Research Article

Mean-Square Almost Periodic Solutions for Impulsive Stochastic Host-Macroparasite Equation on Time Scales

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We consider an impulsive stochastic host-macroparasite equation on time scales. By use of the Banach fixed point theorem and Gronwall-Bellman's inequality technique on time scales, we obtain the existence and exponential stability of mean-square almost periodic solutions for the host-macroparasite equation on time scales. Finally, we give an example to illustrate the feasibility of our results.

1. Introduction

Many important human diseases, particularly in tropical and subtropical regions, arise from infection by macroparasites or metazoan organisms. These organisms tend to have much larger generation times and more complex life cycles than microparasites. In life cycles, there are two or more obligatory host species together with the final host (humans). Macroparasitic infections are generally chronic in form and they are more a cause of morbidity than mortality and tend to be persistent in character in areas where they are endemic. The final hosts of parasites are usually humans (the hosts in which the parasite attains reproductive maturity) and they gain entry to the definitive host as a consequence of developmental changes which normally occur before the organism arrives at its preferred site and attains reproductive maturity. Because of this fact, the host-macroparasite system has been attracting the attention of many researchers (see [1–4]). A time delay, therefore, exists between entry to the definitive host and the point when the parasite begins the production of eggs or larvae for transmission to other hosts (see [5–7]). In [7], by means of a continuation theorem in coincidence degree theory, the authors considered the oscillation and global attractivity of the nonlinear delay host-macroparasite model with periodic coefficients.

On the one hand, the theory of impulsive differential equations is now being recognized to not only be richer than the corresponding theory of differential equations without impulses, but also represent a more natural framework for mathematical modelling of many real-world phenomena, such as population dynamical models and neural networks. Since many dynamical processes are characterized by the fact that, at certain moments of time, they undergo abrupt changes of state, with the development of the theory of impulsive differential equations (see [8, 9]), various models of impulsive differential equations have been proposed and studied extensively (see [10–14]). For example, authors of [14] considered the nonlinear impulsive delay host-macroparasite model with periodic coefficients:

$$M'(t) = -\beta(t)M(t) + \frac{p(t)M(t)}{[q(t) + M(t - n\omega)]^{n+1}}, \quad t \neq t_k, \quad (1)$$

$$M(t_k^+) = (1 + b_k)M(t_k), \quad t = t_k, \quad k \in \mathbb{N},$$

where m is a positive integer, $q(t) = 1/\delta(t)$, $p(t) = \alpha(t)/\delta^{n+1}(t)$, and $\beta(t)$ are ω -periodic functions. By use of the continuation theorem of coincidence degree, some sufficient

conditions are obtained for the global attractivity and oscillation of positive periodic solutions.

In fact, both continuous and discrete systems are very important in implementation and applications. But it is troublesome to study the dynamical properties for continuous and discrete systems, respectively. Therefore, it is significant to study that on time scales which can unify the continuous and discrete cases. In [15], the author considered the following host-macroparasite equation on time scales:

$$x^\Delta(t) = -a(t)x(t) + \sum_{i=1}^m \frac{b_i(t)x(t - \tau_i(t))}{[1 + x(t - \tau_i(t))]^{N_i+1}}. \quad (2)$$

By using the contraction principle and Gronwall-Bellman's inequality on time scale, some sufficient conditions are obtained for the existence and exponential stability of almost periodic solutions.

On the other hand, as a matter of fact, population systems are often subject to environmental noise; that is, due to environmental fluctuations, parameters involved in population models are not absolute constants, and they may fluctuate around some average values. Based on these factors, more and more people began to be concerned about stochastic population systems (see [16–20]). Meanwhile, almost periodicity is universal than periodicity, and the mean-square almost periodicity is important in probability for investigating stochastic processes. To the best of our knowledge, there exist few results for mean-square almost periodic solutions for impulsive stochastic process models with delays; one can see some results in [21–23]. However, there exists no result on the existence and uniqueness of mean-square almost periodic solutions for impulsive stochastic host-macroparasite equation on time scales. Motivated by the above, we consider the following impulsive stochastic host-macroparasite equation on time scales:

$$\begin{aligned} \Delta x(t) = & \left[-a(t)x(t) + \sum_{i=1}^m \frac{b_i(t)x(t - \tau_i(t))}{[1 + x(t - \tau_i(t))]^{N_i+1}} \right] \Delta t \\ & + \sum_{i=1}^m \delta_i(t, x(t - \eta(t))) \Delta w(t), \quad t \neq t_k, \end{aligned} \quad (3)$$

$$\begin{aligned} \tilde{\Delta}x(t_k) &= x(t_k + 0) - x(t_k - 0) \\ &= \alpha_k x(t_k) + I_k(x(t_k)) + \nu_k, \quad t = t_k, \quad k \in \mathbb{N}, \end{aligned}$$

where $\Delta x(t)$ denotes a Δ -stochastic differential of $x(t)$, $a(t)$, $b_i(t)$, $\tau_i(t)$, $\eta(t) \in \text{PC}_{\text{rd}}(\mathbb{T}, \mathbb{R}^+)$, $N_i > 0$, δ_i is Borel measurable, $i = 1, 2, \dots, m$, $k \in \mathbb{Z}$, $\{t_k\} \in \mathcal{B} = \{\{t_k\} \mid t_k \in \mathbb{R}, t_k < t_{k+1}, \lim_{t \rightarrow \infty} t_k = +\infty\}$, $\alpha_k, \nu_k \in \mathbb{R}$, and $I_k \in C(\mathbb{R}, \mathbb{R})$. Let $(\Omega, \mathbb{F}, \mathbb{P})$ be a complete probability space furnished with a complete family of right continuous increasing sub- σ -algebras $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying $\mathcal{F}_t \subset \mathbb{F}$. $w(t)$ is a standard Brownian motion over $(\Omega, \mathbb{F}, \mathbb{P})$.

Throughout this paper, we assume the following.

