Research Article

Robust Distributed H_∞ Filtering for Nonlinear Systems with Sensor Saturations and Fractional Uncertainties with Digital Simulation

Dong Liu, Guangfu Tang, Zhiyuan He, Yan Zhao, and Hui Pang

State Grid Smart Grid Research Institute, North Zone of Future Sci-Tech City, Beiqijia, Changping District, Beijing 102211, China

Correspondence should be addressed to Dong Liu; liudong@sgri.sgcc.com.cn

Received 13 February 2015; Revised 30 March 2015; Accepted 5 April 2015

Academic Editor: Zidong Wang

Copyright © 2015 Dong Liu et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper is concerned with the robust distributed H_∞ filtering problem for nonlinear systems subject to sensor saturations and fractional parameter uncertainties. A sufficient condition is derived for the filtering error system to reach the required H_∞ performance in terms of recursive linear matrix inequality method. An iterative algorithm is then proposed to obtain the filter parameters recursively by solving the corresponding linear matrix inequality. A numerical example is presented to show the effectiveness of the proposed method.

1. Introduction

The sensor networks have gradually received more and more research interest in the past decades. A typical example is the power grids which have evolved over the past century from a series of small independent community based systems to large-scale and complex systems involving many kinds of intelligent electronic devices [1]. As is well known, the information collected by the sensor networks need to be further processed before used for diagnosis, control, optimization, and other transactions for the smart grids, which requires much more advanced methodologies that are more sensitive, reliable, and economic than those applied in the traditional centralized electricity networks. Due to its clear engineering insights, recently, sensor networks have gained increasing research interests during the past decades in various branches of theoretical research and industrial applications. In particular, the distributed filtering over sensor networks has been ongoing research issue that attracts special attention from researchers in the area. Different from traditional filtering techniques based on single or centralized structured/located sensors, the information available for the filter design on an individual node of the sensor network is not only from its own measurement but also from its neighboring sensors' measurements according to the given topology [2]. As such,

the objective of filtering based on a sensor network can be achieved in a distributed yet collaborative way. Such a problem is usually referred to as the distributed filtering problem. It is noticed that one of the main challenges in designing distributed filters lies in how to cope with the complicated couplings between one sensor and its neighboring sensors by reflecting such couplings in the filter structure specification.

It is worth mentioning that in real-world applications, due to a variety of reasons, such as abrupt changes of working conditions, internal/external disturbances, erosions, equipment aging, the sensor outputs are usually corrupted by these different disadvantages. Such phenomena are always referred to as incomplete information. Up to now, filtering problems with incomplete information have been widely studied and a lot of results have been reported in the literature; see [3–14] for some latest publications. As far as the distributed filtering problems subject to incomplete information are concerned, some recent representative work can be summarized as follows. In [15], the distributed filtering problem was solved when randomly occurring saturations and successive packet dropouts appear over sensor networks. In [16], distributed H_∞ filtering problem was studied for a class of Markovian jump nonlinear time-delay systems over lossy sensor networks. In [17], distributed filtering for a class of time-varying systems was discussed over sensor

networks with quantization errors and successive packet dropouts. The distributed H_∞ state estimation with stochastic parameters and nonlinearities through sensor networks was solved in [18] over the finite-horizon. With randomly varying nonlinearities and missing measurements, [19] developed a distributed filtering method to achieve the performance requirements. In [20], distributed state estimation problem for uncertain Markov-type sensor networks with mode-dependent distributed delays was solved. Using linear matrix inequality approach, the distributed filters were given in [21, 22], respectively, for different types of stochastic systems. However, it is worth mentioning that, up to now, the distributed filtering problem has not been studied for nonlinear time-varying systems subject to randomly occurring sensor failures.

Motivated by the above discussion, in this paper, it is the objective to design a robust filter for a class of discrete time-varying nonlinear stochastic systems subject to sensor saturations and fractional uncertainties. The contribution of this paper is twofold: (i) for the first time, the filtering problem is discussed while taking both sensor saturations and fractional uncertainties into consideration. (ii) An iterative algorithm is developed to solve the robust H_∞ filtering problem and then seek the desired filter parameters step by step. The rest of this paper is organized as follows: Section 2 formulates the distributed event-based filter design problem for nonlinear discrete time-varying stochastic system. The main results are presented in Section 3 where sufficient conditions for the existence of the desired filter are given in terms of recursive linear matrix inequalities. Section 4 gives a numerical example. Section 5 is the conclusion.

2. Problem of Formulation

In this paper, it is assumed that the sensor network has N sensor nodes which are distributed in the space according to a specific interconnection topology characterized by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ denotes the set of sensor nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{L} = [r_{ij}]_{N \times N}$ is the nonnegative adjacency matrix associated with the edges of the graph, that is, $r_{ij} > 0 \Leftrightarrow \text{edge}(i, j) \in \mathcal{E}$, which means that there is information transmission from sensor j to sensor i . If $(i, j) \in \mathcal{E}$, then node j is called one of the neighbors of node i . For all $i \in \mathcal{V}$, denote $\mathcal{N}_i \triangleq \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$, which means that, in the sensor network, sensor node i can receive the information from its neighboring nodes $j \in \mathcal{N}_i$ according to the given network topology.

