H_k^{i,1} Q(t_k, t_k^{i,N_k^i}) H_k^{i,N_k^i} \\ \vdots & \ddots & \vdots \\ H_k^{i,N_k^i} Q(t_k, t_k^{i,N_k^i}) H_k^{i,1} & \dots & R_k^{i,N_k^i} + H_k^{i,N_k^i} Q(t_k, t_k^{i,N_k^i}) H_k^{i,N_k^i} \end{bmatrix}. \tag{17}$$

From (2), (8), and (11), we have

$$\begin{aligned}
&E \left[w(t_k, t_{k-1}) (\xi_k^{i,j} \Lambda_k^{i,j})^T \mid \xi_k^{i,j} \right] \\
&= \xi_k^{i,j} E \left[w(t_k, t_{k-1}) (\bar{v}_k^{i,j} - H_k^{i,j} w(t_k, t_k^{i,j}))^T \right] \\
&= -\xi_k^{i,j} E \left[w(t_k, t_{k-1}) (H_k^{i,j} w(t_k, t_k^{i,j}))^T \right] \\
&= -\xi_k^{i,j} Q(t_k, t_k^{i,j}) (H_k^{i,j})^T.
\end{aligned} \tag{18}$$

Further we have

$$\begin{aligned}
&E \left[w(t_k, t_{k-1}) (\Xi_k^i \mathcal{Z}_k^i)^T \right] \\
&= \left[E \left[w(t_k, t_{k-1}) (\xi_k^{i,j} \Lambda_k^{i,1})^T \right] E \left[w(t_k, t_{k-1}) (\xi_k^{i,j} \Lambda_k^{i,2})^T \right] \right. \\
&\quad \left. \dots E \left[w(t_k, t_{k-1}) (\xi_k^{i,j} \Lambda_k^{i,N_k^i})^T \right] \right]
\end{aligned}$$

$$= \Xi_k^i \mathcal{V}_k^i$$

$$= V_k^i,$$

(19)

where $\mathcal{V}_k^i = [E[w(t_k, t_{k-1})(\Lambda_k^{i,1})^T] \ E[w(t_k, t_{k-1})(\Lambda_k^{i,2})^T] \dots E[w(t_k, t_{k-1})(\Lambda_k^{i,N_k^i})^T]]$.

It can be observed from (19) that the discrete-time process noise and the augmented measurement noise are cross correlated; this correlation comes from the common continuous-time process noise [40].

For sensor i , the asynchronous local estimation problem in $\mathcal{N}_i$ is converted to an estimation problem of the following system:

$$\begin{aligned}
x(t_k) &= \Phi(t_k, t_{k-1}) x(t_{k-1}) + w(t_k, t_{k-1}), \\
Y_k^i &= H_k^i x(t_k) + \Lambda_k^i,
\end{aligned} \tag{20}$$

where $Y_k^i = \mathcal{Y}_k^i$, $H_k^i = \Xi_k^i \mathcal{H}_k^i$, and $\Lambda_k^i = \Xi_k^i \mathcal{L}_k^i$.

Thus a local asynchronous estimation method is summarized in Theorem 1 in an optimal batch asynchronous data fusion fashion [38].

Theorem 1 (local asynchronous estimator). *For sensor i , after it gets asynchronous measurements from sensors in N_i , the local asynchronous estimator at sensor i is summarized as follow: given the local state estimate $\hat{x}_{k-1}^i$ of system (1) and its error covariance matrix M_{k-1}^i at time t_{k-1} , the local estimate and its error covariance matrix at time t_k are given by*

$$\begin{aligned}\bar{x}_k^i &= \Phi(t_k, t_{k-1}) \hat{x}_{k-1}^i, \\ \bar{M}_k^i &= \Phi(t_k, t_{k-1}) M_{k-1}^i \Phi(t_k, t_{k-1}) + Q(t_k, t_{k-1}), \\ M_k^i &= \left\{ \left(\bar{M}_k^i \right)^{-1} + \left[H_k^i + (V_k^i)^T \left(\bar{M}_k^i \right)^{-1} \right]^T \right. \\ &\quad \times \left[R_k^i - (V_k^i)^T \left(\bar{M}_k^i \right)^{-1} V_k^i \right]^{-1} \\ &\quad \times \left. \left[H_k^i + (V_k^i)^T \left(\bar{M}_k^i \right)^{-1} \right] \right\}^{-1}, \\ \hat{x}_k^i &= \bar{x}_k^i + M_k^i \left[H_k^i + (V_k^i)^T \left(\bar{M}_k^i \right)^{-1} \right]^T \\ &\quad \times \left[R_k^i - (V_k^i)^T \left(\bar{M}_k^i \right)^{-1} V_k^i \right]^{-1} \\ &\quad \times \left(Y_k^i - H_k^i \bar{x}_k^i \right),\end{aligned}\quad (21)$$