(H₁) $a, b_i, \tau_i, \eta : \mathbb{T} \rightarrow \mathbb{R}^+$ are all mean-square almost periodic functions, $a(t) \geq \lambda > 0$, $t - \tau_i(t) \in \mathbb{T}$, $t - \eta(t) \in \mathbb{T}$, for $t \in \mathbb{T}$, $\delta_i : \mathbb{T} \times \mathbb{R} \rightarrow \mathbb{R}^+$ is mean-square almost periodic in t

uniformly for $x \in \mathbb{R}$, $\delta_i(t, 0) = 0$, and there exists constant l_i such that

$$\begin{aligned} E |\delta_i(t, x) - \delta_i(t, y)|^2 &\leq l_i E \|x - y\|^2, \\ t \in \mathbb{T}, \quad x, y &\in \text{PC}_{\text{rd}}(\mathbb{T}, L^2(\mathbb{R})), \quad i = 1, 2, \dots, m. \end{aligned} \quad (4)$$

(H₂) The sequences α_k, ν_k are almost periodic in k and the sequences $\{I_k(x)\}$ are almost periodic in k uniformly for $x \in \text{PC}_{\text{rd}}(\mathbb{T}, L^2(\mathbb{R}))$ and there exists constant L such that

$$\begin{aligned} E |I_k(x) - I_k(y)|^2 &\leq LE \|x - y\|^2, \\ t \in \mathbb{T}, \quad x, y &\in \text{PC}_{\text{rd}}(\mathbb{T}, L^2(\mathbb{R})), \quad i = 1, 2, \dots, m. \end{aligned} \quad (5)$$

(H₃) The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k, j \in \mathbb{N}$, are equipotentially almost periodic and $\inf_k t_k^1 = \theta > 0$, where $L^2(\mathbb{R})$ will be introduced in Section 2.

For convenience, we denote

$$\begin{aligned} \bar{f} &= \sup_{t \in \mathbb{R}} |f(t)|, \\ \bar{g} &= \sup_{(t, x) \in \mathbb{T} \times L^2(\mathbb{R})} |g(t, x)|, \end{aligned} \quad (6)$$

where $f(t)$ is a mean-square almost periodic function and $g(t, x)$ is a mean-square uniformly almost periodic function on time scales.

Let $\varphi_0 : (t_0 - \tau, t_0)_{\mathbb{T}} \rightarrow L^2(\mathbb{R})$ be a mean-square almost periodic function. Denote by $x(t) = x(t; t_0, \varphi_0)$ the solution of (3) satisfying the initial conditions:

$$\begin{aligned} x(t; t_0, \varphi_0) &= \varphi_0(t), \quad t \in (t_0 - \tau, t_0)_{\mathbb{T}}, \\ x(t_0^+; t_0, \varphi_0) &= \varphi_0(t_0), \end{aligned} \quad (7)$$

where $\tau = \sup_{t \in \mathbb{T}} \{\max_i |\tau_i(t)|, |\eta(t)|\}$.

Our main purpose of this paper is to study the existence and exponential stability of mean-square almost periodic solutions to (3) by means of the Banach fixed point theorem and Gronwall-Bellman's inequality technique.

2. Preliminaries

In this section, we shall recall some basic definitions and lemmas which are used in what follows.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers; the forward and backward jump operators $\sigma, \rho : \mathbb{T} \rightarrow \mathbb{T}$ and the forward graininess $\mu : \mathbb{T} \rightarrow \mathbb{R}^+$ are defined, respectively, by

$$\begin{aligned} \sigma(t) &:= \inf \{s \in \mathbb{T} : s > t\}, \\ \rho(t) &:= \sup \{s \in \mathbb{T} : s < t\}, \\ \mu(t) &= \sigma(t) - t. \end{aligned} \quad (8)$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered

maximum m , then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m , then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}_k = \mathbb{T}$.

A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be continuous function on $\mathbb{T}$.

For $y : \mathbb{T} \rightarrow \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t)$, $y^\Delta(t)$, to be the number (if it exists) with the property that, for a given $\varepsilon > 0$, there exists a neighborhood U of t such that

$$| [y(\sigma(t)) - y(s)] - y^\Delta(t) [\sigma(t) - s] | < \varepsilon |\sigma(t) - s|, \quad (9)$$

for all $s \in U$.

If y is continuous, then y is right-dense continuous, and if y is delta differentiable at t , then y is continuous at t .

Let f be right-dense continuous; if $F^\Delta(t) = f(t)$, then we define the delta integral by

$$\int_r^s f(t) \Delta t = F(s) - F(r), \quad r, s \in \mathbb{T}. \quad (10)$$

Lemma 1 (see [24]). Assume $f, g : \mathbb{T} \rightarrow \mathbb{R}$ are delta differentiable at $t \in \mathbb{T}$; then,

- (i) $(f + g)^\Delta(t) = f^\Delta(t) + g^\Delta(t)$;
- (ii) $(fg)^\Delta(t) = f^\Delta(t)g(t) + f^\sigma(t)g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g^\sigma(t)$;
- (iii) if $g(t)g^\sigma(t) \neq 0$, then $(f/g)^\Delta = (f^\Delta(t)g(t) - f(t)g^\Delta(t))/g(t)g^\sigma(t)$;
- (iv) if f and f^Δ are continuous, then $(\int_a^t f(t, s) \Delta s)^\Delta = f(\sigma(t), t) + \int_a^t f^\Delta(t, s) \Delta s$.

A function $p : \mathbb{T} \rightarrow \mathbb{R}$ is called regressive provided $1 + \mu(t)p(t) \neq 0$, for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $p : \mathbb{T} \rightarrow \mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T}, \mathbb{R})$. We define the set $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0, \forall t \in \mathbb{T}\}$.

If $r \in \mathcal{R}$, then the generalized exponential function e_r is defined by

$$e_r(t, s) = \exp \left\{ \int_s^t \xi_{\mu(\tau)}(r(\tau)) \Delta \tau \right\}, \quad (11)$$

for all $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\log(1 + hz)}{h}, & h \neq 0, \\ z, & h = 0. \end{cases} \quad (12)$$

Let $p, q : \mathbb{T} \rightarrow \mathbb{R}$ be two regressive functions; we define

$$\begin{aligned} p \oplus q &= p + q + \mu p q, \\ \ominus p &= -\frac{p}{1 + \mu p}, \\ p \ominus q &= p \oplus (\ominus q) = \frac{p - q}{1 + \mu q}. \end{aligned} \quad (13)$$

Then, the generalized exponential function has the following properties.

