Research Article

Fuzzy Availability Assessment for a Repairable Multistate Series-Parallel System

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This paper considers a repairable multistate series-parallel system (RMSSPS) with fuzzy parameters. It is assumed that the system components are independent, and their state transition rates and performance rates are fuzzy values. The fuzzy universal generating function technique is adopted to determine fuzzy state probability and fuzzy performance rate of the system. On the basis of α -cut approach and the extension principle, parametric programming technique is employed to obtain the α -cuts of some indices for the system. The system fuzzy availability is defined as the ability of the system to satisfy fuzzy consumer demand. A special assessment approach is developed for evaluating the fuzzy steady-state availability of the system with the fuzzy demand. A flow transmission system with three components is presented to demonstrate the validity of the proposed method.

1. Introduction

The study of repairable systems is an important topic in engineering systems. System availability is very good evaluation for performance of repairable systems and occupies an increasingly important issue in power plants, manufacturing systems, industrial systems, and transportation systems, and so forth.

In the real-world problems, many repairable systems are designed to perform their intended tasks in a given environment. One type of these repairable systems is repairable multistate system (RMSS). The RMSS is able to perform its task with various distinguished levels of efficiency usually referred to as performance rates. Since the number of RMSS states increases very rapidly with the increase in the number of its components, the universal generating function (UGF) technique was introduced and proved to be efficient in evaluating the reliability of the multistate systems [1–4].

The repairable multistate series-parallel system (RMSSPS) model is frequently used in practice and has been extensively studied for many years. The conventional study for the RMSSPS considers the assumptions of the exact state transition rate and performance rate for each system component. However, in many engineering applications

it is very difficult to obtain accurate and sufficient data to estimate the precise values of the state transition rate and performance rate for each system component. For this reason, the concept of fuzzy reliability has been introduced and developed by several authors [5–9].

Fuzzy set theory proposed by Zadeh [10] is a very good approach to deal with fuzzy uncertainty and has gained successful applications in various fields. It provides useful tools to investigate and analyze imprecision phenomena in queuing systems [11], rock engineering classification systems [12], transport systems [13], manufacturing systems [14], supply chain problems [15], and various optimization problems [16–18]. Wong and Lai [19] provided a survey of applications of the fuzzy set theory technique in production and operations management and pointed out that nearly every application is potentially able to realize some of the benefits of fuzzy set theory. Furthermore, fuzzy set theory also provides useful methodology to analyze the reliability in uncertain systems. It can deal with the problem of lacking of inaccuracy or fluctuation data for system components in reliability analysis of some realistic engineering systems. Thus, it is necessary to introduce fuzzy set theory into the reliability theory to deal with reliability of the system with uncertain parameters. The theory of fuzzy reliability has been developed on the basis of

fuzzy set theory. Many research works on the application of fuzzy set theory to problems in reliability or availability of systems were presented in [20–23], and a systemic review on fuzzy reliability of systems with binary-state was provided by Cai [24].

Recently, fuzzy reliability research has focused on reliability evaluation of fuzzy multistate systems. Ding et al. [25, 26] proposed firstly the concept of fuzzy multistate system and assessed the fuzzy reliability of fuzzy multistate systems with fuzzy demand using fuzzy UGF method. Liu et al. [27, 28] investigated the dynamic fuzzy state probabilities, fuzzy performance rates, and fuzzy availability for fuzzy unrepairable multistate elements and fuzzy unrepairable multistate system according to the parametric programming technique and the extension principle. Bamrungsetthapong and Pongpulpornsak [29] discussed fuzzy confidence interval for the fuzzy reliability of a RMSSPS with a fuzzy failure rate and a fuzzy repair rate and considered the performance of fuzzy confidence interval based on the coverage probability and the expected length.

Steady-state availability of a repairable system, as a system performance measure, is the probability that the system is performing satisfactorily over a reasonable period of time [30]. The RMSS availability is defined as the ability of the system to satisfy consumer demand, which is equal to the sum of the probabilities of occurrence of system states in which the system performance rates satisfy consumer demand. Comparable work on the steady-state availability assessment for the RMSSPS with fuzzy state transition rate and fuzzy performance rate is rarely found in the literature. This motivates us to develop the fuzzy steady-state availability assessment for a RMSSPS with fuzzy parameters. According to [4], in this paper, we assumed that the fuzzy performance rate of each multistate parallel subsystem is the sum of the fuzzy performance rates of its all components, and the fuzzy performance rate of the entire RMSSPS is the minimum of the fuzzy performance rates of all parallel subsystems. The purpose of this study is to utilize the α -cut approach, the extension principle, and parametric programming technique

to determine the fuzzy state probability and fuzzy performance rate of the RMSSPS and to evaluate the fuzzy steady-state availability of the system with the fuzzy consumer demand.

The rest of the paper is organized as follows. Section 2 introduces the description of the system considered here and analyzes the fuzzy state probability of multistate component. The fuzzy state probabilities and fuzzy performance rates of the repairable parallel subsystem and the RMSSPS are presented in Sections 3 and 4, respectively. The fuzzy availability assessment method for the RMSSPS is given in Section 5. An illustrative example is presented in Section 6. Finally, Section 7 gives conclusions.

2. RMSSPS with Fuzzy Parameters

The RMSSPS we considered here is composed of n subsystems connected in series, and each subsystem i consists of m_i components in parallel, $i = 1, 2, \dots, n$. The structure of a RMSSPS is shown in Figure 1.

Each component j of subsystem i has $M_{ij} + 1$ different states corresponding to the fuzzy performance rates which can be represented by ordered fuzzy values $\tilde{g}_{ij} = (\tilde{g}_{ij,0}, \tilde{g}_{ij,1}, \dots, \tilde{g}_{ij,M_{ij}})$, and $\tilde{g}_{ij,k}$, $k = 0, 1, \dots, M_{ij}$, is arbitrary fuzzy number. It is assumed that transitions for each component can only occur between adjacent states, and the state transition diagram is shown in Figure 2. The transition rate from state k to state $k - 1$ is presented as the fuzzy value $\tilde{\lambda}_{ij}(k, k - 1)$, and the transition rate from state $k - 1$ to state k is presented as the fuzzy value $\tilde{\mu}_{ij}(k - 1, k)$, $k = 1, 2, \dots, M_{ij}$, and $\tilde{\lambda}_{ij}(k, k - 1)$ and $\tilde{\mu}_{ij}(k - 1, k)$ are arbitrary fuzzy numbers.

With the fuzzy transition rate, the state probability of the component j of subsystem i for the steady state is also a fuzzy value denoted as $\tilde{p}_{ij,l}$, $l_j = 0, 1, \dots, M_{ij}$; let $\tilde{\mathbf{p}}_{ij} = (\tilde{p}_{ij,0}, \tilde{p}_{ij,1}, \dots, \tilde{p}_{ij,M_{ij}})$.