Let us consider the discrete-time nonlinear stochastic system with N sensors defined on $k \in [0, T]$:

$$\begin{aligned} x_{k+1} &= (A_k + \Delta A_k) x_k + g_k(x_k) + D_k \omega_k, \\ y_{i,k} &= \sigma(C_{i,k} x_k) + E_{i,k} \omega_k, \\ z_k &= M_k x_k, \end{aligned} \quad (1)$$

where x_k , $y_{i,k}$, z_k , and ω_k represent the state, the measured output of the i th sensor, estimated output, and disturbance belonging to l_2 , respectively. A_k , $C_{i,k}$, D_k , $E_{i,k}$, and M_k are

known real time-varying matrices with appropriate dimensions.

Assumption 1. The nonlinear function $g_k(x_k)$ is assumed to obey the following constraint:

$$g_k^T(x_k) S_k^{-1} g_k(x_k) \leq \delta x_k^T x_k, \quad (2)$$

where S_k are known positive definite matrix sequence with appropriate dimensions describing the shape of the ellipsoids with 0 being the center of the ellipsoids and $\delta > 0$ is a known positive scalar.

The nonlinear function $g_k(x_k)$ stands for the nonlinearity that is unknown, bounded, and deterministic but reside within an ellipsoidal set. Such a type of nonlinearity is usually occurring in practical engineering practice and is always a main origin for the degradation of the system performance.

Assumption 2 (see [23]). ΔA_k has the linear fractional form as follows:

$$\Delta A_k = U \Sigma_k (I - J \Sigma_k)^{-1} V \quad (3)$$

with $J^T J < I$ and $\Sigma_k^T \Sigma_k < I$, where U , V , and J are known constant matrices and Σ_k denotes the unknown matrix functions with Lebesgue measurable elements.

Definition 3. A nonlinear function $\psi(\cdot)$ is said to satisfy the sector-bounded condition if

$$(\psi(\theta) - \Theta_1 \theta)^T (\psi(\theta) - \Theta_2 \theta) \leq 0 \quad (4)$$

for some real matrices Θ_1 and Θ_2 , where $\Theta = \Theta_2 - \Theta_1$ is a symmetric positive definite matrix. In this case, we say that $\psi(\cdot)$ belongs to $[\Theta_1, \Theta_2]$.

As discussed in [23], there exist matrices K_1 and K_2 such that $0 \leq K_1 < I \leq K_2$; the sensor saturation function $\sigma(C_{i,k} x_k)$ can be written as

$$\sigma(C_{i,k} x_k) = K_1 C_{i,k} x_k + \varphi_y(C_{i,k} x_k), \quad (5)$$

where $\varphi_y(C_{i,k} x_k)$ is a nonlinear vector-valued function satisfying the sector-bounded condition with $\Theta_1 = 0$ and $\Theta_2 = K$. In this case, $\varphi_y(C_{i,k} x_k)$ can be represented as follows:

$$\varphi_y^T(C_{i,k} x_k) (\varphi_y(C_{i,k} x_k) - K C_{i,k} x_k) \leq 0, \quad (6)$$

where $K = K_2 - K_1$.

The objective of this paper is to design a filter with the following form for system (1):

$$\hat{x}_{i,k+1} = G_{i,k} \hat{x}_{i,k} + \sum_{j \in \mathcal{N}_i} r_{ij} H_{ij,k} (y_{j,k} - C_{j,k} \hat{x}_{j,k}), \quad (7)$$

$$\hat{z}_{i,k} = M_k \hat{x}_{i,k},$$

where $\hat{x}_{i,k}$ is the state estimate, $\hat{z}_k$ is the output to be estimated, and $G_{i,k}$ and $H_{ij,k}$ are the filter parameters to be determined.

From system (1) and filter structure (7), one can obtain

$$\begin{aligned}
 e_{i,k+1} &= x_{k+1} - \hat{x}_{i,k+1} = (A_k + \Delta A_k) x_k + g(x_k) \\
 &+ D_k \omega_k - \left(G_{i,k} \hat{x}_{i,k} + \sum_{j \in \mathcal{N}_i} r_{ij} H_{ij,k} (y_{j,k} - C_{j,k} \hat{x}_{j,k}) \right) \\
 &= (A_k + \Delta A_k) x_k + g(x_k) + D_k \omega_k - G_{i,k} \hat{x}_{i,k} \\
 &- \left(\sum_{j \in \mathcal{N}_i} r_{ij} H_{ij,k} (\sigma(C_{j,k} x_k) + E_{j,k} \omega_k - C_{j,k} \hat{x}_{j,k}) \right) \quad (8) \\
 &= (A_k + \Delta A_k) x_k + g(x_k) + D_k \omega_k - G_{i,k} \hat{x}_{i,k} \\
 &- \left(\sum_{j \in \mathcal{N}_i} r_{ij} H_{ij,k} (K_1 C_{j,k} x_k + \varphi_y(C_{j,k} x_k) + E_{j,k} \omega_k \right. \\
 &\quad \left. - C_{j,k} \hat{x}_{j,k}) \right),
 \end{aligned}$$

which can be rewritten in a more compact form as follows:

