Research Article

The Existence of Exponential Attractor for Discrete Ginzburg-Landau Equation

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This paper studies the following discrete systems of the complex Ginzburg-Landau equation:

\[ i \dot{u}_m - (\alpha - i\epsilon)(2u_m - u_{m+1} - u_{m-1}) + i\sigma u_m + \beta |u_m|^2 u_m = g_m, \quad m \in \mathbb{Z}. \]

Under some conditions on the parameters \( \alpha, \epsilon, \sigma, \beta, \) and \( \sigma \), we prove the existence of exponential attractor for the semigroup associated with these discrete systems.

1. Introduction

In the study of infinite dynamical systems, attractors occupy a central position (see, e.g., Chepyzhov and Vishik [1], Hale [2], Ladyzhenskaya [3], and Temam [4]). Exponential attractors are realistic objects intermediate between the global attractors and the inertial manifolds. There are several approaches for proving the existence of exponential attractors for parabolic and hyperbolic partial differential equations (PDEs) arising from mathematical physics. For example, we can refer to [5–7] for the existence of the exponential attractors for general evolution equations in Banach spaces, to [8] for the exponential attractors for reaction diffusion equations in unbounded domains, to [9] for the exponential attractors of the nonlinear wave equations, and to [10] for the exponential attractor for the generalized 2D Ginzburg-Landau equations. Also there are some references investigating the exponential attractors for lattice dynamical systems (LDSs). We can see [11–13] for the exponential attractors for first-order LDSs; see [14, 15] for the pullback exponential attractors for first- and second-order LDSs; see [16, 17] for second-order nonautonomous LDSs and discrete Zakharov equations for the uniform exponential attractors.

Lattice dynamical systems (LDSs) are currently under active investigation for their wide applications in electrical engineering [18], chemical reaction theory [19, 20], laser systems [21], and biology [22]. There are many references studying the asymptotic behavior of general LDSs. For instance, we can refer to [23–25] for the existence of global attractor, to [26–28] for the uniform attractor, to [11, 14, 15] for the exponential and pullback exponential attractor, and to [29, 30] for the random attractor. Also, there are some concrete applications of the above theory to the discrete PDEs. We can refer to [31–33] for discrete Klein-Gordon-Schrödinger equations, [34] for discrete three-component reversible Gray-Scott model, [35] for discrete coupled nonlinear Schrödinger-Boussinesq equations, [36] for discrete long-wave-short-wave resonance equations, and [37] for the discrete complex Ginzburg-Landau equation.

Lattice systems including coupled ordinary differential equations, coupled map lattices, and cellular automata are spatiotemporal systems with discretization in some variables. In some cases, lattice systems arise as the spatial discretization of partial differential equations on unbounded or bounded domains.
This paper will study the following discrete systems (lattice systems):
\[ iu_m - (\alpha - i\epsilon)(2u_m - u_{m+1} - u_{m-1}) + i\beta|u_m|^{2\sigma}u_m = g_m, \]
\[ u_m(0) = u_{0,m}, \quad m \in \mathbb{Z}, \quad (1) \]
where $i$ is the unit of imaginary numbers and $\alpha, \epsilon, \beta, \sigma$ are parameters. Equation (1) can be regarded as a discrete analogue of the following complex Ginzburg-Landau equation on the real line:
\[ iu + (\alpha - i\epsilon)u_{xx} + i\beta|u|^{2\sigma}u = g, \quad x \in \mathbb{R}. \quad (3) \]
The complex Ginzburg-Landau equation is a simplified mathematical model for various pattern formation systems in mechanics, physics, and chemistry. We can refer to [10, 38, 39] for the detailed significations of the complex Ginzburg-Landau equation.

