Research Article

Dynamics of Uncertain Discrete-Time Neural Network with Delay and Impulses

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The stability of discrete-time impulsive delay neural networks with and without uncertainty is investigated. First, by using Razumikhin-type theorem, a new less conservative condition for the exponential stability of discrete-time neural network with delay and impulse is proposed. Moreover, some new sufficient conditions are derived to guarantee the stability of uncertain discrete-time neural network with delay and impulse by using Lyapunov function and linear matrix inequality (LMI). Finally, several examples with numerical simulation are presented to demonstrate the effectiveness of the obtained results.

1. Introduction

In the last three decades, much attention has been paid to the study of neural networks due to their potential applications in a variety of fields such as signal processing, pattern recognition, associative memory, combinational optimization, and other areas [1–5]. As we all know, it is very important to know the dynamical behaviors of neural networks. In particular, stability properties of dynamical neural networks are very essential in their design for solving practical problems.

It is well known that time delay is always encountered in the implementation of a signal transmission and even small time delay may lead to significantly deteriorated performance or instability for the system. So stability of delayed neural networks has been widely investigated. Impulse is also a common phenomenon in control schemes and evolution processes of dynamic system. For the dynamics of system, impulse can bring about many complex influences. For some stable systems, impulse may play a role as perturbation and may destabilize the system. However, for others, impulse may serve as control power and makes unstable system stabilization. Consequently, it is essential to study how impulse affects the stability of system. Meanwhile, over the past few decades, many results about the stability of neural networks with time delay and impulse were obtained [6–12].

In general, most neural networks were assumed to be continuous time and many considerable results were obtained. However, in the fields of engineering especially in the numerical simulations, many phenomena are often described by discrete-time systems. Generally speaking, there always exist different dynamic properties between continuous-time neural networks and discrete-time neural networks. And results about the discrete-time neural networks are still few [13–32].

Furthermore, uncertainty is unavoidable in many evolution processes, which may destroy the stability of the neural networks. Therefore, it is necessary to analyze the robust stability of neural networks with uncertainty. But for the discrete-time impulsive neural networks with delay and uncertainty, there are very few results. In previous articles, several approaches were proposed to study the stability of impulsive delayed systems, most of which were based on Lyapunov functional, inequality technique, and Razumikhin technique. For instance, in [18], exponential stability for discrete-time impulsive delayed neural networks with and without uncertainty was investigated by using Lyapunov functionals. In [24], the global exponential stability of the discrete-time systems of Cohen-Grossberg neural networks with and without delays was studied by using Lyapunov methods. Global dissipativity criteria of uncertain discrete-time stochastic neural networks with time-varying delays

were obtained in [25] by using linear matrix inequalities. The authors in [26] studied the stability effects of impulses in discrete-time delayed neural networks. In [27], the impulsive stabilization results for neural network without uncertainty were given by using Razumikhin technique, but the authors did not take discrete-time analogs into account. The authors in [28] investigated the existence and global exponential stability of periodic solution for delayed neural networks with impulsive and stochastic effects. The robust exponential stability of discrete-time switched Hopfield neural networks with time delay was studied in [29]. In [30], some new stability results for discrete-time recurrent neural networks with time-varying delay were proposed. And some conditions for robust stability of discrete-time uncertain neural networks with leakage time-varying delay were obtained in [31]. In [32], a new delay-independent condition for global robust stability of neural networks with time delays was given. In [18], although exponential stability for discrete-time impulsive delay neural networks with and without uncertainty was investigated by using Lyapunov functionals, the results were complicated. Consequently, we consider globally exponential stability of discrete-time impulsive delayed neural networks with uncertainty via Razumikhin-type theorems. Besides, different from articles [1, 2, 4, 16, 18–22], we do not require that activation functions in this paper are bounded on R^n .

The rest of this paper is organized as follows. In Section 2, the model description and some useful lemmas are provided. In Section 3, the main results are derived to ensure the exponential stability analysis of discrete-time impulsive delayed neural network with certainties and uncertainties. In Section 4, some numerical examples are presented to demonstrate the effectiveness of the obtained results. The conclusion is given in Section 5.