$$\begin{aligned}
 e_{k+1} &= \mathcal{A}_k \xi_k + \tilde{g}_k + \mathcal{D}_k \tilde{\omega}_k - \mathcal{E}_k \hat{x}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{K}_k \xi_k \\
 &- \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \tilde{\varphi}_y - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{E}_k \tilde{\omega}_k \\
 &+ \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{C}_k \hat{x}_k \quad (9) \\
 &= \left(\mathcal{A}_k - \mathcal{E}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{K}_k + \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{C}_k \right) \xi_k \\
 &+ \left(\mathcal{E}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{C}_k \right) e_k + \tilde{g}_k \\
 &+ \left(\mathcal{D}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{E}_k \right) \tilde{\omega}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \tilde{\varphi}_y,
 \end{aligned}$$

where

$$e_k = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix},$$

$$\xi_k = \begin{bmatrix} x_k \\ x_k \\ \vdots \\ x_k \end{bmatrix},$$

$$\hat{x}_k = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_N \end{bmatrix},$$

$$\tilde{g}_k = \begin{bmatrix} g(x_k) \\ g(x_k) \\ \vdots \\ g(x_k) \end{bmatrix},$$

$$\tilde{\omega}_k = \begin{bmatrix} \omega_k \\ \omega_k \\ \vdots \\ \omega_k \end{bmatrix},$$

$$\tilde{\varphi}_y = \begin{bmatrix} \varphi(C_{1,k} x_k) \\ \varphi(C_{2,k} x_k) \\ \vdots \\ \varphi(C_{N,k} x_k) \end{bmatrix},$$

$$\mathcal{A}_k = \text{diag} \{A_k + \Delta A_k, A_k + \Delta A_k, \dots, A_k + \Delta A_k\},$$

$$\mathcal{D}_k = \text{diag} \{D_k, D_k, \dots, D_k\},$$

$$\mathcal{E}_k = \text{diag} \{G_{1,k}, G_{2,k}, \dots, G_{N,k}\},$$

$$\Phi_i = \text{diag} \left\{ \underbrace{0, \dots, 0}_{i-1}, I, \underbrace{0, \dots, 0}_{N-i} \right\},$$

$$\mathcal{H}_k = \{H_{ij,k}\} = \begin{bmatrix} H_{11,k} & H_{12,k} & \cdots & H_{1N,k} \\ H_{21,k} & H_{22,k} & \cdots & H_{2N,k} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1,k} & H_{N2,k} & \cdots & H_{NN,k} \end{bmatrix},$$

$$\mathcal{K}_k = \text{diag} \{K_1 C_{1,k}, K_1 C_{2,k}, \dots, K_1 C_{N,k}\},$$

$$\mathcal{C}_k = \text{diag} \{C_{1,k}, C_{2,k}, \dots, C_{N,k}\},$$

$$\mathcal{E}_k = \text{diag} \{E_{1,k}, E_{2,k}, \dots, E_{N,k}\}.$$

Define $\tilde{z}_{i,k} = M_k e_{i,k}$; it is the objective of this paper to design the filter parameters $\mathcal{G}_k$ and $\mathcal{H}_k$ to satisfy

$$\sum_{i=1}^T \|\tilde{z}_k\|^2 \leq \gamma^2 \sum_{i=1}^T \|\tilde{\omega}_k\|^2 + e_0^T \Xi e_0 \quad (11)$$

for some given disturbance attenuation level $\gamma > 0$ and given positive matrix $\Xi = \Xi^T$.

3. Main Results

Before giving the main results, some useful lemmas are firstly introduced.

Lemma 4. Let $W_0(\cdot), W_1(\cdot), \dots, W_p(\cdot)$ be quadratic functions of the variable η : $W_i(\eta) \triangleq \eta^T T_i \eta$ with $T_i^T = T_i$ ($i = 0, 1, \dots, p$). If there exist scalars $\tau_1 \geq 0, \tau_2 \geq 0, \dots, \tau_p \geq 0$ such that

$$T_0 - \sum_{i=1}^p \tau_i T_i \leq 0 \quad (12)$$

then the following is true

$$\begin{aligned} W_1(\eta) \leq 0, W_2(\eta) \leq 0, \dots, W_p(\eta) \leq 0 \\ \implies W_0(\eta) \leq 0. \end{aligned} \quad (13)$$

Lemma 5 (Schur Complement Equivalence). Given constant matrices Q_1, Q_2 , and Q_3 , where $Q_1 = Q_1^T$ and $Q_2 = Q_2^T > 0$, then

$$Q_1 + Q_3^T Q_2^{-1} Q_3 < 0 \quad (14)$$

if and only if

$$\begin{bmatrix} Q_1 & Q_3^T \\ Q_3 & -Q_2 \end{bmatrix} < 0 \quad (15)$$

or, equivalently,

$$\begin{bmatrix} -Q_2 & Q_3 \\ Q_3^T & Q_1 \end{bmatrix} < 0. \quad (16)$$

Lemma 6 (see [23]). Let U, V , and Γ be real matrices of appropriate dimensions with Γ satisfying $\Gamma^T \Gamma \leq \lambda I$. Then, for any scalar $\epsilon > 0$, we have

$$UTV + (UTV)^T \leq \epsilon^{-1} U U^T + \epsilon \lambda V^T V. \quad (17)$$

3.1. H_∞ Performance Analysis. The following theorem gives a sufficient condition under which the required H_∞ performance can be satisfied.