The existence of the exponential attractors for continuous complex Ginzburg-Landau equation in two-dimensional space was proved in [10]. Later, under some conditions on $\alpha, \epsilon, \kappa, \beta, \sigma$, and $g_m$, [37] established the existence of global attractor for the semigroup associated with discrete systems (1)-(2). The aim of this paper is to prove the existence of exponential attractors for discrete systems (1)-(2). To this end, we will establish the following three items:

(I) The solution operators associated with (1)-(2) generate a continuous semigroup $\{S(t)\}_{t \geq 0}$ in the phase space $\ell^2$ and $\{S(t)\}_{t \geq 0}$ possesses a bounded and closed positively invariant set $\mathcal{B} \subset \ell^2$. Moreover, for any $T' > 0$, the map $S(t)$ is Lipschitz continuous from $[0, T'] \times \mathcal{B}$ into $\mathcal{B}$.

(II) There exists a time $T_*$ such that the map $S(T_*) := S_* : \mathcal{B} \rightarrow \mathcal{B}$ is an $\alpha$-contraction on $\mathcal{B}$.

(III) The map $S_*$ satisfies the discrete squeezing property on $\mathcal{B}$.

Compared with previous works such as [9], here we no longer require the compactness of the invariant set $\mathcal{B}$ (this fact was first noted by Babin and Nicolaenko [8] and then by Eden et al. [6]), which can usually be obtained by the compact embedding between Sobolev spaces when studying PDEs. Note that the compact embedding theorem of Sobolev spaces seems difficult to be applicable when studying LDSs. This is caused by the discrete characteristics of LDSs which restrict us to choose the phase spaces. Fortunately, the intrinsic characteristics of LDSs enable us to use the $\alpha$-contraction property to compensate the compactness of the invariant set.

2. Positively Invariant Set and Lipschitz Continuity

Set
\[ \ell^2 = \left\{ u = (u_m)_{m \in \mathbb{Z}}, u_m \in \mathbb{C} : \sum_{m \in \mathbb{Z}} |u_m|^2 < +\infty \right\}; \quad (4) \]
and equip it with the inner product and norm as
\[ \langle u, v \rangle = \sum_{m \in \mathbb{Z}} u_m \overline{v}_m, \]
\[ \|u\|^2 = \langle u, u \rangle, \quad (u, v) \in \ell^2, \]
\[ u = (u_m), \quad v = (v_m)_{m \in \mathbb{Z}} \in \ell^2, \]
where $\overline{v}_m$ denotes the conjugate of $v_m$. Then $(\ell^2, \| \cdot \|, (\cdot, \cdot))$ is a separable Hilbert space. We now introduce the operators $A$, $B$, and $B^*$ on $\ell^2$ as follows:
\[ (Au)_m = 2u_m - u_{m+1} - u_{m-1}, \quad \forall m \in \mathbb{Z}, \]
\[ (Bu)_m = u_{m+1} - u_m, \quad \forall m \in \mathbb{Z}, \]
\[ (B^*u)_m = u_{m-1} - u_m, \quad \forall m \in \mathbb{Z}. \]

In fact, $B^*$ is the adjoint operator of $B$ and one can easily check that
\[ (Au, v) = (B^*Bu, v) = (Bu, Bv), \]
\[ (Bu, v) = (u, B^*v), \quad \forall u, v \in \ell^2, \]
\[ \|Au\|^2 \leq 16\|u\|^2, \quad (7) \]
\[ \|Bu\|^2 \leq 4\|u\|^2, \]
\[ \|B^*u\|^2 \leq 4\|u\|^2, \quad \forall u \in \ell^2. \]

Using the notations introduced above, we can write problem (1)-(2) as
\[ iu - (\alpha - i\epsilon)Au + i\beta|u|^{2\sigma}u = g, \quad (8) \]
\[ u(0) = u_0, \quad (9) \]
where $u = (u_m)_{m \in \mathbb{Z}}, |u|^{2\sigma}u = (|u_m|^{2\sigma}u_m)_{m \in \mathbb{Z}}, g = (g_m)_{m \in \mathbb{Z}},$ and $u_0 = (u_0(m))_{m \in \mathbb{Z}}$.

For the well-posedness of problem (1)-(2), we have the following.

