Research Article

Dynamical Behavior of a System of Third-Order Rational Difference Equation

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This paper deals with the boundedness, persistence, and global asymptotic stability of positive solution for a system of third-order rational difference equations $x_{n+1} = A + x_n/y_{n-1}y_{n-2}$, $y_{n+1} = A + y_n/x_{n-1}x_{n-2}$, $n = 0, 1, \dots$, where $A \in (0, \infty)$, $x_{-i} \in (0, \infty)$; $y_{-i} \in (0, \infty)$, $i = 0, 1, 2$. Some examples are given to demonstrate the effectiveness of the results obtained.

1. Introduction

It is known that difference equation appears naturally as discrete analogous and as numerical solutions of differential equation and delay differential equation having many applications in economics, biology, computer science, control engineering, and so forth. The study of discrete dynamical systems described by difference equations has now been paid great attention by many mathematical researchers. Particularly, the persistence, boundedness, local asymptotic stability, global character, and the existence of positive periodic solutions can be discussed in many papers.

In 1998, DeVault et al. [1] proved every positive solution of the difference equation:

$$x_{n+1} = A + \frac{x_n}{x_{n-1}}, \quad n = 0, 1, \dots, \quad (1)$$

where $A \in (0, \infty)$ oscillates about the positive equilibrium $c = 1 + A$ of (1). Moreover every positive solution of (1) is bounded away from zero and infinity. Also the positive equilibrium of (1) is globally asymptotically stable.

In 2003, Abu-Saris and DeVault [2] studied the following recursive difference equation:

$$x_{n+1} = A + \frac{x_n}{x_{n-k}}, \quad n = 0, 1, \dots, \quad (2)$$

where $A \in (1, +\infty)$, $x_{-k}, x_{-k+1}, \dots, x_0$ are positive real numbers. For similar results the reader can refer to [3–9].

Difference equations or discrete dynamical systems are a diverse field which impact almost every branch of pure and applied mathematics. We refer to [10, 11] for basic theory of difference equations and rational difference equations. It is very interesting to investigate the qualitative behavior of the discrete dynamical systems of nonlinear difference equations. Recently there has been a lot of work concerning the global asymptotic stability, the periodicity, and the boundedness of nonlinear difference equations. Moreover similar results in [12–17] have been derived for systems of two nonlinear difference equations.

Papaschinopoulos and Schinas [12] investigated the global behavior for a system of the following two nonlinear difference equations:

$$x_{n+1} = A + \frac{y_n}{x_{n-p}}, \quad y_{n+1} = A + \frac{x_n}{y_{n-q}}, \quad n = 0, 1, \dots, \quad (3)$$

where A is a positive real number, p, q are positive integers, and $x_{-p}, \dots, x_0, y_{-q}, \dots, y_0$ are positive real numbers.

In 2005, Yang [13] studied the global behavior of the following system:

$$x_n = A + \frac{y_{n-1}}{x_{n-p}y_{n-q}}, \quad y_n = A + \frac{x_{n-1}}{x_{n-r}y_{n-s}}, \quad n = 1, 2, \dots, \quad (4)$$

where $p \geq 2, q \geq 2, r \geq 2, s \geq 2$, A is a positive constant, and initial values $x_{1-\max\{p,r\}}, x_{2-\max\{p,r\}}, \dots, x_0, y_{1-\max\{q,s\}}, \dots, y_0$ are positive real numbers.

In 2012, Zhang et al. [14] investigated the global behavior for a system of the following third-order nonlinear difference equations:

$$x_{n+1} = \frac{x_{n-2}}{B + y_{n-2}y_{n-1}y_n}, \quad y_{n+1} = \frac{y_{n-2}}{A + x_{n-2}x_{n-1}x_n}, \quad (5)$$

where $A, B \in (0, \infty)$, and the initial values $x_{-i}, y_{-i} \in (0, \infty)$, $i = 0, 1, 2$.

Motivated by the above mentioned discussion, in this paper we study the behavior of solutions of the following system:

$$\begin{aligned} x_{n+1} &= A + \frac{x_n}{y_{n-1}y_{n-2}}, \\ y_{n+1} &= A + \frac{y_n}{x_{n-1}x_{n-2}}, \\ n &= 0, 1, \dots, \end{aligned} \quad (6)$$

where A is a positive constant. And initial conditions $x_{-i}, y_{-i} \in (0, \infty)$, $i = 0, 1, 2$.

