Research Article

Algorithms for Routing Optimization in Multipoint to Multipoint 4PL System

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Fourth party logistics (4PL) has been receiving great attention and studied by researchers. This paper studies multipoint to multipoint 4PL system routing optimization with reliability constraints. Objective factors which cause disturbances of transportation time are taken into consideration. The mathematical model of the multipoint to multipoint multitask routing optimization in 4PL system with reliability constraints is set up. To solve the model, a Messy Genetic Algorithm (Messy GA) with double arrays encoding method is designed. The experimental result is compared with the Enumeration Algorithm (EA), and it shows that this algorithm can find the satisfactory solution and is a powerful optimization method for solving the problem.

1. Introduction

With the rapid development of global economy, trade between enterprises grows remarkably. Hence the ability to integrate supply chain for logistics companies has to increase. They are eager to a logistics company that owns strong comprehensive strength to provide scheme for the whole supply chain and be able to meet multiproduct distribution services from different production places to different demand places. Then, the fourth party logistics (4PL) appeared [1]. The operation essence and the core superiority of 4PL lie in its ability to integrate the supply chain resources. Then through integrating the most high-quality supply chain resources, the flexibility of the supply chain will get to a full extent, the customers will acquire the personalized and diversified integration solutions, and the fast, high-quality, and low-cost logistics services will be provided [2]. Meanwhile, with the increasing complexity of logistics network, its reliability has increasingly become one focus of business concern. However, the current planning and design of logistics network take little customers' demand of system reliability.

Many researchers have studied the 4PL and it has received increasing attention [3–8]. The routing problem of 4PL is management and optimization process from suppliers to

demanders, which is directly related to the operational efficiency of 4PL. Most of these researches draw on graph-based network methods [9, 10]. In [11], a target system of transporters is put forward and a hierarchy model for 4PL is established. A conceptual model that is based on research of 4PL implementation is presented in [12]. In [13], the authors consider several characteristics and issues related to horizontal collaboration implementation through an exploratory investigation of retailers, suppliers, and logistics service providers in the UK grocery retailing sector, focusing specifically on the role of fourth party logistics service providers. In [14], a tabu search algorithm for model of integration of 4PL is established.

Multipoint to multipoint and multitask problem in 4PL system with route selection and 3PL supplier's selection integrated is considered in this paper. It is a complex system. We take objective factors that cause disturbances of transportation time into consideration, and the mathematical model is set up. Our goal is minimizing the total cost in this multitask problem, and all tasks must satisfy reliability constraints that customer required. Because the multipoint to multipoint 4PL system routing problem is NP-hard two algorithms are designed: Messy Genetic Algorithm (Messy GA) and Enumeration Algorithm (EA). The algorithm is coded in Matlab

FIGURE 1: Directed graph with 8 nodes.

7.0 and run on a Core 2 2.83 GHz computer. The comparisons in simulation testify the algorithms' effectiveness.

2. Description of Problem and Mathematical Model

2.1. Description of Problem. We use a straightforward way to describe the 4PL logistics system, which is multigraph. It can easily tell us all information needed clearly, as Figure 1 shows. The multigraph can be defined by $G(V, A)$. V denotes the set of supply, transshipment, and demand nodes. A denotes the set of the arcs, namely, 3PL providers for transportation between two cities. V_1 and V_2 are supply nodes, V_7 and V_8 are demand nodes, and other nodes are transshipment nodes. There may many ways and 3PL providers between two locations, so multiple arcs exist between two nodes in multigraph. Each arc has the properties of cost, time, and capacity. In this paper, the objective of the problem is to find routes (i.e., Origin-Destination pair) with minimum cost as well as subject to constraints on customer's requirement for reliability and capacity.

2.2. The Reliability of 4PL System. The route of 4PL system is composed of cities and 3PL providers. It forms a parallel system. According to reliability theory, the reliability of 4PL system can be calculated by continued multiplication of the reliability of 3PL. Here, each 3PL is seen as a logistics unit. In this paper, the transportation time of the 3PL providers is uncertain and has normal distribution. Hence, the time reliability of logistics unit can be defined as the probability of the goods delivered in a timely manner. It can be calculated by the following formula:

$$R(t) = P(T \leq t) = \frac{1}{\sqrt{2\pi}\sigma} \int_0^t e^{-(t-\mu)^2/2\sigma^2} dt \quad t > 0. \quad (1)$$

$R(t)$ is the time reliability of logistics unit. T is the transportation time of logistics unit. We assume $T \sim N(\mu, \sigma^2)$. t is the time limit of the customer's requirements.

