Research Article

Dynamical Behaviors of a Stage-Structured Predator-Prey Model with Harvesting Effort and Impulsive Diffusion

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We consider a delayed predator-prey model with harvesting effort and impulsive diffusion between two patches. By the impulsive comparison theorem and the discrete dynamical system determined by the stroboscopic map, we obtain some sufficient conditions on the existence and global attractiveness of predator-eradicated periodic solution for the system. Furthermore, the permanence of the system is derived. The obtained results will modify and improve the ones in some existing publications and give the estimate for the ultimately low and upper boundedness of the systems.

1. Introduction

The effect of spatial factors in population dynamics is an interesting topic since dispersal often occurs between patches in ecological environment [1, 2]. Some models focused on dispersal process in continuous time meaning [3, 4]. However, many species diffuse only during a single period, and the diffusion often occurs in regular pulses. For example, when winter comes, birds will migrate between patches in search for a better environment, whereas they do not diffuse during other seasons. Since the short-time diffusion is often assumed to be in the form of impulses in the modeling process, some mathematical models on impulsive diffusion have been studied by impulsive differential equations (see, e.g., [5–11]). In [5], a single species model with impulsive diffusion was initially formulated by

$$\begin{aligned} x_1'(t) &= x_1(t)(a_1 - b_1 x_1(t)), \\ x_2'(t) &= x_2(t)(a_2 - b_2 x_2(t)), \\ &\quad t \neq n\tau, \\ \Delta x_1(t) &= d_1(x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2(x_1(t) - x_2(t)), \\ &\quad t = n\tau, \quad n \in \mathbb{N}, \end{aligned} \quad (1)$$

where, for the i th patch, x_i is the density of species in the logistic growth; $\Delta x_i(t) = x_i(t^+) - x_i(t^-)$ and $x_i(t) = x_i(t^-)$, $x_i(t^+)$, $x_i(t^-)$ represent the right-hand limit and left-hand one at t ; the parameters $a_i, a_i/b_i$ are the intrinsic growth rates and carrying capacities, respectively; the diffusion occurs at impulsive moments $n\tau$, $n = 0, 1, 2, \dots$, $\tau > 0$; $d_i \in (0, 1)$ is dispersal rate from the j th patch to i th, $i, j = 1, 2$, $i \neq j$. We assume that the net exchange from the j th patch to i th patch is proportional to the difference $x_j - x_i$ of population densities.

On the other hand, almost all species go through the life stage from the immature to the mature stage. Since it is necessary to spend units of time that the immature becomes the mature, time delay plays important role in stage-structured model. Aiello and Freedman [6] introduced the following stage-structured single species model:

$$\begin{aligned} y'(t) &= \alpha z(t) - \omega y(t) - \alpha e^{-\omega\tau} z(t - \tau), \\ z'(t) &= \alpha e^{-\omega\tau} z(t - \tau) - rz^2(t), \end{aligned} \quad (2)$$

where y, z represent the immature and mature populations densities, respectively, α is the birth rate, ω, r are the immature and mature death rate, τ is the maturation time delay, and the term $\alpha e^{-\omega\tau} z(t - \tau)$ is the number of immature populations who were born at time $t - \tau$ and survive at time t , and this term represents the transformation from the immature to the mature stages. Note that the death number of mature

population is of a logistic nature, which is proportional to the square of the population with proportionality constant r .

Recently, some stage-structured models with time delays and impulsive diffusion were investigated in [7–11]. Jiao et al. [7, 9] and Shao et al. [8, 10] investigated a class of predator-prey models with prey-impulsive diffusion between two patches in which the predator is subject to stage-structured effects (delayed effects) only in one of the patches. Dhar and Jatav [11] consider a delayed stage-structured predator-prey model with impulsive diffusion, and both of the predators in two patches are subject to stage-structured effects. Many interesting results in the mentioned publications mainly focused on the global attractiveness of the predator-eradicated periodic solution and the permanence of the systems. To manage effectively the species, we should know or estimate how many members the population has at large time in every patch when a species is uniformly permanent. Mathematically, this corresponds to the ultimately low and upper boundedness of solutions of the systems. Although the permanence was derived by the estimate for the ultimately upper boundedness of solutions, there are some errors or negligence on the estimate in [7–11] (see Remarks 4 and 6 below). In addition, according to Aiello and Freedman [6], it may be more reasonable to take the square function of the mature population as the death numbers of the mature than to take the linear one in [7–11].