Theorem 7. Given a H_∞ performance $\gamma > 0$ and the initial matrix $\Xi > 0$. Let the filter parameters $\mathcal{G}_k$ and $\mathcal{H}_k$ be given. If there exist three sequences of positive scalars τ_{1k}, τ_{2k} ($0 \leq k \leq T$), and a sequence of positive definite matrices P_k ($0 \leq$

$k \leq T + 1$) satisfying $\eta_0^T P_0 \eta_0 \leq \gamma^2 e_0^T \Xi e_0$ such that the following matrix inequality

$$\begin{bmatrix} \Lambda_{11} & 0 & 0 & \tau_{2k} \frac{\bar{C}_k^T K^T}{2} & \bar{\mathcal{A}}_k^T \\ * & -\tau_{1k} S_k^{-1} & 0 & 0 & \bar{F}_k^T \\ * & * & -\gamma^2 I & 0 & \bar{\mathcal{D}}_k^T \\ * & * & * & -\tau_{2k} I & \bar{Y}_k^T \\ * & * & * & * & -P_{k+1}^{-1} \end{bmatrix} \leq 0, \quad (18)$$

where

$$\Lambda_{11} = -P_k + \bar{\mathcal{M}}_k^T \bar{\mathcal{M}}_k + \tau_{1k} \delta I,$$

$$\bar{\mathcal{A}}_k = \begin{bmatrix} \mathcal{A}_k & 0 \\ \bar{\mathcal{A}}_k & \bar{\mathcal{E}}_k \end{bmatrix},$$

$$\bar{\mathcal{A}}_k = \mathcal{A}_k - \mathcal{E}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i (\mathcal{H}_k - \mathcal{E}_k),$$

$$\bar{\mathcal{E}}_k = \mathcal{E}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{E}_k,$$

$$\bar{Y}_k = \begin{bmatrix} 0 \\ \bar{Y}_k \end{bmatrix},$$

$$\bar{Y}_k = -\sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \quad (19)$$

$$\bar{\mathcal{D}}_k = \begin{bmatrix} \mathcal{D}_k \\ \bar{\mathcal{D}}_k \end{bmatrix},$$

$$\bar{\mathcal{D}}_k = \mathcal{D}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i \mathcal{E}_k,$$

$$\bar{F}_k = \begin{bmatrix} I \\ I \end{bmatrix},$$

$$\bar{\mathcal{M}}_k = [0 \quad \mathcal{M}_k],$$

$$\mathcal{M}_k = \text{diag}\{M_k, M_k, \dots, M_k\}$$

is feasible, then H_∞ performance can be satisfied.

Proof. Defining

$$\eta_k = \begin{bmatrix} \xi_k \\ e_k \end{bmatrix} \quad (20)$$

then the filtering error system can be expressed by

$$\eta_{k+1} = \bar{\mathcal{A}}_k \eta_k + \bar{F}_k \tilde{g}_k + \bar{\mathcal{D}}_k \tilde{\omega}_k + \bar{Y}_k \tilde{\varphi}_y, \quad (21)$$

$$\tilde{z}_k = \bar{\mathcal{M}}_k \eta_k.$$

Defining

$$J_k = \eta_{k+1}^T P_{k+1} \eta_{k+1} - \eta_k^T P_k \eta_k \quad (22)$$

one can get from (9) that

$$\begin{aligned}
J_k &= \eta_{k+1}^T P_{k+1} \eta_{k+1} - \eta_k^T P_k \eta_k \\
&= (\bar{\mathcal{A}}_k \eta_k + \bar{F}_k \bar{g}_k + \bar{\mathcal{D}}_k \bar{\omega}_k + \bar{Y}_k \bar{\varphi}_y)^T \\
&\quad \cdot P_{k+1} (\bar{\mathcal{A}}_k \eta_k + \bar{F}_k \bar{g}_k + \bar{\mathcal{D}}_k \bar{\omega}_k + \bar{Y}_k \bar{\varphi}_y) - \eta_k^T P_k \eta_k \\
&= \eta_k^T \bar{\mathcal{A}}_k^T P_{k+1} \bar{\mathcal{A}}_k \eta_k + \bar{g}_k^T \bar{F}_k^T P_{k+1} \bar{F}_k \bar{g}_k \\
&\quad + \bar{\omega}_k^T \bar{\mathcal{D}}_k^T P_{k+1} \bar{\mathcal{D}}_k \bar{\omega}_k + \bar{\varphi}_y^T \bar{Y}_k^T P_{k+1} \bar{Y}_k \bar{\varphi}_y \\
&\quad + 2\eta_k^T \bar{\mathcal{A}}_k^T P_{k+1} \bar{F}_k \bar{g}_k + 2\eta_k^T \bar{\mathcal{A}}_k^T P_{k+1} \bar{\mathcal{D}}_k \bar{\omega}_k \\
&\quad + 2\eta_k^T \bar{\mathcal{A}}_k^T P_{k+1} \bar{Y}_k \bar{\varphi}_y + 2\bar{g}_k^T \bar{F}_k^T P_{k+1} \bar{\mathcal{D}}_k \bar{\omega}_k \\
&\quad + 2\bar{g}_k^T \bar{F}_k^T P_{k+1} \bar{Y}_k \bar{\varphi}_y + 2\bar{\omega}_k^T \bar{\mathcal{D}}_k^T P_{k+1} \bar{Y}_k \bar{\varphi}_y - \eta_k^T P_k \eta_k \\
&= \psi_k^T \Omega_k \psi_k,
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\psi_k &= [\eta_k^T \quad \bar{g}_k^T \quad \bar{\omega}_k^T \quad \bar{\varphi}_y^T]^T, \\
\Omega_k &= \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \bar{F}_k^T P_{k+1} \bar{F}_k & \bar{F}_k^T P_{k+1} \bar{\mathcal{D}}_k & \Omega_{24} \\ * & * & \bar{\mathcal{D}}_k^T P_{k+1} \bar{\mathcal{D}}_k & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix}, \\
\Omega_{11} &= \bar{\mathcal{A}}_k^T P_{k+1} \bar{\mathcal{A}}_k - P_k, \\
\Omega_{12} &= \bar{\mathcal{A}}_k^T P_{k+1} \bar{F}_k, \\
\Omega_{13} &= \bar{\mathcal{A}}_k^T P_{k+1} \bar{\mathcal{D}}_k, \\
\Omega_{14} &= \bar{\mathcal{A}}_k^T P_{k+1} \bar{Y}_k, \\
\Omega_{24} &= \bar{F}_k^T P_{k+1} \bar{Y}_k, \\
\Omega_{34} &= \bar{\mathcal{D}}_k^T P_{k+1} \bar{Y}_k, \\
\Omega_{44} &= \bar{Y}_k^T P_{k+1} \bar{Y}_k.
\end{aligned} \tag{24}$$