The fuzzy transition rate matrix for the component j of subsystem i is given as

$$\tilde{\Delta}_{ij} = \begin{pmatrix} -\tilde{\mu}_{ij}(0, 1) & \tilde{\mu}_{ij}(0, 1) & 0 & \cdots & 0 & 0 & 0 \\ \tilde{\lambda}_{ij}(1, 0) & -(\tilde{\mu}_{ij}(1, 2) + \tilde{\lambda}_{ij}(1, 0)) & \tilde{\mu}_{ij}(1, 2) & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \tilde{\lambda}_{ij}(M_{ij} - 1, M_{ij} - 2) & -(\tilde{\mu}_{ij}(M_{ij} - 1, M_{ij}) + \tilde{\lambda}_{ij}(M_{ij} - 1, M_{ij} - 2)) & \tilde{\mu}_{ij}(M_{ij} - 1, M_{ij}) \\ 0 & 0 & 0 & \cdots & 0 & \tilde{\lambda}_{ij}(M_{ij}, M_{ij} - 1) & -\tilde{\lambda}_{ij}(M_{ij}, M_{ij} - 1) \end{pmatrix}. \quad (1)$$

Based on the state transition diagram (Figure 2) and the fuzzy transition rate matrix, the balance equations for steady state of the component j of subsystem i are given by

$$\begin{aligned} \tilde{\mu}_{ij}(0, 1) \tilde{p}_{ij,0} &= \tilde{\lambda}_{ij}(1, 0) \tilde{p}_{ij,1}, \\ (\tilde{\lambda}_{ij}(l_j, l_j - 1) + \tilde{\mu}_{ij}(l_j, l_j + 1)) \tilde{p}_{ij,l_j} & \end{aligned}$$

$$= \tilde{\lambda}_{ij}(l_j + 1, l_j) \tilde{p}_{ij,l_j+1} + \tilde{\mu}_{ij}(l_j - 1, l_j) \tilde{p}_{ij,l_j-1},$$

$$l_j = 1, 2, \dots, M_{ij} - 1,$$

$$\tilde{\lambda}_{ij}(M_{ij}, M_{ij} - 1) \tilde{p}_{ij,M_{ij}} = \tilde{\mu}_{ij}(M_{ij} - 1, M_{ij}) \tilde{p}_{ij,M_{ij}-1}. \quad (2)$$

FIGURE 1: General structure of a RMSSPS.

FIGURE 2: The state-transition diagram for the component j of subsystem i .

Equations (2) are Chapman-Kolmogorov equations with fuzzy transition rates for steady state. They state that the rate of flow into state l_j equals the rate of flow out of state l_j in the steady state, $l_j = 0, 1, \dots, M_{ij}$. Taking the relation $\sum_{l_j=0}^{M_{ij}} \tilde{p}_{ij,l_j} = 1$ into account, we can obtain the state probability $\tilde{p}_{ij,l_j}$ as function of $\tilde{\lambda}_{ij}(k, k-1)$ and $\tilde{\mu}_{ij}(k-1, k)$, $k = 1, 2, \dots, M_{ij}$; that is

$$\tilde{p}_{ij,l_j} = \tilde{p}_{ij,l_j}(\tilde{\lambda}_{ij}(k, k-1), \tilde{\mu}_{ij}(k-1, k); k = 1, 2, \dots, M_{ij}). \quad (3)$$

According to the state probability distribution of the conventional multistate component, we have

$$\begin{aligned} \tilde{p}_{ij,l_j} &= \frac{\tilde{\mu}_{ij}(l_j-1, l_j) \tilde{\mu}_{ij}(l_j-2, l_j-1) \cdots \tilde{\mu}_{ij}(0, 1)}{\tilde{\lambda}_{ij}(l_j, l_j-1) \tilde{\lambda}_{ij}(l_j-1, l_j-2) \cdots \tilde{\lambda}_{ij}(1, 0)} \tilde{p}_{ij,0}, \quad (4) \\ l_j &= 1, \dots, M_{ij}, \end{aligned}$$

where $\tilde{p}_{ij,0} = 1/(1 + \sum_{k=1}^{M_{ij}} (\tilde{\mu}_{ij}(0, 1) \tilde{\mu}_{ij}(1, 2) \cdots \tilde{\mu}_{ij}(k-1, k) / \tilde{\lambda}_{ij}(1, 0) \tilde{\lambda}_{ij}(2, 1) \cdots \tilde{\lambda}_{ij}(k, k-1)))$.

Let $\eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1))$ and $\eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k))$ denote the membership functions of $\tilde{\lambda}_{ij}(k, k-1)$ and $\tilde{\mu}_{ij}(k-1, k)$, respectively. Definitions for the α -cuts of $\tilde{\lambda}_{ij}(k, k-1)$ and $\tilde{\mu}_{ij}(k-1, k)$ as crisp intervals are as follows:

$$\begin{aligned} \tilde{\lambda}_{ij}(k, k-1)_\alpha &= \left[\min_{\lambda_{ij}(k, k-1) \in \Lambda_{ij}(k, k-1)} \left\{ \lambda_{ij}(k, k-1) \mid \right. \right. \\ &\quad \left. \left. \eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1)) \geq \alpha \right\}, \right. \\ &\quad \left. \max_{\lambda_{ij}(k, k-1) \in \Lambda_{ij}(k, k-1)} \left\{ \lambda_{ij}(k, k-1) \mid \right. \right. \\ &\quad \left. \left. \eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1)) \geq \alpha \right\} \right] = \left[(\lambda_{ij}(k, k-1))_\alpha^L, (\lambda_{ij}(k, k-1))_\alpha^U \right], \end{aligned}$$

$$\begin{aligned} \tilde{\mu}_{ij}(k-1, k)_\alpha &= \left[\min_{\mu_{ij}(k-1, k) \in N_{ij}(k-1, k)} \left\{ \mu_{ij}(k-1, k) \mid \right. \right. \\ &\quad \left. \left. \eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k)) \geq \alpha \right\}, \right. \\ &\quad \left. \max_{\mu_{ij}(k-1, k) \in N_{ij}(k-1, k)} \left\{ \mu_{ij}(k-1, k) \mid \right. \right. \\ &\quad \left. \left. \eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k)) \geq \alpha \right\} \right] = \left[(\mu_{ij}(k-1, k))_\alpha^L, (\mu_{ij}(k-1, k))_\alpha^U \right], \end{aligned} \quad (5)$$

where $\Lambda_{ij}(k, k-1)$ and $N_{ij}(k-1, k)$ are the crisp universal sets of the transition rate from state k to state $k-1$ and the transition rate from state $k-1$ to state k for the component j of subsystem i .