Lemma 1 (see [37]). Let $\alpha, \epsilon, \kappa, \beta, \sigma > 0$ and $g = (g_m)_{m \in \mathbb{Z}} \in \ell^2$:

(i) For any $u_0 \in \ell^2$, problem (8)-(9) has a unique solution $u \in \ell^2([0, T_0]; \ell^2)$ for some $T_0 > 0$. Moreover, if $T_0 < +\infty$, then $\lim_{t \to +\infty} \|u(t)\| = +\infty$.

(ii) For any $u_0 \in \ell^2$, the solution of problem (8)-(9) satisfies
\[ \|u(t)\|^2 \leq \|u_0\|^2 e^{-\kappa t} + \frac{\|g\|^2}{\kappa^2}, \quad \forall t > 0. \quad (10) \]
Lemma 1(i) shows that, for each initial value $u_0 \in \ell^2$, problem (8)-(9) possesses a unique solution. Letting $t \to +\infty$, we see from (10) that, for any $u_0 \in \ell^2$, the corresponding solution $u(t) \in \ell^2$ of problem (8)-(9) is uniformly (with respect to $t$) bounded for all $t \in [0, +\infty)$. Again, by Lemma 1(i), the solution exists globally; that is, problem (8)-(9) is globally well-posed. The above analysis implies that the solution operators

$$S(t): \ell^2 \ni u_0 \mapsto S(t)u_0 = u(t) \in \ell^2$$

(11)

generate a continuous semigroup $\{S(t)\}_{t \geq 0}$ on $\ell^2$. We next investigate the existence of the bounded and closed positively invariant set, as well as the Lipschitz property for the semigroup $\{S(t)\}_{t \geq 0}$.

**Lemma 2.** Let $\alpha, \epsilon, \kappa, \beta, \sigma > 0$ and $g = (g_m)_{m \in \mathbb{Z}} \in \ell^2$. Then the semigroup $\{S(t)\}_{t \geq 0}$ possesses a bounded and closed positively invariant set $B \in \ell^2$.

**Proof.** By (10) we see that the set

$$B := \left\{ u \in \ell^2 : \|u\| \leq \frac{\sqrt{2}\|g\|}{\kappa} \right\}$$

(12)

is a bounded and closed absorbing set for $\{S(t)\}_{t \geq 0}$. Thus, there is a time $t_* := t_*(B)$ such that $S(t)B \subseteq B$ for any $t \geq t_*$. Set

$$B := \bigcup_{t \geq t_*} S(t)B.$$  

(13)

Then $B$ is the bounded and closed positively invariant set for $\{S(t)\}_{t \geq 0}$. The proof is complete.

The positively invariant property of $B$ implies that

$$S(t)B \subseteq B \subseteq B, \quad \forall t \geq 0.$$  

(14)

**Lemma 3.** Let $\alpha, \epsilon, \kappa, \delta > 0$, $\sigma \geq 1/2$, and $g = (g_m)_{m \in \mathbb{Z}} \in \ell^2$. Then the semigroup $\{S(t)\}_{t \geq 0}$ is Lipschitz continuous from $[0, T] \times B$ into $B$ for each $T > 0$.

**Proof.** Let $u_0, v_0 \in B$, $S(t)u_0 = u(t) = (u_m(t))_{m \in \mathbb{Z}}$, $S(t)v_0 = v(t) = (v_m(t))_{m \in \mathbb{Z}}$, and $u(t) - v(t)$. By (8),

$$i\omega - (\alpha - i\epsilon) A\omega + i\kappa \omega + \beta |u|^{2\sigma} u - \beta |v|^{2\sigma} v = 0,$$

$$w(0) = u_0 - v_0.$$  

(15)

(16)

Using $i\omega(t)$ to take inner product $(\cdot, \cdot)$ with both sides of (15) and then taking the real part, we obtain

$$\frac{1}{2} \frac{d}{dt} \|w(t)\|^2 + \epsilon \|Bu\|^2 + \kappa \|w(t)\|^2$$

$$- \text{Im} \beta \sum_{m \in \mathbb{Z}} \left( |u_m|^{2\sigma} u_m - |v_m|^{2\sigma} v_m \right) \overline{w_m} = 0.$$  

(17)