Adding the zero term

$$\bar{z}_k^T \bar{z}_k - \gamma^2 \bar{\omega}_k^T \bar{\omega}_k - (\bar{z}_k^T \bar{z}_k - \gamma^2 \bar{\omega}_k^T \bar{\omega}_k) \tag{25}$$

to both sides of (23), one can get

$$J_k = \psi_k^T \Lambda_k \psi_k - (\bar{z}_k^T \bar{z}_k - \gamma^2 \bar{\omega}_k^T \bar{\omega}_k), \tag{26}$$

where

$$\Lambda_k = \Omega_k + \text{diag} \{ \bar{\mathcal{M}}_k^T \bar{\mathcal{M}}_k, 0, -\gamma^2 I, 0 \}. \tag{27}$$

Next, by summing up (26) on both sides from 0 to N with respect to k , one can have

$$\sum_{k=0}^T J_k = \eta_{T+1}^T P_{T+1} \eta_{T+1} - \eta_0^T P_0 \eta_0 \tag{28}$$

$$= \sum_{k=0}^T \psi_k^T \Lambda_k \psi_k - \sum_{k=0}^T (\bar{z}_k^T \bar{z}_k - \gamma^2 \bar{\omega}_k^T \bar{\omega}_k)$$

which is equivalent to

$$\begin{aligned}
\sum_{k=0}^T \bar{z}_k^T \bar{z}_k &= \sum_{k=0}^T \gamma^2 \bar{\omega}_k^T \bar{\omega}_k + \sum_{k=0}^T \psi_k^T \Lambda_k \psi_k - \eta_{T+1}^T P_{T+1} \eta_{T+1} \\
&\quad + \eta_0^T P_0 \eta_0.
\end{aligned} \tag{29}$$

From (29), since $P_{T+1} > 0$ and the initial condition $\eta_0^T P_0 \eta_0 \leq \gamma^2 e_0^T \Xi e_0$, one can see that the H_∞ performance can be satisfied if $\psi_k^T \Lambda_k \psi_k \leq 0$ is satisfied.

Next, one can see from the ellipsoidal nonlinear function $g_k(x_k)$ (2) that

$$\psi_k^T \Pi_k \psi_k \leq 0, \tag{30}$$

where

$$\Pi_k = \begin{bmatrix} -\delta I & 0 & 0 & 0 \\ * & \mathcal{S}_k^{-1} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \tag{31}$$

with $\mathcal{S}_k = \text{diag}\{S_k, S_k, \dots, S_k\}$.

Moreover, it also can be obtained from the sensor fault constraint (6) that

$$\psi_k^T \bar{\Pi}_k \psi_k \leq 0, \tag{32}$$

where

$$\bar{\Pi}_k = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & -\bar{C}_k^T K^T \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 2I \end{bmatrix} \tag{33}$$

with $\bar{C}_k = [C_{1,k} \ 0 \ C_{2,k} \ 0 \ \dots \ C_{N,k} \ 0]$.

By Schur Complement Equivalence Lemma, the matrix inequality (18) can be rewritten as follows:

$$\Lambda_k - \tau_{1k} \Pi_k - \tau_{2k} \bar{\Pi}_k \leq 0. \tag{34}$$

By Lemma 4, $\psi_k^T \Lambda_k \psi_k \leq 0$ can be satisfied, which means that the required H_∞ performance can be satisfied. The proof is complete. $\square$

Remark 8. By means of iterative matrix inequality method, Theorem 7 presents a sufficient condition for filtering error system to achieve the desired H_∞ specification subject to the sensor saturations. Moreover, notice that the obtained condition is expressed by certain matrix inequalities including fractional parameter uncertainties ΔA_k ; we cannot solve the matrix inequality directly to obtain the explicit parametric expression of the required filter parameters $\mathcal{G}$ and $\mathcal{H}$. As such, in the following, we will provide a theorem with which the addressed problem can be solved and the filter structure can then be obtained step by step.