where $\hat{x}_k^i$ and $\bar{x}_{k+1}^i$ are the state estimate and one-step predicted estimate, respectively, M_k^i and $\bar{M}_k^i$ are the covariance matrices of filtering error and one-step prediction error, respectively, and the initial values of $\bar{x}_{k+1}^i$ and $\bar{M}_k^i$ are given, respectively, by x_0 and M_0 , where x_0 is a normally distributed random variable with mean equal to the initial truth ground of the state x and covariance M_0 .

Proof. For neighborhood $\mathcal{N}_i$, the local asynchronous estimation problem at sensor i is converted to the state estimation problem of system (20). This is a classical filtering problem with noise cross-covariance which can be solved by using the Kalman filter [40] for correlated process and measurement noises. $\square$

Remark 2. The random packets losses have been taken into account in Theorem 1. And in Theorem 1 each sensor in the sensor network generates a local estimate at the fusion time instant by using measurements only from its neighbors. These local estimates are suboptimal in the sense that not all the measurements in the sensor network are used. Therefore, a fusion criterion, which will improve each local estimation performance, weighted by matrices will be developed later in this paper. The main results will be presented in the following subsection.

3.2. CI-Based Fusion Estimator. In this subsection, a fusion criterion weighted by matrices is developed to generate fusion estimate at every sensor, and it takes into consideration the

random packets dropouts. The criterion is first given in the following lemma in [16].

Lemma 3. *Let $\hat{\eta}_1, \hat{\eta}_2, \dots, \hat{\eta}_n$ denote unbiased estimates for the unknown state vector η and let $P_1, P_2, \dots, P_n$ denote the corresponding estimated covariance matrices; the CI fusion estimator is given by the following convex combination:*

$$\begin{aligned}P_0^{-1} &= \sum_{n=1}^N \omega_n P_n^{-1}, \\ P_0^{-1} \eta_0 &= \sum_{n=1}^N \omega_n P_n^{-1} \eta_n,\end{aligned}\quad (22)$$

where $0 \leq \omega_n \leq 1$ and $\omega_1 + \omega_2 + \dots + \omega_n = 1$ hold.

When local estimates calculated by the local estimators in Theorem 1 are available at the sensors in the sensor network, sensor i collects them from its neighborhood $\mathcal{N}_i$ to generate a fused estimate according to the fusion rule based on covariance intersection. For the packet-dropping process, it is also modeled as i.i.d. Bernoulli random process. Denote $\mu_k^{i,r}$ (for all $L^{i,r} = 1$) as the packet transmit indicator. It is also assumed that $\mu_k^{i,i} = 1$.

Theorem 4 (CI-based fusion estimator). *By gathering the local estimates provided by the sensors in neighborhood N_i , the distributed weighted fusion filter at sensor i is given by*

$$\bar{x}_k^i = \sum_{l \in \mathcal{N}_i} S_k^l \hat{x}_k^l, \quad (23)$$

where

$$\begin{aligned}S_k^l &= \mu_k^{i,l} \alpha_k^l (\Psi_k^i)^{-1} (M_k^l)^{-1}, \\ \Psi_k^i &= \sum_{l \in \mathcal{N}_i} \mu_k^{i,l} \alpha_k^l (M_k^l)^{-1}.\end{aligned}\quad (24)$$

α_k^l are subject to $0 \leq \alpha_k^l \leq 1$, $\sum_{l \in \mathcal{N}_i} \mu_k^{i,l} \alpha_k^l = 1$.