According to Zadeh extension principle [10, 31], the α -cut of the fuzzy state probability $\tilde{p}_{ij,l_j} = \tilde{p}_{ij,l_j}(\tilde{\lambda}_{ij}(k, k-1), \tilde{\mu}_{ij}(k-1, k); k = 1, \dots, M_{ij})$ can be obtained as

$$\begin{aligned} (\tilde{p}_{ij,l_j})_\alpha &= \left[\min_{\Psi} \left\{ p_{ij,l_j} \mid \eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1)) \right. \right. \\ &\quad \left. \left. \geq \alpha, \eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k)) \geq \alpha \right\}, \right. \\ &\quad \left. \max_{\Psi} \left\{ p_{ij,l_j} \mid \eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1)) \right. \right. \\ &\quad \left. \left. \geq \alpha, \eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k)) \geq \alpha \right\} \right] = \left[(p_{ij,l_j})_\alpha^L, \right. \\ &\quad \left. (p_{ij,l_j})_\alpha^U \right], \end{aligned} \quad (6)$$

where $\Psi = \{\lambda_{ij}(k, k-1) \in \Lambda_{ij}(k, k-1), \mu_{ij}(k-1, k) \in N_{ij}(k-1, k); k = 1, \dots, M_{ij}\}$. Using the parametric programming technique [27, 28, 32, 33], the lower and upper bounds of the α -cut of $\tilde{p}_{ij,l_j}$ can be obtained as

$$\begin{aligned} (p_{ij,l_j})_\alpha^L &= \min p_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\ \text{s.t.} \quad &\lambda_{ij}(k, k-1)_\alpha^L \leq \lambda_{ij}(k, k-1) \\ &\leq \lambda_{ij}(k, k-1)_\alpha^U \\ &\mu_{ij}(k-1, k)_\alpha^L \leq \mu_{ij}(k-1, k) \\ &\leq \mu_{ij}(k-1, k)_\alpha^U, \\ &k = 1, 2, \dots, M_{ij}, \\ (p_{ij,l_j})_\alpha^U &= \max p_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\ \text{s.t.} \quad &\lambda_{ij}(k, k-1)_\alpha^L \leq \lambda_{ij}(k, k-1) \\ &\leq \lambda_{ij}(k, k-1)_\alpha^U \end{aligned}$$

$$\begin{aligned}
\mu_{ij}(k-1, k)_\alpha^L &\leq \mu_{ij}(k-1, k) \\
&\leq \mu_{ij}(k-1, k)_\alpha^U \\
k &= 1, 2, \dots, M_{ij}.
\end{aligned} \tag{7}$$

3. Fuzzy Performance Rate and Fuzzy State Probability of the Parallel Subsystem

According to the following fuzzy UGF of the component j of subsystem i

$$\begin{aligned}
\tilde{u}_{ij}(z) &= \tilde{p}_{ij,0} \cdot z^{\tilde{g}_{ij,0}} + \tilde{p}_{ij,1} \cdot z^{\tilde{g}_{ij,1}} + \dots + \tilde{p}_{ij,M_{ij}} \cdot z^{\tilde{g}_{ij,M_{ij}}} \\
&= \sum_{l_j=0}^{M_{ij}} \tilde{p}_{ij,l_j} \cdot z^{\tilde{g}_{ij,l_j}},
\end{aligned} \tag{8}$$

where $\tilde{g}_{ij,l_j}$ is the fuzzy performance rate of the component j of subsystem i in its state l_j and $\tilde{p}_{ij,l_j}$ is the fuzzy state probability associated with the state l_j , $l_j = 0, 1, \dots, M_{ij}$.

Using fuzzy UGF technique and fuzzy composition operator $\tilde{\Theta}_{\phi_p}$ [25], we have the fuzzy UGF of subsystem i

$$\begin{aligned}
\tilde{u}_i(z) &= \tilde{\Theta}_{\phi_p} \left(\sum_{l_1=0}^{M_{i1}} \tilde{p}_{i1,l_1} \cdot z^{\tilde{g}_{i1,l_1}}, \dots, \sum_{l_{m_i}=0}^{M_{im_i}} \tilde{p}_{im_i,l_{m_i}} \cdot z^{\tilde{g}_{im_i,l_{m_i}}} \right) \\
&= \sum_{l_1=0}^{M_{i1}} \sum_{l_2=0}^{M_{i2}} \dots \sum_{l_{m_i}=0}^{M_{im_i}} \left(\prod_{j=1}^{m_i} \tilde{p}_{ij,l_j} \cdot z^{\sum_{j=1}^{m_i} \tilde{g}_{ij,l_j}} \right) \\
&= \sum_{l_1=0}^{M_{i1}} \sum_{l_2=0}^{M_{i2}} \dots \sum_{l_{m_i}=0}^{M_{im_i}} \left(\tilde{p}_{i,r_i} \cdot z^{\tilde{g}_{i,r_i}} \right) = \sum_{r_i=0}^{M_i} \tilde{p}_{i,r_i} \cdot z^{\tilde{g}_{i,r_i}},
\end{aligned} \tag{9}$$

where $\tilde{g}_{i,r_i} = \sum_{j=1}^{m_i} \tilde{g}_{ij,l_j}$ is the fuzzy performance rate of the parallel subsystem i in its state r_i and $\tilde{p}_{i,r_i} = \prod_{j=1}^{m_i} \tilde{p}_{ij,l_j}$ is the fuzzy state probability associated with the state r_i , $r_i = 0, 1, \dots, M_i$. As shown in (4), $\tilde{p}_{ij,l_j}$ is a function of $\tilde{\lambda}_{ij}(k, k-1)$ and $\tilde{\mu}_{ij}(k-1, k)$, $k = 1, 2, \dots, M_{ij}$, so the α -cut of $\tilde{p}_{i,r_i}$ can be obtained as

$$\begin{aligned}
(\tilde{p}_{i,r_i})_\alpha &= \left[\min_{\Psi} \left\{ \prod_{j=1}^{m_i} p_{ij,l_j} \mid \eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1)) \right. \right. \\
&\quad \left. \left. \geq \alpha, \eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k)) \geq \alpha \right\}, \right. \\
&\quad \left. \max_{\Psi} \left\{ \prod_{j=1}^{m_i} p_{ij,l_j} \mid \eta_{\tilde{\lambda}_{ij}}(\lambda_{ij}(k, k-1)) \right. \right. \\
&\quad \left. \left. \geq \alpha, \eta_{\tilde{\mu}_{ij}}(\mu_{ij}(k-1, k)) \geq \alpha \right\} \right] = \left[(p_{i,r_i})_\alpha^L, \right. \\
&\quad \left. (p_{i,r_i})_\alpha^U \right],
\end{aligned} \tag{10}$$

where $\Psi = \{\lambda_{ij}(k, k-1) \in \Lambda_{ij}(k, k-1), \mu_{ij}(k-1, k) \in N_{ij}(k-1, k); k = 1, \dots, M_{ij}\}$. The parametric programming to find the lower and upper bounds of the α -cut of $\tilde{p}_{i,r_i}$ is

$$\begin{aligned}
(p_{i,r_i})_\alpha^L &= \min \prod_{j=1}^{m_i} p_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\
\text{s.t.} \quad \lambda_{ij}(k, k-1)_\alpha^L &\leq \lambda_{ij}(k, k-1) \\
&\leq \lambda_{ij}(k, k-1)_\alpha^U \\
\mu_{ij}(k-1, k)_\alpha^L &\leq \mu_{ij}(k-1, k) \\
&\leq \mu_{ij}(k-1, k)_\alpha^U \\
k &= 1, 2, \dots, M_{ij}; \quad j = 1, 2, \dots, m_i,
\end{aligned} \tag{11}$$

$$\begin{aligned}
(p_{i,r_i})_\alpha^U &= \max \prod_{j=1}^{m_i} p_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\
\text{s.t.} \quad \lambda_{ij}(k, k-1)_\alpha^L &\leq \lambda_{ij}(k, k-1) \\
&\leq \lambda_{ij}(k, k-1)_\alpha^U \\
\mu_{ij}(k-1, k)_\alpha^L &\leq \mu_{ij}(k-1, k) \\
&\leq \mu_{ij}(k-1, k)_\alpha^U \\
k &= 1, 2, \dots, M_{ij}; \quad j = 1, 2, \dots, m_i.
\end{aligned}$$

Let $\eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j})$ denote the membership function of the fuzzy performance rate $\tilde{g}_{ij,l_j}$, and definition for the α -cut of $\tilde{g}_{ij,l_j}$ as crisp interval is as follows:

$$\begin{aligned}
(\tilde{g}_{ij,l_j})_\alpha &= \left[\min_{g_{ij,l_j} \in G_{ij,l_j}} \{g_{ij,l_j} \mid \eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j}) \geq \alpha\}, \right. \\
&\quad \left. \max_{g_{ij,l_j} \in G_{ij,l_j}} \{g_{ij,l_j} \mid \eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j}) \geq \alpha\} \right] = \left[(g_{ij,l_j})_\alpha^L, \right. \\
&\quad \left. (g_{ij,l_j})_\alpha^U \right],
\end{aligned} \tag{12}$$

where G_{ij,l_j} is the crisp set of the performance rate of the component j of subsystem i .