Motivated by the above discussion, we propose the following stage-structured predator-prey model with generalized functional response, the harvesting effort of the mature predator, and impulsive diffusion between two predators' territories:

$$\begin{aligned} x_1'(t) &= x_1(t) (a_1 - b_1 x_1(t)) - p_1(x_1(t)) z_1(t), \\ y_1'(t) &= \alpha_1 z_1(t) - \alpha_1 e^{-\omega_1 \tau_1} z_1(t - \tau_1) - \omega_1 y_1(t), \\ z_1'(t) &= \alpha_1 e^{-\omega_1 \tau_1} z_1(t - \tau_1) + \beta_1 p_1(x_1(t)) z_1(t) \\ &\quad - E_1 z_1(t) - r_1 z_1^2(t), \\ x_2'(t) &= x_2(t) (a_2 - b_2 x_2(t)) - p_2(x_2(t)) z_2(t), \\ y_2'(t) &= \alpha_2 z_2(t) - \alpha_2 e^{-\omega_2 \tau_2} z_2(t - \tau_2) - \omega_2 y_2(t), \\ z_2'(t) &= \alpha_2 e^{-\omega_2 \tau_2} z_2(t - \tau_2) + \beta_2 p_2(x_2(t)) z_2(t) \\ &\quad - E_2 z_2(t) - r_2 z_2^2(t), \\ t &\neq n\tau, \end{aligned}$$

$$\begin{aligned} \Delta x_1(t) &= d_1 (x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2 (x_1(t) - x_2(t)), \\ \Delta y_1(t) &= 0, \\ \Delta y_2(t) &= 0, \end{aligned}$$

$$\Delta z_1(t) = 0,$$

$$\Delta z_2(t) = 0,$$

$$t = n\tau, \quad n \in \mathbb{N}, \quad (3)$$

where for the i th patch $i = 1, 2$, x_i, y_i, z_i are prey population density and predator populations density of the immature and the mature at time t , respectively; α_i represents the growth rate from the immature predator to the mature one; ω_i is the death rate of the immature predator; r_i is the death rate of mature predator, which is of a logistic nature; E_i is the harvesting rate of the mature population; β_i is the rate of conversion of nutrients into the reproduction rate of the mature predator; τ_i represents a constant time to maturity; $\tau > 0$ is the period of impulsive diffusion; and $d_i \in (0, 1)$ is dispersal rate; the other parameters have the same biological meaning in system (1). In addition, $p_i(x_i(t))$ represents the functional response including Holling-type and Ivlev-type ones, which satisfies

$$\begin{aligned} p_i \in H \triangleq \{f \in C(R_+, R_+) \mid f(0) = 0, f'(s) > 0, f''(s) \\ \leq 0, s > 0, R_+ = [0, \infty)\}. \end{aligned} \quad (4)$$

Mathematically, the proposed system (3) generalizes some existing models. For example, (3) is the model in [7] when $p_1(\cdot) = 0$, $p_2(x_2) = \beta x_2$, $r_i = 0$, and y_1, z_1 are removed; (3) becomes the model in [11] when $r_i = 0$, $p_i(x_i) = c_i x_i$. In this paper, we will mainly investigate the existence and global attractiveness of periodic solution and the permanence for system (3) by employing the impulsive comparison theorem and the discrete dynamical system determined by the stroboscopic map. The obtained results will modify and improve the ones in some existing publications and give the estimate for the ultimately low and upper boundedness of the systems. Some examples and their simulations are given to illustrate the effectiveness of our results.

2. Preliminaries

Let $x(t) = (x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t))^T$ be the solution of (3) with the initial conditions

$$\begin{aligned} (\phi_1(\zeta), \phi_2(\zeta), \phi_3(\zeta), \phi_4(\zeta), \phi_5(\zeta), \phi_6(\zeta)) \\ \in C([- \tau, 0], \mathbb{R}_+^6), \quad \phi_i(0) > 0, \quad i = 1, 2, 3, 4, 5, 6. \end{aligned} \quad (5)$$

From the theory of impulsive differential equations and the biological meanings, we can obtain the global existence and uniqueness of solution $x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t) \geq 0$.

Note that the variables y_1, y_2 do not appear in the first, third, fifth, and sixth equations in (3). We will simplify

the model and need to restrict our attention to the following system:

$$\begin{aligned}
 x_1'(t) &= x_1(t)(a_1 - b_1 x_1(t)) - p_1(x_1(t)) z_1(t), \\
 z_1'(t) &= \alpha_1 e^{-\omega_1 \tau_1} z_1(t - \tau_1) + \beta_1 p_1(x_1(t)) z_1(t) \\
 &\quad - E_1 z_1(t) - r_1 z_1^2(t), \\
 x_2'(t) &= x_2(t)(a_2 - b_2 x_2(t)) - p_2(x_2(t)) z_2(t), \\
 z_2'(t) &= \alpha_2 e^{-\omega_2 \tau_2} z_2(t - \tau_2) + \beta_2 p_2(x_2(t)) z_2(t) \\
 &\quad - E_2 z_2(t) - r_2 z_2^2(t), \\
 t &\neq n\tau,
 \end{aligned} \tag{6}$$

