Research Article

Robust State Feedback Stabilization of Uncertain Discrete-Time Switched Linear Systems Subject to Actuator Saturation

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The robust stabilization problem is investigated for a class of discrete-time switched linear systems with time-varying norm-bounded uncertainties and saturating actuators by using the multiple Lyapunov functions method. A switching law and a state feedback law are designed to asymptotically stabilize the system with a large domain of attraction. Based on the multiple Lyapunov functions method, sufficient conditions are obtained for robust stabilization. Furthermore, when some parameters are given in advance, the state feedback controllers and the estimation of domain of attraction are presented by solving a convex optimization problem subject to a set of linear matrix inequalities (LMI) constraints. A numerical example is given to show the effectiveness of the proposed technique.

1. Introduction

Switched systems are an important class of hybrid dynamic systems, which consist of a family of continuous-time or discrete-time subsystems and a switching law orchestrating which subsystem to be activated during a certain interval of time. In the last decade, the study of switched systems has received a growing attention in control theory [1–5] and practice [6–9]. The motivation for studying switched systems stems from the fact that many real-world processes and systems can be modeled as switched systems, including chemical processes, computer disk drives, network control, power systems, communication industries, robotic manufacture, and multiple-model systems. A lot of attention is focused on the stability issue of switched systems [1, 2], because stability is the most delicate and important issue in the study. Many analytical approaches and techniques have been developed for this issue, for example, common Lyapunov function approach [10–12], switched Lyapunov function approach [13], multiple Lyapunov functions approach [14–16], and the average dwell-time method [17]. For the switched linear systems, the multiple Lyapunov functions method has been considered largely as one of the most effective tools.

It is well known that nearly all control systems in practice often operate with actuator saturation, since all control devices are limited by inherent physical constraints. Saturation is a source of degradation of system performance, occurrence of limit cycle, more than one equilibrium states some of which may have deterrent stability behavior, and even system instability caused by some large perturbation. Hence, the stability analysis and synthesis for nonswitched control systems subject to actuator saturation nonlinearities have been receiving increased attention for years and decades due to the practical and theoretical importance, which may also be seen in the recent works [18–22]. It is pointed out in [20, 22] that there are two main approaches dealing with actuator saturation nonlinearities. The first strategy is to neglect actuator saturation and design a linear controller that meets the performance specifications in the first stage of the control design process and then add an antiwindup compensator to minimize the influence of saturation [20]. The second strategy is to take actuator saturation into account at the outset of the control design and then design a linear controller which guarantees the asymptotic stability of the system [22]. The main goal in these two strategies is to obtain

a larger estimate of the domain of attraction, despite the presence of saturation.

For switched systems with actuator saturation, the analysis and synthesis become more difficult owing to the phenomena of interacting switching and actuator saturation nonlinearities. In turn, the results for switched systems with actuator saturation are very few. Via utilizing multiple Lyapunov functions method, [23] considered the design of switching scheme for a class of switched linear systems in the presence of actuator saturation. In [24], a switching antiwindup design was proposed for a linear system subject to actuator saturation, which is based on the min-function of multiple quadratic Lyapunov functions. Robust stabilization problem for a class of switched linear systems with saturating actuators and uncertainties was investigated in [25] also via the multiple Lyapunov functions method. These papers are devoted to the continuous-time switched systems subject to input saturation. Reference [26] investigated the stabilization problem for a class of discrete-time linear switched systems subject to actuator saturations via a composite Lyapunov function method. References [27, 28] extended the result in [26] by showing that the same conditions also allow that the union of all the corresponding level sets of functions constitutes a large region of asymptotic stability of the switched system. All the results mentioned above are about the stabilization problem of discrete-time linear switched systems under arbitrary switchings. To the best of the authors' awareness, for uncertain discrete-time switched linear systems subject to actuator saturation no results on the problem of the robust stabilization with resort to the multiple Lyapunov functions method have been reported in the existing literature, which motivates the present study.

This paper studies the problems of robust stability analysis and robustly stabilizing control synthesis for a class of uncertain discrete-time switched linear systems subject to actuator saturation via multiple Lyapunov functions method. Sufficient conditions of asymptotic stability are obtained. Furthermore, the problem of designing linear state feedback controllers which guarantee the stability of the closed-loop system and a larger estimation of domain of attraction is formulated and solved in terms of the solution of a set of linear matrix inequalities (LMIs).

