Research Article

An Impulsively Controlled Three-Species Prey-Predator Model with Stage Structure and Birth Pulse for Predator

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We investigate the dynamic behaviors of a two-prey one-predator system with stage structure and birth pulse for predator. By using the Floquet theory of linear periodic impulsive equation and small amplitude perturbation method, we show that there exists a globally asymptotically stable two-prey eradication periodic solution when the impulsive period is less than some critical value. Further, we study the permanence of the investigated model. Our results provide valuable strategy for biological economics management. Numerical analysis is also inserted to illustrate the results.

1. Introduction

In the natural world, the predator-prey relationship is one of the important interactions among species, and it has been extensively studied by many authors [1–7] because of its universal existence. The following two-prey, one-predator model was studied by many works [8–10]:

$$\begin{aligned}\dot{x}_1 &= x_1(b_1 - x_1 - \alpha x_2 - r_1 y), \\ \dot{x}_2 &= x_2(b_2 - \beta x_1 - x_2 - r_2 y), \\ \dot{y} &= y_1(-b_3 + dr_1 x_1 + dr_2 x_2),\end{aligned}\quad (1)$$

where $x_i(t)$ ($i = 1, 2$) and $y(t)$ are densities of the prey population and the predator population, respectively. $r_i > 0$ ($i = 1, 2$) are coefficients of decrease of prey due to predation; $b_i > 0$ ($i = 1, 2, 3$) are intrinsic rates of increase or decrease. $\alpha > 0$, $\beta > 0$ are parameters representing competitive effects between two prey species; $d > 0$ is an equal transformation rate of the predator.

As we know, life history often occurs in natural ecological environments which has significant morphological and behavioral differences between immature and mature species; the dynamics of stage-structured prey-predator system has

been widely studied [11–13]. Recently, many impulsive differential equation models or hybrid dynamical systems have been proposed to model the introduction of a periodic IPM strategy [14, 15]; Xiang and Song [16] proposed impulsive prey-dependent consumption two-prey one-predator models with stage structure for the predator, which combined the biological control and chemical control:

$$\begin{aligned}\dot{x}_1(t) &= b_1 x_1(t)(1 - x_1(t)) - \alpha x_1(t)x_2(t) \\ &\quad - r_1 x_1(t)y_2(t),\end{aligned}$$

$$\begin{aligned}\dot{x}_2(t) &= b_2 x_2(t)(1 - x_2(t)) - \beta x_1(t)x_2(t) \\ &\quad - r_2 x_2(t)y_2(t),\end{aligned}$$

$$\begin{aligned}\dot{y}_1(t) &= \frac{\lambda_1 r_1 x_1(t)y_2(t)}{1 + h_1 r_1 x_1(t)} + \frac{\lambda_2 r_2 x_2(t)y_2(t)}{1 + h_2 r_2 x_2(t)} \\ &\quad - (m + \mu)y_1(t),\end{aligned}$$

$$\dot{y}_2(t) = m y_1(t) - \mu y_2(t),$$

$$t \neq nT,$$

$$\Delta x_1(t) = -E_1 x_1(t),$$

$$\Delta x_2(t) = -E_2 x_2(t),$$

$$\Delta y_1(t) = p_1,$$

$$\Delta y_2(t) = p_2,$$

$$t = nT. \quad (2)$$

The biological meanings of the parameters in (2) can be seen in [16]. As far as the population dynamics is concerned, most models often considered that the population reproduces throughout all year. However, many species give birth seasonally or in regular pulses. Particular examples of birth pulse type are $\Delta N = B(N)N$. The model discussed in [17] considered the birth pulse $\Delta N = B(N)N$, $B(N) = b - CN^\theta$. A birth pulse type $\Delta N = B(N)N$ with $B(N) = be^N$ and $B(N) = p/(q + N^n)$ are assumed in [18]. Comparatively, we consider birth pulse $\Delta N = N(a - bN)$.

In view of birth pulse and impulsive control strategy, we formulate the following two prey-predator models with stage-structure and birth pulse for predator:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)(b_1 - x_1(t)) - \alpha x_1(t)x_2(t) \\ &\quad - r_1 x_1(t)y_1(t) - r_2 x_1(t)y_2(t), \\ \dot{x}_2(t) &= x_2(t)(b_2 - x_2(t)) - \beta x_1(t)x_2(t) \\ &\quad - r_3 x_2(t)y_1(t) - r_4 x_2(t)y_2(t), \\ \dot{y}_1(t) &= \lambda_1 r_1 x_1(t)y_1(t) + \lambda_3 r_3 x_2(t)y_1(t) \\ &\quad - (m + \mu)y_1(t), \\ \dot{y}_2(t) &= \lambda_2 r_2 x_1(t)y_2(t) + \lambda_4 r_4 x_2(t)y_2(t) + my_1(t) \\ &\quad - \mu y_2(t), \\ t &\neq (n+l)T, \quad t \neq (n+1)T, \\ \Delta x_1(t) &= -p_1 x_1(t), \\ \Delta x_2(t) &= -p_2 x_2(t), \\ \Delta y_1(t) &= -p_3 y_1(t), \\ \Delta y_2(t) &= -p_4 y_2(t), \\ t &= (n+l)T, \quad n = 1, 2, \dots \\ \Delta x_1(t) &= 0, \\ \Delta x_2(t) &= 0, \\ \Delta y_1(t) &= y_2(t)(a - by_2(t)), \\ \Delta y_2(t) &= 0, \\ t &= (n+1)T, \quad n = 1, 2, \dots, \end{aligned} \quad (3)$$

where $x_i(t)$ ($i = 1, 2$) are densities of the prey population and $y_1(t)$ and $y_2(t)$ represent the densities of the immature and the mature predator population, respectively. $r_i > 0$ ($i = 1, 2, 3, 4$) are coefficients of decrease of prey due to predation, $\alpha > 0$, $\beta > 0$ are parameters representing competitive effects between two prey species, and $\lambda_i > 0$ ($i = 1, 2, 3, 4$) represent the rate of conversion of nutrients into the predators. The maturity is m , which determines the mean length of the juvenile period; μ is the natural death rate of the predator population. $\Delta y_1(t) = y_2(t)(a - by_2(t))$ is birth pulse at $t = (n+1)T$ as intrinsic rate of natural increase and density dependence rate of predator population are denoted by a , b , respectively. a/b is the carrying capacity of the predator population. $0 \leq l \leq 1$, $0 \leq p_i \leq 1$ ($i = 1, 2, 3, 4$) represent the fraction of prey and predators which die due to the pesticide at $t = (n+l)T$, $n \in \mathbb{Z}_+$.

The organization of this paper is as follows. In the next section, some important lemmas are presented. Sections 3 and 4 give the global asymptotical stability of the prey eradication periodic solution and permanence for system (3). Numerical analysis is displayed in Section 5. Finally, a brief discussion is given to conclude this work.

2. Preliminaries

In this section, we will give some definitions, notations, and some lemmas which will be useful for our main results.

Let $R_+ = [0, \infty)$, $R_+^4 = \{x \in R^4 : x > 0\}$. Thus $\Omega = \text{int } R_+^4$; let $\mathbb{Z}_+$ be the set of all nonnegative integers. Denote $f = (f_1, f_2, f_3, f_4)$, the map defined by the right hand side of the first four equations of system (3). Let $V : R_+ \times R_+^4 \mapsto R_+$; then V is said to belong to class $V \in V_0$ if

- (i) V is continuous in $(nT, (n+l)T] \times R_+^4$ and $((n+l)T, (n+1)T] \times R_+^4$ for each, $x \in R_+^4$, $n \in \mathbb{Z}_+$, $\lim_{(t,y) \rightarrow ((n+l)T^+, x)} V(t, y) = V((n+l)T^+, x)$ and $\lim_{(t,y) \rightarrow ((n+1)T^+, x)} V(t, y) = V((n+1)T^+, x)$ existing;

- (ii) V is locally Lipschitzian in x .