$$\Delta x_1(t) = d_1(x_2(t) - x_1(t)),$$

$$\Delta x_2(t) = d_2(x_1(t) - x_2(t)),$$

$$\Delta z_1(t) = 0,$$

$$\Delta z_2(t) = 0,$$

$$t = n\tau, \quad n \in \mathbb{N} = \{1, 2, 3, \dots\}.$$

Suppose that $z_1(t) = 0, z_2(t) = 0$; the above subsystem becomes model (1). Integrating and solving the first two equations of (1) between impulsive moments, we have

$$\begin{aligned}
 x_i(t) &= \left[\frac{b_i}{a_i} + \left(\frac{1}{x_i(n\tau^+)} - \frac{b_i}{a_i} \right) e^{-a_i(t-n\tau)} \right]^{-1}, \\
 n\tau < t \leq (n+1)\tau, \quad i = 1, 2.
 \end{aligned} \tag{7}$$

Combining with the impulsive diffusion, we obtain the following stroboscopic map:

$$\begin{aligned}
 x_{1,(n+1)\tau} &= \frac{x_{1,n\tau}}{h_1 + c_1 x_{1,n\tau}} \\
 &\quad + d_1 \left(\frac{x_{2,n\tau}}{h_2 + c_2 x_{2,n\tau}} - \frac{x_{1,n\tau}}{h_1 + c_1 x_{1,n\tau}} \right), \\
 x_{2,(n+1)\tau} &= \frac{x_{2,n\tau}}{h_2 + c_2 x_{2,n\tau}} \\
 &\quad + d_2 \left(\frac{x_{1,n\tau}}{h_1 + c_1 x_{1,n\tau}} - \frac{x_{2,n\tau}}{h_2 + c_2 x_{2,n\tau}} \right).
 \end{aligned} \tag{8}$$

Here $x_{i,n\tau} = x_i(n\tau^+)$, $c_i = (b_i/a_i)(1 - e^{-a_i\tau}) > 0$, and $h_i = e^{-a_i\tau} < 1, i = 1, 2, n \in \mathbb{N}$.

We define a map $F: R_+^2 \rightarrow R_+^2$ such that

$$\begin{aligned}
 F_1(x) &= \frac{x_1}{h_1 + c_1 x_1} + d_1 \left(\frac{x_2}{h_2 + c_2 x_2} - \frac{x_1}{h_1 + c_1 x_1} \right), \\
 F_2(x) &= \frac{x_2}{h_2 + c_2 x_2} + d_2 \left(\frac{x_1}{h_1 + c_1 x_1} - \frac{x_2}{h_2 + c_2 x_2} \right).
 \end{aligned} \tag{9}$$

The set of all iterations of the map is equivalent to the set of all density sequences generated by system (8).

By using the theory of monotone dynamical systems in [12], Hui and Chen [5] gave the following lemma.

Lemma 1. *There exists a unique positive fixed point $q = (q_1, q_2)$ of map F in (9), $F^n(x) \rightarrow q$ as $n \rightarrow \infty$. Furthermore, all nontrivial trajectories of system (1) approach the positive periodic solution $(\tilde{x}_1(t), \tilde{x}_2(t))$ with period τ , where*

$$\begin{aligned}
 \tilde{x}_i(t) &= \left[\frac{b_i}{a_i} + \left(\frac{1}{q_i} - \frac{b_i}{a_i} \right) e^{-a_i(t-n\tau)} \right]^{-1}, \\
 n\tau < t \leq (n+1)\tau, \quad i = 1, 2.
 \end{aligned} \tag{10}$$

To obtain our results, we also need the following lemma.

Lemma 2 (see [13]). *Consider the following delay differential equation:*

$$x'(t) = ax(t - \tau) - bx(t) - cx^2(t), \tag{11}$$

where a, b, c , and τ are positive constants and $x(t) > 0$ for $t \in [-\tau, 0]$. One has the following:

(i) if $a > b$, then $\lim_{t \rightarrow +\infty} x(t) = (a - b)/c$;

(ii) if $a < b$, then $\lim_{t \rightarrow +\infty} x(t) = 0$.

3. Extinction of the Predator-Eradicated Periodic Solution

It is clear that system (6) has a predator-eradicated periodic solution $(\tilde{x}_1(t), 0, \tilde{x}_2(t), 0)$, where the positive periodic functions $\tilde{x}_1(t), \tilde{x}_2(t)$ with period τ are given in (10). In the following, we will show that it is globally attractive.