Another important influence factor of reliability is transportation cost. It can be calculated by the following formula:

$$R(c) = e^{-1/c}. \quad (2)$$

Here the reliability of logistics unit function is given as follows:

$$R(t, c) = \alpha_1 R(t) + \alpha_2 R(c) \quad \alpha_1 + \alpha_2 = 1. \quad (3)$$

α_1, α_2 are the weighting coefficients of the transportation time and the transportation cost separately.

The reliability of 4PL system can be calculated by the following formula:

$$R_S = \prod_{i=1}^n R_i(t, c). \quad (4)$$

R_S is the reliability of 4PL system. n is the number of the logistics unit in the 4PL system.

2.3. Assumption of Model

- (i) There is not only one task between the supply and demand nodes.
- (ii) Each path on the 4PL system cannot be reused.
- (iii) The transportation time of 3PL providers is uncertain and has normal distribution. It is independent from each other.

2.4. Establishment of Model. Variables and parameters are defined as follows:

V : set of nodes including suppliers, transit facilities, and demanders, $V = O \cup W \cup D$;

O : sets of suppliers;

W : set of transit nodes;

D : set of demanders;

q_{ij} : the number of 3PL provider between nodes i and j ;

a_{ijk} : the k th 3PL provider between nodes i and j ($i, j = 1, 2, \dots, n$) ($k = 1, 2, \dots, q_{ij}$);

$$X_{ijk}^m = \begin{cases} 1 & \text{if the } k\text{th arc between nodes } i \text{ and } j \\ & \text{is selected for task } m \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$

$$Y_i^m = \begin{cases} 1 & \text{if node } i \text{ is selected for task } m \\ 0 & \text{otherwise;} \end{cases}$$

C_{ijk} : the unit transportation cost of the k th 3PL provider between nodes i and j ;

C_i' : the unit cost of node i ;

f^{odm} : the total number of goods transported from the o th supply node to d th demand node for task m ;

P_{ijk} : the transportation capacity of the k th 3PL provider between nodes i and j ;

Q_i : processing capacity of node i ;

T_{ijk} : time of 3PL provider represented by a_{ijk} , $T_{ijk} \sim N(\mu_{ijk}, \sigma_{ijk}^2)$;

t_{ij} : time limit of nodes i to j ;

R_{ijk} : reliability of a_{ijk} ; $R_{ijk} = \alpha_1(1/\sqrt{2\pi}\sigma_{ijk}) \int_0^{t_{ij}} e^{-(t-\mu_{ijk})^2/2\sigma_{ijk}^2} dt + \alpha_2 e^{-1/C_{ijk}}$, $\alpha_1 + \alpha_2 = 1$;

R_{OD} : reliability of OD pair; $R_{OD} = \prod_{i=1}^n \prod_{j=1}^n \prod_{k=1}^{q_{ij}} [\alpha_1(1/\sqrt{2\pi}\sigma_{ijk}) \int_0^{t_{ij}} e^{-(t-\mu_{ijk})^2/2\sigma_{ijk}^2} dt + \alpha_2 e^{-1/C_{ijk}}] \cdot X_{ijk}$;

R_0 : desired reliability level.

The mathematical model for 4PL system routing optimization with reliability constraints can be described as follows:

$$\min \left[\sum_{o=1}^O \sum_{d=1}^D \sum_{m=1}^M \left(\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{q_{ij}} C_{ijk} \cdot f^{odm} \cdot X_{ijk}^m + \sum_{i=1}^n C_i' \cdot f^{odm} \cdot Y_i^m \right) \right], \quad (6)$$

$$\text{s.t.} \quad \prod_{i=1}^n \prod_{j=1}^n \prod_{k=1}^{q_{ij}} \left[\alpha_1 \frac{1}{\sqrt{2\pi}\sigma_{ijk}} \int_0^{t_{ij}} e^{-(t-\mu_{ijk})^2/2\sigma_{ijk}^2} dt + \alpha_2 e^{-1/C_{ijk}} \right] \cdot X_{ijk} \geq R_0$$