Definition 1. Consider $V \in V_0$; then, for $(t, x) \in (nT, (n+l)T] \times R_+^4$ and $(t, x) \in ((n+l)T, (n+1)T] \times R_+^4$, the upper right derivative of $V(t, x)$ with respect to the impulsive differential system (3) is defined as $D^+V(t, x) = \lim_{h \rightarrow 0^+} \sup (1/h)[V(t+h, x+hf(t, x)) - V(t, x)]$.

The solution of system (3), denoted by $x(t) = (x_1(t), x_2(t), y_1(t), y_2(t)) : R_+ \mapsto R_+^4$, is continuously differentiable on $(nT, (n+l)T]$ and $((n+l)T, (n+1)T]$, $n \in \mathbb{Z}_+$, $0 \leq l \leq 1$.

Obviously the smoothness properties of f guarantee the global existence and uniqueness of solutions of system (3); for details see [19]. The following lemma is obvious.

Lemma 2. Suppose $x(t)$ is a solution of (3) with $x(0^+) \geq 0$; then $x(t) \geq 0$ for all $t \geq 0$. And further $x(t) > 0$, $t \geq 0$, if $x(0^+) > 0$.

Make a notation as

$$p_4^* = \frac{[e^{-mT}(1-p_3) + p_3(e^{-mT} - e^{-mlT})a] - e^{\mu T} + 1 + a(1 - e^{-mT}) - (1-p_3)e^{-(m+\mu)T}}{1 + a(1 - e^{-mT}) - (1-p_3)e^{-(m+\mu)T}}. \quad (4)$$

If $x_1(t) = 0$, $x_2(t) = 0$, then system (3) can be rewritten to

$$\dot{y}_1(t) = -(m+\mu)y_1,$$

$$\dot{y}_2(t) = my_1(t) - \mu y_2(t),$$

$$t \neq (n+l)T, t \neq (n+1)T,$$

$$\Delta y_1(t) = -p_3 y_1(t),$$

$$\Delta y_2(t) = -p_4 y_2(t),$$

$$t = (n+l)T,$$

$$\Delta y_1(t) = y_2(t)(a - by_2(t)),$$

$$\Delta y_2(t) = 0,$$

$$t = (n+1)T.$$

(5)

It is easy to obtain the analytic solution of system (5) between pulses:

$$y_1(t) = \begin{cases} y_1(nT^+)e^{-(m+\mu)(t-nT)}, & t \in (nT, (n+l)T], \\ y_1((n+l)T^+)e^{-(m+\mu)(t-(n+l)T)}, & t \in ((n+l)T, (n+1)T], \end{cases} \quad (6)$$

$$y_2(t) = \begin{cases} e^{-\mu(t-nT)}[y_2(nT^+) + y_1(nT^+)(1 - e^{-m(t-nT)})], & t \in (nT, (n+l)T], \\ e^{-\mu(t-(n+l)T)}[y_2((n+l)T^+) + y_1((n+l)T^+)(1 - e^{-m(t-(n+l)T})], & t \in ((n+l)T, (n+1)T]. \end{cases}$$

Considering (5), we have the stroboscopic map of system

$$\begin{aligned} y_1((n+1)T^+) &= ae^{-\mu T}(1-p_4)y_2(nT^+) \\ &+ \{a[(1-p_4)e^{-\mu T}(1-e^{-mlT}) \\ &+ (1-p_3)e^{-(\mu+ml)T}(1-e^{(1-l)mT})] + (1-p_3) \\ &\cdot e^{-(m+\mu)T}\} y_1(nT^+) - b\{e^{-\mu T}(1-p_4)y_2(nT^+) \\ &+ [(1-p_4)e^{-\mu T}(1-e^{-mlT}) \\ &+ (1-p_3)e^{-(\mu+ml)T}(1-e^{(1-l)mT})] y_1(nT^+)\}, \end{aligned} \quad (7)$$

$$\begin{aligned} y_2((n+1)T^+) &= e^{-\mu T}(1-p_4)y_2(nT^+) + [(1-p_4) \\ &\cdot e^{-\mu T}(1-e^{-mlT}) + (1-p_3)e^{-(\mu+ml)T}(1 \\ &- e^{(1-l)mT})] y_1(nT^+). \end{aligned}$$

For convenience, we choose $A = e^{-\mu T}(1-p_4) > 0$, $B = (1-p_3)e^{-(\mu+ml)T} > 0$, and $C = (1-p_4)e^{-\mu T}(1-e^{-mlT}) + (1-p_3)e^{-(\mu+ml)T}(1-e^{(1-l)mT}) > 0$; the following two equivalence relations are calculated to be

$$\begin{aligned} p_4 > p_4^* &\iff B + aC - 1 - AB + A < 0, \\ p_4 < p_4^* &\iff B + aC - 1 - AB + A > 0. \end{aligned} \quad (8)$$

The two fixed points of (5) are obtained as $G_0(0,0)$ and $G(y_1^*, y_2^*)$, where

$$\begin{aligned} y_1^* &= \frac{(B + aC - 1 - AB + A)(1-A)}{bC^2}, \quad p_4 < p_4^*, \\ y_2^* &= \frac{(B + aC - 1 - AB + A)}{bC}, \quad p_4 < p_4^*. \end{aligned} \quad (9)$$

Lemma 3. (i) If $p_4 > p_4^*$, the fixed point $G_0(0,0)$ is globally asymptotically stable.

(ii) If $p_4 < p_4^*$, the fixed point $G(y_1^*, y_2^*)$ is globally asymptotically stable.

Proof. For convenience, we make a notation as $(y_1^n, y_2^n) = (y_1^*, y_2^*)$. Linear form (5) can be written as

$$\begin{pmatrix} y_1^{n+1} \\ y_2^{n+1} \end{pmatrix} = J \begin{pmatrix} y_1^n \\ y_2^n \end{pmatrix}. \quad (10)$$

Obviously, the near dynamics of $G_0(0,0)$ and $G(y_1^*, y_2^*)$ are determined by linear system (10). The stables of $G_0(0,0)$ and $G(y_1^*, y_2^*)$ are determined by the eigenvalue of J less than 1. If J satisfies the Jury criteria [20], we can know that the eigenvalue of J is less than 1:

$$1 - \text{tr} J + \det J > 0, \quad (11)$$

where

$$J = \begin{pmatrix} B + aC - 2bC^2y_1^* - 2bACy_2^* & aA - 2bA^2y_2^* - 2bACy_1^* \\ C & A \end{pmatrix}. \quad (12)$$

(i) If $p_4 > p_4^*$, then $B + aC - 1 - AB + A < 0$, $G_0(0, 0)$ is the unique fixed point of system (5); we have

$$J_1 = J|_{(0,0)} \begin{pmatrix} B + aC & aA \\ C & A \end{pmatrix}. \quad (13)$$

Calculating $1 - \text{tr}J_1 + \det J_1 = 1 - A - B + AB - aC > 0$, and from the Jury criteria $G_0(0, 0)$ is locally stable, and then it is globally asymptotically stable.

(ii) If $p_4 < p_4^*$, then $B + aC - 1 - AB + A > 0$, $G_0(0, 0)$ is unstable. For $B + aC - 1 - AB + A > 0$, $G(y_1^*, y_2^*)$ exists, we have

$$J_2 = J|_{(y_1^*, y_2^*)} \begin{pmatrix} B + aC - 2bC(Cy_1^* + Ay_2^*) & aA - 2bA(Cy_1^* + Ay_2^*) \\ C & A \end{pmatrix}. \quad (14)$$

Consider $1 - \text{tr}J + \det J = 1 - A - B + AB - aC > 0$. From the Jury criteria $G(y_1^*, y_2^*)$ is locally stable, and then it is globally asymptotically stable. The proof is completed. $\square$

Summarizing the above results, we have the following theorems.