Theorem 3. *Assume that*

$$\begin{aligned}
 \alpha_i e^{-\omega_i \tau_i} + \beta_i p_i(X_i^*) &< E_i, \\
 X_i^* &:= \left[\frac{b_i}{a_i} + \left(\frac{1}{q_i} - \frac{b_i}{a_i} \right) e^{-a_i \Delta_i \tau} \right]^{-1}, \\
 i &= 1, 2,
 \end{aligned} \tag{12}$$

where $\Delta_i = |\min\{0, \operatorname{sgn}(q_i - a_i/b_i)\}|$, (q_1, q_2) is a unique positive fixed point of map F defined in (9) with $c_i = (b_i/a_i)(1 - e^{-a_i\tau})$ and $h_i = e^{-a_i\tau}$. Then the predator-eradicated periodic solution $(\tilde{x}_1(t), 0, \tilde{x}_2(t), 0)$ of (6) is globally attractive.

Proof. From (12), we can choose $\varepsilon_0 > 0$ sufficiently small such that

$$\begin{aligned}
 \alpha_1 e^{-\omega_1 \tau_1} + \beta_1 p_1(X_1^* + \varepsilon_0) &< E_1, \\
 \alpha_2 e^{-\omega_2 \tau_2} + \beta_2 p_2(X_2^* + \varepsilon_0) &< E_2.
 \end{aligned} \tag{13}$$

It follows from system (6) that

$$\begin{aligned} x_1'(t) &\leq x_1(t) (a_1 - b_1 x_1(t)), \\ x_2'(t) &\leq x_2(t) (a_2 - b_2 x_2(t)), \\ t &\neq n\tau, \\ \Delta x_1(t) &= d_1 (x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2 (x_1(t) - x_2(t)), \\ t &= n\tau, \quad n \in \mathbb{N}. \end{aligned} \quad (14)$$

By Lemma 1 and the comparison theorem of impulsive differential equation, we have $\limsup_{t \rightarrow \infty} x_i(t) \leq \tilde{x}_i(t)$, where $\tilde{x}_i(t)$ are defined in (10). So, for given $\varepsilon_0 > 0$, there exists an n_1 such that for $n \geq n_1$

$$\begin{aligned} x_1(t) &\leq \tilde{x}_1(t) + \varepsilon_0 \leq X_1^* + \varepsilon_0, \\ x_2(t) &\leq \tilde{x}_2(t) + \varepsilon_0 \leq X_2^* + \varepsilon_0, \\ n\tau < t \leq (n+1)\tau. \end{aligned} \quad (15)$$

From the second and fourth equations in system (6), we get

$$\begin{aligned} z_1'(t) &\leq \alpha_1 e^{-\omega_1 \tau_1} z_1(t - \tau_1) \\ &\quad - (E_1 - \beta_1 p_1(X_1^* + \varepsilon_0)) z_1(t) - r_1 z_1^2(t), \\ t &> n\tau + \tau_1, \quad n > n_1, \\ z_2'(t) &\leq \alpha_2 e^{-\omega_2 \tau_2} z_2(t - \tau_2) \\ &\quad - (E_2 - \beta_2 p_2(X_2^* + \varepsilon_0)) z_2(t) - r_2 z_2^2(t), \\ t &> n\tau + \tau_2, \quad n > n_1. \end{aligned} \quad (16)$$

By using the comparison theorem, it follows from Lemma 2 and the inequalities (13) that $\lim_{t \rightarrow \infty} z_1(t) = 0$ and $\lim_{t \rightarrow \infty} z_2(t) = 0$. Therefore, for any sufficiently small positive number ε_1 , there is an n_2 such that $z_1(t) < \varepsilon_1$, $z_2(t) < \varepsilon_1$ for all $t > n_2\tau$. Since $p_i \in H$ implies that $p_i(x_i(t)) \leq p_i'(0)x_i(t)$, from system (6) we have

$$\begin{aligned} x_1'(t) &\geq x_1(t) [(a_1 - p_1'(0)\varepsilon_1) - b_1 x_1(t)], \\ x_2'(t) &\geq x_2(t) [(a_2 - p_2'(0)\varepsilon_1) - b_2 x_2(t)], \\ t &\neq n\tau, \\ \Delta x_1(t) &= d_1 (x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2 (x_1(t) - x_2(t)), \\ t &= n\tau, \quad n \in \mathbb{N}. \end{aligned} \quad (17)$$