According to Zadeh extension principle, the α -cut of the fuzzy performance rate $\tilde{g}_{i,r_i} = \sum_{j=1}^{m_i} \tilde{g}_{ij,l_j}$ of the parallel subsystem i can be obtained as

$$\begin{aligned}
(\tilde{g}_{i,r_i})_\alpha &= \left[\min_Y \left\{ g_{i,r_i} = \sum_{j=1}^{m_i} g_{ij,l_j} \mid \eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j}) \geq \alpha, \right. \right. \\
&\quad \left. \left. j = 1, 2, \dots, m_i \right\}, \max_Y \left\{ g_{i,r_i} \right. \right.
\end{aligned}$$

$$\begin{aligned}
&= \left\{ \sum_{j=1}^{m_i} g_{ij,l_j} \mid \eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j}) \geq \alpha, \quad j = 1, 2, \dots, m_i \right\} \\
&= \left[(g_{i,r_i})_{\alpha}^L, (g_{i,r_i})_{\alpha}^U \right],
\end{aligned} \tag{13}$$

where $Y = \{g_{ij,l_j} \in G_{ij,l_j}; \quad j = 1, 2, \dots, m_i\}$. The lower bound $(g_{i,r_i})_{\alpha}^L$ and the upper bound $(g_{i,r_i})_{\alpha}^U$ of the α -cut of $\tilde{g}_{i,r_i}$ can be obtained using the following parametric programming:

$$\begin{aligned}
(g_{i,r_i})_{\alpha}^L &= \min \sum_{j=1}^{m_i} g_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\
\text{s.t.} \quad (g_{ij,l_j})_{\alpha}^L &\leq g_{ij,l_j} \leq (g_{ij,l_j})_{\alpha}^U \\
&\quad j = 1, 2, \dots, m_i, \\
(g_{i,r_i})_{\alpha}^U &= \max \sum_{j=1}^{m_i} g_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\
\text{s.t.} \quad (g_{ij,l_j})_{\alpha}^L &\leq g_{ij,l_j} \leq (g_{ij,l_j})_{\alpha}^U \\
&\quad j = 1, 2, \dots, m_i.
\end{aligned} \tag{14}$$

4. Fuzzy Performance Rate and Fuzzy State Probability of the Entire System

Using the fuzzy composition operator $\tilde{\Theta}_{\phi_s}$ [25], we can obtain the fuzzy UGF of the RMSSPS, which is denote by $\tilde{u}_{SP}(z)$, based on the fuzzy UGF of its parallel subsystems

$$\begin{aligned}
\tilde{u}_{SP}(z) &= \tilde{\Theta}_{\phi_s}(\tilde{u}_1(z), \tilde{u}_2(z), \dots, \tilde{u}_n(z)) \\
&= \tilde{\Theta}_{\phi_s} \left(\sum_{r_1=0}^{M_1} \tilde{p}_{1,r_1} \cdot z^{\tilde{g}_{1,r_1}}, \sum_{r_2=0}^{M_2} \tilde{p}_{2,r_2} \cdot z^{\tilde{g}_{2,r_2}}, \dots, \right. \\
&\quad \left. \sum_{r_n=0}^{M_n} \tilde{p}_{n,r_n} \cdot z^{\tilde{g}_{n,r_n}} \right) \\
&= \sum_{r_1=0}^{M_1} \sum_{r_2=0}^{M_2} \dots \sum_{r_n=0}^{M_n} \left(\prod_{i=1}^n \tilde{p}_{i,r_i} \cdot z^{\min\{\tilde{g}_{1,r_1}, \tilde{g}_{2,r_2}, \dots, \tilde{g}_{n,r_n}\}} \right) \\
&= \sum_{l=0}^M \tilde{p}_l \cdot z^{\tilde{g}_l},
\end{aligned} \tag{15}$$

where M is the highest possible state for the entire system, $\tilde{p}_l = \prod_{i=1}^n \tilde{p}_{i,r_i} = \prod_{i=1}^n \prod_{j=1}^{m_i} \tilde{p}_{ij,l_j}$ is the fuzzy probability of the system in the state l , and $\tilde{g}_l = \min\{\tilde{g}_{1,r_1}, \tilde{g}_{2,r_2}, \dots, \tilde{g}_{n,r_n}\} = \min\{\sum_{j=1}^{m_1} \tilde{g}_{1j,l_j}, \sum_{j=1}^{m_2} \tilde{g}_{2j,l_j}, \dots, \sum_{j=1}^{m_n} \tilde{g}_{nj,l_j}\}$ is the fuzzy performance rate of the system in state l , $l = 0, 1, \dots, M$.

The α -cut of $\tilde{p}_l$ can be determined as

$$\begin{aligned}
(\tilde{p}_l)_{\alpha} &= \left[\min_{\Psi} \left\{ \prod_{i=1}^n \prod_{j=1}^{m_i} p_{ij,l_j} \mid \eta_{\tilde{\lambda}_{ij}(k,k-1)}(\lambda_{ij}(k,k-1)) \right. \right. \\
&\quad \left. \left. \geq \alpha, \eta_{\tilde{\mu}_{ij}(k-1,k)}(\mu_{ij}(k-1,k)) \geq \alpha \right\}, \right. \\
&\quad \left. \max_{\Psi} \left\{ \prod_{i=1}^n \prod_{j=1}^{m_i} p_{ij,l_j} \mid \eta_{\tilde{\lambda}_{ij}(k,k-1)}(\lambda_{ij}(k,k-1)) \right. \right. \\
&\quad \left. \left. \geq \alpha, \eta_{\tilde{\mu}_{ij}(k-1,k)}(\mu_{ij}(k-1,k)) \geq \alpha \right\} \right] = [(p_l)_{\alpha}^L, \\
&\quad (p_l)_{\alpha}^U],
\end{aligned} \tag{16}$$

where $\Psi = \{\lambda_{ij}(k,k-1) \in \Lambda_{ij}(k,k-1), \mu_{ij}(k-1,k) \in N_{ij}(k-1,k); \quad k = 1, \dots, M_{ij}\}$. The parametric programming to find the lower and upper bounds of the α -cut of $\tilde{p}_l$ is