Compared with the existing results for switched systems subject to actuator saturation, there are two distinct features. Firstly, the multiple Lyapunov functions method is used to consider the stability problem of discrete-time switched systems with input saturation for the first time and this problem does not require solvability for every subsystem, while in the existing literature, this problem has been studied by using the switched Lyapunov function method and the solvability of the problem for every subsystem is required; secondly, controllers are designed based on the multiple Lyapunov functions method to achieve better performance (such as large the estimation of domain of attraction) for discrete-time switched systems with input saturation, while the existing literature either only focuses on the analysis, not design, or investigates the design issue that uses the switched Lyapunov function and dwell time methods.

Further this paper is organized as follows: Section 2 introduces the problem formulation. Section 3 presents the main results which consist of the stability analysis, the controller design, and the estimation of domain of attraction. An example is shown in Section 4 to illustrate the feasibility of our results. Finally, conclusions end the paper in Section 5.

2. Problem Statement and Preliminaries

We consider the following class of uncertain discrete-time switched linear systems with input saturation:

$$\begin{aligned} x(k+1) = & (A_{\sigma(k)} + \Delta A_{\sigma(k)})x(k) \\ & + (B_{\sigma(k)} + \Delta B_{\sigma(k)})\text{sat}(u(k)), \end{aligned} \quad (1)$$

where $k \in \mathbb{Z}^+$, $x(k) \in \mathbb{R}^n$ is the state vector, and $u(k) \in \mathbb{R}^m$ is the control input vector. The function $\text{sat} : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is the vector valued standard saturation function defined as

$$\begin{aligned} \text{sat}(u) &= [\text{sat}(u^1), \dots, \text{sat}(u^m)]^T, \\ \text{sat}(u^j) &= \text{sign}(u^j) \min\{1, |u^j|\}, \quad j \in Q_m = \{1, \dots, m\}. \end{aligned} \quad (2)$$

The function $\sigma(k) : \mathbb{Z}^+ \rightarrow I_N = \{1, \dots, N\}$ is a piecewise constant switching signal; $\sigma(k) = i$ means that the i th subsystem is active. A_i, B_i are constant matrices of appropriate dimensions that describe the set of nominal systems. $\Delta A_i, \Delta B_i$ denote unknown matrices with time-varying parameter uncertainties appearing in system matrices and having the following form:

$$[\Delta A_i, \Delta B_i] = T_i \Gamma(k) [F_{1i}, F_{2i}], \quad i \in I_N, \quad (3)$$

where T_i, F_{1i} , and F_{2i} are given constant matrices with proper dimensions which characterize the structure of uncertainties. Matrix $\Gamma(k)$ denotes the unknown time-varying matrix function satisfying

$$\Gamma^T(k) \Gamma(k) \leq I. \quad (4)$$

In this paper, we consider the following linear state feedback control laws:

$$u(k) = F_i x(k), \quad i \in I_N, \quad (5)$$

where F_i are matrices either given beforehand or are to be designed. Then the closed-loop system is

$$x(k+1) = (A_i + \Delta A_i)x(k) + (B_i + \Delta B_i)\text{sat}(F_i x(k)). \quad (6)$$

The following lemmas will be used to develop the main results.

Lemma 1 (see [29]). *Let Y, U, V be given matrices of appropriate dimensions; then for any matrix Γ satisfying $\Gamma^T \Gamma \leq I$,*

$$Y + UTV + V^T \Gamma^T U^T < 0, \quad (7)$$

if and only if there exists a constant $\lambda > 0$ such that

$$Y + \lambda U U^T + \lambda^{-1} V^T V < 0. \quad (8)$$

For a positive definite matrix $P \in R^{n \times n}$ and a scalar $\rho > 0$, an ellipsoid $\Omega(P, \rho)$ is defined as

$$\Omega(P, \rho) = \{x \in R^n : x^T P x \leq \rho\}. \quad (9)$$

Let F^j be the j th row of the matrix $F \in R^{m \times n}$. We define the symmetric polyhedron

$$L(F) = \{x \in R^n : |F^j x| \leq 1, j \in Q_m\}. \quad (10)$$

Let D be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. For example, if $m = 2$, then

$$D = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}. \quad (11)$$

There are 2^m elements in D . Suppose that each element of D is labeled as D_s , $s \in Q = \{1, 2, \dots, 2^m\}$, and denote $D_s^- = I - D_s$. Clearly, D_s^- is also an element of D if $D_s \in D$.