Now set $f(x) = x^{2\sigma}$, $x \in \mathbb{R}_+$. Since $\sigma \geq 1/2$, $f'(x) = 2\sigma x^{2\sigma-1}$ is continuous and increasing on $\mathbb{R}_+$. By Cauchy inequality,

$$- \beta \sum_{m \in \mathbb{Z}} \left( |u_m|^{2\sigma} u_m - |v_m|^{2\sigma} v_m \right) \overline{w_m}$$

$$\leq \frac{\kappa}{2} \|w(t)\|^2$$

(18)

Using mean value theorem, (12), and (14), we get

$$\sum_{m \in \mathbb{Z}} \left( f(|u_m|) - f(|v_m|) \right) |v_m|^2$$

$$\leq \left( 2 + 8\sigma^2 \right) \left( \frac{2\|g\|^2}{\kappa^2} \right)^{2\sigma} \|w\|^2.$$  

(19)

It then follows from (17)-(19) that

$$\frac{d}{dt} \|w(t)\|^2 + (\kappa - \delta) \|w(t)\|^2 \leq 0,$$  

(20)

where

$$\delta = \delta(\kappa, \beta, \sigma, \|g\|) := \frac{2\beta^2}{\kappa} \cdot (1 + 4\sigma^2) \left( \frac{2\|g\|^2}{\kappa^2} \right)^{2\sigma}.$$  

(21)

Applying Gronwall inequality to (20) yields

$$\|w(t)\|^2 \leq \|S(t)u_0 - S(t)v_0\|^2 \leq \|w(0)\|^2 e^{(\delta-\kappa)t},$$

(22)

and, for any $T > 0$,

$$\|S(t)u_0 - S(t)v_0\| \leq \|u_0 - v_0\| e^{(\delta-\kappa)T/2},$$

(23)

\forall t \in [0, T].

The proof is complete.

**3. Existence of Exponential Attractor**

For each positive number $M$, we define the orthogonal projection $P_M : \ell^2 \mapsto \ell^2$ as

$$P_M u_m = \begin{cases} u_m, & |m| \leq M; \\ 0, & |m| > M \end{cases}$$  

(24)

and set $Q_M = I - P_M$, where $I$ is the identity operator on $\ell^2$.

We next make some assumptions on the numbers $\alpha, \kappa, \beta, \sigma$, and $g_m$:

$$\text{(H)} \quad \text{Assume } g = (g_m)_{m \in \mathbb{Z}} \in \ell^2, \quad \alpha, \kappa, \beta \text{ are positive, }$$

$$\sigma \geq 1/2, \quad \kappa > \delta,$$  

(25)

where $\delta$ is defined by (21).

The definitions of $\alpha$-contraction and discrete squeezing property can be found in [2, 6].
Lemma 4. Let assumption (H) hold. Then there exists a time $T_*$ such that the operator $S(t_*) := S_* : \mathcal{B} \to \mathcal{B}$ is an $\alpha$-contraction on $\mathcal{B}$.

Proof. Let $u_0, v_0 \in \mathcal{B}, S(t)u_0 = u(t) = (u_m(t))_{m \in \mathbb{Z}}, S(t)v_0 = v(t) = (v_m(t))_{m \in \mathbb{Z}},$ and $w(t) = u(t) - v(t)$. By (20), we have for any $M \in \mathbb{N}$ that

$$\frac{d}{dt} \|w(t)\|^2 + \kappa \|w(t)\|^2 \leq \delta \|w(t)\|^2 \tag{26}$$

which, together with (25), gives

$$\frac{d}{dt} \|w(t)\|^2 + (\kappa - \delta) \|w(t)\|^2 \leq \delta \|P_M w(t)\|^2, \quad t \geq t_* \tag{27}$$

Thus we have

$$\frac{d}{dt} \left( e^{(\kappa - \delta)t} \|w(t)\|^2 \right) \leq \delta e^{(\kappa - \delta)t} \|P_M w(t)\|^2, \quad t \geq t_* \tag{28}$$