2. Preliminary

Consider the following discrete-time neural network with delay and impulses:

$$\begin{aligned} y(n+1) &= Ay(n) + B\bar{f}(y(n)) + C\bar{g}(y(n-\tau)) + D, \\ n &\neq n_k - 1, \\ y(n_k) &= I_k(y(n_k - 1)), \quad k \in Z^+, \\ y(n_0 + s) &= \psi(s), \quad s \in J, \end{aligned} \quad (1)$$

where $y(n) = (y_1(n), y_2(n), \dots, y_n(n))^T$, $n \geq n_0$, $n \in N$, N denotes the set of nonnegative integers, that is, $N = \{0, 1, 2, \dots\}$, $y \in R^n$, and $y_i(n)$ denotes the state associated with the i th neuron at time n , $A \in R^{n \times n}$, $B \in R^{n \times n}$, $C \in R^{n \times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $\bar{f}(\cdot)$, $\bar{g}(\cdot)$ denote the neuron activation functions, and $\tau \in Z^+$, Z^+ denotes the set of positive integers, and $\bar{f}(y(n)) = (\bar{f}_1(y(n)), \bar{f}_2(y(n)), \dots, \bar{f}_n(y(n)))^T$, $\bar{g}(y(n-\tau)) = (\bar{g}_1(y(n-\tau)), \bar{g}_2(y(n-\tau)), \dots, \bar{g}_n(y(n-\tau)))^T$, $D = (d_1, d_2, \dots, d_n)^T$ means a constant input vector, I_k determines the size of the jump, and $I_k : R^n \rightarrow R^n$, $J = \{-\tau, -\tau + 1, \dots, -1, 0\}$, $\psi \in C(J, R^n)$, $\{n_k\} \subseteq N$ are

impulsive moments satisfying $0 \leq n_0 < n_1 < \dots < n_k < \dots$ and $\lim_{k \rightarrow \infty} n_k = +\infty$.

A vector $y^* = (y_1^*, y_2^*, \dots, y_n^*)^T$ is said to be an equilibrium point of system (1) if it satisfies

$$\begin{aligned} y^* &= Ay^* + B\bar{f}(y^*) + C\bar{g}(y^*) + D, \\ y^* &= I_k(y^*). \end{aligned} \quad (2)$$

In this paper, we assume that there exists a unique equilibrium point for system (1), and $I_k(y(n_k - 1)) = C_k(y(n_k - 1)) + (I - C_k)y^*$, where $C_k = \{c_{ij}\} \in R^{n \times n}$. Set $x(\cdot) = y(\cdot) - y^*$; then system (1) becomes

$$\begin{aligned} x(n+1) &= Ax(n) + Bf(x(n)) + Cg(x(n-\tau)), \\ n &\neq n_k - 1, \\ x(n_k) &= C_k x(n_k - 1), \quad k \in Z^+, \end{aligned} \quad (3)$$

$$x(n_0 + s) = \varphi(s), \quad s \in J,$$

where $f(x(n)) = \bar{f}(y(n)) - \bar{f}(y^*) = (f_1(x(n)), f_2(x(n)), \dots, f_n(x(n)))^T$, $g(x(n-\tau)) = \bar{g}(y(n-\tau)) - \bar{g}(y^*) = (g_1(x(n-\tau)), g_2(x(n-\tau)), \dots, g_n(x(n-\tau)))^T$, $\varphi(s) = \psi(s) - y^*$.

When parameter uncertainty is considered in system (3), the system becomes an impulsive discrete-time neural network with delay and uncertainty, which can be formulated as follows:

$$\begin{aligned} x(n+1) &= (A + \Delta A(n))x(n) \\ &\quad + (B + \Delta B(n))f(x(n)) \\ &\quad + (C + \Delta C(n))g(x(n-\tau)), \\ n &\neq n_k - 1, \end{aligned} \quad (4)$$

$$x(n_k) = C_k x(n_k - 1), \quad k \in Z^+,$$

$$x(n_0 + s) = \varphi(s), \quad s \in J.$$

For interval matrices A, B, C , there exist known real constant matrices $\bar{D}, F(n), \bar{A}, \bar{B}, \bar{C}$ such that

$$[\Delta A(n) \quad \Delta B(n) \quad \Delta C(n)] = \bar{D}F(n) [\bar{A} \quad \bar{B} \quad \bar{C}], \quad (5)$$

with $F^T(n)F(n) < I$.

In order to obtain our main results, the following assumptions are always made through this paper.