Lemma 2 (see [24]). Assume that $p, q : \mathbb{T} \rightarrow \mathbb{R}$ are two regressive functions; then,

- (i) $e_0(t, s) \equiv 1$ and $e_p(t, t) \equiv 1$;
- (ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;
- (iii) $e_p(t, s) = 1/e_p(s, t) = e_{\ominus p}(s, t)$;
- (iv) $e_p(t, s)e_p(s, r) = e_p(t, r)$;
- (v) $e_p(t, s)e_q(t, s) = e_{p \oplus q}(t, s)$;
- (vi) $e_p(t, s)/e_q(t, s) = e_{p \ominus q}(t, s)$;
- (vii) $(1/e_p(t, s))^\Delta = -p(t)/e_p^\sigma(t, s)$.

In the following, we will present some basic concepts and results concerning stochastic differential equations on time scales which will be essential to prove our main results. For more details, the reader may want to consult [17].

Definition 3 (see [25]). For a function $f : [0, 1]_{\mathbb{T}} \rightarrow \mathbb{R}$, one can define the extension $\tilde{f} : [0, 1] \rightarrow \mathbb{R}$ by $\tilde{f}(t) = f(\sup[0, t]_{\mathbb{T}})$, for all $t \in [0, 1]$.

Let $(\Omega, \mathbb{F}, \mathbb{P})$ be a probability space, and $L^2(\mathbb{R})$ stands for a space that consists of all $\mathbb{R}$ -valued random variables x with the norm

$$\mathbb{E} \|x\|^2 = \int_{\mathbb{W}} |x|^2 d\mathbb{P}. \quad (14)$$

Let w be a standard Wiener process and suppose $\{w(t+h) - w(t) : h \geq 0\}$ is independent of $\mathcal{F}_t := \sigma\{w(s) : 0 \leq s \leq t\}$, where $\mathbb{F}_{\mathbb{R}} = \{\mathcal{F}_t : t \in \mathbb{R}\}$ is a filtration on $\mathbb{R}$, and with $\sigma\{\cdot\}$, we mean the σ -algebra generated by $\cdot$. We denote Δ -stochastic integral on $[0, 1]_{\mathbb{T}}$, by $\int_0^1 f(t) \Delta w(t)$.

Definition 4 (see [25]). One can say that the random process $f : \mathbb{T} \times \Omega \rightarrow \mathbb{R}$ belongs to the class $L^2([0, 1]_{\mathbb{T}})$ if the following conditions hold:

- (i) f is adapted to $\mathbb{F}_{\mathbb{T}}$; that is, $f(t, \cdot)$ is $\mathcal{F}_t$ -measurable, for all $t \in [0, 1]_{\mathbb{T}}$.

$$(ii) \mathbb{P}(\int_0^1 |f(t, w)|^2 \Delta t < \infty) = 1.$$

Definition 5 (see [25]). One can say that the random process $f \in L^2([0, 1]_{\mathbb{T}})$ has a Δ -stochastic integral on $[0, 1]_{\mathbb{T}}$, provided the corresponding process $\tilde{f}$ has a stochastic integral in the common sense on $[0, 1]$ and then one can have $\int_0^1 f(t) \Delta w(t) = \int_0^1 \tilde{f}(t) d\omega(t)$.

Lemma 6 (see [25]). *The Δ -stochastic integral has the following properties:*

(i) *If $f_1, f_2 \in L^2([0, 1]_{\mathbb{T}})$ and $c_1, c_2 \in \mathbb{R}$, then*

$$\begin{aligned} & \int_0^1 (c_1 f_1(t) + c_2 f_2(t)) \Delta w(t) \\ &= c_1 \int_0^1 f_1(t) \Delta w(t) + c_2 \int_0^1 f_2(t) \Delta w(t). \end{aligned} \quad (15)$$

(ii) *If $\mathbb{E}(\int_0^1 |f(t)|^2 \Delta t) < +\infty$, then $\mathbb{E}(\int_0^1 f(t) \Delta w(t)) = 0$ and the Itô-isometry holds; that is,*

$$\mathbb{E} \left(\left(\int_0^1 f(t) \Delta w(t) \right)^2 \right) = \mathbb{E} \left(\int_0^1 f^2(t) \Delta t \right). \quad (16)$$

Definition 7 (see [25]). If, for all $t_1, t_2 \in [0, 1]_{\mathbb{T}}$ and $t_1 < t_2$, such that

$$X(t_2) - X(t_1) = \int_{t_1}^{t_2} b(t, X) \Delta t + \int_{t_1}^{t_2} B(t, X) \Delta w(t), \quad (17)$$

where b is Lebesgue integrable on $[0, 1]_{\mathbb{T}}$ and $B \in L^2([0, 1]_{\mathbb{T}})$, then we say that the process X has a Δ -stochastic differential indicated by the notation

$$\Delta X(t) = b(t, X) \Delta t + B(t, X) \Delta w(t). \quad (18)$$

We consider the impulsive stochastic dynamic system on $[t_0, a]_{\mathbb{T}} \subset \mathbb{T}$:

$$\begin{aligned} \Delta X(t) &= b(t) X(t) \Delta t + b(X, t) \Delta t + B(X, t) \Delta w(t), \\ t &\neq t_k, \end{aligned} \quad (19)$$

$$\tilde{\Delta} X(t_k) = \alpha_k X(t_k) + I_k(X(t_k)), \quad t = t_k,$$

with initial value $X(t_0) = X_0$, where X_0 is a random variable, independent of $w(t)$.