Integrating both sides of (28) over $[t_*, T]$ with $T > t_*$ and then using (22), we obtain

$$\|w(T)\|^2 \leq e^{-\delta(T - t_*)} \|w(0)\|^2 + \frac{\delta}{\kappa - \delta} \max_{s \in [t_*, T]} \|P_M w(s)\|^2, \quad T \geq t_* \tag{29}$$

Now we choose

$$T_* = \max \left\{ t_* + \frac{\ln 256}{\delta} + \frac{(\kappa + \delta) t_*}{\kappa} \ln \frac{256}{\kappa - \delta}, \frac{\ln [(\kappa + \delta)/2048\beta\sigma] (\kappa/2 \|g\|)^{2\gamma}}{\kappa - \delta} \right\} \tag{30}$$

and it follows from (29) that

$$\|w(T_*)\|^2 \leq \frac{\|w(0)\|^2}{256} + \frac{\delta}{\kappa - \delta} \max_{s \in [t_*, T]} \|P_M w(s)\|^2 \tag{31}$$

Proceding as that as [11] did, we can show $\sqrt{\delta/\kappa} \max_{s \in [t_*, T]} \|P_M w(s)\|$ is a precompact pseudometric on $\mathcal{B}$, which, together with (31) and [2, Lemma 2.3.6], gives the desired result. $\square$

Remark 5. Since Lemma 4 holds for any $M \in \mathbb{N}$, we can specify some $M_*$ (see (40)). Then $T_*$ is chosen such that both $e^{(\kappa - \delta)t_*} \leq 1/256$ and (41) hold.

Lemma 6. Let assumption (H) hold. Then the operator $S_* : \mathcal{B} \to \mathcal{B}$ satisfies the discrete squeezing property on $\mathcal{B}$.

Proof. Define a smooth function $\chi(x) \in \mathcal{C}(\mathbb{R}_+, [0, 1])$ (see, e.g., [33]) such that

$$\chi(x) = \begin{cases} 0, & 0 \leq x \leq 1; \\ 1, & x \geq 2, \end{cases} \tag{32}$$

and $\chi'(x) \leq \chi_0$ (constant), $\forall x \in \mathbb{R}_+$.

Let $u_0, v_0 \in \mathcal{B}, S(t)u_0 = u(t) = (u_m(t))_{m \in \mathbb{Z}}, S(t)v_0 = v(t) = (v_m(t))_{m \in \mathbb{Z}}$, and $w(t) = u(t) - v(t)$. Set $y_m = \chi(|m|/M_*)w_m$ for each $m \in \mathbb{Z}$ and $y = (y_m)_{m \in \mathbb{Z}}$, where $M_*$ is a positive integer that will be specified later. Using $\langle y(t), \cdot \rangle$ with both sides of (15) and then taking the real part, we obtain

$$\frac{d}{dt} \left( \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) |w_m|^2 \right) + \text{Re} (i\alpha + \varepsilon) \sum_{m \in \mathbb{Z}} (Bw)_m (B\bar{y})_m$$

$$- \text{Im} \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) \left( |u_m|^{2\gamma} u_m - |v_m|^{2\gamma} v_m \right) \bar{w}_m$$

$$+ \kappa \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) |w_m|^2 = 0. \tag{33}$$

By (31) and [37, (4.8)], we have for any $t \geq t_*$ that

$$\text{Re} (i\alpha + \varepsilon) \sum_{m \in \mathbb{Z}} (Bw)_m (B\bar{y})(t)_m \geq - \frac{2\alpha \chi_0}{M_*} \|w\|^2,$$

$$- \text{Im} \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) \left( f(|u_m|) u_m - f(|v_m|) v_m \right) \bar{w}_m \tag{34}$$

$$\geq -2\beta\sigma \frac{\|g\|}{\kappa} \|w\|^2.$$

Taking (33)-(34) into account, we obtain

$$\frac{d}{dt} \left( \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) |w_m|^2 \right) + 2\kappa \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) |w_m|^2$$

$$\leq C \|w\|^2, \quad \forall t \geq t_* \tag{35}$$

where $C := 4\chi_0\alpha/M_* + 4\beta\sigma(2\|g\|/\kappa)^{2\gamma}$. By (22) and (35), we have for any $t \geq t_*$ that

$$\frac{d}{dt} \left( \sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) e^{2\gamma t} |w_m|^2 \right) \leq C e^{(\delta + \lambda)\gamma T} \|w(0)\|^2. \tag{37}$$