3.2. Filter Design

Theorem 9. Let the H_∞ performance $\gamma > 0$ and the initial condition matrix $S_k > 0$ be given. If there exist a sequence of matrices $\mathcal{G}_k$ ($0 \leq k \leq T$), a sequence of matrices $\mathcal{H}_k$ ($0 \leq k \leq T$), three sequences of positive scalars τ_{1k} , τ_{2k} , and ϵ_k ($0 \leq k \leq T$), a sequence of positive definite matrices X_k ($0 \leq k \leq T+1$), and a sequence of positive definite matrices Z_k ($0 \leq k \leq T+1$) satisfying $\eta_0^T \text{diag}\{X_0^{-1}, Z_0^{-1}\} \eta_0 \leq \gamma^2 e_0 \Xi e_0$ such that

$$\begin{bmatrix} \bar{\Lambda}_k + \epsilon_k \lambda \mathcal{V}^T \mathcal{V} & \mathcal{U} \\ \mathcal{U}^T & -\epsilon_k I \end{bmatrix} \leq 0, \quad (35)$$

where

$$\bar{\Lambda}_k = \begin{bmatrix} -\mathcal{X}_k + \delta \tau_{1k} I & 0 & 0 & 0 & \frac{1}{2} \tau_{2k} \tilde{C}_k^T K^T & \tilde{\mathcal{A}}_k^T & \tilde{\mathcal{A}}_k^T \\ * & -\mathcal{X}_k + \mathcal{M}_k^T \mathcal{M}_k + \delta \tau_{1k} I & 0 & 0 & 0 & 0 & \tilde{\mathcal{G}}_k^T \\ * & * & -\tau_{1k} \mathcal{S}_k^{-1} & 0 & 0 & I & I \\ * & * & * & -\gamma^2 I & 0 & \mathcal{D}_k^T & \bar{\mathcal{D}}_k^T \\ * & * & * & * & -\tau_{2k} I & 0 & \bar{Y}_k^T \\ * & * & * & * & * & -X_{k+1} & 0 \\ * & * & * & * & * & * & -Z_{k+1} \end{bmatrix},$$

$$\tilde{\mathcal{A}}_k = \text{diag}\{A_k, A_k, \dots, A_k\},$$

$$\tilde{\mathcal{A}}_k = \tilde{\mathcal{A}}_k - \mathcal{G}_k - \sum_{i=1}^N \Phi_i \mathcal{H}_k R_i (\mathcal{H}_k - \mathcal{C}_k),$$

$$\mathcal{U} = [0 \ 0 \ 0 \ 0 \ 0 \ \bar{U}^T \ \bar{U}^T]^T,$$

$$\bar{U} = \text{diag}\{U, U, \dots, U\},$$

$$\mathcal{V} = [\bar{V} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0],$$

$$\bar{V} = \text{diag}\{V, V, \dots, V\},$$

(36)

with $\mathcal{X}_k = X_k^{-1}$ and $\mathcal{Z}_k = Z_k^{-1}$, and λ is the solution to the following optimization problem

$$\begin{aligned} \min_{\Sigma_k} \quad & \lambda = (I - \Sigma_k^T J^T)^{-1} (I - J \Sigma_k)^{-1} \\ \text{s.t.} \quad & \Sigma_k^T \Sigma_k < 1, \end{aligned} \quad (37)$$

then the addressed H_∞ filtering problem is solvable and the desired filter parameters at each time k can be obtained by solving the corresponding matrix inequality.

Proof. The proof is quite straightforward based on Theorem 7. First, define $P_k^{-1} = \text{diag}\{\mathcal{X}_k^{-1}, \mathcal{Z}_k^{-1}\} = \text{diag}\{X_k, Z_k\}$. One can obtain that the matrix inequality (18) is equivalent to the following inequality:

$$\begin{bmatrix} -\mathcal{X}_k + \delta\tau_{1k}I & 0 & 0 & 0 & \frac{1}{2}\tau_{2k}\tilde{C}_k^T K^T & \mathcal{A}_k^T & \overline{\mathcal{A}}_k^T \\ * & -\mathcal{Z}_k + \mathcal{M}_k^T \mathcal{M}_k + \delta\tau_{1k}I & 0 & 0 & 0 & 0 & \overline{\mathcal{Z}}_k^T \\ * & * & -\tau_{1k}\mathcal{O}_k^{-1} & 0 & 0 & I & I \\ * & * & * & -\gamma^2 I & 0 & \mathcal{D}_k^T & \overline{\mathcal{D}}_k^T \\ * & * & * & * & -\tau_{2k}I & 0 & \overline{Y}_k^T \\ * & * & * & * & * & -X_{k+1} & 0 \\ * & * & * & * & * & * & -Z_{k+1} \end{bmatrix} \leq 0. \quad (38)$$