(H₁) The activation function $f_i(\cdot)$ is a continuous function on R and there exist scalars $\underline{k}_i, \bar{k}_i$, ($i = 1, 2, \dots$) such that, for any x, y , $x \neq y$,

$$\underline{k}_i \leq \frac{\bar{f}_i(x) - \bar{f}_i(y)}{x - y} \leq \bar{k}_i. \quad (6)$$

(H₂) The activation function $g_i(\cdot)$ is a continuous function on R and there exist scalars $\underline{l}_i, \bar{l}_i$ ($i = 1, 2, \dots$) such that, for any x, y , $x \neq y$,

$$\underline{l}_i \leq \frac{\bar{g}_i(x) - \bar{g}_i(y)}{x - y} \leq \bar{l}_i. \quad (7)$$

For convenience, the following notations are defined:

$$\begin{aligned} L_1 &= \text{diag} \{ \bar{l}_1, \bar{l}_2, \dots, \bar{l}_n \}, \\ K_1 &= \text{diag} \{ \bar{k}_1, \bar{k}_2, \dots, \bar{k}_n \}, \\ L_2 &= \text{diag} \left\{ \frac{\bar{l}_1 + \bar{l}_1}{2}, \frac{\bar{l}_2 + \bar{l}_2}{2}, \dots, \frac{\bar{l}_n + \bar{l}_n}{2} \right\}, \\ K_2 &= \text{diag} \left\{ \frac{\bar{k}_1 + \bar{k}_1}{2}, \frac{\bar{k}_2 + \bar{k}_2}{2}, \dots, \frac{\bar{k}_n + \bar{k}_n}{2} \right\}. \end{aligned} \quad (8)$$

We introduce the following lemmas.

Lemma 1 (see [11]). Let $M, N, F(n)$, and X be real matrices of appropriate dimensions with X satisfying $X = X^T$; then

$$X + MF(n)N + N^T F^T(n)M^T < 0 \quad (9)$$

for all $F^T(n)F(n) < I$ if and only if there exists $\varepsilon > 0$ such that

$$X + \varepsilon^{-1}MM^T + \varepsilon N^T N < 0. \quad (10)$$

Lemma 2 (Schur's complement). The linear matrix inequality

$$\begin{pmatrix} A(x) & B(x) \\ B^T(x) & C(x) \end{pmatrix} < 0, \quad (11)$$

where $A^T(x) = A(x)$, $C^T(x) = C(x)$, is equivalent to either of the following conditions:

- (1) $A(x) < 0$ and $C(x) - B^T(x)A^{-1}(x)B(x) < 0$.
- (2) $C(x) < 0$ and $A(x) - B(x)C^{-1}(x)B^T(x) < 0$.

Lemma 3 (see [17]). Let $\lambda_0 < 1$, $\lambda_1 < 1$, $c_1, c_2, \gamma > 1$; suppose there exists a Lyapunov function $V(n, x)$, such that

- (1) $c_1 \|x\|^p \leq V(n, x) \leq c_2 \|x\|^p$, for all $n \geq n_0 - \tau$, $x \in R^n$;
- (2) $V(n+1, x(n+1)) \leq \lambda_0 V(n, x(n)) + \lambda_1 V(n-\tau, x(n-\tau))$ holds for all $n \in [n_k, n_{k+1} - 1]$;
- (3) $V(n_k, x(n_k)) \leq \gamma V(n_k - 1, x(n_k - 1))$, $k \in Z^+$.

Then, if $(\lambda_0 + \lambda_1)\gamma < 1$, the trivial solution of system (3) is globally exponentially stable.

Lemma 4 (see [17]). Let $\lambda_0 < 1$, $\lambda_1 < 1$, $c_1, c_2, \gamma < 1$, $\rho = \sup_{k \in Z^+} \{n_k - n_{k-1}\} < \infty$ be all positive numbers; suppose there exists a Lyapunov function $V(n, x)$, such that

- (1) $c_1 \|x\|^p \leq V(n, x) \leq c_2 \|x\|^p$, for all $n \geq n_0 - \tau$, $x \in R^n$;
- (2) $V(n+1, x(n+1)) \leq \lambda_0 V(n, x(n)) + \lambda_1 V(n-\tau, x(n-\tau))$ holds for all $n \in [n_k, n_{k+1} - 1]$;
- (3) $V(n_k, x(n_k)) \leq \gamma V(n_k - 1, x(n_k - 1))$, $k \in Z^+$.

Then, if $\lambda_0 + \lambda_1(1/\gamma) < (1/\gamma)^{1/\rho}$, the trivial solution of system (3) is globally exponentially stable.

3. Main Results

Theorem 5. Suppose that (H_1) – (H_2) hold. If there exist positive numbers α and β satisfying $\alpha + \beta < 1$, $\gamma > 1$ and positive diagonal matrices $W_i = \text{diag}(w_{i1}, w_{i2}, \dots, w_{in})$ ($i = 1, 2$) such that

$$(1)$$

$$\Lambda = \begin{pmatrix} A^T A - W_1 K_1 - \alpha I & A^T B + W_1 K_2 & A^T C & 0 \\ * & B^T B - W_1 & B^T C & 0 \\ * & * & C^T C - W_2 & W_2 L_2 \\ * & * & * & -\beta I - W_2 L_1 \end{pmatrix} < 0, \quad (12)$$

where the symbol $*$ represents the elements below the main diagonal of the symmetric matrix;

- (2) $(\alpha + \beta)\gamma < 1$, where $\gamma = \max\{c_{ij}^2\}$,

then the trivial solution of system (3) is globally exponentially stable.