$$\begin{aligned}
(p_l)_{\alpha}^L &= \min \prod_{i=1}^n \prod_{j=1}^{m_i} p_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\
\text{s.t.} \quad \lambda_{ij}(k,k-1)_{\alpha}^L &\leq \lambda_{ij}(k,k-1) \\
&\leq \lambda_{ij}(k,k-1)_{\alpha}^U \\
\mu_{ij}(k-1,k)_{\alpha}^L &\leq \mu_{ij}(k-1,k) \\
&\leq \mu_{ij}(k-1,k)_{\alpha}^U \\
i &= 1, 2, \dots, n, \quad j = 1, 2, \dots, m_i, \quad k = 1, \dots, M_{ij}, \\
(p_l)_{\alpha}^U &= \max \prod_{i=1}^n \prod_{j=1}^{m_i} p_{ij,l_j}, \quad (0 \leq \alpha \leq 1) \\
\text{s.t.} \quad \lambda_{ij}(k,k-1)_{\alpha}^L &\leq \lambda_{ij}(k,k-1) \\
&\leq \lambda_{ij}(k,k-1)_{\alpha}^U \\
\mu_{ij}(k-1,k)_{\alpha}^L &\leq \mu_{ij}(k-1,k) \\
&\leq \mu_{ij}(k-1,k)_{\alpha}^U \\
i &= 1, 2, \dots, n, \quad j = 1, 2, \dots, m_i, \quad k = 1, \dots, M_{ij}.
\end{aligned} \tag{17}$$

The α -cut of the fuzzy performance rate $\tilde{g}_l$ can be determined as

$$\begin{aligned}
(\tilde{g}_l)_{\alpha} &= \left[\min_Y \left\{ \min \left\{ \sum_{j=1}^{m_1} g_{1j,l_j}, \dots, \sum_{j=1}^{m_n} g_{nj,l_j} \right\} \mid \right. \right. \\
&\quad \left. \left. \eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j}) \geq \alpha, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m_i \right\} \right],
\end{aligned}$$

$$\begin{aligned} & \max_Y \left\{ \min \left\{ \sum_{j=1}^{m_1} g_{1j,l_j}, \dots, \sum_{j=1}^{m_n} g_{nj,l_j} \right\} \mid \eta_{\tilde{g}_{ij,l_j}}(g_{ij,l_j}) \right. \\ & \left. \geq \alpha, i=1, 2, \dots, n, j=1, 2, \dots, m_i \right\} = [(g_l)_\alpha^L, \\ & (g_l)_\alpha^U], \end{aligned} \quad (18)$$

where $Y = \{g_{ij,l_j} \in G_{ij,l_j}; j = 1, 2, \dots, m_i\}$. The lower bound $(g_l)_\alpha^L$ and the upper bound $(g_l)_\alpha^U$ of the α -cut of $\tilde{g}_l$ can be obtained using the following parametric programming:

$$\begin{aligned} (g_l)_\alpha^L &= \min \left\{ \min \left\{ \sum_{j=1}^{m_1} g_{1j,l_j}, \sum_{j=1}^{m_2} g_{2j,l_j}, \dots, \sum_{j=1}^{m_n} g_{nj,l_j} \right\} \right\}, \\ & \quad (0 \leq \alpha \leq 1) \\ \text{s.t.} \quad & (g_{ij,l_j})_\alpha^L \leq g_{ij,l_j} \leq (g_{ij,l_j})_\alpha^U \\ & \quad i = 1, 2, \dots, n, j = 1, 2, \dots, m_i, \\ (g_l)_\alpha^U &= \max \left\{ \min \left\{ \sum_{j=1}^{m_1} g_{1j,l_j}, \sum_{j=1}^{m_2} g_{2j,l_j}, \dots, \sum_{j=1}^{m_n} g_{nj,l_j} \right\} \right\}, \\ & \quad (0 \leq \alpha \leq 1) \\ \text{s.t.} \quad & (g_{ij,l_j})_\alpha^L \leq g_{ij,l_j} \leq (g_{ij,l_j})_\alpha^U \\ & \quad i = 1, 2, \dots, n, j = 1, 2, \dots, m_i. \end{aligned} \quad (19)$$

5. Fuzzy Steady-State Availability Assessment for the RMSSPS

For the RMSSPS, the system availability is defined as the ability of the system to satisfy consumer demand. Here, the consumer demand is presented as fuzzy value $\tilde{\omega}$, and the steady-state availability of the system is the fuzzy probability that the fuzzy performance rate of the system satisfies the fuzzy demand $\tilde{\omega}$. So, we have

$$\tilde{A}(\tilde{\omega}) = \sum_{l=0}^M \tilde{p}_l \cdot p(\tilde{g}_l \geq \tilde{\omega}), \quad (20)$$

where $p(\tilde{g}_l \geq \tilde{\omega})$ denote the possibility degree of $\tilde{g}_l \geq \tilde{\omega}$.

For crisp performance rate g_l and consumer demand ω , $p(g_l \geq \omega) = 1$ if $g_l \geq \omega$, and $p(g_l \geq \omega) = 0$ if $g_l < \omega$. In the fuzzy system model, the relationship between $\tilde{g}_l$ and $\tilde{\omega}$ is illustrated in Figure 3 under triangular fuzzy number. It can be seen from Figure 3 that $p(\tilde{g}_l \geq \tilde{\omega}_1) = 0$, $p(\tilde{g}_l \geq \tilde{\omega}_2) = 1$, and $0 < p(\tilde{g}_l \geq \tilde{\omega}_{3(4)}) < 1$. For $0 < p(\tilde{g}_l \geq \tilde{\omega}_{3(4)}) < 1$, we let $\tilde{r}_l = \tilde{g}_l - \tilde{\omega} = \{r_l, \eta_{\tilde{r}_l}(r_l) \mid r_l = g_l - \omega, g_l \in G_l, \omega \in W\}$, $\tilde{r}_l \geq 0$ represents $\{r_l, \eta_{\tilde{r}_l}(r_l) \mid r_l \geq 0, r_l \in R_l, \eta_{\tilde{r}_l}(r_l) = \sup_{\Omega} \min\{\eta_{\tilde{g}_l}(g_l), \eta_{\tilde{\omega}}(\omega) \mid r_l = g_l - \omega\} (\Omega = \{g_l \in G_l, \omega \in W\})$ is the membership function of $\tilde{r}_l$, G_l , W and R_l are the

crisp universal sets of the system performance rate, consumer demand and r_l , respectively.