Theorem 4. *The triviality periodic solution $G_0(0, 0)$ is stable if $p_4 > p_4^*$; otherwise, if $p_4 < p_4^*$, the triviality periodic solution $G_0(0, 0)$ is unstable, and the periodic solution $G(y_1^*, y_2^*)$ is globally asymptotically stable.*

Correspondingly, system (5) has a globally asymptotically stable positive periodic solution $(\tilde{y}_1(t), \tilde{y}_2(t))$, where

$$\begin{aligned} \tilde{y}_1(t) &= \begin{cases} y_1^* e^{-(m+\mu)(t-nT)}, & t \in (nT, (n+1)T], \\ y_1^* e^{-(m+\mu)lT} e^{-(m+\mu)(t-(n+1)T)}, & t \in ((n+1)T, (n+2)T], \end{cases} \\ \tilde{y}_2(t) &= \begin{cases} e^{-\mu(t-nT)} [y_2^* + y_1^* (1 - e^{-m(t-nT)})], & t \in (nT, (n+1)T], \\ e^{-\mu(t-(n+1)T)} [e^{-\mu lT} [y_2^* + y_1^* (1 - e^{-mlT})] + y_1^* e^{-(m+\mu)lT} (1 - e^{-m(t-(n+1)T})], & t \in ((n+1)T, (n+2)T]. \end{cases} \end{aligned} \quad (15)$$

Lemma 5. *There exists a constant $M > 0$ such that $x_i(t) \leq M$, $y_i(t) \leq M$ ($i = 1, 2$), for each positive solution $x(t) = (x_1(t), x_2(t), y_1(t), y_2(t))$ of (5) with all t large enough.*

Proof. Define a function $V(t, x)$ such that $V(t, x) = \sum_{i=1}^2 (\lambda x_i(t) + y_i(t))$, where $\lambda = \max\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$. When $t \neq (n+1)T$, $t \neq (n+2)T$, we have

$$\begin{aligned} D^+V(t, x) + \mu V(t, x) &\leq -b_1 \left(x_1 - \frac{b_1 + \mu}{2b_1} \right)^2 + \frac{(b_1 + \mu)^2}{4b_1} \\ &\quad - b_2 \left(x_1 - \frac{b_2 + \mu}{2b_2} \right)^2 + \frac{(b_2 + \mu)^2}{4b_2} \\ &\leq \frac{(b_1 + \mu)^2}{4b_1} + \frac{(b_2 + \mu)^2}{4b_2} \triangleq M_1. \end{aligned} \quad (16)$$

When $t = (n+1)T$,

$$\begin{aligned} V((n+1)T^+) &= \lambda x_1((n+1)T^+) + \lambda x_2((n+1)T^+) \\ &\quad + y_1((n+1)T^+) + y_2((n+1)T^+) \\ &\leq V((n+1)T), \end{aligned} \quad (17)$$

and when $t = (n+1)T$,

$$\begin{aligned} V((n+1)T^+) &= \lambda x_1((n+1)T^+) + \lambda x_2((n+1)T^+) \\ &\quad + y_1((n+1)T^+) + y_2((n+1)T^+) \\ &\leq V((n+1)T) + \frac{a^2}{4b}. \end{aligned} \quad (18)$$

It follows from the comparison theorem of impulsive differential equations (see Lemma 2.2 [21], page 23) that, for $t \in (nT, (n+1)T)$, we have

$$\begin{aligned} V(t, x) &\leq V(0^+) e^{-\mu t} + \frac{M_1}{\mu} (1 - e^{-\mu t}) + \frac{a^2}{4b} \frac{e^{-\mu(t-T)}}{1 - e^{-\mu T}} \\ &\quad + \frac{a^2}{4b} \frac{e^{\mu T}}{e^{\mu T} - 1} \longrightarrow \frac{M_1}{\mu} + \frac{a^2}{4b} \frac{e^{\mu T}}{e^{\mu T} - 1}, \end{aligned} \quad (19)$$

$t \longrightarrow \infty$.

So $V(t, x)$ is ultimately bounded. Hence, by the definition of $V(t, x)$, there exists a constant $M > 0$, such that $x_i(t) \leq M$, $y_i(t) \leq M$, $i = 1, 2$, for t large enough. The proof is complete. $\square$

Definition 6. System (3) is said to be permanent if there are constants $m, M > 0$ and a finite time T_0 such that all solutions

$x_i(t)$, $y_i(t)$ with all initial values $x_i(0^+) > 0$, $y_i(0^+) > 0$, $m \leq x_i(t) \leq M$, and $m \leq y_i(t) \leq M$ hold for all $t \geq T_0$.

3. Extinction

In this section, we investigate the stability of the two-pest prey eradication periodic solution as a solution of system (3). We give the condition which assures the asymptotical stability of the prey eradication periodic solution $(0, 0, \tilde{y}_1(t), \tilde{y}_2(t))$.

Theorem 7. Let $(x_1(t), x_2(t), y_1(t), y_2(t))$ be any solution of (3); then $(0, 0, \tilde{y}_1(t), \tilde{y}_2(t))$ is globally asymptotically stable if

$$p_4 < p_4^*,$$

$$T < \min \left\{ \frac{\ln(1/(1-p_1)) + r_1 y_1^*/(m+\mu) + r_2 (y_1^* + y_2^*)/\mu}{b_1}, \frac{\ln(1/(1-p_2)) + r_3 y_1^*/(m+\mu) + r_4 (y_1^* + y_2^*)/\mu}{b_2} \right\}, \quad (20)$$

hold true.

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} b_1 - r_1 \tilde{y}_1(t) - r_2 \tilde{y}_2(t) & 0 & 0 & 0 \\ 0 & b_2 - r_3 \tilde{y}_1(t) - r_4 \tilde{y}_2(t) & 0 & 0 \\ \lambda_1 r_1 \tilde{y}_1(t) & \lambda_3 r_3 \tilde{y}_1(t) & -(m+\mu) & 0 \\ \lambda_2 r_2 \tilde{y}_2(t) & \lambda_4 r_4 \tilde{y}_2(t) & m & -\mu \end{pmatrix} \Phi(t), \quad (23)$$

with $\Phi(0) = I$, where I is the identity matrix.

$$\Phi(t) = \begin{pmatrix} \exp\left(\int_0^t H_1 ds\right) & 0 & 0 & 0 \\ * & \exp\left(\int_0^t H_2 ds\right) & 0 & 0 \\ * & * & \exp(-(m+\mu)t) & 0 \\ * & * & \exp(-\mu t) - \exp(-(\mu+m)t) & \exp(-\mu t) \end{pmatrix}, \quad (24)$$

where $H_1 = b_1 - r_1 \tilde{y}_1(s) - r_2 \tilde{y}_2(s)$ and $H_2 = b_2 - r_3 \tilde{y}_1(s) - r_4 \tilde{y}_2(s)$. There is no need to calculate the exact form of (*) as it is not required in the analysis that follows. The resetting impulsive conditions of (3) become

$$\begin{pmatrix} u_1((n+l)T^+) \\ u_2((n+l)T^+) \\ v_1((n+l)T^+) \\ v_2((n+l)T^+) \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 & 0 \\ 0 & 1-p_2 & 0 & 0 \\ 0 & 0 & 1-p_3 & 0 \\ 0 & 0 & 0 & 1-p_4 \end{pmatrix} \begin{pmatrix} u_1((n+l)T) \\ u_2((n+l)T) \\ v_1((n+l)T) \\ v_2((n+l)T) \end{pmatrix}, \quad (25)$$

Proof. Firstly, we prove the local stability of T -period solution $(0, 0, \tilde{y}_1(t), \tilde{y}_2(t))$ which may be determined by considering the behavior of small-amplitude perturbations $(u_1(t), u_2(t), v_1(t), v_2(t))$ of the solution.