Clearly, if $A > 0$, system (6) has always positive equilibrium. Consider

$$(c, c) = \left(\frac{A + \sqrt{A^2 + 4}}{2}, \frac{A + \sqrt{A^2 + 4}}{2} \right). \quad (7)$$

If $1 < A < 2/\sqrt{3}$, system (6) has two additional positive equilibria:

$$\begin{aligned} (a_1, b_1) &= \left(\frac{2}{A - \sqrt{4 - 3A^2}}, \frac{2}{A + \sqrt{4 - 3A^2}} \right), \\ (a_2, b_2) &= \left(\frac{2}{A + \sqrt{4 - 3A^2}}, \frac{2}{A - \sqrt{4 - 3A^2}} \right). \end{aligned} \quad (8)$$

Our results in this paper are listed below:

- (i) If $A > 1$, every positive solution of system (6) is bounded.
- (ii) If $A > 2/\sqrt{3}$, (c, c) is locally asymptotically stable.
- (iii) If $A > \sqrt{3}$, every positive solution of (6) approaches (c, c) .
- (iv) If $1 < A < 2/\sqrt{3}$, (a_1, b_1) and (a_2, b_2) are locally asymptotically stable.

2. Boundedness

Theorem 1. Let $\{(x_n, y_n)\}$ be a positive solution of (6). Then the following statements hold:

- (i) $x_n \geq A$ and $y_n \geq A$ for all $n \geq 1$.
- (ii) If $A > 1$, then, for $k \geq 4$, we have the following:

$$\begin{aligned} x_k &\leq \frac{1}{(A^2)^{k-3}} \left(x_3 - \frac{A^3}{A^2 - 1} \right) + \frac{A^3}{A^2 - 1}, \\ y_k &\leq \frac{1}{(A^2)^{k-3}} \left(y_3 - \frac{A^3}{A^2 - 1} \right) + \frac{A^3}{A^2 - 1}. \end{aligned} \quad (9)$$

Proof. Assertion (i) is obviously true. We now prove assertion (ii). From (6) and in view of (i), we have, for all $k \geq 4$, that

$$\begin{aligned} x_k &= A + \frac{x_{k-1}}{y_{k-2}y_{k-3}} \leq A + \frac{1}{A^2}x_{k-1}, \\ y_k &= A + \frac{y_{k-1}}{x_{k-2}x_{k-3}} \leq A + \frac{1}{A^2}y_{k-1}. \end{aligned} \quad (10)$$

Let v_k, w_k be the solution of following system, respectively,

$$v_k = A + \frac{1}{A^2}v_{k-1}, \quad w_k = A + \frac{1}{A^2}w_{k-1}, \quad (11)$$

such that

$$x_3 = v_3, \quad y_3 = w_3. \quad (12)$$

We prove by induction that

$$x_k \leq v_k, \quad y_k \leq w_k, \quad k \geq 4. \quad (13)$$

Suppose that (13) is true for $k = m \geq 4$. Then from (10) we get the following:

$$\begin{aligned} x_{m+1} &\leq A + \frac{1}{A^2}x_m \leq A + \frac{1}{A^2}v_m = v_{m+1}, \\ y_{m+1} &\leq A + \frac{1}{A^2}y_m \leq A + \frac{1}{A^2}w_m = w_{m+1}. \end{aligned} \quad (14)$$

Therefore (13) is true. From (11) we have the following:

$$\begin{aligned} v_k &= \frac{1}{(A^2)^{k-3}} \left(x_3 - \frac{A^3}{A^2 - 1} \right) + \frac{A^3}{A^2 - 1}, \\ w_k &= \frac{1}{(A^2)^{k-3}} \left(y_3 - \frac{A^3}{A^2 - 1} \right) + \frac{A^3}{A^2 - 1}, \end{aligned} \quad (15)$$

$k \geq 4$.

Then from (10), (13), and (14) the proof of the relation (9) follows immediately. $\square$

3. Stability

Theorem 2. Assume that $A > 2/\sqrt{3}$. Then the equilibrium $(c, c) = ((A + \sqrt{A^2 + 4})/2, (A + \sqrt{A^2 + 4})/2)$ is locally asymptotically stable.