Lemma 2 (see [22]). Let $F, H \in R^{m \times n}$ be given. For any $x \in R^n$, if $x \in L(H)$, then

$$\text{sat}(Fx) \in \text{co} \{D_s Fx + D_s^- Hx, s \in Q\}, \quad (12)$$

where $\text{co}\{\cdot\}$ denotes the convex hull of a set. Consequently, $\text{sat}(Fx)$ can be expressed as

$$\text{sat}(Fx) = \sum_{s=1}^{2^m} \eta_s (D_s F + D_s^- H) x, \quad (13)$$

where $\sum_{s=1}^{2^m} \eta_s = 1$, $0 \leq \eta_s \leq 1$. We note that the parameters η_s in (13) are functions of the state x .

Lemma 3 (Schur's complements). Given the symmetric matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$, the following statements are equivalent:

- (1) $A < 0$;
- (2) $A_{11} < 0$, $A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0$;
- (3) $A_{22} < 0$, $A_{11} - A_{12} A_{22}^{-1} A_{12}^T < 0$.

The objective of this paper is to design a switching law and a state feedback law of the form (5) such that the resulting closed-loop system (6) is locally asymptotically stable at the origin of the state space with a domain of attraction as large as possible.

3. Main Results

In this section, we will develop stability conditions and design stabilizing controllers.

3.1. Stability Analysis. In this subsection, we assume the control laws $u_i = F_i x$ are known beforehand. Then, we give a sufficient condition for stability of switched system (6) by means of the multiple Lyapunov functions method.

Theorem 4. Suppose that there exist positive definite matrices P_i , matrices H_i , and a set of scalars $\beta_{ir} \geq 0$, $\lambda_i > 0$ such that

$$\begin{bmatrix} -P_i + \sum_{r=1, r \neq i}^N \beta_{ir} (P_r - P_i) & * & * & * \\ P_i [A_i + B_i (D_s F_i + D_s^- H_i)] & -P_i & * & * \\ 0 & T_i^T P_i & -\lambda_i^{-1} I & * \\ F_{1i} + F_{2i} (D_s F_i + D_s^- H_i) & 0 & 0 & -\lambda_i I \end{bmatrix} < 0, \quad (14)$$

$i \in I_N, \quad s \in Q.$

Moreover, assume $\Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$. Then, the switched system (6) is robustly asymptotically stable at the origin with $\cup_{i=1}^N (\Omega(P_i, 1) \cap \Phi_i)$ contained inside the domain of attraction under the state dependent switching law

$$\sigma(k) = \text{argmin} \{x^T(k) P_i x(k), i \in I_N\}, \quad (15)$$

where $\Phi_i = \{x(k) \in R^n : x^T(k) (P_r - P_i) x(k) \geq 0, \forall r \in I_N, r \neq i\}$.

Proof. By virtue of Lemma 2, for every $x(k) \in \Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$,

$$\text{sat}(F_i x(k)) \in \text{co} \{D_s F_i x(k) + D_s^- H_i x(k), s \in Q\}. \quad (16)$$

It follows that

$$\begin{aligned} & (A_i + \Delta A_i) x(k) + (B_i + \Delta B_i) \text{sat}(F_i x(k)) \\ & \in \text{co} \{ (A_i + \Delta A_i) x(k) + (B_i + \Delta B_i) \\ & \quad \cdot (D_s F_i x(k) + D_s^- H_i x(k)), s \in Q \}. \end{aligned} \quad (17)$$

In view of the switching law (15), for $\forall x(k) \in \Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$, the i th subsystem is active.

Choose the Lyapunov function candidate for system (6) as

$$V(x(k)) = V_{\sigma(k)}(x(k)) = x^T(k) P_{\sigma(k)} x(k). \quad (18)$$

We split the proof into two parts.