Definition 8 (see [23]). One can say that a random process $X(t)$ with $X(t_0) = X_0$ is a solution of the impulsive stochastic dynamic system (19) on $[t_0, a]_{\mathbb{T}}$ if the following conditions hold:

- (i) X is adapted to the filtration $\mathbb{F}$.
- (ii) For all $t \in [t_0, a]_{\mathbb{T}}$, we have almost surely

$$\begin{aligned} X(t) &= W(t, t_0) X_0 + \int_{t_0}^t W(t, \sigma(s)) b(X(s), s) \Delta s \\ &+ \int_{t_0}^t W(t, \sigma(s)) B(X(s), s) \Delta w(s) \\ &+ \sum_{t_0 \leq t_k < t} W(t, t_k) I_k(X(t_k)), \end{aligned} \quad (20)$$

where $W(t, s)$ is the Cauchy matrix of the following system:

$$\begin{aligned} \Delta X(t) &= b(t) X(t) \Delta t, \quad t \neq t_k, \\ \tilde{\Delta} X(t_k) &= \alpha_k X(t_k), \quad t = t_k. \end{aligned} \quad (21)$$

Definition 9 (see [23]). One can say $\varphi : \mathbb{T} \rightarrow L^2(\mathbb{R})$ is rd-piecewise continuous with respect to a sequence $\{t_k\} \subset \mathbb{T}$ which satisfies $t_k < t_{k+1}$, $k \in \mathbb{Z}$, if $\varphi(t)$ is continuous on $[t_k, t_{k+1})_{\mathbb{T}}$ and rd-continuous on $\mathbb{T} \setminus \{t_k\}$. Furthermore, $[t_k, t_{k+1})_{\mathbb{T}}$, $k \in \mathbb{Z}$, are called intervals of continuity of the function $\varphi(t)$.

For convenience, $\text{PC}_{\text{rd}}(\mathbb{T}, L^2(\mathbb{R}))$ denotes the set of all piecewise continuous functions with respect to a sequence $\{t_k\}$, $k \in \mathbb{Z}$. For any integers k and j , denote $t_k^j = t_{k+j} - t_k$.

Definition 10 (see [26]). A time scale $\mathbb{T}$ is called an almost periodic time scale if

$$\Pi = \{\tau \in \mathbb{R} : t \pm \tau \in \mathbb{T}, \forall t \in \mathbb{T}\} \neq \{0\}. \quad (22)$$

Definition 11 (see [10]). For any $\varepsilon > 0$, let $\Gamma_\varepsilon \subset \Pi$ be a set of real numbers and $\{t_k\} \subset \mathbb{T}$. One can say $\{t_k^j\}$, $k, j \in \mathbb{Z}$, is equipotentially almost periodic on an almost periodic time scale $\mathbb{T}$ if, for $r \in \Gamma_\varepsilon \subset \Pi$, there exists at least one integer q such that

$$|t_k^q - r| < \varepsilon, \quad \forall k \in \mathbb{Z}. \quad (23)$$

Definition 12 (see [23]). Let $\mathbb{T}$ be an almost periodic time scale and assume that $\{t_k\} \subset \mathbb{T}$ satisfying the derived sequence $\{t_k^j\}$, $k, j \in \mathbb{Z}$, is equipotentially almost periodic. One can call a stochastic process $\varphi \in \text{PC}_{\text{rd}}(\mathbb{T}, L^2(\mathbb{R}))$ mean-square almost periodic if

- (i) for any $\varepsilon > 0$, there is a positive number $\delta = \delta(\varepsilon)$ such that if the points t' and t'' belong to the same interval of continuity and $|t' - t''| < \delta$, then $\mathbb{E} \|\varphi(t') - \varphi(t'')\|^2 < \varepsilon$;
- (ii) for any $\varepsilon > 0$, there is relative dense set Γ such that if $\tau \in \Gamma$, then $\mathbb{E} \|\varphi(t + \tau) - \varphi(t)\|^2 < \varepsilon$, for all $t \in \mathbb{T}$, which satisfy the condition $|t - t_i| > \varepsilon$, $i \in \mathbb{Z}$.

Definition 13 (see [23]). Let $\mathbb{T}$ be an almost periodic time scale and assume that $\{t_k\} \subset \mathbb{T}$ satisfying the derived sequence $\{t_k^j\}$, $k, j \in \mathbb{Z}$, is equipotentially almost periodic. One can call a stochastic process $\varphi \in \text{PC}_{\text{rd}}(\mathbb{T} \times \Omega, L^2(\mathbb{R}))$ mean-square almost periodic in t uniformly for $x \in \Omega$ if for any $\varepsilon > 0$ and for each compact subset S of Ω there existed the following:

- (i) for any $\varepsilon > 0$, there is a positive number $\delta = \delta(\varepsilon, S)$ such that if the points t' and t'' belong to the same interval of continuity and $|t' - t''| < \delta$, then $\mathbb{E} \|\varphi(t', x) - \varphi(t'', x)\|^2 < \varepsilon$;
- (ii) for any $\varepsilon > 0$, there is relative dense set Γ such that if $\tau \in \Gamma$, then $\mathbb{E} \|\varphi(t + \tau, x) - \varphi(t, x)\|^2 < \varepsilon$, for all $(t, x) \in \mathbb{T} \times \Omega$, which satisfy the condition $|t - t_k| > \varepsilon$, $k \in \mathbb{Z}$,

where $\Omega \subset L^2(\mathbb{R})$ or $\Omega = L^2(\mathbb{R})$.

Definition 14. The equation (3) is said to be exponentially stable if, for all $c > 0$, there exist $\alpha = \alpha(c) > 0$ and $K > 0$ such that if $\|u_{t_0} - v_{t_0}\| \leq c$, then, for all $t \geq t_0$,

$$\begin{aligned} & E \|u(t; t_0, u_{t_0}) - v(t; t_0, v_{t_0})\|^2 \\ & < K \left(E \|u_{t_0} - v_{t_0}\|^2 \right) e^{-\alpha(t-t_0)}. \end{aligned} \quad (24)$$

In order to study (3), we consider the linear system:

$$\begin{aligned} \Delta x(t) &= -a(t) x(t) \Delta t, \quad t \neq t_k, \\ \tilde{\Delta} x(t_k) &= \alpha_k x(t_k), \quad t = t_k, \quad k \in \mathbb{N}. \end{aligned} \quad (25)$$

Now let us consider the equations

$$\Delta x(t) = -a(t) x(t) \Delta t, \quad t_{k-1} < t \leq t_k \quad (26)$$

and their solutions

$$x(t) = x(s) e_{-a}(t, s), \quad t_{k-1} < s \leq t \leq t_k. \quad (27)$$

Then, by [10], the Cauchy matrix of the linear equation (25) is

$$\begin{aligned} W(t, s) &= \begin{cases} e_{-a}(t, s), & t_{k-1} < s \leq t \leq t_k, \\ \prod_{i=r}^{k+1} (1 + \alpha_i) e_{-a}(t, s), & t_{r-1} < s \leq t_r < t_k < t \leq t_{k+1} \end{cases} \end{aligned} \quad (28)$$

and the solutions of (25) are in the following form:

$$x(t; t_0, \varphi_0) = W(t, t_0) \varphi_0, \quad \varphi_0 \in \mathbb{R}. \quad (29)$$

Similar to the proofs of Lemma 3.1 and Lemma 3.2 in [10], one can easily show the following two lemmas, respectively.