$$o = 1, 2, \dots, O \quad d = 1, 2, \dots, D \quad m = 1, 2, \dots, M, \quad (7)$$

$$\sum_{o=1}^O \sum_{d=1}^D \sum_{m=1}^M f^{odm} \cdot X_{ijk}^m \leq P_{ijk} \quad (8)$$

$$i, j \in \{1, 2, \dots, n\}, \quad k \in \{1, 2, \dots, q_{ij}\},$$

$$\sum_{o=1}^O \sum_{d=1}^D \sum_{m=1}^M f^{odm} \cdot Y_i^m \leq Q_i, \quad i \in \{1, 2, \dots, n\}, \quad (9)$$

$$\sum_{i=1}^n f^{odm} \cdot X_{ijk}^m - \sum_{i=1}^n f^{odm} \cdot X_{jik}^m = \begin{cases} -f^{odm} & \text{if } j = 1 \\ f^{odm} & \text{if } i = n \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

$$j \in \{1, 2, \dots, n\}, \quad k \in \{1, 2, \dots, q_{ij}\},$$

$$o = 1, 2, \dots, O \quad d = 1, 2, \dots, D,$$

$$m = 1, 2, \dots, M,$$

$$\sum_{i=1}^n \sum_{k=1}^{q_{ij}} X_{ijk}^m = Y_j^m \quad j \in \{2, 3, \dots, n\}, \quad (11)$$

$$m = 1, 2, \dots, M,$$

$$\sum_{i=1}^n \sum_{k=1}^{q_{ij}} X_{ijk}^m = Y_i^m \quad i \in \{1, 2, \dots, n-1\}, \quad (12)$$

$$m = 1, 2, \dots, M,$$

$$X_{ijk}^m = 0 \text{ or } 1, \quad i, j \in \{1, 2, \dots, n\},$$

$$k \in \{1, 2, \dots, q_{ij}\}, \quad o = 1, 2, \dots, O \quad (13)$$

$$d = 1, 2, \dots, D \quad m = 1, 2, \dots, M,$$

$$Y_i^m = 0 \text{ or } 1, \quad i \in \{1, 2, \dots, n\}, \quad o = 1, 2, \dots, O$$

$$d = 1, 2, \dots, D \quad m = 1, 2, \dots, M, \quad (14)$$

where (6) is the objective function containing the costs of all nodes and arcs on the route; constraint (7) indicates the reliability of OD pair must meet customer's requirement; constraint (8) represents capacity of the selected 3PL provider must be not less than the transportation capacity f^{odm} required by customer; constraint (9) denotes capacity of nodes in a route must be not less than customer required capacity. Equation (10) means to keep a balance of the network flow. Equations (11) and (12) ensure that the selected nodes and arcs should be made up of routes from the source to the destination. Equations (13) and (14) represent that X_{ijk}^m and Y_i^m are 0-1 decision variables, respectively.

3. Algorithm Design

The multipoint to multipoint 4PL system routing optimization is a typical NP-hard problem. According to the characteristics of the model, a Messy Genetic Algorithm (Messy GA) with double arrays encoding method is designed. In the following, we first describe the Messy GA in detail and then introduce an Enumeration Algorithm (EA), which is used to check the quality of the solutions obtained by the Messy GA.

3.1. The Messy GA. According to the characteristics of the model, a Messy GA with double arrays encoding method is designed. Messy GA differ from normal genetic algorithms in that they allow variable-length strings. The detailed description of the process can be written as below.

Step 1 (encoding). For the problem's characteristic, the double arrays coding method is adopted where optional nodes in graph are the elements of one array coded by natural number and 3PL providers make up the other array coded by integer number and the length of individuals is variant. The bold line in Figure 1 can be encoded as $[V_1 V_3 V_5 V_7 a_2 a_1]$.

Step 2 (initialization). Generate the initial population of PS chromosomes randomly. Population size is PS.

Step 3 (fitness function). In this paper, reliability constraint is used as a penalty added to the objective function, and the ω is a penalty factor. The fitness function is represented by (15).