Proof. Consider the Lyapunov function

$$V(n, x(n)) = x^T(n)x(n) = \|x(n)\|^2. \quad (13)$$

When $n \in [n_k, n_{k+1} - 1]$, we have

$$\begin{aligned} V(n+1, x(n+1)) &= x^T(n+1)x(n+1) \\ &= [Ax(n) + Bf(x(n)) + Cg(x(n-\tau))]^T \end{aligned}$$

$$\begin{aligned} &\cdot [Ax(n) + Bf(x(n)) + Cg(x(n-\tau))] = x^T(n) \\ &\cdot A^T Ax(n) + 2x^T(n)A^T Bf(x(n)) + 2x^T(n) \\ &\cdot A^T Cg(x(n-\tau)) + f^T(x(n))B^T Bf(x(n)) \\ &+ 2f^T(x(n))B^T Cg(x(n-\tau)) + g^T(x(n-\tau)) \\ &\cdot C^T Cg(x(n-\tau)). \end{aligned} \quad (14)$$

By using (H_1) and (H_2) , for any $x, y, x \neq y$, we obtain

$$\begin{aligned} \bar{k}_i &\leq \frac{f_i(x) - f_i(y)}{x - y} \leq \bar{k}_i, \\ \bar{l}_i &\leq \frac{g_i(x) - g_i(y)}{x - y} \leq \bar{l}_i. \end{aligned} \quad (15)$$

Thus,

$$\begin{aligned}
0 \leq & \sum_{i=1}^n w_{1i} (\bar{k}_i x_i(n) - f_i(x_i(n))) (f_i(x_i(n)) \\
& - \bar{k}_i x_i(n)) + \sum_{i=1}^n w_{2i} (\bar{l}_i x_i(n-\tau) - g_i(x_i(n-\tau))) \\
& \cdot (g_i(x_i(n-\tau)) - \bar{l}_i x_i(n-\tau)) = 2x^T(n) \\
& \cdot W_1 K_2 f(x(n)) - f^T(x(n)) W_1 f(x(n)) - x^T(n) \\
& \cdot W_1 K_1 x(n) + 2x^T(n-\tau) W_2 L_2 g(x(n-\tau)) \\
& - g^T(x(n-\tau)) W_2 g(x(n-\tau)) - x^T(n-\tau) \\
& \cdot W_2 L_1 x(n-\tau).
\end{aligned} \tag{16}$$

Substituting these into (14),

$$\begin{aligned}
V(n+1, x(n+1)) & \leq x^T(n) (A^T A - W_1 K_1 - \alpha I) x(n) \\
& + 2x^T(n) (A^T B + W_1 K_2) f(x(n)) \\
& + 2x^T(n) A^T C g(x(n-\tau)) \\
& + f^T(x(n)) (B^T B - W_1) f(x(n)) \\
& + 2f^T(x(n)) B^T C g(x(n-\tau)) \\
& + 2x^T(n-\tau) W_2 L_2 g(x(n-\tau)) \\
& + g^T(x(n-\tau)) (C^T C - W_2) g(x(n-\tau)) \\
& + \alpha x^T(n) x(n) \\
& + x^T(n-\tau) (-\beta I - W_2 L_1) x(n-\tau) \\
& + \beta x^T(n-\tau) x(n-\tau) \\
& = \xi^T(n) \Lambda \xi(n) + \alpha x^T(n) x(n) \\
& + \beta x^T(n-\tau) x(n-\tau),
\end{aligned} \tag{17}$$

where $\xi^T(n) = (x^T(n), f^T(x(n)), g^T(x(n-\tau)), x^T(n-\tau))$.