Define

$$\begin{aligned} x_1(t) &= u_1(t), \\ x_2(t) &= u_2(t), \\ y_1(t) &= v_1(t) + \tilde{y}_1(t), \\ y_2(t) &= v_2(t) + \tilde{y}_2(t), \end{aligned} \quad (21)$$

where $u_1(t)$, $u_2(t)$, $v_1(t)$, and $v_2(t)$ are small perturbations; they may be written as

$$\begin{pmatrix} u_1(t) \\ u_2(t) \\ v_1(t) \\ v_2(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u_1(0) \\ u_2(0) \\ v_1(0) \\ v_2(0) \end{pmatrix}, \quad (22)$$

where $\Phi(t)$ satisfy

Hence the fundamental solution matrix is

$$\begin{pmatrix} u_1((n+1)T^+) \\ u_2((n+1)T^+) \\ v_1((n+1)T^+) \\ v_2((n+1)T^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1((n+1)T^+) \\ u_2((n+1)T^+) \\ v_1((n+1)T^+) \\ v_2((n+1)T^+) \end{pmatrix}. \quad (26)$$

Hence, the stability of the periodic solution $(0, 0, \tilde{y}_1(t), \tilde{y}_2(t))$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1-p_1 & 0 & 0 & 0 \\ 0 & 1-p_2 & 0 & 0 \\ 0 & 0 & 1-p_3 & 0 \\ 0 & 0 & 0 & 1-p_4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{pmatrix} \Phi(T) \quad (27)$$

which are

$$\begin{aligned}
 \lambda_1 &= (1 - p_1) \exp \left(\int_0^T (b_1 - r_1 \tilde{y}_1(s) - r_2 \tilde{y}_2(s)) ds \right), \\
 \lambda_2 &= (1 - p_2) \exp \left(\int_0^T (b_2 - r_3 \tilde{y}_1(s) - r_4 \tilde{y}_2(s)) ds \right), \\
 \lambda_3 &= (1 - p_3) \left[e^{-(\mu+m)T} + a e^{-\mu T} (1 - e^{-mT}) \right] < 1, \\
 \lambda_4 &= (1 - p_4) e^{-\mu T} < 1, \\
 \lambda_1 &< 1 \text{ if } T \\
 &< \frac{\ln(1/(1-p_1)) + r_1 y_1^*/(m+\mu) + r_2(y_1^* + y_2^*)/\mu}{b_1}, \\
 \lambda_2 &< 1 \text{ if } T \\
 &< \frac{\ln(1/(1-p_2)) + r_3 y_1^*/(m+\mu) + r_4(y_1^* + y_2^*)/\mu}{b_2}.
 \end{aligned} \tag{28}$$

According to the Floquet theory (see [21]), the two-pest eradication solution $(0, 0, \tilde{y}_1(t), \tilde{y}_2(t))$ is locally asymptotically stable.

In the following, we prove the global attractivity. Choose $\varepsilon_1, \varepsilon_2 > 0$ such that

$$\begin{aligned}
 \eta &= (1 - p_3) \\
 &\cdot \left(\exp \left(\int_0^T (b_1 - r_1 (\tilde{y}_1(t) - \varepsilon_1) - r_2 (\tilde{y}_2(t) - \varepsilon_2)) ds \right) \right) \\
 &< 1.
 \end{aligned} \tag{29}$$

Noting that $\dot{y}_1(t) \geq -(m+\mu)y_1(t)$, $\dot{y}_2(t) \geq my_1(t) - \mu y_2(t)$, consider the following impulsive differential equation:

$$\begin{aligned}
 \dot{z}_1(t) &= -(m+\mu)z_1(t), \\
 \dot{z}_2(t) &= mz_1(t) - \mu z_2(t), \\
 &t \neq (n+l)T, \quad t \neq (n+1)T, \\
 \Delta z_1(t) &= -p_3 z_1(t), \\
 \Delta z_2(t) &= -p_4 z_2(t), \\
 &t = (n+l)T, \\
 \Delta z_1(t) &= z_2(t)(a - bz_2(t)), \\
 \Delta z_2(t) &= 0, \\
 &t = (n+1)T.
 \end{aligned} \tag{30}$$

It follows from impulsive differential equation (30) that $y_1(t) \geq z_1(t)$, $y_2(t) \geq z_2(t)$ and $z_1(t) \rightarrow \tilde{y}_1(t)$, $z_2(t) \rightarrow \tilde{y}_2(t)$ as $t \rightarrow \infty$. This is

$$\begin{aligned}
 y_1(t) &\geq z_1(t) \geq \tilde{y}_1(t) - \varepsilon_1, \\
 y_2(t) &\geq z_2(t) \geq \tilde{y}_2(t) - \varepsilon_2,
 \end{aligned} \tag{31}$$

for all t large enough. For convenience, we may assume that (31) holds. From (3) and (31) we have

$$\begin{aligned}
 \dot{x}_1(t) &\leq x_1(t)(b_1 - r_1(\tilde{y}_1(t) - \varepsilon_1) - r_2(\tilde{y}_2(t) - \varepsilon_2)), \\
 &t \neq (n+l)T, \tag{32}
 \end{aligned}$$

$$x_1(t^+) = (1 - p_1)x_1(t), \quad t = (n+l)T,$$

holding for all t large enough. Integrate (32) on $((n+l)T, (n+1)T]$, which yields $x_1((n+l)T) \leq x_1(nT^+)(1 - p_1) \exp(\int_{nT}^{(n+l)T} (b_1 - r_1(\tilde{y}_1(s) - \varepsilon_1) - r_2(\tilde{y}_2(s) - \varepsilon_2)) ds)$. Thus $x_1((n+l)T) \leq x_1(lT^+)\eta^n$ and $x_1((n+l)T) \rightarrow 0$ as $t \rightarrow \infty$. Therefore $x_1(t) \rightarrow 0$ as $t \rightarrow \infty$, since $0 < x_1(t) < x_1((n+l)T)(1 - p_1) \exp(b_1 T)$ for $(n+l)T \leq t \leq (n+1)T$. By the same method we can prove $x_2(t) \rightarrow 0$, as $t \rightarrow \infty$, so we omit it.

Next, we will prove that $y_1(t) \rightarrow \tilde{y}_1(t)$, $y_2(t) \rightarrow \tilde{y}_2(t)$, as $t \rightarrow \infty$. If $\lim_{t \rightarrow \infty} x_1(t) = 0$ and $\lim_{t \rightarrow \infty} x_2(t) = 0$, for $0 < \varepsilon \leq (m+\mu)/(\lambda_1 r_1 + \lambda_3 r_3)$, there exists $t_0 > 0$ such that $0 < x_1(t) < \varepsilon_1$ and $0 < x_2(t) < \varepsilon$, for all $t > t_0$; then we have

$$\begin{aligned}
 -(m+\mu)y_1(t) &\leq \dot{y}_1(t) \\
 &\leq -[(m+\mu) - \lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon] y_1(t), \\
 my_1(t) - \mu y_2(t) &\leq \dot{y}_2(t) \\
 &\leq my_1(t) - (\mu - \lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon) y_2(t).
 \end{aligned} \tag{33}$$

From the left hand inequality of (33), it follows from impulsive differential equation (5) that $y_1(t) \rightarrow \tilde{y}_1(t)$ and $y_2(t) \rightarrow \tilde{y}_2(t)$ as $t \rightarrow \infty$. For the right hand inequality, we consider the following impulsive differential equation:

$$\begin{aligned}
 \dot{z}_3(t) &= -[(m+\mu) - \lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon] z_3(t), \\
 \dot{z}_4(t) &= mz_3(t) - (\mu - \lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon) z_4(t), \\
 &t \neq (n+l)T, \quad t \neq (n+1)T, \\
 \Delta z_3(t) &= -p_3 z_3(t), \\
 \Delta z_4(t) &= -p_4 z_4(t), \\
 &t = (n+l)T, \\
 \Delta z_3(t) &= z_4(t)(a - bz_4(t)), \\
 \Delta z_4(t) &= 0, \\
 &t = (n+1)T.
 \end{aligned} \tag{34}$$