By using the comparison theorem and Lemma 1, we can obtain that $x_i(t) \geq s_i(t)$, $i = 1, 2$, where

$$\begin{aligned} s_i &= \left[\frac{b_i}{a_i - p_i'(0)\varepsilon_1} \right. \\ &\quad \left. + \left(\frac{1}{\bar{q}_i} - \frac{b_i}{a_i - p_i'(0)\varepsilon_1} \right) e^{-(a_i - p_i'(0)\varepsilon_1)\tau} \right]^{-1}, \end{aligned} \quad (18)$$

and $(\bar{q}_1, \bar{q}_2)$ is a unique positive fixed point of map F defined in (9) with $c_i = (b_i/(a_i - p_i'(0)\varepsilon_1))(1 - e^{-(a_i - p_i'(0)\varepsilon_1)\tau})$ and $h_i = e^{-(a_i - p_i'(0)\varepsilon_1)\tau}$. From (15) and (18), letting $\varepsilon_0, \varepsilon_1 \rightarrow 0$ and $t \rightarrow \infty$, then

$$\tilde{x}_i(t) \geq x_i(t) \geq s_i(t) \longrightarrow \tilde{x}_i(t), \quad i = 1, 2. \quad (19)$$

The proof is complete. $\square$

Remark 4. X_i^* is the maximum of the periodical function $\tilde{x}_i(t)$ defined in (10), which is dependent on the sign of the constant $1/\bar{q}_i - b_i/a_i$, $i = 1, 2$. However, the authors neglected the details when they derived the global attractivity of predator-eradicated periodic solution in [7, 8, 10, 11].

4. Permanence

Firstly, we can estimate the ultimately upper boundedness as follows.

Lemma 5. *If*

$$\beta_i p_i(X_i^*) < E_i < \alpha_i e^{-\omega_i \tau_i} + \beta_i p_i(X_i^*), \quad (20)$$

then $\limsup_{t \rightarrow \infty} x_i(t) \leq X_i^*$, $\limsup_{t \rightarrow \infty} z_i(t) \leq Z_i^*$, $i = 1, 2$, in system (6), where X_i^* is given in (12) and

$$Z_i^* = \frac{\alpha_i e^{-\omega_i \tau_i} - (E_i - \beta_i p_i(X_i^*))}{r_i}. \quad (21)$$

Proof. It follows from system (6) that

$$\begin{aligned} x_1'(t) &\leq x_1(t) (a_1 - b_1 x_1(t)), \\ x_2'(t) &\leq x_2(t) (a_2 - b_2 x_2(t)), \\ t &\neq n\tau, \\ \Delta x_1(t) &= d_1 (x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2 (x_1(t) - x_2(t)), \\ t &= n\tau, \quad n \in \mathbb{N}. \end{aligned} \quad (22)$$

By Lemma 1 and the comparison theorem of impulsive differential equation, we have $\limsup_{t \rightarrow \infty} x_i(t) \leq \tilde{x}_i(t) \leq X_i^*$. From the second and fourth equations in system (6),

$$\begin{aligned} z_1'(t) &\leq \alpha_1 e^{-\omega_1 \tau_1} z_1(t - \tau_1) \\ &\quad - (E_1 - \beta_1 p_1(X_1^* + \varepsilon_0)) z_1(t) - r_1 z_1^2(t), \\ t &> n\tau + \tau_1, \quad n > n_2, \\ z_2'(t) &\leq \alpha_2 e^{-\omega_2 \tau_2} z_2(t - \tau_2) \\ &\quad - (E_2 - \beta_2 p_2(X_2^* + \varepsilon_0)) z_2(t) - r_2 z_2^2(t), \\ t &> n\tau + \tau_2, \quad n > n_2. \end{aligned} \quad (23)$$

It follows from Lemma 2 and (20) that

$$\begin{aligned} \limsup_{t \rightarrow \infty} z_1(t) &\leq \frac{\alpha_1 e^{-\omega_1 \tau_1} - (E_1 - \beta_1 p_1(X_1^* + \varepsilon_0))}{r_1}, \\ \limsup_{t \rightarrow \infty} z_2(t) &\leq \frac{\alpha_2 e^{-\omega_2 \tau_2} - (E_2 - \beta_2 p_2(X_2^* + \varepsilon_0))}{r_2}. \end{aligned} \quad (24)$$

From the arbitrariness of ε_0 , we obtain the conclusion. The proof is complete. $\square$

Remark 6. In [7–11], the authors gave the estimate of the ultimately upper boundedness of solutions to derive the persistence of the system by constructing the V -functions. Nevertheless, there are errors on the estimate since the constructed V -functions do not satisfy $V(n\tau^+) \leq V(n\tau)$ when $t = n\tau, n = 1, 2, \dots$. In Lemma 5, we make a modification for the estimate.