Case 1. When $\sigma(k+1) = \sigma(k) = i$, for $\forall x(k) \in \Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$, it follows that

$$\begin{aligned} \Delta V(x(k)) &= V_i(x(k+1)) - V_i(x(k)) \\ &= x^T(k+1) P_i x(k+1) - x^T(k) P_i x(k) \\ &\leq \max_{s \in Q} x^T(k) [(A_i + \Delta A_i) + (B_i + \Delta B_i) \\ &\quad \cdot (D_s F_i + D_s^- H_i)]^T \\ &\quad \cdot P_i [(A_i + \Delta A_i) + (B_i + \Delta B_i) \\ &\quad \cdot (D_s F_i + D_s^- H_i)] x(k) - x^T(k) P_i x(k). \end{aligned} \quad (19)$$

Case 2. When $\sigma(k) = i$, $\sigma(k+1) = r$, and $i \neq r$, for $\forall x(k) \in \Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$, using switching law (15), we have

$$\begin{aligned} \Delta V(x(k)) &= x^T(k+1) P_r x(k+1) - x^T(k) P_i x(k) \\ &\leq x^T(k+1) P_i x(k+1) - x^T(k) P_i x(k). \end{aligned} \quad (20)$$

From Cases 1 and 2, we obtain

$$\begin{aligned} \Delta V(x(k)) &\leq \max_{s \in Q} x^T(k) [(A_i + \Delta A_i) + (B_i + \Delta B_i) \\ &\quad \cdot (D_s F_i + D_s^- H_i)]^T \\ &\quad \cdot P_i [(A_i + \Delta A_i) + (B_i + \Delta B_i) \\ &\quad \cdot (D_s F_i + D_s^- H_i)] x(k) - x^T(k) P_i x(k), \\ &\quad \forall x(k) \in \bigcup_{i=1}^N (\Omega(P_i, 1) \cap \Phi_i). \end{aligned} \quad (21)$$

Then, by Lemma 3, (14) is equivalent to

$$\begin{aligned} &\begin{bmatrix} -P_i + \sum_{r=1, r \neq i}^N \beta_{ir} (P_r - P_i) & * \\ P_i [A_i + B_i (D_s F_i + D_s^- H_i)] & -P_i \end{bmatrix} + \lambda_i \begin{bmatrix} 0 \\ P_i T_i \end{bmatrix} \begin{bmatrix} 0 & T_i^T P_i \end{bmatrix} \\ &\quad + \lambda_i^{-1} \begin{bmatrix} [F_{1i} + F_{2i} (D_s F_i + D_s^- H_i)]^T \\ 0 \end{bmatrix} \\ &\quad \cdot [F_{1i} + F_{2i} (D_s F_i + D_s^- H_i) \quad 0] < 0. \end{aligned} \quad (22)$$

Thus, using Lemma 1 we have

$$\begin{bmatrix} -P_i + \sum_{r=1, r \neq i}^N \beta_{ir} (P_r - P_i) & * \\ P_i [(A_i + \Delta A_i) + (B_i + \Delta B_i) (D_s F_i + D_s^- H_i)] & -P_i \end{bmatrix} < 0. \quad (23)$$

Again, from the Lemma 3, we get

$$\begin{aligned} &[(A_i + \Delta A_i) + (B_i + \Delta B_i) (D_s F_i + D_s^- H_i)]^T \\ &\quad \cdot P_i [(A_i + \Delta A_i) + (B_i + \Delta B_i) (D_s F_i + D_s^- H_i)] \end{aligned} \quad (24)$$

$$-P_i + \sum_{r=1, r \neq i}^N \beta_{ir} (P_r - P_i) < 0.$$

Then, in view of the switching law (15),

$$\begin{aligned} \Delta V(x(k)) &\leq \max_{s \in Q} x^T(k) \\ &\quad \cdot [(A_i + \Delta A_i) + (B_i + \Delta B_i) \\ &\quad \cdot (D_s F_i + D_s^- H_i)]^T \\ &\quad \cdot P_i [(A_i + \Delta A_i) + (B_i + \Delta B_i) \\ &\quad \cdot (D_s F_i + D_s^- H_i)] x(k) - x^T(k) P_i x(k) \\ &< - \sum_{r=1, r \neq i}^N \beta_{ir} x^T(k) (P_r - P_i) x(k) \\ &\leq 0, \end{aligned} \quad (25)$$

which indicates $\Delta V(x(k)) < 0$.