System (34) has a globally asymptotically stable positive periodic solution $(\tilde{z}_3(t), \tilde{z}_4(t))$, where

$$\begin{aligned}\bar{z}_3(t) &= \begin{cases} z_3^* e^{-(m+\mu-\lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon)(t-nT)}, & t \in (nT, (n+l)T], \\ z_3^* e^{-(m+\mu-\lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon)lT} e^{-(m+\mu-\lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon)(t-(n+l)T)}, & t \in ((n+l)T, (n+1)T], \end{cases} \\ \bar{z}_4(t) &= \begin{cases} e^{-(\mu-\lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon)(t-nT)} [z_4^* + z_3^* (1 - e^{-m(t-nT)})], & t \in (nT, (n+l)T], \\ e^{-(\mu-\lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon)(t-(n+l)T)} \{e^{-(\mu-\lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon)lT} [z_4^* + z_3^* (1 - e^{-mlT})] + z_3^* e^{-(m+\mu-\lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon)lT} (1 - e^{-m(t-(n+l)T)})\}, & t \in ((n+l)T, (n+1)T], \end{cases}\end{aligned}\quad (35)$$

where

$$z_4^* = \frac{(B_1 + aC_1 - 1 - A_1 B_1 + A_1)}{bC_1}, \quad p_4 < p_4^*, \quad (36)$$

$$z_3^* = \frac{(B_1 + aC_1 - 1 - A_1 B_1 + A_1)(1 - A_1)}{bC_1^2},$$

$$p_4 < p_4^*,$$

$$\text{and } A_1 = e^{-(\mu-\lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon)T} (1 - p_4) > 0, \quad B_1 = (1 - p_3) e^{-(m+\mu-\lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon)T} > 0, \quad \text{and } C_1 = (1 - p_4) e^{-(\mu-\lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon)T} (1 - e^{-mlT}) + (1 - p_3) e^{-(m+\mu-\lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon)T} (1 - e^{-(1-l)mT}) > 0.$$

Set

$$p_4^* = \frac{e^{-\theta_1 T} e^{\theta_2 T} (1 - p_3) + (1 - \mu_1) (1 - e^{-mT - mlT}) a - e^{\theta_2 T} + 1 + a (1 - e^{-mT}) - (1 - p_3) e^{-\theta_1 T}}{1 + a (1 - e^{-mT}) - (1 - p_3) e^{-\theta_1 T}}, \quad (37)$$

where $\theta_1 = m + \mu - \lambda_1 r_1 \varepsilon - \lambda_3 r_3 \varepsilon$ and $\theta_2 = \mu - \lambda_2 r_2 \varepsilon - \lambda_4 r_4 \varepsilon$. Therefore, there exists $\varepsilon_1 > 0$ such that

$$\bar{z}_1(t) - \varepsilon_1 < y_1(t) < \bar{z}_3(t) + \varepsilon_1 \quad (38)$$

for t large enough; let $\varepsilon \rightarrow 0$; then we get $\bar{z}_3(t) \rightarrow \bar{y}_1(t)$, $\bar{z}_1(t) \rightarrow \bar{y}_1(t)$. Hence $y_1(t) \rightarrow \bar{y}_1(t)$ as $t \rightarrow \infty$. Similarly, we can prove $y_2(t) \rightarrow \bar{y}_2(t)$ as $t \rightarrow \infty$, so we omit it. This proof is complete. $\square$

4. Permanence

In this section, we will investigate the permanence of system (3). In biological terms, the permanence implies that preys and predators will coexist, none of them facing extinction or growing indefinitely.

Theorem 8. *System (3) is permanent if $p_4 < p_4^*$, $b_1 > \alpha b_2$, and $b_2 > \beta b_1$,*

$$\begin{aligned}& - \frac{r_1 y_1^* e^{-(m+\mu-r_3 \lambda_3 b_2)lT} (e^{-(m+\mu-r_3 \lambda_3 b_2)T} - 1)}{-(m + \mu - r_3 \lambda_3 b_2)} - \frac{r_2 y_1^* e^{-(m+\mu-r_3 \lambda_3 b_2)lT} (e^{-(\mu-r_4 \lambda_4 b_2+m)T} - 1)}{-(\mu - r_4 \lambda_4 b_2) - m} - \ln \frac{1}{1 - p_1} \\& - \frac{r_2 (e^{-(\mu-r_4 \lambda_4 b_2)T} - 1) \{e^{-(\mu-r_4 \lambda_4 b_2)lT} [y_1^* (1 - e^{-mlT}) + y_2^*] + y_1^* e^{-(m+\mu-r_3 \lambda_3 b_2)lT}\}}{-(\mu - r_4 \lambda_4 b_2)} + (b_1 - \alpha b_2) T > 0, \\& - \frac{r_3 y_1^* e^{-(m+\mu-r_3 \lambda_3 b_2)lT} (e^{-(m+\mu-r_3 \lambda_3 b_2)T} - 1)}{-(m + \mu - r_3 \lambda_3 b_2)} - \frac{r_4 y_1^* e^{-(m+\mu-r_3 \lambda_3 b_2)lT} (e^{-(\mu-r_4 \lambda_4 b_2+m)T} - 1)}{-(\mu - r_4 \lambda_4 b_2) - m} - \ln \frac{1}{1 - p_2} \\& - \frac{r_4 (e^{-(\mu-r_4 \lambda_4 b_2)T} - 1) \{e^{-(\mu-r_4 \lambda_4 b_2)lT} [y_1^* (1 - e^{-mlT}) + y_2^*] + y_1^* e^{-(m+\mu-r_3 \lambda_3 b_2)lT}\}}{-(\mu - r_4 \lambda_4 b_2)} + (b_2 - \beta b_1) T > 0.\end{aligned}\quad (39)$$

Proof. Let us suppose a solution $x(t) = (x_1(t), x_2(t), y_1(t), y_2(t))$ of (3) with initial value $x(t) > 0$. From Lemma 5, we have proved that there exists a constant $M > 0$ such that $x_i(t) \leq M$, $y_i(t) \leq M$ ($i = 1, 2$), and $M > \max\{b_1/(r_1 + r_2), b_2/(r_3 + r_4)\}$ for $t > 0$. Note that

$$\begin{aligned} \dot{x}_1(t) &\leq x_1(t)(b_1 - x_1(t)), \quad t \neq (n+l)T, \\ \Delta x_1(t) &= -p_1 x_1(t), \quad t = (n+l)T, \\ x_1(0^+) &\geq 0. \end{aligned} \quad (40)$$

Consider the following comparison equation:

$$\begin{aligned} \dot{\omega}_1(t) &= \omega_1(t)(b_1 - \omega_1(t)), \quad t \neq (n+l)T, \\ \Delta \omega_1(t) &= -p_1 \omega_1(t), \quad t = (n+l)T, \\ \omega_1(0^+) &= \omega_1(0^+) \geq 0. \end{aligned} \quad (41)$$

We have $x_1(t) \leq \omega_1(t)$ and $\omega_1(t) \rightarrow b_1$ as $t \rightarrow \infty$; thus there exists ε'_3 such that $x_1(t) < b_1 + \varepsilon'_3$ for t large enough. Without loss of generality, we may assume $x_1(t) < b_1 + \varepsilon'_3$ for $t > 0$. Similarly, we may set $x_2(t) < b_2 + \varepsilon'_4$. From (31), we know $y_1(t) > \tilde{z}_1(t) - \varepsilon$, $y_2(t) > \tilde{z}_2(t) - \varepsilon$ for all t large enough ($\varepsilon > 0$ is enough small), so that $y_1(t) \geq y_1^* e^{-(m+\mu)T} - \varepsilon \triangleq m_3$, $y_2(t) \geq e^{-\mu T}(y_2^* + y_1^*(1 - e^{-mT})) + y_1^* e^{-(\mu+m)T}(1 - e^{-m(1-l)T}) - \varepsilon \triangleq m_4$ for t large enough. Thus we only need to find $\bar{m}_1 > 0$, $\bar{m}_2 > 0$ such that $x_1(t) \geq \bar{m}_1$, $x_2(t) \geq \bar{m}_2$ for t large enough. We will do it in the following two steps.