Based on the analysis above, we can replace $\tilde{g}_l \geq \tilde{\omega}$ with $\tilde{r}_l \geq 0$; that is $p(\tilde{g}_l \geq \tilde{\omega}) = p(\tilde{r}_l \geq 0)$. According to [27, 28], the possibility degree of $\tilde{r}_l \geq 0$ is defined as

$$p(\tilde{r}_l \geq 0) = \frac{\int_{\substack{r_l \in R_l \\ r_l \geq 0 \\ \eta_{\tilde{r}_l}(r_l) \geq \alpha}} \eta_{\tilde{r}_l}(r_l) dr_l}{\int_{\substack{r_l \in R_l \\ \eta_{\tilde{r}_l}(r_l) \geq \alpha}} \eta_{\tilde{r}_l}(r_l) dr_l}. \quad (21)$$

However, the definition of the possibility degree of $\tilde{r}_l \geq 0$ in (21) is given by the membership function of $\tilde{r}_l$. When the performance rates of the components and consumer demand are triangular fuzzy numbers, the membership function of $\tilde{r}_l$ can be easily obtained. If system has a lot of components, and the performance rates of the components and consumer demand are arbitrary fuzzy representation, it is difficult to obtain the explicit expression of the membership function of $\tilde{r}_l$, and some approximation techniques may be developed to calculate the membership function. Here, we tackle the problem by using the α -cut approach. Based on the α -cuts of $\tilde{g}_l$ and $\tilde{\omega}$, the α -cut of $\tilde{r}_l$ can be easily determined. Let $[(r_l)_\alpha^L, (r_l)_\alpha^U]$ denote the α -cut of $\tilde{r}_l$; we define the possibility degree of $\tilde{r}_l \geq 0$ as

$$p(\tilde{r}_l \geq 0) = \begin{cases} 1, & (r_l)_\alpha^L \geq 0 \\ \frac{(r_l)_\alpha^U - 0}{(r_l)_\alpha^U - (r_l)_\alpha^L}, & (r_l)_\alpha^L < 0, (r_l)_\alpha^U > 0 \\ 0, & (r_l)_\alpha^U < 0. \end{cases} \quad (22)$$

Hence, the fuzzy steady-state availability of the system for a given fuzzy demand $\tilde{\omega}$ can be rewritten as

$$\tilde{A}(\tilde{\omega}) = \sum_{l=0}^M \tilde{p}_l \cdot p(\tilde{r}_l \geq 0). \quad (23)$$

We denote the α -cut of $\tilde{A}(\tilde{\omega})$ by $(\tilde{A}(\tilde{\omega}))_\alpha = [(A(\omega))_\alpha^L, (A(\omega))_\alpha^U]$; the lower and upper bounds of $(\tilde{A}(\tilde{\omega}))_\alpha$ can be determined using the following parametric programming:

$$(A(\omega))_\alpha^L = \min \sum_{l=0}^M p_l \cdot p(\tilde{r}_l \geq 0), \quad (0 \leq \alpha \leq 1)$$

$$\text{s.t.} \quad (p_l)_\alpha^L \leq p_l \leq (p_l)_\alpha^U,$$

$$l = 0, 1, \dots, M,$$

$$\sum_{l=0}^M p_l = 1,$$

FIGURE 3: Fuzzy performance rate and fuzzy demand.

$$\begin{aligned}
 (A(\omega))_\alpha^U &= \max \sum_{l=0}^M p_l \cdot p(\tilde{r}_l \geq 0), \quad (0 \leq \alpha \leq 1) \\
 \text{s.t.} \quad (p_l)_\alpha^L &\leq p_l \leq (p_l)_\alpha^U, \\
 l &= 0, 1, \dots, M, \\
 \sum_{l=0}^M p_l &= 1.
 \end{aligned} \tag{24}$$

In order to obtain rational outcome of the system steady-state availability, the equality constraint $\sum_{l=0}^M p_l = 1$ is added into the parametric programming.

6. Illustrative Example

Consider a flow transmission multistate series-parallel system with three repairable components. In this example, the system consists of $n = 2$ parallel subsystems connected in series, and subsystems 1 and 2 consist of $m_1 = 2$ and $m_2 = 1$ components, respectively. The parameters for the components are represented as triangular fuzzy values. The fuzzy transition rates and fuzzy performance rates of each component are presented in Tables 1 and 2, respectively. Suppose that the fuzzy demand $\tilde{\omega}$ is a triangular fuzzy number (1.9, 2.1, 2.3).

TABLE 1: The fuzzy transition rates for the components of each subsystem.

Subsystem 1		Subsystem 2	
$\tilde{\lambda}_{11}(1, 0)$ (0.04, 0.05, 0.06)	$\tilde{\lambda}_{12}(1, 0)$ (0.03, 0.04, 0.05)	$\tilde{\lambda}_{21}(2, 1)$ (0.02, 0.03, 0.04)	$\tilde{\lambda}_{21}(1, 0)$ (0.01, 0.02, 0.03)
$\tilde{\mu}_{11}(0, 1)$ (0.4, 0.5, 0.6)	$\tilde{\mu}_{12}(0, 1)$ (0.25, 0.3, 0.35)	$\tilde{\mu}_{21}(1, 2)$ (0.2, 0.3, 0.4)	$\tilde{\mu}_{21}(0, 1)$ (0.15, 0.2, 0.25)

TABLE 2: The fuzzy performance rates for the components of each subsystem.

State	Subsystem 1	Subsystem 2
0	$\tilde{g}_{11,0}$ 0	$\tilde{g}_{12,0}$ 0
1	$\tilde{g}_{11,1}$ (1.5, 1.6, 1.7)	$\tilde{g}_{12,1}$ (1.8, 2.0, 2.2)
2	$\backslash$ $\backslash$	$\tilde{g}_{21,2}$ (3.6, 3.7, 3.8)

According to the method of determining the α -cut of the triangular fuzzy number, the α -cuts of the fuzzy transition rates for each component are

$$\begin{aligned}
 &[\lambda_{11}(1, 0)_\alpha^L, \lambda_{11}(1, 0)_\alpha^U] \\
 &= [0.04 + 0.01\alpha, 0.06 - 0.01\alpha],
 \end{aligned}$$

$$\begin{aligned}
[\mu_{11}(0,1)_\alpha^L, \mu_{11}(0,1)_\alpha^U] &= [0.4 + 0.1\alpha, 0.6 - 0.1\alpha], & [\mu_{21}(1,2)_\alpha^L, \mu_{21}(1,2)_\alpha^U] &= [0.2 + 0.1\alpha, 0.4 - 0.1\alpha], \\
[\lambda_{12}(1,0)_\alpha^L, \lambda_{12}(1,0)_\alpha^U] &= [0.03 + 0.01\alpha, 0.05 - 0.01\alpha], & [\lambda_{21}(1,0)_\alpha^L, \lambda_{21}(1,0)_\alpha^U] &= [0.01 + 0.01\alpha, 0.03 - 0.01\alpha], \\
[\mu_{12}(0,1)_\alpha^L, \mu_{12}(0,1)_\alpha^U] &= [0.25 + 0.05\alpha, 0.35 - 0.05\alpha], & [\mu_{21}(0,1)_\alpha^L, \mu_{21}(0,1)_\alpha^U] &= [0.15 + 0.05\alpha, 0.25 - 0.05\alpha]. \\
[\lambda_{21}(2,1)_\alpha^L, \lambda_{21}(2,1)_\alpha^U] &= [0.02 + 0.01\alpha, 0.04 - 0.01\alpha], & &
\end{aligned} \tag{25}$$