Integrating both sides of (37) over $[t_*, T]$ with $T \geq t_*$, we then get

$$\sum_{m \in \mathbb{Z}} \chi \left( \frac{|m|}{M_*} \right) e^{2\gamma t} |w_m(T)|^2$$

$$\leq e^{2\gamma t} \|w(t_*)\|^2 + \frac{C}{\kappa + \delta} e^{(\delta + \lambda)\gamma T} \|w(0)\|^2. \tag{38}$$
Again from (22), we obtain that
\[
\|Q_M w(T)\|^2 \\
\leq \left( e^{(\delta + \kappa)T - 2\kappa T} + \frac{C}{\kappa + \delta} e^{\delta - \kappa T} \right) \|w(0)\|^2 ,
\]
(39)
\[T \geq t^* .\]

We now take 
\[M^* = \frac{2048 \chi_0 \alpha}{\kappa + \delta}, \]
(40)
and then from (25), (30), and (39), we have
\[
\frac{4\alpha \beta}{\kappa + \delta} \left( \frac{2\|g\|^2}{\kappa} \right) e^{(\delta - \kappa)T^*} \leq 1 \]
(41)
Thus \(\|Q_{2M} w(T^*)\|^2 = \|Q_{2M}(S^* u_0 - S^* v_0)\|^2 \leq (1/128)\|w(0)\|^2 = (1/128)\|u_0 - v_0\|^2 .\) Therefore, we can claim that if \(\|P_{2M}(S^* u_0 - S^* v_0)\| \leq \|Q_{2M}(S^* u_0 - S^* v_0)\|,\) then
\[
\|S^* u_0 - S^* v_0\| = \sum_{|m| \leq 2M} |w_m(T^*)|^2 \\
+ \sum_{|m| > 2M} |w_m(T^*)|^2 \\
= \|P_{2M}(S^* u_0 - S^* v_0)\|^2 \\
+ \|Q_{2M}(S^* u_0 - S^* v_0)\|^2 \\
\leq 2 \|Q_{2M}(S^* u_0 - S^* v_0)\|^2 \\
\leq \frac{1}{64} \|u_0 - v_0\|^2 .
\]

The proof is complete. \(\square\)

Taking Lemmas 2, 3, 4, and 6 and [7, Theorem 3.1] into account, we now can state the main result of this paper as follows.

**Theorem 7.** Let assumption (H) hold. Then, one has the following:

1. \(S^*\) has an exponential attractor \(\mathcal{A}^*\) on \(\mathcal{B}\) which satisfies the following:
   - (i) \(\mathcal{M} \subset \mathcal{A}^* \subset \mathcal{B}\), where \(\mathcal{M}\) is the global attractor of \([S(t)]_{t \geq 0}\);
   - (ii) \(S^* \mathcal{A}^* \subset \mathcal{A}^* ; \) that is, \(\mathcal{A}^*\) is positively invariant under \(S^* ;\)
   - (iii) \(\mathcal{A}^*\) has finite fractal dimension \(\text{Dim}_f(\mathcal{A}^*) ;\)
   - (iv) there exist two constants \(c_1\) and \(c_2\) such that, for each \(u \in \mathcal{B}\) and every positive integer \(k, \)
     \[\text{Dist}(S^*_k u, \mathcal{A}^*) \leq c_1 e^{-c_2 k} ;\]

2. \(\mathcal{A} = \bigcup_{t \geq t^*} S(t) \mathcal{A}^*\) is an exponential attractor for \([S(t)]_{t \geq 0}\) on \(\mathcal{B}\) and \(\text{Dim}_f(\mathcal{A}) \leq \text{Dim}_f(\mathcal{A}^*) + 1 .\)

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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