Therefore, in view of multiple Lyapunov functions methodology, the switched system (6) is asymptotically stable for all initial states $x_0 \in \bigcup_{i=1}^N (\Omega(P_i, 1) \cap \Phi_i)$ and admissible uncertainties described by (3) and (4). This completes the proof. $\square$

3.2. Controller Design. In this subsection, we study how to design state feedback controllers such that the class of switched system with actuator saturation (6) is robustly stable.

Theorem 5. If there exist positive definite matrices X_i , matrices M_i , N_i , and a set of scalars $\beta_{ir} \geq 0$, $\delta_{ir} > 0$, and $\lambda_i > 0$ such that

$$\begin{bmatrix} -X_i - \sum_{r=1, r \neq i}^N \beta_{ir} X_i & * & * & * & * & * & * \\ A_i X_i + B_i (D_s M_i + D_s^- N_i) & -X_i & * & * & * & * & * \\ 0 & T_i^T & -\lambda_i^{-1} I & * & * & * & * \\ F_{1i} X_i + F_{2i} (D_s M_i + D_s^- N_i) & 0 & 0 & -\lambda_i I & * & * & * \\ X_i & 0 & 0 & 0 & -\beta_{i1}^{-1} X_1 & * & * \\ X_i & 0 & 0 & 0 & 0 & \ddots & * \\ X_i & 0 & 0 & 0 & 0 & 0 & -\beta_{iN}^{-1} X_N \end{bmatrix} < 0, \quad (26)$$

$$\begin{bmatrix} X_i + \sum_{r=1, r \neq i}^N \delta_{ir} X_i & * & * & * & * \\ N_i^j & 1 & * & * & * \\ X_i & 0 & \delta_{i1}^{-1} X_1 & * & * \\ X_i & 0 & 0 & \ddots & * \\ X_i & 0 & 0 & 0 & \delta_{iN}^{-1} X_N \end{bmatrix} \geq 0, \quad i \in I_N, s \in Q, j \in Q_m, \quad (27)$$

where N_i^j denotes the j th row of N_i , then, under the controllers $F_i = M_i X_i^{-1}$ and the state dependent switching law

$$\sigma(k) = \operatorname{argmin} \{x^T(k) X_i^{-1} x(k), i \in I_N\}, \quad (28)$$

the closed-loop switched system (6) with input saturation is robustly asymptotical stable at the origin $\forall x_0 \in \cup_{i=1}^N (\Omega(X_i^{-1}, 1) \cap \Phi_i)$.

Proof. By Lemma 3, (26) is equivalent to

$$\begin{bmatrix} -X_i + \sum_{r=1, r \neq i}^N \beta_{ir} (X_i X_N^{-1} X_i - X_i) & * & * & * \\ A_i X_i + B_i (D_s M_i + D_s^- N_i) & -X_i & * & * \\ 0 & T_i^T & -\lambda_i^{-1} I & * \\ F_{1i} X_i + F_{2i} (D_s M_i + D_s^- N_i) & 0 & 0 & -\lambda_i I \end{bmatrix} < 0. \quad (29)$$

Let $M_i = F_i X_i$, $N_i = H_i X_i$, and $X_i = P_i^{-1}$. Then, pre- and postmultiplying both sides of inequality (29) by *block-diagonal* $\{P_i, P_i, I, I\}$, we have

$$\begin{bmatrix} -P_i + \sum_{r=1, r \neq i}^N \beta_{ir} (P_r - P_i) & * & * & * \\ P_i [A_i + B_i (D_s F_i + D_s^- H_i)] & -P_i & * & * \\ 0 & T_i^T P_i & -\lambda_i^{-1} I & * \\ F_{1i} + F_{2i} (D_s F_i + D_s^- H_i) & 0 & 0 & -\lambda_i I \end{bmatrix} < 0, \quad (30)$$

which is exactly (14) in Theorem 4.

Applying a similar method to inequality (27), we can also obtain

$$\begin{bmatrix} 1 & H_i^j \\ * & P_i - \sum_{r=1, r \neq i}^N \delta_{ir} (P_r - P_i) \end{bmatrix} \geq 0, \quad (31)$$

where H_i^j denotes the j th row of H_i .