Step 1. Denote

$$0 < m_1 < \min \left\{ \frac{(m + \mu) - r_3 \lambda_3 (b_2 + \varepsilon'_4)}{r_1 \lambda_1}, \frac{\mu - m - r_4 \lambda_4 (b_2 + \varepsilon'_4)}{r_2 \lambda_2} \right\},$$

$$\begin{aligned} \dot{z}_6(t) &= -\omega_1 z_6(t), \\ \dot{z}_7(t) &= m z_7(t) - \omega_2 z_7(t), \end{aligned}$$

$$\begin{aligned} \Delta z_6(t) &= -p_3 z_6(t), \\ \Delta z_7(t) &= -p_4 z_7(t), \end{aligned}$$

$$\begin{aligned} \Delta z_6(t) &= z_7(t)(a - b z_7(t)), \\ \Delta z_7(t) &= 0, \end{aligned}$$

$$\begin{aligned} \tilde{z}_6(t) &= \begin{cases} z_6^* e^{-\omega_1(t-nT)}, & t \in (nT, (n+l)T], \\ z_6^* e^{-\omega_1 l T} e^{-\omega_1(t-(n+l)T)}, & t \in ((n+l)T, (n+1)T], \end{cases} \\ \tilde{z}_7(t) &= \begin{cases} e^{-\omega_2(t-nT)} [z_7^* + z_6^* (1 - e^{-m(t-nT)})], & t \in (nT, (n+l)T], \\ e^{-\omega_2(t-(n+l)T)} \{e^{-\omega_2 l T} [z_7^* + z_6^* (1 - e^{-m l T})] + z_6^* e^{-\omega_1 l T} (1 - e^{-m(t-(n+l)T)})\}, & t \in ((n+l)T, (n+1)T], \end{cases} \end{aligned}$$

$$0 < m_2 < \min \left\{ \frac{(m + \mu) - r_1 \lambda_1 (b_1 + \varepsilon'_4)}{r_3 \lambda_3}, \frac{\mu - m - r_2 \lambda_2 (b_1 + \varepsilon'_4)}{r_4 \lambda_4} \right\}, \quad (42)$$

and $r_1 \lambda_1 m_1 + r_3 \lambda_3 m_2 > m + \mu$. We will prove that there exist $t_1, t_2 \in (0, \infty)$, such that $x_1(t_1) \geq m_1$ and $x_2(t_2) \geq m_2$. Otherwise there will be three cases:

- (i) There exists $t_2 > 0$, such that $x_2(t_2) \geq m_2$ and $x_1(t) < m_1$ for all $t > 0$.
- (ii) There exists $t_1 > 0$, such that $x_1(t_1) \geq m_1$ and $x_2(t) < m_2$ for all $t > 0$.
- (iii) Consider $x_1(t) < m_1$, $x_2(t) < m_2$, for all $t > 0$.

Firstly, consider case (i). Let $\varepsilon_1 > 0$ be small enough, so that

$$\begin{aligned} \sigma &= (b_1 - m_1 - \alpha(b_2 + \varepsilon'_4) - r_1 \varepsilon_1 - r_2 \varepsilon_1) T \\ &\quad - \frac{r_1 z_6^* e^{-\omega_1 l T} (e^{-\omega_1 T} - 1)}{(-\omega_1)} - \ln \frac{1}{1 - p_1} > 0 \\ &\quad - \frac{(e^{-\omega_2 T} - 1) \{e^{-\omega_2 l T} [z_6^* (1 - e^{-m l T}) + z_7^*] + z_6^* e^{-\omega_1 l T}\}}{(-\omega_2)} \\ &\quad - \frac{z_6^* e^{-\omega_1 l T} (e^{-(\omega_2 + m)T} - 1)}{(-\omega_2 - m)}, \end{aligned} \quad (43)$$

where

$$\begin{aligned} \omega_1 &= (m + \mu) - r_1 \lambda_1 m_1 - r_3 \lambda_3 (b_2 + \varepsilon'_4), \\ \omega_2 &= \mu - r_2 \lambda_2 m_1 - r_4 \lambda_4 (b_2 + \varepsilon'_4). \end{aligned} \quad (44)$$

According to the above assumption, it is easy to get that $\dot{y}_1(t) \leq -y_1(t)\omega_1$ and $\dot{y}_2(t) \leq m y_1(t) - \omega_2 y_2(t)$. Then we consider the following impulsive differential equation:

$$t \neq (n+l)T, \quad t \neq (n+1)T,$$

$$t = (n+l)T,$$

$$(45)$$

$$t = (n+1)T,$$

where

$$z_6^* = \frac{(B_2 + aC_2 - 1 - A_2B_2 + A_2)(1 - A_2)}{bC_2^2}, \quad p_4 < p_4^*, \quad (46)$$

$$z_7^* = \frac{(B_2 + aC_2 - 1 - A_2B_2 + A_2)}{bC_2}, \quad p_4 < p_4^*,$$

and $A_2 = e^{-\bar{\omega}_1 T}(1 - p_4) > 0$, $B_2 = (1 - p_3)e^{-\bar{\omega}_1 T} > 0$, $C_2 = (1 - p_4)e^{-\bar{\omega}_2 T}(1 - e^{-mT}) + (1 - p_3)e^{-\bar{\omega}_1 T}(1 - e^{-(1-l)mT}) > 0$, and

$$p_4^* = \frac{e^{-\bar{\omega}_1 T} e^{\bar{\omega}_2 T} (1 - p_3) + (1 - \mu_1) (1 - e^{-mT - mlT}) a - e^{\bar{\omega}_2 T} + 1 + a (1 - e^{-mT}) - (1 - p_3) e^{-\bar{\omega}_1 T}}{1 + a (1 - e^{-mT}) - (1 - p_3) e^{-\bar{\omega}_1 T}}. \quad (47)$$

Therefore, there exists $T_1 > 0$ such that

$$\begin{aligned} y_1(t) &\leq z_6(t) \leq \tilde{z}_6(t) + \varepsilon_1, \\ y_2(t) &\leq z_7(t) \leq \tilde{z}_7(t) + \varepsilon_1. \end{aligned} \quad (48)$$

Thus we can easily get

$$\dot{x}_1(t) \geq x_1(t) (b_1 - m_1 - \alpha(b_2 + \varepsilon_4') - r_1(\tilde{z}_6(t) + \varepsilon_1))$$

$$-r_2(\tilde{z}_7(t) + \varepsilon_1)), \quad t \neq (n+l)T,$$

$$\Delta x_1(t) = -p_1 x_1(t), \quad t = (n+l)T, \quad (49)$$

for $t > T_1$. Set $(N+l-1)T \geq T_1$, $N \in \mathbb{Z}_+$. Integrating (49) on $[(n+l)T, (n+1)T]$, $n \geq N$, then we obtain

$$\begin{aligned} x_1((n+l+1)T) &\geq x_1((n+l)T^+) (1 - p_1) \exp \left(\int_{(n+l)T}^{(n+l+1)T} (b_1 - m_1 - \alpha(b_2 + \varepsilon_4') - r_1(\tilde{z}_6(t) + \varepsilon_1) - r_2(\tilde{z}_7(t) + \varepsilon_1)) dt \right) \\ &= x_1((n+l)T) e^{\sigma}; \end{aligned} \quad (50)$$

consequently $x_1((N+n+l)T) \geq x_1((N+l)T) e^{n\sigma} \rightarrow \infty$ as $n \rightarrow \infty$, which contradicts to the boundedness of $x_1(t)$.

Similarly, cases (ii) and (iii) can be analyzed as in case (i). Here we omit it. From the above three cases, we conclude that there exist $t_1 > 0$, $t_2 > 0$ such that $x_1(t_1) \geq m_1$, $x_2(t_2) \geq m_2$.

Step 2. If $x_1(t) \geq m_1$ for all $t \geq t_1$, then our aim is obtained. Otherwise, $x_1(t) < m_1$ for any $t \geq t_1$. Let $\tilde{t} = \inf_{t \geq t_1} \{x_1(t) < m_1\}$; there are two possible cases for $\tilde{t}$.