Theorem 7. Let X_i^*, Z_i^* be given in Lemma 5 and

$$\begin{aligned} x_i^* &:= \left[\frac{b_i}{a_i - p_i'(0) Z_i^*} \right. \\ &\quad \left. + \left(\frac{1}{\bar{q}_i} - \frac{b_i}{a_i - p_i'(0) Z_i^*} \right) e^{-(a_i - p_i'(0) Z_i^*) \bar{\Delta}_i \tau} \right]^{-1}, \\ z_i^* &:= \frac{\alpha_i e^{-\omega_i \tau_i} - (E_i - \beta_i p_i(x_i^*))}{r_i}, \end{aligned} \quad (25)$$

where $\bar{\Delta}_i = |\max\{0, \text{sgn}(\bar{q}_i - (a_i - p_i'(0) Z_i^*)/b_i)\}|$; $(\bar{q}_1, \bar{q}_2)$ is a unique positive fixed point of map F defined in (9) with $c_i = b_i(1 - e^{-(a_i - p_i'(0) Z_i^*)})/(a_i - p_i'(0) Z_i^*)$ and $h_i = e^{-(a_i - p_i'(0) Z_i^*) \tau}$, $i = 1, 2$. If

$$\begin{aligned} a_i &> p_i'(0) Z_i^*, \\ \beta_i p_i(X_i^*) &< E_i < \alpha_i e^{-\omega_i \tau_i} + \beta_i p_i(x_i^*), \end{aligned} \quad (26)$$

$i = 1, 2,$

then system (6) is uniformly persistent in the following meaning:

$$\begin{aligned} x_i^* &\leq \liminf_{t \rightarrow \infty} x_i(t) \leq \limsup_{t \rightarrow \infty} x_i(t) \leq X_i^*, \\ z_i^* &\leq \liminf_{t \rightarrow \infty} z_i(t) \leq \limsup_{t \rightarrow \infty} z_i(t) \leq Z_i^*. \end{aligned} \quad (27)$$

Proof. We can see that (26) implies (20). Therefore, $\limsup_{t \rightarrow \infty} x_i(t) \leq X_i^*$, $\limsup_{t \rightarrow \infty} z_i(t) \leq Z_i^*$, $i = 1, 2$. Furthermore, we can choose $\varepsilon > 0$ sufficiently small such that

$$\begin{aligned} \alpha_1 e^{-\omega_1 \tau_1} &> E_1 - \beta_1 p_1(x_1^* - \varepsilon), \\ \alpha_2 e^{-\omega_2 \tau_2} &> E_2 - \beta_2 p_2(x_2^* - \varepsilon). \end{aligned} \quad (28)$$

Note that $p_i(x_i(t)) \leq p_i'(0)x_i(t)$ since $p_i \in H$. From system (6) and Theorem 7, there is an n_3 such that

$$\begin{aligned} x_1'(t) &\geq x_1(t) \left[(a_1 - p_1'(0) Z_1^*) - b_1 x_1(t) \right], \\ x_2'(t) &\geq x_2(t) \left[(a_2 - p_2'(0) Z_2^*) - b_2 x_2(t) \right], \\ t &\neq n\tau, \end{aligned} \quad (29)$$

$$\Delta x_1(t) = d_1(x_2(t) - x_1(t)),$$

$$\Delta x_2(t) = d_2(x_1(t) - x_2(t)),$$

$$t = n\tau, \quad n > n_3, \quad n \in \mathbb{N}.$$

By Lemma 1 and the comparison theorem, there exists an $n_4 > n_3$ such that

$$\begin{aligned} x_1(t) &\geq \bar{l}_1(t) - \varepsilon \geq x_1^* - \varepsilon, \\ x_2(t) &\geq \bar{l}_2(t) - \varepsilon \geq x_2^* - \varepsilon, \\ n\tau < t \leq (n+1)\tau, \quad n > n_4, \end{aligned} \quad (30)$$

where

$$\begin{aligned} \bar{l}_i &= \left[\frac{b_i}{a_i - p_i'(0) Z_i^*} \right. \\ &\quad \left. + \left(\frac{1}{\bar{q}_i} - \frac{b_i}{a_i - p_i'(0) Z_i^*} \right) e^{-(a_i - p_i'(0) Z_i^*)(t - n\tau)} \right]^{-1} \end{aligned} \quad (31)$$

and $\bar{q}_i$ can be confirmed homoplastically to q_i . Letting $\sigma_1 = x_1^* - \varepsilon$, $\sigma_2 = x_2^* - \varepsilon$, from the second and fourth equations in system (6), we have

$$\begin{aligned} z_1'(t) &\geq \alpha_1 e^{-\omega_1 \tau_1} z_1(t - \tau_1) - (E_1 - \beta_1 p_1(\sigma_1)) z_1(t) \\ &\quad - r_1 z_1^2(t), \quad t > n\tau + \tau_1, \quad n > n_4, \\ z_2'(t) &\geq \alpha_2 e^{-\omega_2 \tau_2} z_2(t - \tau_2) - (E_2 - \beta_2 p_2(\sigma_2)) z_2(t) \\ &\quad - r_2 z_2^2(t), \quad t > n\tau + \tau_2, \quad n > n_4. \end{aligned} \quad (32)$$