Lemma 15. For system (3), let (H_1) – (H_3) hold. Then, for each $\varepsilon > 0$, there exist $\varepsilon_1 > 0$, $0 < \varepsilon_1 < \varepsilon$, a relative dense set $\bar{T} \subset \Pi$ of real numbers and $\mathbb{Q}$, such that the following relations are fulfilled:

- (a) $E \|a(t+\tau) - a(t)\|^2 < \varepsilon$, $E \|b_i(t+\tau) - b_i(t)\|^2 < \varepsilon$, $E \|\tau_i(t+\tau) - \tau_i(t)\|^2 < \varepsilon$, $E \|\xi(t+\tau) - \xi(t)\|^2 < \varepsilon$, $E \|\delta_i(t+\tau, x) - \delta_i(t, x)\|^2 < \varepsilon$, $t \in \mathbb{T}$, $\tau \in \bar{T}$, $x \in PC_{rd}(\mathbb{T}, L^2(\mathbb{R}))$, $|t - t_k| > \varepsilon$, $k \in \mathbb{Z}$, $i = 1, 2, \dots, m$;
- (b) $|\alpha_{k+q} - \alpha_k| < \varepsilon$, $|I_{k+q}(x) - I_k(x)| < \varepsilon$, $|v_{k+q} - v_k| < \varepsilon$, $\forall x \in PC_{rd}(\mathbb{T}, L^2(\mathbb{R}))$, $q \in \mathbb{Q}$, $k \in \mathbb{Z}$, $i = 1, 2, \dots, m$;
- (c) $|t_k^q - \tau| < \varepsilon_1$, $q \in \mathbb{Q}$, $\tau \in \bar{T}$, $k \in \mathbb{Z}$.

Lemma 16. For (3), let (H_1) – (H_3) hold. Then, the Cauchy matrix $W(t, s)$ of (3) satisfies the inequality

$$\|W(t, s)\| \leq K e^{-\lambda(t-s)}, \quad t \geq s \quad (30)$$

and for any $\varepsilon > 0$, $t, s \in \mathbb{T}$, $|t - t_k| > \varepsilon$, $|s - t_k| > \varepsilon$, $k \in \mathbb{Z}$, there exists a relatively dense set $\bar{T} \subset \Pi$ of ε -almost periods of the function $a(t)$ and a positive constant Γ such that, for $r \in \bar{T}$,

$$\|W(t+r, s+r) - W(t, s)\| \leq \varepsilon \Gamma e^{-(\lambda/2)(t-s)}, \quad t \geq s. \quad (31)$$

Definition 17. The equation (3) is said to be exponentially stable if, for all $c > 0$, there exist $\alpha = \alpha(c) > 0$ and $M > 0$ such that if $\|u_{t_0} - v_{t_0}\| \leq c$, then, for all $t \geq t_0$,

$$\begin{aligned} & E \|u(t; t_0, u_{t_0}) - v(t; t_0, v_{t_0})\|^2 \\ & < M \left(E \|u_{t_0} - v_{t_0}\|^2 \right) e_{\Theta\alpha}(t, t_0). \end{aligned} \quad (32)$$

3. Existence of Almost Periodic Solution

In this section, we will study the existence and exponential stability of mean-square almost periodic solutions of (3) by using the Banach fixed point theorem and Gronwall-Bellman's inequality technique on time scales.

Theorem 18. Assume that (H_1) – (H_3) hold and there exist two positive constants d , N such that the following conditions hold. (H_4) $\gamma < 1$, where

$$\begin{aligned} \gamma &= \left\{ \frac{3K^2(1 + \bar{\mu}\lambda)^2}{\lambda^2} \left[\left(\sum_{i=1}^m \bar{b}_i M_i \right)^2 + \left(\sum_{i=1}^m \bar{\delta}_i \right)^2 \right] \right. \\ &\quad \left. + 3 \left(\frac{KL}{1 - e^{-\lambda\theta}} \right)^2 \right\}, \end{aligned} \quad (33)$$

$$M_i = \frac{1}{N_i(1 + 1/N_i)^{1+N_i}}, \quad i = 1, 2, \dots, m.$$

(H_5) $(\Theta\lambda) \oplus p < 0$, where

$$p = \frac{4K^2(1 + \lambda\bar{\mu})^2}{\lambda} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right]. \quad (34)$$

Then,

- (1) There exists a unique square-mean almost periodic solution $x(t)$ of (3) in the region $B^* = \{\varphi \mid \varphi \in PC_{rd}(\mathbb{T}, L^2(\mathbb{R})), E\|\varphi(t)\|^2 \leq d^2, t \in \mathbb{T}\}$.
- (2) The solution $x(t)$ in the region B^* is exponentially stable.

Proof. We define an operator T in B^* as

$$\begin{aligned} T\varphi &= \int_{-\infty}^t W(t, \sigma(s)) S_\varphi(s) \Delta s \\ &\quad + \int_{-\infty}^t W(t, \sigma(s)) L_\varphi(s) \Delta w(s) \\ &\quad + \sum_{t_k < t} W(t, t_k) h_\varphi^k, \end{aligned} \quad (35)$$

where

$$\begin{aligned} S_\varphi(s) &= \sum_{i=1}^m \frac{b_i(s) \varphi(s - \tau_i(s))}{[1 + \varphi(s - \tau_i(s))]^{N_i+1}}, \\ L_\varphi(s) &= \sum_{i=1}^m \delta_i(s, x(s - \eta(s))), \\ h_\varphi^k &= I_k(\varphi(t_k)) + v_k, \quad k \in \mathbb{Z}. \end{aligned} \quad (36)$$