Case 1. Consider $\tilde{t} = (n_1 + l)T$, $n_1 \in \mathbb{Z}_+$. Then $x_1(t) \geq m_1$ for $t \in [t_1, \tilde{t}]$ and $(1 - p_1)m_1 \leq$

$x_1(\tilde{t}^+) = (1 - p_1)x_1(\tilde{t}) < m_1$. Choose $n_2, n_3 \in \mathbb{Z}_+$ such that

$$(n_2 - 1)T > \max \left\{ \frac{\ln(\varepsilon_1/M)}{-\bar{\omega}_1}, \frac{\ln((\varepsilon_1/M))}{-\bar{\omega}_2} \right\}, \quad (51)$$

$$(1 - p_1)^{n_2} e^{\sigma n_3} e^{(n_2 n_3 T)} > 1.$$

Set $T' = n_2 T + n_3 T$; we claim that there must exist $t_2 \in (\tilde{t}, \tilde{t} + T')$ such that $x_1(t_2) > m_1$. In view of (45), with $z_6(n_1 T^+) = y_1(n_1 T^+)$, $z_7(n_1 T^+) = y_2(n_1 T^+)$, we have

$$\begin{aligned} z_6(t) &= \begin{cases} z_6(n_1 T^+) e^{-(\mu + \alpha)r_1 \lambda_1 m_1 + r_3 \lambda_3 (b_2 + \varepsilon_4')(t - nT)} + \tilde{z}_6(t), & t \in (nT, (n+l)T], \\ z_6((n_1 + l)T^+) e^{-(\mu + \alpha)r_1 \lambda_1 m_1 + r_3 \lambda_3 (b_2 + \varepsilon_4')(t - (n+l)T)} + \tilde{z}_6(t), & t \in ((n+l)T, (n+1)T], \end{cases} \\ z_7(t) &= \begin{cases} e^{-(\mu - r_2 \lambda_2 m_1 - r_4 \lambda_4 (b_2 + \varepsilon_4')(t - nT))} [z_7(n_1 T^+) + z_6(n_1 T^+) (1 - e^{-m(t - nT)})] + \tilde{z}_7(t), & t \in (nT, (n+l)T], \\ e^{-(\mu - r_2 \lambda_2 m_1 - r_4 \lambda_4 (b_2 + \varepsilon_4')(t - (n+l)T))} [z_7((n_1 + l)T^+) + z_6((n_1 + l)T^+) (1 - e^{-m(t - (n+l)T)})] + \tilde{z}_7(t), & t \in ((n+l)T, (n+1)T]. \end{cases} \end{aligned} \quad (52)$$

And $n_1 + 1 \leq n \leq n_1 + n_2 + n_3$. Then

$$\begin{aligned} |z_6(t) - \tilde{z}_6(t)| &< M \\ &\cdot \exp\left(-\left((m+\mu) + r_1\lambda_1 m_1 + r_3\lambda_3(b_2 + \varepsilon'_4)\right)\right. \\ &\cdot (t - nT)\big) < \varepsilon_1, \end{aligned} \quad (53)$$

$$\begin{aligned} |z_7(t) - \tilde{z}_7(t)| &< M \\ &\cdot \exp\left(-\left(\mu - r_2\lambda_2 m_1 - r_4\lambda_4(b_2 + \varepsilon'_4)\right)\right) < \varepsilon_1; \end{aligned}$$

thus $y_1(t) \leq z_6(t) < \tilde{z}_6(t) + \varepsilon_1$ and $y_2(t) \leq z_7(t) < \tilde{z}_7(t) + \varepsilon_1$ for $n_1T + (n_2 - 1)T \leq t \leq \tilde{t} + T'$, which implies that (49) holds for $\tilde{t} + n_2T \leq t \leq \tilde{t} + T'$. So as in step 1, we get

$$x_1(\tilde{t} + T') \geq x_1(\tilde{t} + n_2T) e^{\sigma^{n_3}}. \quad (54)$$

From system (3), we get

$$\begin{aligned} \dot{x}_1(t) &\geq x_1(t) \left(b_1 - m_1 - \alpha(b_2 + \varepsilon'_4) - r_1M - r_2M \right), \\ &t \neq (n+l)T, \end{aligned} \quad (55)$$

$$\Delta x_1(t) = -p_1 x_1(t), \quad t = (n+l)T.$$

Integrating (55) on $[\tilde{t}, \tilde{t} + n_2T]$, we have

$$x_1(\tilde{t} + n_2T) \geq m_1 (1 - p_1)^{n_2} \exp(n_2 \eta_3 T). \quad (56)$$

Therefore, we deduce that

$$x_1(\tilde{t} + T') \geq m_1 (1 - p_1)^{n_2} e^{\sigma^{n_3}} e^{(n_2 \eta_3 T)} > m_1, \quad (57)$$

which is a contradiction.

Let $\tilde{t} = \inf_{t > \tilde{t}} \{x_1(t) \geq m_1\}$; then for $t \in (\tilde{t}, \tilde{t})$, $x_1(t) < m_1$ and $x_1(\tilde{t}) = m_1$. For $t \in (\tilde{t}, \tilde{t})$, set $t \in (\tilde{t} + (r-1)T, \tilde{t} + rT]$, $r \in \mathbb{Z}_+$, $r \leq n_2 + n_3$; from (55), we have

$$\begin{aligned} x_1(t) &\geq x_1(\tilde{t}^+) (1 - p_1)^{r-1} \exp((r-1)\eta_3 T) \\ &\cdot \exp(\eta_3(t - (\tilde{t} + (r-1)T))) \geq m_1 (1 - p_1)^r \end{aligned}$$

$$\begin{aligned} &\cdot \exp(r\eta_3 T) \geq m_1 (1 - p_1)^{n_2+n_3} \\ &\cdot \exp((n_2 + n_3)\eta_3 T). \end{aligned} \quad (58)$$

Let $\widehat{m}_1 = m_1(1 - p_1)^{n_2+n_3} \exp((n_2 + n_3)\eta_3 T)$, so we have $x_1(t) \geq \widehat{m}_1$ for $t \in (\tilde{t}, \tilde{t})$. For $t > \tilde{t}$, the same argument can be continued since $x_1(\tilde{t}) \geq m_1$.

Case 2. Consider $\tilde{t} \neq (n+l)T$, $n \in \mathbb{Z}_+$. Then $x_1(t) \geq m_1$ for $t \in [t_1, \tilde{t})$ and $x_1(\tilde{t}) = m_1$. Let $\tilde{t} \in ((n'_1 + l)T, (n'_1 + 1)T)$, $n'_1 \in \mathbb{Z}_+$. There are two possible cases for $t \in (\tilde{t}, (n'_1 + l)T)$.

If $x_1(t) < m_1$ for all $t \in (\tilde{t}, (n'_1 + 1)T)$, there must be $t'_2 \in [(n'_1 + 1)T, (n'_1 + 1)T + T']$ such that $x_1(t'_2) \geq m_1$. Here, we omit it.

Let $\tilde{t} = \inf_{t > \tilde{t}} \{x_1(t) \geq m_1\}$; then for $t \in (\tilde{t}, \tilde{t})$, $x_1(t) < m_1$ and $x_1(\tilde{t}) = m_1$. For $t \in (\tilde{t}, \tilde{t})$, let $t \in ((n'_1 + l)T + (r' - 1)T, (n'_1 + l)T + r'T]$, $r' \in \mathbb{Z}_+$, $r' \leq n_2 + n_3$; from (55), we have

$$\begin{aligned} x_1(t) &\geq m_1 (1 - p_1)^{r'} \exp((r' + 1)\eta_3 T) \\ &\geq m_1 (1 - p_1)^{n_2+n_3} \exp((n_2 + n_3 + 1)\eta_3 T). \end{aligned} \quad (59)$$

Let $\overline{m}_1 = m_1(1 - p_1)^{n_2+n_3} \exp((n_2 + n_3 + 1)\eta_3 T)$, so we have $x_1(t) \geq \overline{m}_1$ for $t \in (\tilde{t}, \tilde{t})$. For $t > \tilde{t}$, the same argument can be continued since $x_1(\tilde{t}) \geq m_1$.