In what follows, we will try to prove that inequality (38) can be satisfied if inequality (35) is fulfilled. To this end, rewrite (38) as follows:

$$\overline{\Lambda}_k + \mathcal{U}\tilde{\Xi}\mathcal{V} + (\mathcal{U}\tilde{\Sigma}\mathcal{V})^T \leq 0, \quad (39)$$

where

$$\tilde{\Sigma} = \text{diag} \{ (I - J\Sigma)^{-1}, (I - J\Sigma)^{-1}, \dots, (I - J\Sigma)^{-1} \}. \quad (40)$$

From Lemma 6, it follows that

$$\mathcal{U}\tilde{\Xi}\mathcal{V} + (\mathcal{U}\tilde{\Xi}\mathcal{V})^T \leq \epsilon^{-1}\mathcal{U}\mathcal{U}^T + \epsilon\lambda\mathcal{V}^T\mathcal{V}. \quad (41)$$

Furthermore, one can obtain that

$$\overline{\Lambda}_k + \epsilon^{-1}\mathcal{U}\mathcal{U}^T + \epsilon\lambda\mathcal{V}^T\mathcal{V} \leq 0, \quad (42)$$

which, by Schur Complement Lemma, is equivalent to the linear matrix inequality (35). Thus, inequality (18) is satisfied if (35) is met, and, according to Theorem 7, the required H_∞ filtering problem is solved. $\square$

Remark 10. Theorem 9 outlines the methodology with which the addressed robust H_∞ distributed filtering problem is solved for nonlinear systems subject to sensor saturations and fractional uncertainties. An iterative linear matrix inequality approach has been developed to calculate the desired filter parameters step by step. It should be pointed out that it will inevitably lead to the larger computing burdens. However, the algorithm we proposed in this paper is capable of dealing with the time-varying case, and the increase in computing burden can be largely reduced with the fast development of modern computer technology. In what follows, we will present a computing algorithm with which the numerical values of filter parameters can be obtained in each time step.

3.3. Computational Algorithm

Algorithm 11 (H_∞ Filter Design Algorithm).

Step 1. Set the required γ , the initial condition Ξ , the initial η_0 , and T . Choose properly the values of U , V , and J . Select the

initial values for $\mathcal{X}_0$ and $\mathcal{Z}_0$ satisfying the initial condition constraint.

Step 2. Set $k = 0$. Solve (35) for $\mathcal{G}_0$, $\mathcal{H}_0$, X_1 , and Z_1 . Then the $\mathcal{X}_1$ and $\mathcal{Z}_1$ can also be obtained.

Step 3. Set $k = k + 1$. Using the obtained $\mathcal{X}_k$ and $\mathcal{Z}_k$, solve (35) for $\mathcal{G}_k$, $\mathcal{H}_k$, X_{k+1} , and Z_{k+1} .

Step 4. If $k = N + 1$, then stop. Else go to Step 3.

The H_∞ Filter Design Algorithm gives a recursive way to obtain the numerical values of the desired filter parameters at each time point k . It should be pointed out that the existence of the filter is expressed as the feasibility of certain linear matrix inequalities that can be solved forward in time. The possible research topic in the future is to consider more performance indices and give the filtering schemes that are able to satisfy multiple requirements simultaneously.

4. Simulation Example

In this section, an illustrative example is presented to show the effectiveness of the proposed H_∞ Filtering Design Algorithm.

Let us consider the nonlinear system (1) with the parameters given below:

$$\begin{aligned} A_k &= \begin{bmatrix} 0 & -0.4 \\ 0.5 & 0.7 \sin(6k - 6) \end{bmatrix}, \\ D_k &= \begin{bmatrix} 0.5 \\ -0.4 \end{bmatrix}, \\ C_{1,k} &= [0.3 \quad 0.1 * \sin(6k - 6)], \\ C_{2,k} &= [0.3 \quad 0.2 * \sin(6k - 6)], \\ C_{3,k} &= [0.3 \quad 0.3 * \sin(6k - 6)], \\ E_{1,k} &= E_{2,k} = E_{3,k} = 0.3, \\ M_k &= [1.5 \quad 0.75]. \end{aligned} \quad (43)$$

FIGURE 1: The state x_k^1 and its estimate.

FIGURE 2: The state x_k^2 and its estimate.

Let $g(x_k) = [\sin(10x_k) \quad 0.5\cos(10x_k)]^T$ and set $S_k = 2$. Let $\omega_k = e^{-k/35}$.

Choose $U = 0.1$, $V = 0.2$, and $J = 0.25$, and we solve the optimization problem given in Theorem 9 using Matlab toolbox and obtain $\lambda = 0.7$. Set $\gamma = 1$, $\xi_0 = [0.3 \quad 0.25 \quad 0 \quad 0]^T$, $\Xi = I$, and $N = 3$. Set the initial value of the system state and its estimate by $x_0 = [0.3 \quad 0.25]^T$ and $\hat{x}_0 = [0 \quad 0]^T$, and the initial values of X_0 and Z_0 are selected to be $X_0 = Z_0 = 2I$. We can see the effectiveness and applicability of the proposed filtering algorithm from the simulation results (Figures 1-2).