For any $\varphi \in B^*$, by means of the proof of Theorem 1 in [19], we have

$$\begin{aligned} \|S_\varphi\| &= \sup_{s \in \mathbb{T}} \left| \sum_{i=1}^m \frac{b_i(s) \varphi(s - \tau_i(s))}{[1 + \varphi(s - \tau_i(s))]^{N_i+1}} \right| \\ &\leq \sup_{s \in \mathbb{T}} \left| \sum_{i=1}^m \frac{\bar{b}_i \varphi(s - \tau_i(s))}{[1 + \varphi(s - \tau_i(s))]^{N_i+1}} \right| \leq \sum_{i=1}^m \bar{b}_i M_i. \end{aligned} \quad (37)$$

Firstly, we will show that T is a self-mapping from B^* to B^* . In fact, for any $\varphi \in B^*$, by use of (37), we have

$$\begin{aligned} \mathbb{E} \|T\varphi\|^2 &\leq 3\mathbb{E} \left(\int_{-\infty}^t \|W(t, \sigma(s))\| \|S_\varphi(s)\| \Delta s \right)^2 \\ &\quad + 3\mathbb{E} \left(\left\| \int_{-\infty}^t W(t, \sigma(s)) L_\varphi(s) \Delta w(s) \right\| \right)^2 \\ &\quad + 3\mathbb{E} \left(\sum_{t_k < t} \|W(t, t_k)\| \|h_\varphi^k\| \right)^2 \\ &\leq \frac{3K^2(1 + \bar{\mu}\lambda)^2}{\lambda^2} \left[\left(\sum_{i=1}^m \bar{b}_i M_i \right)^2 + \left(\sum_{i=1}^m \bar{\delta}_i \right)^2 \right] \\ &\quad + 3 \left(\frac{K}{1 - e^{-\lambda\theta}} \sup_{k \in \mathbb{Z}} (\bar{I}_k + |v_k|) \right)^2 := d^2. \end{aligned} \quad (38)$$

On the other hand, let $\tau \in \bar{T}$ and $q \in \mathbb{Q}$, where the set $\bar{T}$ is determined in Lemma 15. Then,

$$\begin{aligned} &\|T\varphi(t + \tau) - T\varphi(t)\| \\ &\leq \left\| \int_{-\infty}^t [W(t + \tau, \sigma(s + \tau)) - W(t, \sigma(s))] \right. \\ &\quad \cdot S_\varphi(s + \tau) \Delta s + \int_{-\infty}^t W(t, \sigma(s)) \\ &\quad \cdot [S_\varphi(s + \tau) - S_\varphi(s)] \Delta s \\ &\quad + \int_{-\infty}^t [W(t + \tau, \sigma(s + \tau)) - W(t, \sigma(s))] \\ &\quad \cdot L_\varphi(s + \tau) \Delta w(s) + \int_{-\infty}^t W(t, \sigma(s)) \end{aligned}$$

$$\begin{aligned} &\cdot [L_\varphi(s + \tau) - L_\varphi(s)] \Delta w(s) + \sum_{t_k < t + \tau} W(t + \tau, t_k) \\ &\cdot h_\varphi^k - \sum_{t_k < t} W(t, t_k) h_\varphi^k \Big\| \leq \left\| \sum_{i=1}^m \bar{b}_i M_i \right. \\ &\cdot \int_{-\infty}^t [W(t + \tau, \sigma(s + \tau)) - W(t, \sigma(s))] \Delta s \\ &+ \int_{-\infty}^t W(t, \sigma(s)) [S_\varphi(s + \tau) - S_\varphi(s)] \Delta s + \sum_{i=1}^m \bar{\delta}_i \\ &\cdot \int_{-\infty}^t [W(t + \tau, \sigma(s + \tau)) - W(t, \sigma(s))] \Delta w(s) \\ &+ \int_{-\infty}^t W(t, \sigma(s)) [L_\varphi(s + \tau) - L_\varphi(s)] \Delta w(s) \\ &+ \sum_{t_k < t} [W(t + \tau, t_{k+q}) - W(t, t_k)] h_\varphi^{k+q} \\ &\left. + \sum_{t_k < t} W(t, t_k) (h_\varphi^{k+q} - h_\varphi^k) \right\| = \|J_1 + J_2 + J_3\|, \end{aligned} \quad (39)$$

where

$$\begin{aligned} J_1 &= \sum_{i=1}^m \bar{b}_i M_i \\ &\cdot \int_{-\infty}^t [W(t + \tau, \sigma(s + \tau)) - W(t, \sigma(s))] \Delta s \\ &+ \int_{-\infty}^t W(t, \sigma(s)) [S_\varphi(s + \tau) - S_\varphi(s)] \Delta s, \\ J_2 &= \sum_{i=1}^m \bar{\delta}_i \\ &\cdot \int_{-\infty}^t [W(t + \tau, \sigma(s + \tau)) - W(t, \sigma(s))] \Delta w(s) \\ &+ \int_{-\infty}^t W(t, \sigma(s)) [L_\varphi(s + \tau) - L_\varphi(s)] \Delta w(s), \\ J_3 &= \sum_{t_k < t} [W(t + \tau, t_{k+q}) - W(t, t_k)] h_\varphi^{k+q} \\ &+ \sum_{t_k < t} W(t, t_k) (h_\varphi^{k+q} - h_\varphi^k). \end{aligned} \quad (40)$$

Since $(a + b + c)^2 \leq 3a^2 + 3b^2 + 3c^2$, it follows that

$$\begin{aligned} \mathbb{E} \|T\varphi(t + \tau) - T\varphi(t)\|^2 &\leq 3\mathbb{E} \|J_1\|^2 + 3\mathbb{E} \|J_2\|^2 + 3\mathbb{E} \|J_3\|^2. \end{aligned} \quad (41)$$

By conditions (H_1) – (H_3) and Lemmas 15 and 16, we can obtain that

$$\mathbb{E} \|T\varphi(t + \tau) - T\varphi(t)\|^2 \leq 3(c_1^2 + c_2^2 + c_3^2) \varepsilon, \quad (42)$$

where

$$\begin{aligned} c_1 &= \frac{\Gamma(2 + \bar{\mu}\lambda)}{\lambda} \sum_{i=1}^m \bar{b}_i M_i + \frac{K(1 + \bar{\mu}\lambda)}{\lambda}, \\ c_2 &= \frac{\Gamma(2 + \bar{\mu}\lambda)}{\lambda} \sum_{i=1}^m \bar{\delta}_i + \frac{K(1 + \bar{\mu}\lambda)}{\lambda}, \\ c_3 &= \frac{\Gamma}{1 - e^{-(\lambda/2)\theta}} \sup_{k \in \mathbb{Z}} (\bar{I}_k + |v_k|) + \frac{K}{1 - e^{-\lambda\theta}}. \end{aligned} \quad (43)$$

Consequently, by (39) and (42), we obtain that $T\varphi \in B^*$.