Then, we can show that $\Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$ is implied by (31). In fact, if we let

$$G_i = P_i - \sum_{r=1, r \neq i}^N \delta_{ir} (P_r - P_i), \quad (32)$$

since $x^T G_i x \leq 1$ and $H_i^j G_i^{-1} H_i^{jT} \leq 1$, it holds that

$$2x^T H_i^j T \leq x^T G_i x + H_i^j G_i^{-1} H_i^{jT} \leq 2; \quad (33)$$

therefore (31) implies $\Omega(P_i, 1) \cap \Phi_i \subset L(H_i)$.

Since $P_i = X_i^{-1}$, the switching rule (28) is the same as (15) of Theorem 4. Thus, the proof is completed. $\square$

Remark 6. When each subsystem of (6) is unstable under actuator saturation, stability of (6) can still be achieved as long as the conditions of Theorem 4 or Theorem 5 are satisfied. This obviously enlarges the scope of systems that can be stabilized subject to actuator saturation.

3.3. Estimation of Domain of Attraction. In this subsection, we will design state feedback controllers and choose P_i 's such that the estimated domain of attraction of the closed-loop system (6) is maximized with respect to a given shape reference set X_R .

Let $X_R \subset R^n$ be a prescribed bounded convex set containing the origin. For a set $\Xi \subset R^n$ which contains the origin, define [22]

$$\alpha_R(\Xi) := \sup \{\alpha > 0 : \alpha X_R \subset \Xi\}. \quad (34)$$

Obviously, if $\alpha_R(\Xi) \geq 1$, then $X_R \subset \Xi$. Thus, $\alpha_R(\Xi)$ provides a kind of measure of the estimated domain of attraction. Two typical types of X_R are the ellipsoid

$$X_R = \{x \in R^n : x^T R x \leq 1, R > 0\} \quad (35)$$

and the polyhedron

$$X_R = \operatorname{co} \{x_1, x_2, \dots, x_l\}, \quad (36)$$

where $x_1, x_2, \dots, x_l$ are a priori given points in R^n .

As a result, the determination of the largest $\cup_{i=1}^N (\Omega(X_i^{-1}, 1) \cap \Phi_i)$ inside the domain of attraction can be formulated as the following constrained optimization problem:

$$\begin{aligned} & \sup_{X_i, M_i, N_i, \beta_{ir}, \delta_{ir}, \lambda_i} \alpha, \\ & \text{s.t.} \quad (a) \quad \alpha X_R \subset \Omega(X_i^{-1}, 1), \quad i \in I_N, \\ & \quad (b) \quad \text{inequality (26)}, \quad i \in I_N, s \in Q, \\ & \quad (c) \quad \text{inequality (27)}, \quad i \in I_N, j \in Q_m. \end{aligned} \quad (37)$$

Here, we choose X_R as an ellipsoid; then (a) is equivalent to

$$\begin{bmatrix} \frac{1}{\alpha^2} R & I \\ I & X_i \end{bmatrix} \geq 0. \quad (38)$$

Let $1/\alpha^2 = \gamma$; the optimization problem (37) can be rewritten as

$$\begin{aligned} & \inf_{X_i, M_i, N_i, \beta_{ir}, \delta_{ir}, \lambda_i} \gamma, \\ \text{s.t.} \quad & (a) \begin{bmatrix} \gamma R & I \\ I & X_i \end{bmatrix} \geq 0, \quad i \in I_N, \\ & (b) \text{ inequality (26)}, \quad i \in I_N, s \in Q, \\ & (c) \text{ inequality (27)}, \quad i \in I_N, j \in Q_m. \end{aligned} \quad (39)$$

Remark 7. If the parameters β_{ir} , δ_{ir} , and λ_i are given in advance, the state feedback controllers and the estimation of domain of attraction are formulated and solved as a set of linear matrix inequality (LMI) optimization problem with respect to other unknown matrix variables.

4. An Illustrative Example

In the section, an example is given to illustrate the validity of the results in Section 3.