If there exists $t \in (\tilde{t}, (n'_1 + l)T)$ such that $x_1(t) \geq m_1$, let $\bar{t} = \inf_{t > \tilde{t}} \{x_1(t) \geq m_1\}$; then $x_1(t) < m_1$ for $t \in (\tilde{t}, \bar{t})$ and $x_1(\bar{t}) = m_1$. For $t \in (\tilde{t}, \bar{t})$, (55) holds; integrating (55) on $(\tilde{t}, \bar{t})$, we have

$$x_1(t) \geq x_1(\tilde{t}) \exp(\eta_3(t - \tilde{t})) \geq m_1 \exp(\eta_3 T) > \overline{m}_1. \quad (60)$$

Since $x_1(\bar{t}) \geq m_1$ for $t > \bar{t}$, the same argument can be continued. Hence, we have $x_1(t) \geq \overline{m}_1$ for all $t > t_1$. Similarly, we can prove $x_2(t) \geq \overline{m}_2$ for all $t \geq t_2$. The proof is complete. $\square$

From the proof of Theorems 7 and 8, we can derive the following results.

Corollary 9. Let $(x_1(t), x_2(t), y_1(t), y_2(t))$ be any solution of (3); then y_i , $i = 1, 2$, are permanent; $x_i(t) \rightarrow 0$ as $t \rightarrow \infty$ provided, $b_1 > \alpha b_2$, $b_2 > \beta b_1$,

$$\begin{aligned} T &< \frac{\ln(1/(1 - p_2)) + r_3 y_1^* / (m + \mu) + r_4 (y_1^* + y_2^*) / \mu}{b_2}, \\ & - \frac{r_1 y_1^* e^{-(m+\mu-r_3\lambda_3 b_2)T} (e^{-(m+\mu-r_3\lambda_3 b_2)T} - 1)}{-(m + \mu - r_3 \lambda_3 b_2)} - \frac{r_2 y_1^* e^{-(m+\mu-r_3\lambda_3 b_2)T} (e^{-(\mu-r_4\lambda_4 b_2)+m)T} - 1)}{-(\mu - r_4 \lambda_4 b_2) - m} - \ln \frac{1}{1 - p_1} \\ & - \frac{r_2 (e^{-(\mu-r_4\lambda_4 b_2)T} - 1) \{y_1^* (1 - e^{-mT}) + y_2^*\} + y_1^* e^{-(m+\mu-r_3\lambda_3 b_2)T}}{-(\mu - r_4 \lambda_4 b_2)} + (b_1 - \alpha b_2) T > 0. \end{aligned} \quad (61)$$

FIGURE 1: Globally asymptotically stable prey eradication solution of system (3). (a, b) Time series of prey populations $x_1(t)$, $x_2(t)$ and (c, d) time series of predator populations $y_1(t)$, $y_2(t)$ with initial values $x_1(0) = 0.5$, $x_2(0) = 0.5$, $y_1(0) = 0.5$, and $y_2(0) = 0.5$.

Corollary 10. Let $(x_1(t), x_2(t), y_1(t), y_2(t))$ be any solution of (3); then x_1 and y_i , $i = 1, 2$, are permanent; $x_2(t) \rightarrow 0$ as $t \rightarrow \infty$ provided, $b_1 > \alpha b_2$, $b_2 > \beta b_1$,

$$\begin{aligned}
 T &< \frac{\ln(1/(1-p_1)) + r_1 y_1^*/(m+\mu) + r_2 (y_1^* + y_2^*)/\mu}{b_1}, \\
 &- \frac{r_3 y_1^* e^{-(m+\mu-r_3\lambda_3 b_2)lT} (e^{-(m+\mu-r_3\lambda_3 b_2)T} - 1)}{-(m+\mu-r_3\lambda_3 b_2)} - \frac{r_4 y_1^* e^{-(m+\mu-r_3\lambda_3 b_2)lT} (e^{-(\mu-r_4\lambda_4 b_2+m)T} - 1)}{-(\mu-r_4\lambda_4 b_2)-m} - \ln \frac{1}{1-p_2} \\
 &- \frac{r_4 (e^{-(\mu-r_4\lambda_4 b_2)T} - 1) \{e^{-(\mu-r_4\lambda_4 b_2)lT} [y_1^* (1 - e^{-mlT}) + y_2^*] + y_1^* e^{-(m+\mu-r_3\lambda_3 b_2)lT}\}}{-(\mu-r_4\lambda_4 b_2)} + (b_2 - \beta b_1) T > 0.
 \end{aligned} \tag{62}$$

5. Numerical Analysis

In this section, we carry out numerical simulations of system (3) to illustrate and support our analytical results.

(1) Consider the following choice of parametric values: $\alpha = 0.3$, $\beta = 0.2$, $b_1 = 1$, $b_2 = 1$, $r_1 = 0.2$, $r_2 = 0.5$, $r_3 = 0.3$, $r_4 = 0.4$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\lambda_3 = 0.3$, $\lambda_4 = 0.4$, $m = 0.2$,

FIGURE 2: The permanence of system (3). (a, b) Time series of prey populations $x_1(t)$, $x_2(t)$ and (c, d) time series of predator populations $y_1(t)$, $y_2(t)$ with initial values $x_1(0) = 0.5$, $x_2(0) = 0.5$, $y_1(0) = 0.5$, and $y_2(0) = 0.5$.

$\mu = 0.1$, $d = 0.4$, $p_1 = 0.01$, $p_2 = 0.01$, $p_3 = 0.01$, $p_4 = 0.01$, $a = 2$, $b = 1$, $l = 0.6$, and $T = 2$.

Correspondingly, $\min\{(\ln(1/(1-p_1)) + r_1 y_1^*/(m+\mu) + r_2(y_1^* + y_2^*)/\mu)/b_1, (\ln(1/(1-p_2)) + r_3 y_1^*/(m+\mu) + r_4(y_1^* + y_2^*)/\mu)/b_2\} \approx 5.07 > T = 2$ and $p_4 = 0.01 < p_4^* \approx 0.4$. According to Theorem 7, we know that the prey eradication periodic solution of system (3) is globally asymptotically stable for this case (see Figure 1).

(2) Consider the following set of parameters: $\alpha = 0.3$, $\beta = 0.2$, $b_1 = 1.5$, $b_2 = 1.5$, $r_1 = 0.2$, $r_2 = 0.5$, $r_3 = 0.3$, $r_4 = 0.4$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\lambda_3 = 0.3$, $\lambda_4 = 0.4$, $m = 0.2$, $\mu = 0.1$, $d = 0.4$, $p_1 = 0.01$, $p_2 = 0.01$, $p_3 = 0.01$, $p_4 = 0.01$, $a = 2.45$, $b = 1$, $l = 0.6$, and $T = 2$.

Since conditions of Theorem 8 hold for this choice, system (3) is permanent (see Figure 2).

6. Discussion

In this paper, we study an impulsively controlled three-species prey-predator model with stage structure and birth pulse for predator. Using the comparison theorems, we have

shown that there exists a globally asymptotically stable two-pest eradication periodic solution when the impulsive period $T < \min\{(\ln(1/(1-p_1)) + r_1 y_1^*/(m+\mu) + r_2(y_1^* + y_2^*)/\mu)/b_1, (\ln(1/(1-p_2)) + r_3 y_1^*/(m+\mu) + r_4(y_1^* + y_2^*)/\mu)/b_2\}$. In biological terms, the pest population is eradicated totally and the predator population will tend to a stable level. However, the prey population is eradicated totally, which is not most desired for us from the biological point and saving resources; we hope to control the species under the economic threshold level or drive the target prey population to extinction and let the nontarget pest (or harmless) be permanent. Therefore, we give the sufficient conditions for the permanence of the system. What is more, the sufficient conditions for the extinction of one of the two prey species and permanence of the remaining other preys and predators are given in Corollaries 9 and 10.

Compared to earlier modeling studies on the prey-predator concerning chemical control for pest control at different fixed time, our model considers the predator given birth in regular pulses at $t = (n+1)T$, $n = 1, 2, \dots$. To avoid the fact that pesticides impact on the newborn predator,

pesticide applications and birth pulse in predator occur at different time in this paper. All these results show that the dynamical behaviors of system (3) become more complex under different time periodically impulsive effect. Our results suggest a new approach in pest control. We are more interested in the following: how different patterns of insecticide applications affect the results, for instance, applying pesticide after predator is born and applying pesticide when predator is born. We also can study the results of two cases and find the better control way. These works will be done in our next consideration.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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