Next, we prove that the mapping T is a contraction mapping of B^* . In fact, in view of (H_1) – (H_3) , for any $\varphi, \psi \in B^*$, we can easily obtain

$$\mathbb{E} \|S_\varphi(s) - S_\psi(s)\|^2 \leq \left(\sum_{i=1}^m \bar{b}_i \right)^2 \mathbb{E} \|\varphi - \psi\|^2, \quad (44)$$

$$\mathbb{E} \|L_\varphi(s) - L_\psi(s)\|^2 \leq \left(\sum_{i=1}^m l_i \right)^2 \mathbb{E} \|\varphi - \psi\|^2.$$

Notice that

$$\begin{aligned} &E \|T(\varphi) - T(\psi)\|^2 \\ &\leq 3E \left\| \int_{-\infty}^t W(t, \sigma(s)) [S_\varphi(s) - S_\psi(s)] \Delta s \right\|^2 \\ &+ 3E \left\| \int_{-\infty}^t W(t, \sigma(s)) [L_\varphi(s) - L_\psi(s)] \Delta w(s) \right\|^2 \\ &+ 3E \left\| \sum_{t_k < t} W(t, t_k) [h_\varphi^k - h_\psi^k] \right\|^2 \\ &\leq \left\{ \frac{3K^2(1 + \bar{\mu}\lambda)^2}{\lambda^2} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right] \right. \\ &\quad \left. + 3 \left(\frac{KL}{1 - e^{-\lambda\theta}} \right)^2 \right\} \mathbb{E} \|\varphi - \psi\|^2 = \gamma \mathbb{E} \|\varphi - \psi\|^2. \end{aligned} \quad (45)$$

This implies that the mapping T is a contraction mapping. Hence, there exists a unique mean-square almost periodic solution of (3) by the Banach fixed point theorem. Suppose that $y(t)$ is an arbitrary solution of (3) with initial conditions:

$$\begin{aligned} y(t; t_0, \phi_0) &= \phi_0(t), \quad t \in (t_0 - \bar{\tau}, t_0), \\ y(t_0^+; t_0, \phi_0) &= \phi_0(t_0). \end{aligned} \quad (46)$$

Since every solution of system (3) can be represented as

$$\begin{aligned} x(t) &= W(t, t_0) \phi_0 + \int_{t_0}^t W(t, \sigma(s)) S_x(s) \Delta s \\ &+ \int_{t_0}^t W(t, \sigma(s)) L_x(s) \Delta w(s) \\ &+ \sum_{t_0 \leq t_k < t} W(t, t_k) h_x^k, \end{aligned} \quad (47)$$

we obtain that

$$\begin{aligned} &y(t) - x(t) \\ &= W(t, t_0) (\phi_0 - \varphi_0) \\ &+ \int_{t_0}^t W(t, \sigma(s)) (S_y(s) - S_x(s)) \Delta s \\ &+ \int_{t_0}^t W(t, \sigma(s)) (L_y(s) - L_x(s)) \Delta w(s) \\ &+ \sum_{t_0 \leq t_k < t} W(t, t_k) (h_y^k - h_x^k). \end{aligned} \quad (48)$$

Since $(a + b + c + d)^2 \leq 4a^2 + 4b^2 + 4c^2 + 4d^2$, it follows that

$$\begin{aligned} &\mathbb{E} \|y(t) - x(t)\|^2 \leq 4K^2 e^{-\lambda(t-t_0)} \mathbb{E} \|\phi_0 - \varphi_0\|^2 \\ &+ 4K^2 \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right] \left(\int_{-\infty}^t e^{-\lambda(t-\sigma(s))} \Delta s \right) \\ &\cdot \left(\int_{t_0}^t e^{-\lambda(t-\sigma(s))} \mathbb{E} \|y(s) - x(s)\|^2 \Delta s \right) + 4(KL)^2 \\ &\cdot \left(\sum_{-\infty \leq t_k < t} e^{-\lambda(t-t_k)} \right) \\ &\cdot \sum_{t_0 \leq t_k < t} e^{-\lambda(t-t_k)} \mathbb{E} \|y(t_k) - x(t_k)\|^2 \\ &\leq 4K^2 e_{\Theta\lambda}(t, t_0) \mathbb{E} \|\phi_0 - \varphi_0\|^2 \\ &+ \frac{4K^2(1 + \lambda\bar{\mu})}{\lambda} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right] \\ &\cdot \left(\int_{t_0}^t e_{\Theta\lambda}(t, \sigma(s)) \mathbb{E} \|y(s) - x(s)\|^2 \Delta s \right) \\ &+ \frac{4(KL)^2}{1 - e^{-\lambda\theta}} \sum_{t_0 \leq t_k < t} e_{\Theta\lambda}(t, t_k) \mathbb{E} \|y(t_k) - x(t_k)\|^2 \\ &\leq 4K^2 e_{\Theta\lambda}(t, t_0) \mathbb{E} \|\phi_0 - \varphi_0\|^2 \\ &+ \frac{4K^2(1 + \lambda\bar{\mu})^2}{\lambda} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right] \\ &\cdot \left(\int_{t_0}^t e_{\Theta\lambda}(t, s) \mathbb{E} \|y(s) - x(s)\|^2 \Delta s \right) + \frac{4(KL)^2}{1 - e^{-\lambda\theta}} \\ &\cdot \sum_{t_0 \leq t_k < t} e_{\Theta\lambda}(t, t_k) \mathbb{E} \|y(t_k) - x(t_k)\|^2. \end{aligned} \quad (49)$$