The values of ω range from 1 to 2, depending on the actual situation:

$$\begin{aligned}
 f(X, Y) &= \sum_{o=1}^O \sum_{d=1}^D \sum_{m=1}^M \left(\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{q_{ij}} C_{ijk} \cdot f^{odm} \cdot X_{ijk}^m \right. \\
 &\quad \left. + \sum_{i=1}^n C'_i \cdot f^{odm} \cdot Y_i^m \right) \\
 &+ \left\{ \sum_{o=1}^O \sum_{d=1}^D \sum_{m=1}^M \left(\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{q_{ij}} C_{ijk} \cdot f^{odm} \cdot X_{ijk}^m \right. \right. \\
 &\quad \left. \left. + \sum_{i=1}^n C'_i \cdot f^{odm} \cdot Y_i^m \right) \right\} \quad (15) \\
 &\cdot \left\{ \lambda + R_0 \right. \\
 &\quad \left. - \prod_{i=1}^n \prod_{j=1}^n \prod_{k=1}^{q_{ij}} \left[\alpha_1 \frac{1}{\sqrt{2\pi}\sigma_{ijk}} \int_0^{t_{ij}} e^{-(t-\mu_{ijk})^2/2\sigma_{ijk}^2} dt \right. \right. \\
 &\quad \left. \left. + \alpha_2 e^{-1/C_{ijk}} \right] \cdot X_{ijk} \right\}.
 \end{aligned}$$

Step 4 (selection mechanism). The selection process is based on spinning the roulette wheel.

Step 5 (crossover operation). Perform crossover operations on pairs of selected chromosomes with a crossover probability P_c . After cut operator and splicing operator, chromosomes A'' and B'' may be invalid. Therefore, repairing operation is necessary. The crossover procedure is shown in Figure 2. We need to adjust the chromosome to ensure that it is a connective route. Detailed description of the process can be written as below.

Step 5.1. If there are same nodes on the route, delete one node and the adjacent arc. If not, go to Step 5.2.

Step 5.2. If the new one is still a route from the source to the destination, quit with success. If not, add all the nodes after the crossover point into the Set_p . Define the node before the crossover point as the tracker node N_t . Define the genes before the crossover point as the effective route L .

Step 5.3. If the node $N_t \in \text{Set}_p$, splice the route L with the route after the node which is $N_t \in \text{Set}_p$, and if the new chromosome is the legal route, quit with success. If not, select

one node adjacent to the tracker node N_t as the new tracker node N_{t+1} , and add the new arc to the effective route L .

Step 5.4. Determine if the trace node N_{t+1} is the destination node D ; if it is a valid route, L becomes the new chromosome, so quit with success; if not, turn to Step 5.3.

Step 6 (mutation operation). For performing mutation operation in the feasible solution space, select a chromosome randomly, and change the arcs of the adjacent nodes in P_m probability of mutation operation.

Step 7 (termination criterion). Check whether iteration times are over NG. If it is true, stop the process and output the optimum solution; else turn to Step 3.

3.2. The Enumeration Algorithm. To check the quality of the solutions obtained by the Messy GA, the Enumeration Algorithm (EA) is used. Because the solution space is very large, the EA is described as follows.

Step 1. Produce all the routes from the supply node O and the demand node D on the direct graphs that satisfy the requirements of capacity and reliability.

Step 2. Calculate the fitness function value of every route. The fitness function is designed according to the objective function and reliability constraint of the model. And it is similar to the fitness function of the Messy GA in (15).

Step 3. All the combinations of routes are checked and ordered according to the fitness function. The combination with the lowest fitness function value is the best solution.

4. Numerical Analysis

Assume that a 4PL company undertakes a transnational transportation. In this transportation, we need to transport 3 tons of food and 5 tons of coal from the supply nodes V_1 and V_2 to the demand nodes V_7 and V_8 . The topological structure is shown in Figure 1.