Proof. Theorem 4 follows directly from Lemma 3. $\square$

Remark 5. Different choices of α_k^l can be used to optimize the update with respect to different performance criteria, such as minimizing the trace or determinant of Ψ_k^i . However such optimization is nonlinear and a high computation load is required for computing the optimal α_k^l [41]. Thus, several fast CI algorithms that produce suboptimal solutions have been proposed in terms of trace or determinant minimization [41, 42]. In what follows, we choose to minimize the trace of Ψ_k^i . For sensor i , a suboptimal noniterative weighting coefficients solution for fast CI fusion is given by

$$\alpha_k^l = \frac{1 / \text{tr}(M_k^l)}{\sum_{j \in \mathcal{N}_i} \mu_k^{i,j} (1 / \text{tr}(M_k^j))}, \quad l \in \mathcal{N}_i. \quad (25)$$

For clarity, the asynchronous distributed fusion algorithm is summarized in Algorithm 1.

- (1) Initialization: $\bar{M}_k^i = M_0$, $\bar{x}_{k+1}^i = x_0$.
- (2) Obtain local measurements $z_k^{i,p}$.
- (3) Broadcast message $m_k^{1,i} = \{z_k^{i,p}\} (p = 1, 2, \dots, n_k^i)$ to neighbors.
- (4) Receive messages $m_k^{2,l}, l \in \mathcal{N}_i$, from neighbors.
- (5) Compute Y_k^i, H_k^i, R_k^i , and V_k^i by using (9)–(19).
- (6) Calculate the local estimate $\hat{x}_k^i$ by using Theorem 1.
- (7) Broadcast message $m_k^{2,i} = \{M_k^i, \hat{x}_k^i\}$ to neighbors.
- (8) Receive messages $m_k^{2,l}, l \in \mathcal{N}_i$, from neighbors.
- (9) Calculate the fusion estimate $\underline{x}_k^i$ by using Theorem 4 and (25).

ALGORITHM 1

Remark 6. In the distributed asynchronous fusion filter presented in Algorithm 1, each sensor in sensor network broadcasts its local measurement information and local estimate to its neighboring sensors; for every sensor, more information from different sensors, not only the neighboring ones, will be utilized to generate the fusion estimates, which helps improve the local estimate performance and reduce the disagreement of local estimates. Further, not all target information in the sensor network is used to generate the fusion estimate at one sensor; meanwhile the method we used to compute coefficient α_k^l is suboptimal; therefore the fusion estimate is a suboptimal estimate, but it is an improved one compared with the local one.

4. Simulation

In this section, the effectiveness of the proposed estimator design method is demonstrated by presenting simulations of a target tracking system. Consider the following continuous-time dynamic system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t), \quad (26)$$

where $x(t) \in \mathcal{R}^2$ and $w(t)$ is zero-mean white Gaussian noise with variance $q = 1$; by discretizing the continuous-time linear system (26), we have

$$x(t_k) = \begin{bmatrix} 1 & t_k - t_{k-1} \\ 0 & 1 \end{bmatrix} x(t_{k-1}) + w(t_k, t_{k-1}), \quad (27)$$

where the process noise $w(t_k)$ is a white Gaussian noise sequence with zero mean and covariance

$$Q(t_k, t_{k-1}) = \begin{bmatrix} \frac{(t_k - t_{k-1})^3}{3} & \frac{(t_k - t_{k-1})^2}{2} \\ \frac{(t_k - t_{k-1})^2}{2} & t_k - t_{k-1} \end{bmatrix}. \quad (28)$$

The state of system (27) is observed by a sensor network with 12 sensor nodes, and the topology of the sensor network is shown in Figure 3. The observation equations of the sensors are given by (3) with measurement matrices $h_k^{i,j} = [1 \ 0]$,

FIGURE 1: The network topology with 12 sensor nodes.

$i = 1, \dots, 12$ and $j = 1, \dots, n_k^i$; measurement noise variances $R_k^{i,j} = 25$, $i = 1, \dots, 12$ and $j = 1, \dots, n_k^i$; and sampling periods $T_1 = T_2 = T_3 = 0.3s$, $T_4 = T_5 = T_6 = T_7 = T_8 = 0.4s$, and $T_9 = T_{10} = T_{11} = T_{12} = 0.6s$, respectively. The fusion operation is performed with period $T_F = 1s$ (i.e., $t_k - t_{k-1} = 1$) and initial time instant $t_F = 1s$.