We consider the following uncertain discrete-time switched linear system subject to actuator saturation:

$$\begin{aligned} x(k+1) = & (A_{\sigma(k)} + \Delta A_{\sigma(k)})x(k) \\ & + (B_{\sigma(k)} + \Delta B_{\sigma(k)})\text{sat}(u(k)), \end{aligned} \quad (40)$$

where $\sigma(k) \in I_2 = \{1, 2\}$,

$$A_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad (41)$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

the uncertain term $[\Delta A_i, \Delta B_i] = T_i \Gamma(k) [F_{1i}, F_{2i}]$ with

$$\begin{aligned} T_1 &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad F_{11} = \begin{bmatrix} 0.15 \\ -0.3 \end{bmatrix}^T, \\ F_{12} &= \begin{bmatrix} -0.3 \\ 0.2 \end{bmatrix}^T, \end{aligned} \quad (42)$$

$$F_{12} = -7, \quad F_{22} = -0.2, \quad \Gamma(K) = \sin(k).$$

The two subsystems are obviously unstable, and it is also easy to see that none of two subsystems subject to actuator saturation can be individually stabilized via state feedback for all admissible uncertainties. Now, we are going to design

FIGURE 1: The state response of subsystem 1.

a switching law and state feedback controllers to stabilize the switched system (40) with a large domain of attraction.

Let $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\beta_1 = \beta_2 = 30$, $\delta_1 = \delta_2 = 10$, and $\lambda_1 = \lambda_2 = 10$. Solving the optimization problem (39) results in

$$\alpha = 18.6842,$$

$$X_1 = \begin{bmatrix} 10.9278 & -0.0026 \\ 0.0026 & 10.8016 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 10.7819 & 0.0038 \\ 0.0038 & 10.9450 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.1890 \\ -0.4744 \end{bmatrix}^T, \quad M_2 = \begin{bmatrix} -0.5439 \\ -1.7525 \end{bmatrix}^T,$$

$$N_1 = \begin{bmatrix} 0.1109 \\ -0.3708 \end{bmatrix}^T, \quad N_2 = \begin{bmatrix} -0.3472 \\ 0.1630 \end{bmatrix}^T,$$

$$P_1 = \begin{bmatrix} 0.0915 & 0.0000 \\ 0.0000 & 0.0926 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.0927 & -0.0000 \\ -0.0000 & 0.0914 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.0173 \\ -0.0439 \end{bmatrix}^T, \quad F_2 = \begin{bmatrix} -0.0504 \\ -0.1601 \end{bmatrix}^T,$$

$$H_1 = \begin{bmatrix} 0.0101 \\ -0.0343 \end{bmatrix}^T, \quad H_2 = \begin{bmatrix} -0.0322 \\ 0.0149 \end{bmatrix}^T. \quad (43)$$

It is easy to verify that none of two controllers makes the associated subsystem stabilizable (see Figures 1 and 2). However, the switched system (40) is stabilizable under the designed switching law together with the state feedback controllers. The state trajectory with the initial state $x(0) = [1 \ -1]^T$ and the input signal are depicted in Figures 3 and 4, respectively.

FIGURE 2: The state response of subsystem 2.

FIGURE 3: The state response of system (40).

In addition, it is worth noticing that if we neglect actuator saturation and design a linear controller, the obtained optimal solutions are

$$\alpha = 10.4807, \quad (44)$$

$$F_1 = \begin{bmatrix} 0.0088 \\ -0.0421 \end{bmatrix}^T, \quad F_2 = \begin{bmatrix} -0.0043 \\ -0.0244 \end{bmatrix}^T,$$

which indicates that, by using the proposed method, the estimation of domain of attraction of the closed loop system is enlarged.

FIGURE 4: The input signal of system (40).

5. Conclusions

This paper has studied the robust stability and robust stabilization for a class of uncertain discrete-time switched linear systems subject to actuator saturation. Via multiple Lyapunov functions approach, sufficient conditions of asymptotic stability have been established. Moreover, if some parameters are selected in advance, the design problem of the robust state feedback control law that maximizes an estimation of the domain of attraction of the considered systems is presented by solving a convex optimization problem with a set of LMI constraints.

It should be noted that due to the complexity of dealing with simultaneous switching and actuator saturation nonlinearity, the proposed method may also cause some conservatism. Thus, how to design a switching law and controllers and to select some parameters to improve the performance of switched systems is a challenging issue which deserves further study in the future.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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