It follows from (28), Lemma 2, and comparison differential system that

$$\begin{aligned} \liminf_{t \rightarrow \infty} z_1(t) &\geq \frac{\alpha_1 e^{-\omega_1 \tau_1} - (E_1 - \beta_1 p_1(\sigma_1))}{r_1}, \\ \liminf_{t \rightarrow \infty} z_2(t) &\geq \frac{\alpha_2 e^{-\omega_2 \tau_2} - (E_2 - \beta_2 p_2(\sigma_2))}{r_2}. \end{aligned} \quad (33)$$

So there is an n_5 sufficiently large such that $z_1(t) \geq z_1^* > 0$, $z_2(t) \geq z_2^* > 0$ for all $t > n_5\tau$. Hence, by Theorem 7 and the above discussion, we get that system (6) is permanent. The proof is complete. $\square$

Remark 8. Theorem 7 derives the persistence of the system and estimates how many members the population has at large time in every patch. This is important to manage effectively the species, but the estimate may be conservative and further results need to be developed in this direction.

5. Examples and Their Simulations

In this section, we will give some examples and their simulations to illustrate the effectiveness of the obtained results.

Example 1. Consider system (6) with Holling II-type functional response $p_i(x_i) = \mu_i x_i / (1 + \nu_i x_i)$ and $\tau = 1$,

$$\begin{aligned}
 a_1 &= 1, \\
 b_1 &= 0.2, \\
 \alpha_1 &= 0.5, \\
 \beta_1 &= 0.6, \\
 r_1 &= 0.5, \\
 \omega_1 &= 0.5, \\
 \tau_1 &= 1, \\
 \mu_1 &= 2, \\
 \nu_1 &= 3, \\
 d_1 &= 0.4, \\
 E_1 &= 0.8, \\
 a_2 &= 0.8, \\
 b_2 &= 0.1, \\
 \alpha_2 &= 0.2, \\
 \beta_2 &= 0.5, \\
 r_2 &= 0.8, \\
 \omega_2 &= 1, \\
 \tau_2 &= 1, \\
 \mu_2 &= 1, \\
 \nu_2 &= 2, \\
 d_2 &= 0.3, \\
 E_2 &= 0.8.
 \end{aligned} \tag{34}$$

We can compute the unique positive fixed point $(q_1, q_2) = (6.1952, 6.8051)$ and

$$\begin{aligned}
 \alpha_1 e^{-\omega_1 \tau_1} + \beta_1 p_1(X_1^* + \varepsilon_0) - E_1 &= -0.1201 < 0, \\
 \alpha_2 e^{-\omega_2 \tau_2} + \beta_2 p_2(X_2^* + \varepsilon_0) - E_2 &= -0.4922 < 0.
 \end{aligned} \tag{35}$$

It follows from Theorem 3 that the predator-eradicated periodic solution $(\tilde{x}_1(t), 0, \tilde{x}_2(t), 0)$ of (6) is globally attractive. Figure 1 shows the asymptotical behaviors when taking the initial values $x_i(s) = 0.8, z_i(s) = 0.2, -1 \leq s \leq 0, i = 1, 2$.

Example 2. Consider system (6) with Holling II-type functional response $p_i(x_i) = \mu_i(1 - e^{-\nu_i x_i})$ and $\tau = 2$,

$$\begin{aligned}
 a_1 &= 1, \\
 b_1 &= 0.2, \\
 \alpha_1 &= 0.5, \\
 \beta_1 &= 0.6, \\
 r_1 &= 0.01, \\
 \omega_1 &= 1, \\
 \tau_1 &= 1, \\
 \mu_1 &= 0.5, \\
 \nu_1 &= 0.03, \\
 d_1 &= 0.4, \\
 E_1 &= 0.05, \\
 a_2 &= 1.2, \\
 b_2 &= 0.1, \\
 \alpha_2 &= 0.2, \\
 \beta_2 &= 0.5, \\
 r_2 &= 0.01, \\
 \omega_2 &= 1, \\
 \tau_2 &= 1, \\
 \mu_2 &= 0.4, \\
 \nu_2 &= 0.02, \\
 d_2 &= 0.3, \\
 E_2 &= 0.1.
 \end{aligned} \tag{36}$$