For performance comparison, root mean square errors (RMSE) of position and velocity given by [40]

$$RMSE_k(i) = \sqrt{\frac{1}{M} \sum_{m=1}^M [x_k^{m,e}(i) - x_k^{m,true}(i)]^2} \quad (29)$$

are used, where $M = 100$ is the Monte-Carlo simulation times, $x_k^{m,true}(i)$ is the ground truth of the i th component of the state vector in the m th run, and $x_k^{m,e}(i)$ is the corresponding estimate; the disagreement potential of the estimates in the neighborhood $\mathcal{N}_i$ is defined as

$$\Delta(t_k) = \frac{1}{2\mathcal{D}_{i,j \in \mathcal{N}_i}} \sum \left\| x_k^{e,i} - x_k^{e,j} \right\|^2, \quad (30)$$

where $x_k^{e,i}$ is the estimate of sensor i at time instant t_k , P_k^i is the corresponding error covariance, and the simulation ends after 20 seconds.

We present the simulation results without packet losses firstly. As shown in Figure 1, sensors 2 and 12 are directly

FIGURE 2: Performance comparison of local estimates, fusion estimates and centralized asynchronous fusion estimate.

connected to sensor 4, thus neighborhood $\mathcal{N}_4$ consists of sensors 2, 4, and 12, with sampling periods $T_2 = T_4 = 0.3s$ and $T_{12} = 0.6s$, respectively. We take estimate performance of sensors in neighborhood $\mathcal{N}_4$ to testify the effectiveness of the proposed algorithm. At each fusion instant t_k , local estimate at sensor 4 is generated by collecting measurements from itself and sensors 2 and 12, and then sensor 4 collects local estimates from itself and sensors 2 and 12 to form fused estimates by using Theorem 4 and (25). Since trends of the simulation results are the same for both components, only the first component of the state vector is shown and analyzed below, and the simulation results of the proposed algorithm are compared with the optimal linear minimum mean-squared error centralized asynchronous fusion algorithm in [22].

Figure 2 compares estimate performance of the proposed local asynchronous estimator and fusion estimator with the centralized asynchronous fusion estimator proposed in [22]. Although the performance of local estimator and fusion estimator is close to the centralized one, the centralized asynchronous fusion algorithm outperforms them both. This verifies our conclusion that the proposed local asynchronous estimator and fusion estimator are suboptimal as we stated in Remarks 2 and 6.

Figure 3 compares the performance of the proposed local asynchronous estimator and fusion estimator. It can be observed that the RMSE of fusion estimates is smaller than that of the corresponding local estimates; this verifies that the fusion estimates improve the performance of local estimates.

Figure 4 compares the disagreement of estimates obtained by the proposed local asynchronous estimator

FIGURE 3: Performance comparison of local estimates and fusion estimates at sensors 2, 4, and 12.

FIGURE 4: Disagreement potential comparison of the estimates (each curve is determined by averaging 100 random runs of Algorithm 1).

and fusion estimator; the fusion estimator gains smaller disagreement potential than the corresponding local one. This verifies our conclusion that fusion estimator reduces the disagreement among different sensors.

Figures 5 and 6 shows the filtering performances of the sensors in $\mathcal{N}_4$ with different PLPs. It can be observed that

FIGURE 5: Local estimator performances with PLP = 0, 0.1, 0.4.

FIGURE 6: Fusion estimator performances with PLP = 0, 0.1, 0.4.

packet dropout degrades estimation performance. The fusion estimator performance still outperforms local estimator and both estimators RMSEs converge. This demonstrates the effectiveness of the proposed distributed asynchronous fusion algorithm.

5. Conclusion

A CI-based distributed asynchronous fusion algorithm has been proposed for decentralized asynchronous sensor networks with random packet dropouts in this paper. The proposed distributed asynchronous fusion algorithm is applicable to decentralized asynchronous sensor networks; it is capable of improving estimates and reducing disagreement in comparison with the local ones; meanwhile it is also able to deal with random packet losses. The simulation results demonstrate the effectiveness of the proposed distributed asynchronous fusion algorithm.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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