From condition (1), we know

$$\begin{aligned}
V(n+1, x(n+1)) & \leq \alpha x^T(n) x(n) \\
& + \beta x^T(n-\tau) x(n-\tau) \\
& = \alpha V(n, x(n)) \\
& + \beta V(n-\tau, x(n-\tau)).
\end{aligned} \tag{18}$$

Moreover, we have

$$\begin{aligned}
V(n_k, x(n_k)) & = \|x(n_k)\|^2 = c_k^2 \|x(n_k-1)\| \\
& \leq \gamma \|x(n_k-1)\|^2 \\
& = \gamma V(n_k-1, x(n_k-1)).
\end{aligned} \tag{19}$$

From the above proving procedure, one observes that all conditions of Lemma 3 are satisfied, which implies that the trivial solution of system (3) is globally exponentially stable. This completes the proof. $\square$

Remark 6. The parameter γ in inequality $V(n_k, x(n_k)) \leq \gamma V(n_k-1, x(n_k-1))$ describes the influence of impulses on the stability of discrete-time neural networks. In Theorem 5, when $\gamma > 1$, this means the impulses have destabilizing effects, and we give the bound of γ which keep the stability property of the system. In reality, $\gamma > 1$ is not a necessary condition, and if $\gamma \leq 1$, it means that both impulse-free system and its effects are stabilizing the dynamic system.

Theorem 7. Suppose that (H_1) – (H_2) hold. If there exist positive numbers α and β satisfying $\alpha + \beta < 1$, $\gamma < 1$, and $\rho = \sup_{k \in \mathbb{Z}^+} \{n_k - n_{k-1}\} < \infty$ and positive diagonal matrices $W_i = \text{diag}(w_{i1}, w_{i2}, \dots, w_{in})$ ($i = 1, 2$), such that

$$(1)$$

$$\Lambda = \begin{pmatrix} A^T A - W_1 K_1 - \alpha I & A^T B + W_1 K_2 & A^T C & 0 \\ * & B^T B - W_1 & B^T C & 0 \\ * & * & C^T C - W_2 & W_2 L_2 \\ * & * & * & -\beta I - W_2 L_1 \end{pmatrix} < 0, \tag{20}$$

where the symbol $*$ represents the elements below the main diagonal of the symmetric matrix;

$$(2) \alpha + \beta(1/\gamma) < (1/\gamma)^{1/\rho}, \text{ where } \gamma = \max_{ij} \{c_{ij}^2\},$$

then the trivial solution of system (3) is globally exponentially stable.

Proof. Consider the Lyapunov function

$$V(n, x(n)) = x^T(n) x(n) = \|x(n)\|^2. \tag{21}$$

Similar to the proof of Theorem 5, for all $n \in [n_k, n_{k+1}-1]$, we have

$$\begin{aligned}
V(n+1, x(n+1)) & \leq \xi^T(n) \Lambda \xi(n) + \alpha x^T(n) x(n) \\
& + \beta x^T(n-\tau) x(n-\tau) \\
& \leq \alpha V(n, x(n)) \\
& + \beta V(n-\tau, x(n-\tau)),
\end{aligned} \tag{22}$$

where $\xi^T(n) = (x^T(n), f^T(x(n)), g^T(x(n-\tau)), x^T(n-\tau))$.

Moreover, we get

$$\begin{aligned} V(n_k, x(n_k)) &= \|x(n_k)\|^2 = c_k^2 \|x(n_k - 1)\|^2 \\ &\leq \gamma \|x(n_k - 1)\|^2 \\ &= \gamma V(n_k - 1, x(n_k - 1)). \end{aligned} \quad (23)$$

Then all conditions of Lemma 4 are satisfied, which indicates that the trivial solution of system (3) is globally exponentially stable. This completes the proof. $\square$

Remark 8. In [18], in order to obtain the condition of globally exponential stability of the equilibrium point of (1), Theorems 5 and 7 are given. In these theorems, the parameters $d_j > 0$ ($j \in \mathbb{Z}^+$), $0 < \mu < 1$, and positive definite symmetric matrices P_k ($k = 0, 1, \dots, \tau$) and $D_i = \text{diag}\{d_{i1}, d_{i2}, \dots, d_{in}\}$ ($i = 1, 2$) are introduced. And in Theorem 5, the matrix $\Xi \in R^{(\tau+3)n \times (\tau+3)n}$ ($\tau \geq 1$), and under some other conditions, the equilibrium point of (1) is globally exponentially stable. The conditions in Theorem 7 are similar. However, in this paper, we only introduce the positive parameters α, β and positive diagonal matrices W_i ($i = 1, 2$). In Theorem 5, we choose parameters α and β such that $(\alpha + \beta)\gamma < 1$, in which $\gamma = \max\{c_{ij}^2\}$ is known, and $\Lambda < 0$ where $\Lambda \in R^{4n \times 4n}$. In Theorem 7, we choose parameters α and β such that $\alpha + \beta(1/\gamma) < (1/\gamma)^{1/\rho}$ and $\Lambda < 0$, where $\Lambda \in R^{4n \times 4n}$. Comparing the conditions of Theorems 5 and 7 of this paper with those given in [18], we find that the conditions in this paper are easier implementation in practice.