In accordance with (4), we can obtain the fuzzy state probabilities of the components of subsystems 1 and 2

$$\begin{aligned}
\tilde{p}_{11,0} &= \frac{\tilde{\lambda}_{11}(1,0)}{\tilde{\lambda}_{11}(1,0) + \tilde{\mu}_{11}(0,1)}, \\
\tilde{p}_{11,1} &= \frac{\tilde{\mu}_{11}(0,1)}{\tilde{\lambda}_{11}(1,0) + \tilde{\mu}_{11}(0,1)}, \\
\tilde{p}_{12,0} &= \frac{\tilde{\lambda}_{12}(1,0)}{\tilde{\lambda}_{12}(1,0) + \tilde{\mu}_{12}(0,1)}, \\
\tilde{p}_{12,1} &= \frac{\tilde{\mu}_{12}(0,1)}{\tilde{\lambda}_{12}(1,0) + \tilde{\mu}_{12}(0,1)}, \\
\tilde{p}_{21,0} &= \frac{1}{(1 + \tilde{\mu}_{21}(0,1)/\tilde{\lambda}_{21}(1,0) + \tilde{\mu}_{21}(0,1)\tilde{\mu}_{21}(1,2)/\tilde{\lambda}_{21}(1,0)\tilde{\lambda}_{21}(2,1))}, \\
\tilde{p}_{21,1} &= \frac{(\tilde{\mu}_{21}(0,1)/\tilde{\lambda}_{21}(1,0))}{(1 + \tilde{\mu}_{21}(0,1)/\tilde{\lambda}_{21}(1,0) + \tilde{\mu}_{21}(0,1)\tilde{\mu}_{21}(1,2)/\tilde{\lambda}_{21}(1,0)\tilde{\lambda}_{21}(2,1))}, \\
\tilde{p}_{21,2} &= \frac{(\tilde{\mu}_{21}(1,2)\tilde{\mu}_{21}(0,1)/\tilde{\lambda}_{21}(2,1)\tilde{\lambda}_{21}(1,0))}{(1 + \tilde{\mu}_{21}(0,1)/\tilde{\lambda}_{21}(1,0) + \tilde{\mu}_{21}(0,1)\tilde{\mu}_{21}(1,2)/\tilde{\lambda}_{21}(1,0)\tilde{\lambda}_{21}(2,1))}.
\end{aligned} \tag{26}$$

The fuzzy UGFs of the components 1 and 2 in subsystem 1 can be described as follows:

$$\begin{aligned}
\tilde{u}_{11}(z) &= \tilde{p}_{11,0} \cdot z^0 + \tilde{p}_{11,1} \cdot z^{(1.5,1.6,1.7)}, \\
\tilde{u}_{12}(z) &= \tilde{p}_{12,0} \cdot z^0 + \tilde{p}_{12,1} \cdot z^{(1.8,2.2,2.2)}.
\end{aligned} \tag{27}$$

The fuzzy UGF of the component 1 in subsystem 2 can be written as

$$\begin{aligned}
\tilde{u}_{21}(z) &= \tilde{p}_{21,0} \cdot z^0 + \tilde{p}_{21,1} \cdot z^{(2.1,2.4,2.7)} + \tilde{p}_{21,2} \\
&\quad \cdot z^{(3.6,3.7,3.8)}.
\end{aligned} \tag{28}$$

Based on the individual fuzzy UGFs of the components 1 and 2 in subsystem 1 given above, we can get the fuzzy

UGF of the parallel subsystem 1 by using operator $\tilde{\Theta}_{\phi_p}$ as follows:

$$\begin{aligned}
\tilde{u}_1(z) &= \tilde{\Theta}_{\phi_p}(\tilde{u}_{11}(z), \tilde{u}_{12}(z)) \\
&= \tilde{p}_{11,0}\tilde{p}_{12,0} \cdot z^0 + \tilde{p}_{11,1}\tilde{p}_{12,0} \cdot z^{(1.5,1.6,1.7)} \\
&\quad + \tilde{p}_{11,0}\tilde{p}_{12,1} \cdot z^{(1.8,2.2,2.2)} + \tilde{p}_{11,1}\tilde{p}_{12,1} \\
&\quad \cdot z^{(3.3,3.6,3.9)}.
\end{aligned} \tag{29}$$

The fuzzy UGF of subsystem 2 with one component is

$$\begin{aligned}
\tilde{u}_2(z) &= \tilde{u}_{21}(z) \\
&= \tilde{p}_{21,0} \cdot z^0 + \tilde{p}_{21,1} \cdot z^{(2.1,2.4,2.7)} + \tilde{p}_{21,2} \\
&\quad \cdot z^{(3.6,3.7,3.8)}.
\end{aligned} \tag{30}$$

TABLE 3: The α -cuts of the fuzzy state probabilities at 11 α values.

α	$[(p_0)_\alpha^L, (p_0)_\alpha^U]$	$[(p_1)_\alpha^L, (p_1)_\alpha^U]$	$[(p_2)_\alpha^L, (p_2)_\alpha^U]$	$[(p_3)_\alpha^L, (p_3)_\alpha^U]$	$[(p_4)_\alpha^L, (p_4)_\alpha^U]$
0.0	[0.0068, 0.0533]	[0.0664, 0.1559]	[0.0504, 0.1199]	[0.0342, 0.1430]	[0.5844, 0.8208]
0.1	[0.0076, 0.0482]	[0.0700, 0.1504]	[0.0530, 0.1152]	[0.0371, 0.1334]	[0.6008, 0.8122]
0.2	[0.0085, 0.0436]	[0.0737, 0.1449]	[0.0556, 0.1106]	[0.0402, 0.1246]	[0.6166, 0.8034]
0.3	[0.0095, 0.0394]	[0.0774, 0.1396]	[0.0583, 0.1062]	[0.0435, 0.1164]	[0.6317, 0.7944]
0.4	[0.0106, 0.0357]	[0.0812, 0.1345]	[0.0610, 0.1020]	[0.0469, 0.1088]	[0.6462, 0.7850]
0.5	[0.0117, 0.0323]	[0.0851, 0.1294]	[0.0639, 0.0979]	[0.0506, 0.1017]	[0.6602, 0.7754]
0.6	[0.0130, 0.0292]	[0.0891, 0.1245]	[0.0668, 0.0939]	[0.0544, 0.0950]	[0.6736, 0.7655]
0.7	[0.0145, 0.0265]	[0.0932, 0.1197]	[0.0698, 0.0901]	[0.0585, 0.0888]	[0.6865, 0.7553]
0.8	[0.0160, 0.0240]	[0.0974, 0.1150]	[0.0730, 0.0865]	[0.0628, 0.0829]	[0.6990, 0.7448]
0.9	[0.0177, 0.0217]	[0.1016, 0.1105]	[0.0762, 0.0829]	[0.0674, 0.0774]	[0.7110, 0.7339]
1.0	[0.0196, 0.0196]	[0.1060, 0.1060]	[0.0795, 0.0795]	[0.0723, 0.0723]	[0.7226, 0.7226]

In order to find the fuzzy UGF of the entire system, the operator $\tilde{\Theta}_{\phi_s}$ is applied to $\tilde{u}_1(z)$ and $\tilde{u}_2(z)$ based on the composition algorithm in Section 4. We have

$$\begin{aligned}
\tilde{u}_{sp}(z) = \tilde{\Theta}_{\phi_s}(\tilde{u}_1(z), \tilde{u}_2(z)) = & (\tilde{p}_{21,0}(\tilde{p}_{11,0}\tilde{p}_{12,0} \\
& + \tilde{p}_{11,1}\tilde{p}_{12,0} + \tilde{p}_{11,0}\tilde{p}_{12,1} + \tilde{p}_{11,1}\tilde{p}_{12,1}) \\
& + \tilde{p}_{11,0}\tilde{p}_{12,0}(\tilde{p}_{21,1} + \tilde{p}_{21,2})) \cdot z^0 + \tilde{p}_{11,1}\tilde{p}_{12,0}(\tilde{p}_{21,1} \\
& + \tilde{p}_{21,2}) \cdot z^{(1.5, 1.6, 1.7)} + \tilde{p}_{11,0}\tilde{p}_{12,1}(\tilde{p}_{21,1} + \tilde{p}_{21,2}) \\
& \cdot z^{(1.8, 2, 2.2)} + \tilde{p}_{21,1}\tilde{p}_{11,1}\tilde{p}_{12,1} \cdot z^{(2.1, 2.4, 2.7)} \\
& + \tilde{p}_{21,2}\tilde{p}_{11,1}\tilde{p}_{12,1} \cdot z^{(3.3, 3.6, 3.8)}.
\end{aligned} \quad (31)$$