5. Conclusion

The problem of distributed H_∞ filtering for nonlinear stochastic systems in the presence of sensor saturations and fractional uncertainties. A sufficient condition is proposed under which the required H_∞ performance can be satisfied even if the randomly occurring sensor failures happen. A recursive algorithm is given in order to obtain the required filter parameters iteratively. Finally, an illustrative example is presented to show the effectiveness of the proposed method. Noting that the nonlinearities considered in this paper are relatively simpler, one of the possible future research directions may be the robust distributed H_∞ filtering problems with much more complex nonlinearities.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 51261130471 and the “One Thousand Plan” special support project of state grid corporation of China ((2013)1111).

References

- [1] European Commission, “European technology platform smart-grids: vision and strategy for Europe’s electricity networks of the future,” 2009, http://ec.europa.eu/research/participants/data/ref/h2020/wp/2014_2015/main/h2020-wp1415-energy_en.pdf.
- [2] V. Ugrinovskii, “Distributed robust filtering with H_∞ consensus of estimates,” *Automatica*, vol. 47, no. 1, pp. 1–13, 2011.
- [3] H. Gao and T. Chen, “ H_∞ estimation for uncertain systems with limited communication capacity,” *IEEE Transactions on Automatic Control*, vol. 52, no. 11, pp. 2070–2084, 2007.
- [4] M. Sahebsara, T. Chen, and S. L. Shah, “Optimal H_2 filtering in networked control systems with multiple packet dropout,” *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1508–1513, 2007.
- [5] X. Lu, L. Xie, H. Zhang, and W. Wang, “Robust Kalman filtering for discrete-time systems with measurement delay,” *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 54, no. 6, pp. 522–526, 2007.
- [6] L. Ma, Z. Wang, J. Hu, Y. Bo, and Z. Guo, “Robust variance-constrained filtering for a class of nonlinear stochastic systems with missing measurements,” *Signal Processing*, vol. 90, no. 6, pp. 2060–2071, 2010.
- [7] L. Ma, Z. Wang, Y. Bo, and Z. Guo, “Robust H_∞ sliding mode control for nonlinear stochastic systems with multiple data packet losses,” *International Journal of Robust and Nonlinear Control*, vol. 22, no. 5, pp. 473–491, 2012.
- [8] J. C. Delvenne, “An optimal quantized feedback strategy for scalar linear systems,” *IEEE Transactions on Automatic Control*, vol. 51, no. 2, pp. 298–303, 2006.

- [9] N. Elia and S. K. Mitter, "Stabilization of linear systems with limited information," *IEEE Transactions on Automatic Control*, vol. 46, no. 9, pp. 1384–1400, 2001.
- [10] D. Liberzon, "Hybrid feedback stabilization of systems with quantized signals," *Automatica*, vol. 39, no. 9, pp. 1543–1554, 2003.
- [11] E. Fridman and M. Dambrine, "Control under quantization, saturation and delay: an LMI approach," *Automatica*, vol. 45, no. 10, pp. 2258–2264, 2009.
- [12] D. Ding, Z. Wang, B. Shen, and H. Dong, "Envelope-constrained H_∞ filtering with fading measurements and randomly occurring nonlinearities: the finite horizon case," *Automatica*, vol. 55, pp. 37–45, 2015.
- [13] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1453–1464, 2004.
- [14] R. W. Brockett and D. Liberzon, "Quantized feedback stabilization of linear systems," *IEEE Transactions on Automatic Control*, vol. 45, no. 7, pp. 1279–1289, 2000.
- [15] H. Dong, Z. Wang, J. Lam, and H. Gao, "Distributed filtering in sensor networks with randomly occurring saturations and successive packet dropouts," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 12, pp. 1743–1759, 2014.
- [16] H. Dong, Z. Wang, and H. Gao, "Distributed H_∞ filtering for a class of markovian jump nonlinear time-delay systems over lossy sensor networks," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 10, pp. 4665–4672, 2013.
- [17] H. Dong, Z. Wang, and H. Gao, "Distributed filtering for a class of time-varying systems over sensor networks with quantization errors and successive packet dropouts," *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 3164–3173, 2012.
- [18] D. Ding, Z. Wang, H. Dong, and H. Shu, "Distributed H_∞ state estimation with stochastic parameters and nonlinearities through sensor networks: the finite-horizon case," *Automatica*, vol. 48, no. 8, pp. 1575–1585, 2012.
- [19] J. Liang, Z. Wang, and X. Liu, "Distributed state estimation for discrete-time sensor networks with randomly varying nonlinearities and missing measurements," *IEEE Transactions on Neural Networks*, vol. 22, no. 3, pp. 486–496, 2011.
- [20] J. Liang, Z. Wang, and X. Liu, "Distributed state estimation for uncertain Markov-type sensor networks with mode-dependent distributed delays," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 3, pp. 331–346, 2012.
- [21] B. Shen, Z. Wang, Y. S. Hung, and G. Chesi, "Distributed H_∞ filtering for polynomial nonlinear stochastic systems in sensor networks," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 5, pp. 1971–1979, 2011.
- [22] B. Shen, Z. Wang, and Y. S. Hung, "Distributed H_∞ -consensus filtering in sensor networks with multiple missing measurements: the finite-horizon case," *Automatica*, vol. 46, no. 10, pp. 1682–1688, 2010.
- [23] X. Kan, Z. Wang, and H. Shu, "State estimation for discrete-time delayed neural networks with fractional uncertainties and sensor saturations," *Neurocomputing*, vol. 117, pp. 64–71, 2013.