Proof. We can obtain easily that the linearized system of (6) about the positive equilibrium (c, c) is

$$\Psi_{n+1} = B\Psi_n, \quad (16)$$

where

$$\Psi_n = \begin{pmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ y_n \\ y_{n-1} \\ y_{n-2} \end{pmatrix},$$

$$B = \begin{pmatrix} c^{-2} & 0 & 0 & 0 & -c^{-2} & -c^{-2} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -c^{-2} & -c^{-2} & c^{-2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

(17)

Let $\lambda_1, \lambda_2, \dots, \lambda_6$ denote the eigenvalues of matrix B , let $D = \text{diag}(d_1, d_2, \dots, d_6)$ be a diagonal matrix, where $d_1 = d_4 = 1$, $d_k = 1 - k\varepsilon$ ($k = 2, 3, 5, 6$), and

$$0 < \varepsilon < \min \left\{ \frac{1}{6}, \frac{c^2 - 3}{6c^2} \right\}. \quad (18)$$

Clearly, D is invertible. Computing matrix DBD^{-1} , we obtain that

$$DBD^{-1} = \begin{pmatrix} c^{-2} & 0 & 0 & 0 & -c^{-2}d_1d_5^{-1} & -c^{-2}d_1d_6^{-1} \\ d_2d_1^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & d_3d_2^{-1} & 0 & 0 & 0 & 0 \\ 0 & -c^{-2}d_4d_2^{-1} & -c^{-2}d_4d_3^{-1} & c^{-2} & 0 & 0 \\ 0 & 0 & 0 & d_5d_4^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_6d_5^{-1} & 0 \end{pmatrix}. \quad (19)$$

From $d_1 > d_2 > d_3 > 0$ and $d_4 > d_5 > d_6 > 0$ imply that

$$d_2d_1^{-1} < 1, \quad d_3d_2^{-1} < 1, \quad d_5d_4^{-1} < 1, \quad d_6d_5^{-1} < 1. \quad (20)$$

Furthermore,

$$\begin{aligned} c^{-2} + c^{-2}d_1d_5^{-1} + c^{-2}d_1d_6^{-1} &= c^{-2}(1 + d_1d_5^{-1} + d_1d_6^{-1}) \\ &= c^{-2}\left(1 + \frac{1}{1-5\varepsilon} + \frac{1}{1-6\varepsilon}\right) \\ &< c^{-2}\frac{3}{1-6\varepsilon} < 1, \\ c^{-2} + c^{-2}d_4d_2^{-1} + c^{-2}d_4d_3^{-1} &= c^{-2}(1 + d_4d_2^{-1} + d_4d_3^{-1}) \\ &= c^{-2}\left(1 + \frac{1}{1-2\varepsilon} + \frac{1}{1-3\varepsilon}\right) \\ &< c^{-2}\frac{3}{1-3\varepsilon} < c^{-2}\frac{3}{1-6\varepsilon} < 1. \end{aligned} \quad (21)$$

It is well known that B has the same eigenvalues as DBD^{-1} : we have that

$$\begin{aligned} \max_{1 \leq i \leq 6} |\lambda_i| &\leq \|DBD^{-1}\|_{\infty} \\ &= \max \{d_2d_1^{-1}, d_3d_2^{-1}, d_5d_4^{-1}, d_6d_5^{-1}, \\ &\quad c^{-2}(1 + d_1d_5^{-1} + d_1d_6^{-1}), \\ &\quad c^{-2}(1 + d_4d_2^{-1} + d_4d_3^{-1})\} < 1. \end{aligned} \quad (22)$$

This implies that the equilibrium (c, c) of (6) is locally asymptotically stable.

To examine the global attractivity of (c, c) , we need the following result. $\square$

Lemma 3. Assume $\alpha > \beta > \sqrt{3}$. Then $\alpha^3/(\alpha^2 - 1) > \beta^3/(\beta^2 - 1)$.

Proof. Consider the function $f(x) = x^3/(x^2 - 1)$. For $x > \sqrt{3}$, we have

$$f'(x) = \frac{x^2(x + \sqrt{3})(x - \sqrt{3})}{(x^2 - 1)^2} > 0. \quad (23)$$

So $f(x)$ is strictly increasing for $x > \sqrt{3}$. We derive that $\alpha^3/(\alpha^2 - 1) > \beta^3/(\beta^2 - 1)$. $\square$

Theorem 4. Assume that $A > \sqrt{3}$. Then every positive solution of (6) converges to (c, c) .