The fuzzy UGF of the entire system $\tilde{u}_{sp}(z)$ can be written as

$$\tilde{u}_{sp}(z) = \sum_{l=0}^4 \tilde{p}_l \cdot z^{\tilde{g}_l}. \quad (32)$$

That is, the fuzzy performance rates of the system are

$$\begin{aligned}
\tilde{g}_0 &= 0, \\
\tilde{g}_1 &= (1.5, 1.6, 1.7), \\
\tilde{g}_2 &= (1.8, 2, 2.2), \\
\tilde{g}_3 &= (2.1, 2.4, 2.7), \\
\tilde{g}_4 &= (3.3, 3.6, 3.8).
\end{aligned} \quad (33)$$

The fuzzy probabilities corresponding to the fuzzy performance rates are

$$\begin{aligned}
\tilde{p}_0 &= \tilde{p}_{21,0}\tilde{p}_{11,0}\tilde{p}_{12,0} + \tilde{p}_{21,0}\tilde{p}_{11,1}\tilde{p}_{12,0} + \tilde{p}_{21,0}\tilde{p}_{11,0}\tilde{p}_{12,1} \\
&+ \tilde{p}_{21,0}\tilde{p}_{11,1}\tilde{p}_{12,1} + \tilde{p}_{11,0}\tilde{p}_{12,0}\tilde{p}_{21,1} \\
&+ \tilde{p}_{11,0}\tilde{p}_{12,0}\tilde{p}_{21,2},
\end{aligned}$$

$$\tilde{p}_1 = \tilde{p}_{11,1}\tilde{p}_{12,0}\tilde{p}_{21,1} + \tilde{p}_{11,1}\tilde{p}_{12,0}\tilde{p}_{21,2},$$

$$\tilde{p}_2 = \tilde{p}_{11,0}\tilde{p}_{12,1}\tilde{p}_{21,1} + \tilde{p}_{11,0}\tilde{p}_{12,1}\tilde{p}_{21,2},$$

$$\tilde{p}_3 = \tilde{p}_{21,1}\tilde{p}_{11,1}\tilde{p}_{12,1},$$

$$\tilde{p}_4 = \tilde{p}_{21,2}\tilde{p}_{11,1}\tilde{p}_{12,1}.$$

(34)

The α -cuts of $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$) can be obtained by (17). Due to the complicated form of the objective function, it is impossible to represent the optimal solution $(p_l)_\alpha^L$ and $(p_l)_\alpha^U$ in terms of α , $l = 0, 1, 2, 3, 4$. With the help of Mathematica program system, the α -cuts of $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$) can be computed at some distinct α values. The rough shapes of the membership functions for $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$) can be determined according to different α values.

Table 3 presents the α -cuts of $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$) at 11 distinct α values: 0.0, 0.1, ..., 1.0. Based on the results shown in Table 3, we can see that it is impossible for the values of $\tilde{p}_4$ to fall below 0.5844 or exceed 0.8208 though the state probability is fuzzy, and the value of $\tilde{p}_4$ is equal to the result regarding transition rates and performance rates as crisp values when $\alpha = 1$. Similarly, one can read the rest of the results for $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$) presented in Table 3. Figure 4 plots the rough shapes of the membership functions for $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$) constructed from 51 values of α : 0, 0.02, 0.04, ..., 1.00.

According to (22)–(24), the α -cut of the fuzzy steady-state availability $\tilde{A}(\tilde{\omega})$ can be computed and the rough shape of the membership function for $\tilde{A}(\tilde{\omega})$ can be plotted at different α values. The α -cut and the rough shape of the membership function for $\tilde{A}(\tilde{\omega})$ at different α values according to Liu's method [28] and our proposed method are presented in Table 4 and Figure 5, respectively.

It can be seen from Table 4 that the α -cut of the fuzzy steady-state availability calculated by our proposed method is slightly different with that calculated by Liu's method [28] when $0.0 \leq \alpha < 0.3$ and $0.6 \leq \alpha < 0.8$, it is great when $0.3 \leq \alpha < 0.6$, and the results are the same when $0.8 \leq \alpha \leq 1.0$. The proposed method need not to solve the membership function

FIGURE 4: The membership functions for fuzzy state probabilities $\tilde{p}_l$ ($l = 0, 1, 2, 3, 4$).TABLE 4: The α -cut of the fuzzy steady-state availability $\tilde{A}(\tilde{\omega})$ at 11 α values.

α	Liu's method $[(A(\omega))_\alpha^L, (A(\omega))_\alpha^U]$	Proposed method $[(A(\omega))_\alpha^L, (A(\omega))_\alpha^U]$
0.0	[0.6977, 0.8861]	[0.6986, 0.8842]
0.1	[0.7119, 0.8799]	[0.7136, 0.8790]
0.2	[0.7257, 0.8737]	[0.7284, 0.8740]
0.3	[0.7392, 0.8676]	[0.7430, 0.8692]
0.4	[0.7522, 0.8618]	[0.7576, 0.8650]
0.5	[0.7608, 0.8526]	[0.7649, 0.8553]
0.6	[0.7673, 0.8417]	[0.7700, 0.8436]
0.7	[0.7701, 0.8275]	[0.7712, 0.8283]
0.8	[0.7745, 0.8136]	[0.7745, 0.8136]
0.9	[0.7849, 0.8045]	[0.7849, 0.8045]
1.0	[0.7949, 0.7949]	[0.7949, 0.7949]

FIGURE 5: The membership function for fuzzy steady-state availability $\tilde{A}(\tilde{\omega})$.

of $\tilde{r}_l$ (see Section 5), and only need to determine the α -cut of $\tilde{r}_l$. According to our proposed method, the shape of the membership function for the fuzzy steady-state availability $\tilde{A}(\tilde{\omega})$ is presented in Figure 5 compared with the one using the Liu's method [28].

7. Conclusions

This paper applies the concept of α -cut and Zadeh's principle to a RMSSPS with fuzzy state transition rates and fuzzy performance rates, and then analyzes the system indices of interest. The fuzzy UGF technique works efficiently for the computation of the fuzzy state probabilities and fuzzy performance rates of the system. The parametric programming formulas are presented to find the α -cuts of the fuzzy state probabilities, the fuzzy performance rates, and

the fuzzy steady-state availability of the entire system. A special availability evaluation approach is introduced when the system performance rates and the consumer demand are fuzzy values. Numerical technique method is used to determine the membership functions of the fuzzy state probabilities and the fuzzy availability for the entire system. An illustrative example is provided to show the validity of the proposed method. In future work, we will concern the development of fuzzy optimal design for the RMSSPS.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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