Theorem 9. Suppose that (H_1) – (H_2) hold. If there exist constants $\varepsilon > 0$, $\gamma > 1$, α , and β satisfying $\alpha + \beta < 1$ and positive diagonal matrices $W_i = \text{diag}(w_{i1}, w_{i2}, \dots, w_{in})$ ($i = 1, 2$), such that

(1)

$$\begin{pmatrix} \Xi & \varepsilon \tilde{A}^T \tilde{B} + W_1 K_2 & \varepsilon \tilde{A}^T \tilde{C} & 0 & A^T & 0 \\ * & \varepsilon \tilde{B}^T \tilde{B} - W_1 & \varepsilon \tilde{B}^T \tilde{C} & 0 & B^T & 0 \\ * & * & \varepsilon \tilde{C}^T \tilde{C} - W_2 & W_2 L_2 & C^T & 0 \\ * & * & * & -\beta I - W_2 L_1 & 0 & 0 \\ * & * & * & * & -I & \tilde{D} \\ * & * & * & * & * & -\varepsilon I \end{pmatrix} < 0, \quad (24)$$

where $\Xi = \varepsilon \tilde{A}^T \tilde{A} - W_1 K_1 - \alpha I$ and the symbol $*$ represents the elements below the main diagonal of the symmetric matrix;

$$(2) \quad (\alpha + \beta)\gamma < 1, \text{ where } \gamma = \max\{c_{ij}^2\},$$

then the trivial solution of system (4) is globally exponentially stable.

Proof. Consider the Lyapunov function

$$V(n, x(n)) = x^T(n) x(n) = \|x(n)\|^2. \quad (25)$$

Similar to the proof of Theorem 5, for all $n \in [n_k, n_k - 1]$, we have

$$\begin{aligned} V(n+1, x(n+1)) &\leq \xi^T(n) \bar{\Lambda} \xi(n) + \alpha x^T(n) x(n) \\ &\quad + \beta x^T(n-\tau) x(n-\tau), \end{aligned} \quad (26)$$

where $\xi^T(n) = (x^T(n), f^T(x(n)), g^T(x(n-\tau)), x^T(n-\tau))$ and

$$\bar{\Lambda} = \begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 \\ * & * & \Omega_{33} & W_2 L_2 \\ * & * & * & -\beta I - W_2 L_1 \end{pmatrix}. \quad (27)$$

Consider $\Omega_{11} = (A + \Delta A(n))^T (A + \Delta A(n)) - W_1 K_1 - \alpha I$, $\Omega_{12} = (A + \Delta A(n))^T (B + \Delta B(n)) + W_1 K_2$, $\Omega_{13} = (A + \Delta A(n))^T (C + \Delta C(n))$, $\Omega_{22} = (B + \Delta B(n))^T (B + \Delta B(n)) - W_1$, $\Omega_{23} = (B + \Delta B(n))^T (C + \Delta C(n))$, $\Omega_{33} = (C + \Delta C(n))^T (C + \Delta C(n)) - W_2$.

By using Lemma 2, the condition $\bar{\Lambda} < 0$ can be equivalently expressed as

$$\begin{pmatrix} -W_1 K_1 - \alpha I & W_1 K_2 & 0 & 0 & (A + \Delta A(n))^T \\ * & -W_1 & 0 & 0 & (B + \Delta B(n))^T \\ * & * & -W_2 & W_2 L_2 & (C + \Delta C(n))^T \\ * & * & * & -\beta I - W_2 L_1 & 0 \\ * & * & * & 0 & -I \end{pmatrix} < 0. \quad (28)$$

Inequality (28) can be rewritten in the following form:

$$X + MF(n)N + N^T F^T(n)M^T < 0, \quad (29)$$

where

$$X = \begin{pmatrix} -W_1 K_1 - \alpha I & W_1 K_2 & 0 & 0 & A^T \\ * & -W_1 & 0 & 0 & B^T \\ * & * & -W_2 & W_2 L_2 & C^T \\ * & * & * & -\beta I - W_2 L_1 & 0 \\ * & * & * & 0 & -I \end{pmatrix}, \quad (30)$$

$$M^T = (0 \ 0 \ 0 \ 0 \ \bar{D}^T),$$

$$N = (\bar{A} \ \bar{B} \ \bar{C} \ 0 \ 0).$$

According to Lemma 1, inequality (29) holds if and only if there exists a scalar $\varepsilon > 0$ such that

$$X + \varepsilon^{-1} M M^T + \varepsilon N^T N < 0, \quad (31)$$

and again by using Schur's complement, inequality (31) is equivalent to condition (1). Thus, we obtain

$$\begin{aligned} V(n+1, x(n+1)) &\leq \alpha x^T(n) x(n) \\ &+ \beta x^T(n-\tau) x(n-\tau). \end{aligned} \quad (32)$$