We can compute the positive fixed points $(q_1, q_2) = (7.8602, 9.8111)$, $(\tilde{q}_1, \tilde{q}_2) = (7.7286, 9.6796)$, and

$$\begin{aligned}
 0.0252 &= \beta_1 p_1(X_1^*) < E_1 = 0.05 \\
 &< \alpha_1 e^{-\omega_1 \tau_1} + \beta_1 p_1(x_1^*) = 0.0907, \\
 0.0417 &= \beta_2 p_2(X_2^*) < E_2 = 0.1 \\
 &< \alpha_2 e^{-\omega_2 \tau_2} + \beta_2 p_2(x_2^*) = 0.1088.
 \end{aligned} \tag{37}$$

It follows from Theorem 7 that system (6) is permanent, and we obtained the estimate of the ultimately low and upper boundedness of solutions as follows:

$$\begin{aligned}
 3.1273 = x_1^* &\leq \liminf_{t \rightarrow \infty} x_1(t) \leq \limsup_{t \rightarrow \infty} x_1(t) \leq X_1^* \\
 &= 7.8602,
 \end{aligned}$$

FIGURE 1: Time response in Example 1.

$$\begin{aligned}
 4.6796 &= x_2^* \leq \liminf_{t \rightarrow \infty} x_2(t) \leq \limsup_{t \rightarrow \infty} x_2(t) \leq X_2^* \\
 &= 11.7619, \\
 3.0685 &= z_1^* \leq \liminf_{t \rightarrow \infty} z_1(t) \leq \limsup_{t \rightarrow \infty} z_1(t) \leq Z_1^* \\
 &= 4.8784, \\
 0.8777 &= z_2^* \leq \liminf_{t \rightarrow \infty} z_2(t) \leq \limsup_{t \rightarrow \infty} z_2(t) \leq Z_2^* \\
 &= 1.5499.
 \end{aligned}$$

(38)

Figure 2 shows the permanence of the system when taking the initial values $x_1(s) = z_1(s) = 0.8$, $x_2(s) = z_2(s) = 0.2$, $-2 \leq s \leq 0$.

The following example and its simulation show that the conclusions in Theorems 3 and 7 may not hold when their conditions are not satisfied and point out some errors in existing publications.

Example 3. Consider system (6) with Holling I-type functional response $p_i(x_i) = \mu_i x_i$ and $\tau = 3$,

$$\begin{aligned}
 a_1 &= 1, \\
 b_1 &= 1, \\
 \alpha_1 &= 0,
 \end{aligned}$$

$$\begin{aligned}
 \beta_1 &= 0, \\
 r_1 &= 0, \\
 \omega_1 &= 0, \\
 \tau_1 &= 5, \\
 \mu_1 &= 0, \\
 d_1 &= 0.2, \\
 E_1 &= 0.8, \\
 a_2 &= 1, \\
 b_2 &= 1.6, \\
 \alpha_2 &= 0.2, \\
 \beta_2 &= 1, \\
 r_2 &= 0, \\
 \omega_2 &= 2, \\
 \tau_2 &= 10, \\
 \mu_2 &= 4, \\
 d_2 &= 0.9, \\
 E_2 &= 2.6,
 \end{aligned}$$

(39)

FIGURE 2: Time response in Example 2.

FIGURE 3: Time response in Example 3.

which becomes system (9) given in [7] when z_1 are removed. We can compute the unique positive fixed points $(q_1, q_2) = (0.9239, 0.9599)$, $(X_1^*, X_2^*) = (0.9959, 0.6360)$,

$$\begin{aligned}\alpha_1 e^{-\omega_1 \tau_1} + \beta_1 p_1(X_1^*) - E_1 &= -0.8 < 0, \\ \alpha_2 e^{-\omega_2 \tau_2} + \beta_2 p_2(X_2^*) - E_2 &= 1.2397 > 0.\end{aligned}\quad (40)$$

Clearly, those conditions in both Theorems 3 and 7 are not satisfied. So our results cannot verify the permanence and the global attractiveness of the predator-eradicated periodic solution for the system. In fact, Figure 3 shows that the system is not permanent and the predator-eradicated periodic solution is not globally attractive when taking the initial values $x_1(s) = 0.8$, $x_2(s) = 0.5$, $z_1(s) = z_2(s) = 0.1$, $-3 \leq s \leq 0$. However, according to Theorem 4.1 in [7], the predator-eradicated periodic solution is globally attractive for the above system without the variable z_1 because

$$\begin{aligned}\alpha_2 e^{-\omega_2 \tau_2} + \beta_2 \mu_2 \left[\frac{b_i}{a_i} + \left(\frac{1}{q_i} - \frac{b_i}{a_i} \right) e^{-a_i \tau} \right]^{-1} - E_2 \\ = -0.0558 < 0.\end{aligned}\quad (41)$$

This leads to the false conclusion from the simulation and Remark 4, and some similar errors can be found in [8, Theorem 3.1], [10, Theorem 3.1], and [11, Theorem 4.1].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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