Let $\mathbb{E}\|u(t)\|^2 = \mathbb{E}\|y(t) - x(t)\|^2 e_\lambda(t, t_0)$; then,

$$\begin{aligned} \mathbb{E}\|u(t)\|^2 &\leq 4\mathbb{E}\|u(t_0)\|^2 \\ &+ \frac{4K^2(1+\lambda\bar{\mu})^2}{\lambda} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right] \\ &\cdot \int_{t_0}^t \mathbb{E}\|u(s)\|^2 \Delta s + \frac{4(KL)^2}{1-e^{-\lambda\theta}} \sum_{t_0 \leq t_k < t} \mathbb{E}\|u(t_k)\|^2. \end{aligned} \quad (50)$$

From Gronwall-Bellman's inequality on time scales, one has

$$\begin{aligned} \mathbb{E}\|u(t)\|^2 &\leq 4K^2 \mathbb{E}\|u(t_0)\|^2 \prod_{t_0 < t_k < t} \left(1 + \frac{4(KL)^2}{1-e^{-\lambda\theta}} \right) e_p(t, t_0); \end{aligned} \quad (51)$$

that is,

$$\begin{aligned} \mathbb{E}\|y(t) - x(t)\|^2 &\leq \mathbb{E}\|\phi_0 - \varphi_0\|^2 \prod_{t_0 < t_k < t} \left(1 + \frac{4(KL)^2}{1-e^{-\lambda\theta}} \right) e_{(\ominus\lambda)\oplus p}(t, t_0), \end{aligned} \quad (52)$$

where

$$p = \frac{4K^2(1+\lambda\bar{\mu})^2}{\lambda} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m l_i \right)^2 \right]. \quad (53)$$

Thus, by condition (H_5) of the theorem, the mean-square almost periodic solution of (3) is exponentially stable. This completes the proof. $\square$

Remark 19. According to the conditions of Theorem 18, we can find that the uniqueness and exponential stability of the mean-square almost periodic solution for the impulsive stochastic host-macroparasite equation on time scales are independent of the magnitude of delays but are dependent on the magnitude of noise and impulse.

Remark 20. It is the first time that the sufficient conditions for the existence and exponential stability of piecewise mean-square almost periodic solutions for the impulsive stochastic host-macroparasite equation on time scales are investigated. The obtained results are essentially new. Without considering impulsive and stochastic effects on (3), then (3) reduce to (2).

4. An Example

Consider the following impulsive stochastic host-macroparasite equation on time scales:

$$\begin{aligned} \Delta x(t) &= \left[-a(t)x(t) + \sum_{i=1}^m \frac{b_i(t)x(t-\tau_i(t))}{[1+x(t-\tau_i(t))]^{N_i+1}} \right] \Delta t \\ &+ \sum_{i=1}^m \delta_i(t, x(t-\eta(t))) \Delta \omega(t), \quad t \neq t_k, \end{aligned} \quad (54)$$

$$\begin{aligned} \tilde{\Delta}x(t_k) &= x(t_k+0) - x(t_k-0) \\ &= \alpha_k x(t_k) + I_k(x(t_k)) + \nu_k, \quad t = t_k, \quad k \in \mathbb{N}, \end{aligned}$$

where $i = 2, t_k = k, k \in \mathbb{N}$, and

$$\begin{aligned} a(t) &= 8 + \cos \sqrt{2}t, \\ b_1(t) &= 0.03(1 + |\cos \sqrt{3}t|), \\ b_2(t) &= 0.05(1 + |\sin \sqrt{2}t|), \\ \delta_1(t, x) &= 0.04 |\sin \sqrt{5}tx|, \\ \delta_2(t, x) &= 0.02 |\sin \sqrt{2}tx|, \\ \alpha_k &= -0.06 |\cos k|, \\ I_k(x) &= 0.06 |\sin kx|, \\ \nu_k &= 0.07, \\ N_1 &= 1, \\ N_2 &= 2. \end{aligned} \quad (55)$$

By calculating, we have

$$\begin{aligned} \bar{a} &= 9, \\ \bar{b}_1 &= 0.06, \\ \bar{b}_2 &= 0.1, \\ \bar{\delta}_1 &= l_1 = 0.04, \\ \bar{\delta}_2 &= l_2 = 0.02, \\ L &= 0.06, \\ M_1 &= 0.25, \\ M_2 &= \frac{4}{27}. \end{aligned} \quad (56)$$

It is easy to see that $K = \theta = 1$, $\lambda = 5$; thus,

$$\begin{aligned} & \frac{3K^2(1+\bar{\mu}\lambda)^2}{\lambda^2} \left[\left(\sum_{i=1}^m \bar{b}_i M_i \right)^2 + \left(\sum_{i=1}^m \bar{\delta}_i \right)^2 \right] \\ & + 3 \left(\frac{KL}{1-e^{-\lambda\theta}} \right)^2 \approx 0.046 < 1, \\ p - \lambda &= \frac{4K^2(1+\lambda\bar{\mu})^2}{\lambda} \left[\left(\sum_{i=1}^m \bar{b}_i \right)^2 + \left(\sum_{i=1}^m \bar{l}_i \right)^2 \right] - \lambda \\ &\approx 0.841 - 5 = -4.159 < 0. \end{aligned} \quad (57)$$

Hence, $(\ominus\lambda) \oplus p < 0$. According to Theorem 18, the impulsive stochastic equation (54) has a unique piecewise mean-square almost periodic solution which is exponentially stable in the region B^* , where

$$\begin{aligned} B^* &= \left\{ \varphi \mid \varphi \in \text{PC}_{\text{rd}}(\mathbb{T}, L^2(\mathbb{R})), E \|\varphi(t)\|^2 \leq 1, t \right. \\ &\left. \in \mathbb{T} \right\}. \end{aligned} \quad (58)$$

5. Conclusion

In this paper, we have investigated the host-macroparasite equation with impulsive and stochastic effects on time scales. By employing the Banach fixed point theorem, some stochastic analysis techniques, and Gronwall-Bellman's inequality technique on time scales, we have obtained some sufficient conditions ensuring the existence and exponential stability of mean-square almost periodic solutions for impulsive host-macroparasite equation on time scales. These sufficient conditions can be easily checked by simple algebraic methods. An example has been given to demonstrate the effectiveness of the presented results. The methods used in paper can be applied to study square-mean almost periodic problems of many other types of impulsive stochastic models on time scales.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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