The algorithm is coded in Matlab 7.0 and run on a Core 2 2.83 GHz computer [15, 16]. We analyze the performance of Messy GA and compare the result with that of Enumeration Algorithm (EA). The parameters affecting the performance include population size (PS), number of generation (NG), crossover probability (P_c), and scaling factor of mutation (P_m). Desired reliability level (R_0) is 0.85. The costs of the nodes and the arcs in the following are recorded by 1000 dollars. The capacity is recorded by ton, and the time of 3PL provider is recorded by hour. The parameters are set below:

$$\begin{aligned}
 \omega &= 3, & PS &= 30, & NG &= 300, & P_c &= 0.5, \\
 P_m &= 0.15, & \alpha_1 &= \alpha_2 = 0.5.
 \end{aligned} \quad (16)$$

The node and arc data are given in Tables 1 and 2, where " V_i " and " V_j " mean the start and end nodes, respectively.

The experimental results are shown in Table 3.

The 4PL company needs to transport food and coal from supply nodes to demand nodes with the reliability not less than 0.85. From Table 3, it can also be seen that both Messy

FIGURE 2: Crossover operation.

TABLE 1: The data of nodes for the 8-node problem.

Node	Cost (1000 dollars)	Capacity (ton)
V_1	5	9
V_2	4	8
V_3	3	10
V_4	6	12
V_5	4	9
V_6	3	8
V_7	2	10
V_8	2	12

GA and EA can obtain optimal result, but Messy GA can solve the problem quickly.

5. Conclusions

The routing optimization in 4PL system is one of the most important problems in supply chain optimization. This study set up the mathematical model of the multipoint to multipoint 4PL system routing optimization with reliability constraints based on the reliability theory. The Messy GA and the EA are proposed to solve the model. Numerical experiments are carried out to investigate the performance of the proposed Messy GA. The experiment results show that the Messy GA can obtain the same best result as the EA.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

TABLE 2: The data of arcs for the 8-node problem.

V_i	V_j	a_{ijk}	C_{ijk}	P_{ijk}	$T_{ijk} \sim N(\mu_{ijk}, \sigma_{ijk}^2)$	t_{ij}	$R(t)$	$R(c)$	R_{ijk}
1	3	1	18	12	$N(18, 5)$	24	0.88493	0.945959	0.915445
		2	18	10	$N(16, 6)$	24	0.908789	0.945959	0.927374
2	3	1	18	11	$N(16, 8)$	24	0.841345	0.945959	0.893652
		2	18	12	$N(17, 6)$	24	0.878327	0.945959	0.912143
	4	1	17	10	$N(18, 4)$	24	0.933193	0.942873	0.938033
		2	20	12	$N(16, 7)$	24	0.873451	0.951229	0.91234
3	5	1	18	9	$N(20, 5)$	24	0.788145	0.945959	0.867052
		2	17	10	$N(20, 6)$	36	0.99617	0.942873	0.969521
4	5	1	18	15	$N(25, 7)$	36	0.941958	0.945959	0.943959
		2	12	12	$N(12, 4)$	18	0.933193	0.920044	0.926619
	6	1	13	14	$N(11, 4)$	18	0.959941	0.925961	0.942951
		2	16	13	$N(13, 2)$	18	0.99379	0.939413	0.966602
5	7	1	17	11	$N(15, 3)$	18	0.841345	0.942873	0.892109
		2	10	14	$N(14, 5)$	20	0.88493	0.904837	0.894884
	8	1	12	16	$N(13, 3)$	20	0.990185	0.920044	0.955115
		2	12	17	$N(13, 4)$	20	0.959941	0.920044	0.939993
6	8	1	15	10	$N(16, 5)$	24	0.945201	0.935507	0.940354
		2	10	9	$N(13, 4)$	24	0.99702	0.904837	0.950929

TABLE 3: Comparison of two algorithms.

Algorithm	Total cost (1000 dollars)	Optimal route	Reliability of the route	Run time (second)
Messy GA	357	$V_1 V_3 V_5 V_7 a_2 a_1$ (FOOD)	0.859 (FOOD)	<1
		$V_2 V_4 V_6 V_8 a_1 a_2$ (COAL)	0.862 (COAL)	
EA	357	$V_1 V_3 V_5 V_7 a_2 a_1$ (FOOD)	0.859 (FOOD)	5689
		$V_2 V_4 V_6 V_8 a_1 a_2$ (COAL)	0.862 (COAL)	

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