Moreover, we get

$$\begin{aligned} V(n_k, x(n_k)) &= \|x(n_k)\|^2 = C_k^2 \|x(n_k-1)\|^2 \\ &\leq \gamma \|x(n_k-1)\|^2 \\ &= \gamma V(n_k-1, x(n_k-1)). \end{aligned} \quad (33)$$

Then all conditions of Lemma 3 are satisfied, which implies that the trivial solution of system (4) is globally exponentially stable. This completes the proof. $\square$

Theorem 10. Suppose that (H_1) – (H_2) hold. If there exist constants $\varepsilon > 0$, $\gamma < 1$, α , and β satisfying $\alpha + \beta < 1$, $\rho = \sup_{k \in \mathbb{Z}^+} \{n_k - n_{k-1}\} < \infty$ and positive diagonal matrices $W_i = \text{diag}(w_{i1}, w_{i2}, \dots, w_{in})$ ($i = 1, 2$), such that

(1)

$$\begin{pmatrix} \Xi & \varepsilon \bar{A}^T \bar{B} + W_1 K_2 & \varepsilon \bar{A}^T \bar{C} & 0 & A^T & 0 \\ * & \varepsilon \bar{B}^T \bar{B} - W_1 & \varepsilon \bar{B}^T \bar{C} & 0 & B^T & 0 \\ * & * & \varepsilon \bar{C}^T \bar{C} - W_2 & W_2 L_2 & C^T & 0 \\ * & * & * & -\beta I - W_2 L_1 & 0 & 0 \\ * & * & * & * & -I & \bar{D} \\ * & * & * & * & * & -\varepsilon I \end{pmatrix} \quad (34)$$

< 0,

where $\Xi = \varepsilon \bar{A}^T \bar{A} - W_1 K_1 - \alpha I$ and the symbol $*$ represents the elements below the main diagonal of a symmetric matrix.

$$(2) \ \alpha + \beta(1/\gamma) < (1/\gamma)^{1/\rho}, \text{ where } \gamma = \max\{c_{ij}^2\},$$

then the trivial solution of system (4) is globally exponentially stable.

Proof. Consider the Lyapunov function

$$V(n, x(n)) = x^T(n) x(n) = \|x(n)\|^2. \quad (35)$$

Similar to the proof of Theorem 5, Theorem 10 can be similarly proved; thus we omit it here. $\square$

Remark 11. From the conditions of Theorem 10, the original impulse-free neural network may not be globally stable. Theorem 10 actually derives the impulsive stabilization result for discrete-time delayed neural networks with uncertainty, and the impulses are key in stabilizing the uncertain system.

Remark 12. Compared with the existing works in [18], the conditions of Theorems 9 and 10 in this paper are easier implementation.

4. Numerical Examples

Example 1. Consider the following discrete-time impulsive neural network:

$$x(n+1) = Ax(n) + Bf(x(n)) + Cg(x(n-\tau)),$$

$$n \neq n_k - 1, \quad (36)$$

$$x(n_k) = C_k x(n_k - 1), \quad k \in \mathbb{Z}^+.$$

The matrices are given as follows:

$$\begin{aligned} A &= \begin{pmatrix} 0.02 & 0 \\ 0 & 0.02 \end{pmatrix}, \quad B = \begin{pmatrix} -0.5 & -0.3 \\ -0.4 & -0.7 \end{pmatrix}, \\ C &= \begin{pmatrix} 0.1 & 0.2 \\ 0.02 & 0.04 \end{pmatrix}, \quad C_k = \begin{pmatrix} 0.69 & 0 \\ 0 & 0.69 \end{pmatrix}, \end{aligned} \quad (37)$$

$$\begin{aligned} x(n) &= (x_1(n), x_2(n))^T, \quad f(x(n)) = (f(x_1(n)), f(x_2(n)))^T, \\ g(x(n-\tau)) &= (g(x_1(n-\tau)), g(x_2(n-\tau)))^T, \quad f_i(s) = g_i(s) = \tanh(s), \quad \tau = 3. \end{aligned}$$

It is easy to obtain that $K_1 = L_1 = 0$, $K_2 = L_2 = 0.5I$. We choose $n_k - n_{k-1} = 4$, and solving LMI (1) in Theorem 7, we get

$$\begin{aligned} W_1 &= \begin{pmatrix} 0.6 & 0 \\ 0 & 1.6 \end{pmatrix}, \quad W_2 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.3 \end{pmatrix}, \\ \alpha &= 0.64, \quad \beta = 0.22, \quad \gamma = 0.461. \end{aligned} \quad (38)$$

Then condition (2) of Theorem 7 holds. Thus, the trivial solution of system (36) is globally exponentially stable. Figure 1 depicts the state trajectories $x_1(n)$ and $x_2(n)$ of system (36) without impulsive effects. Figure 2 depicts the state variables $x_1(n)$ and $x_2(n)$ with impulsive effects.

FIGURE 1: The response curves of system (36) without impulsive effects.

FIGURE 2: The response curves of system (36) with impulsive effects ($\gamma = 0.4761$).

Example 2. Consider the following discrete-time impulsive neural network:

$$\begin{aligned} x(n+1) = & (A + \Delta A(n))x(n) \\ & + (B + \Delta B(n))f(x(n)) \\ & + (C + \Delta C(n))g(x(n-\tau)), \end{aligned} \quad (39)$$

$$n \neq n_k - 1,$$

$$x(n_k) = C_k x(n_k - 1), \quad k \in \mathbb{Z}^+.$$

The matrices are given as follows:

$$\begin{aligned} A &= \begin{pmatrix} 0.1 & 0.1 \\ 0 & 0.2 \end{pmatrix}, & B &= \begin{pmatrix} 0.02 & 0.1 \\ 0 & -0.1 \end{pmatrix}, \\ C &= \begin{pmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{pmatrix}, & C_k &= \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}, \\ \tilde{A} &= \begin{pmatrix} 0.1 & 0 \\ 0 & 0 \end{pmatrix}, & \tilde{B} &= \begin{pmatrix} 0.01 & 0 \\ 0 & 0.1 \end{pmatrix}, \\ \tilde{C} &= \begin{pmatrix} 0 & 0.1 \\ 0 & 0.1 \end{pmatrix}, & \tilde{D} &= \begin{pmatrix} 1 & 0.1 \\ 0 & 1 \end{pmatrix}, \end{aligned} \quad (40)$$

$$\begin{aligned} x(n) &= (x_1(n), x_2(n))^T, \quad f(x(n)) = (f(x_1(n)), f(x_2(n)))^T, \\ g(x(n-\tau)) &= (g(x_1(n-\tau)), g(x_2(n-\tau)))^T, \quad f_i(s) = g_i(s) = \tanh(s), \quad \tau = 2. \end{aligned}$$

FIGURE 3: The response curves of system (39) without impulsive effects.

FIGURE 4: The response curves of system (39) with impulsive effects ($\gamma = 0.0256$).

FIGURE 5: The response curves of system (39) with impulsive effects ($\gamma = 2.7556$).

It is easy to see that $K_1 = L_1 = 0$, $K_2 = L_2 = 0.5I$. We choose $n_k - n_{k-1} = 5$, and solving LMI (1) in Theorem 9, we get

$$W_1 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad W_2 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}. \quad (41)$$

By choosing $c_1 = 0.16$, $c_2 = 0.15$, $\alpha = 0.21$, $\beta = 0.14$, and $\varepsilon = 3.25$, we get $\gamma = 0.0256$. Also by choosing $c_1 = 1.66$ and $c_2 = 1.65$, we derive $\gamma = 2.7556$. Whether $\gamma = 0.0256$ or $\gamma = 2.7556$, condition (2) of Theorem 9 always holds. Thus the trivial solution of system (39) is globally exponentially stable. Figure 3 depicts the state trajectories $x_1(n)$ and $x_2(n)$ of system (39) without impulsive effects. Figures 4 and 5 depict the state variables $x_1(n)$ and $x_2(n)$ with different impulsive effects, respectively.

Remark 13. In Example 1, compared with Figures 1 and 2, we get that impulse effects will make unstable system stable if conditions of Theorem 7 hold. Similarly, according to Example 2, uncertain systems with delay and impulse will also be stable if the conditions of Theorem 9 hold. Consequently, through two examples, we have the following results. On the one hand, when the original dynamic system is stable, we figure out feasible regions of impulses that can keep the stability property of the system. On the other hand, when the original dynamic system is unstable, we use impulses to stabilize it.

5. Conclusions

In this paper, the stability of discrete-time impulsive delayed neural network with and without uncertainty has been investigated. Uncertain parameters take values in some intervals. By applying Lyapunov functions and Razumikhin-type theorems, as well as linear matrix inequalities (LMIs), some new results for achieving exponential stability have been derived. Meanwhile, two examples together with their simulations have been presented to show the effectiveness and the advantage of the presented results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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