Proof. Let $\{x_n, y_n\}$ be an arbitrary positive solution of (6). Let

$$\begin{aligned}\Lambda_1 &= \lim_{n \rightarrow \infty} \sup \{x_n, x_{n+1}, \dots\}, \\ \lambda_1 &= \lim_{n \rightarrow \infty} \inf \{x_n, x_{n+1}, \dots\}, \\ \Lambda_2 &= \lim_{n \rightarrow \infty} \sup \{y_n, y_{n+1}, \dots\}, \\ \lambda_2 &= \lim_{n \rightarrow \infty} \inf \{y_n, y_{n+1}, \dots\}.\end{aligned}\quad (24)$$

From Theorem 1, we have $0 < A \leq \lambda_1 \leq \Lambda_1 < \infty$, $0 < A \leq \lambda_2 \leq \Lambda_2 < \infty$. The previous and (6) imply that

$$\begin{aligned}\Lambda_1 &\leq A + \frac{\Lambda_1}{\lambda_1^2}, & \Lambda_2 &\leq A + \frac{\Lambda_2}{\lambda_2^2}, \\ \lambda_1 &\geq A + \frac{\lambda_1}{\Lambda_1^2}, & \lambda_2 &\geq A + \frac{\lambda_2}{\Lambda_2^2},\end{aligned}\quad (25)$$

which can derive that

$$\begin{aligned}\Lambda_1 \Lambda_2^2 (\lambda_2^2 - 1) &\leq \lambda_1 \lambda_2^2 (\Lambda_2^2 - 1), \\ \Lambda_2 \Lambda_1^2 (\lambda_1^2 - 1) &\leq \lambda_2 \lambda_1^2 (\Lambda_1^2 - 1).\end{aligned}\quad (26)$$

It follows that $\Lambda_1^3 \Lambda_2^3 (\lambda_1^2 - 1)(\lambda_2^2 - 1) \leq \lambda_1^3 \lambda_2^3 (\Lambda_1^2 - 1)(\Lambda_2^2 - 1)$; namely,

$$\frac{\Lambda_1^3 \Lambda_2^3}{(\Lambda_1^2 - 1)(\Lambda_2^2 - 1)} \leq \frac{\lambda_1^3 \lambda_2^3}{(\lambda_1^2 - 1)(\lambda_2^2 - 1)}.\quad (27)$$

If $\Lambda_1 > \lambda_1$ and $\Lambda_2 > \lambda_2$, it follows from Lemma 3 and the condition $A > \sqrt{3}$ that $\Lambda_1^3 \Lambda_2^3 / (\Lambda_1^2 - 1)(\Lambda_2^2 - 1) > \lambda_1^3 \lambda_2^3 / (\lambda_1^2 - 1)(\lambda_2^2 - 1)$. Then a contradiction occurs. Thus we have either $\Lambda_1 = \lambda_1$ or $\Lambda_2 = \lambda_2$.

We assume $\Lambda_1 = \lambda_1$ (the discussion for the case $\Lambda_2 = \lambda_2$ is similar). Then $\lim_{n \rightarrow \infty} x_n$ exists. Using (27) and in view of $\Lambda_1 = \lambda_1$, it is obvious that $\Lambda_2 \leq \lambda_2$, so $\Lambda_2 = \lambda_2$. Then $\lim_{n \rightarrow \infty} y_n$ exists. From the uniqueness of the positive

equilibrium (c, c) of (6), we conclude that $\lim_{n \rightarrow \infty} x_n = c$, $\lim_{n \rightarrow \infty} y_n = c$.

Combining Theorems 2 and 4, we obtain the following theorem. $\square$

Theorem 5. Assume that $A > \sqrt{3}$. Then the positive equilibrium (c, c) of (6) is globally asymptotically stable for all positive solutions.

Theorem 6. If $1 < A < 2/\sqrt{3}$, then the equilibria (a_1, b_1) and (a_2, b_2) are locally asymptotically stable.

Proof. The proof is similar to the proof of Theorem 2. We can obtain easily that the linearized system of (6) about the positive equilibrium (a_1, b_1) is

$$\Psi_{n+1} = H\Psi_n, \quad (28)$$

where

$$\begin{aligned}\Psi_n &= \begin{pmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ y_n \\ y_{n-1} \\ y_{n-2} \end{pmatrix}, \\ H &= \begin{pmatrix} a_1^{-2} & 0 & 0 & 0 & -a_1 b_1^{-3} & -a_1 b_1^{-3} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -b_1 a_1^{-3} & -b_1 a_1^{-3} & a_1^{-2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.\end{aligned}\quad (29)$$

Let $\lambda'_1, \lambda'_2, \dots, \lambda'_6$ denote the eigenvalues of matrix H , let $D = \text{diag}(d_1, d_2, \dots, d_6)$ be a diagonal matrix, where $d_1 = d_4 = 1$, $d_k = 1 - k\varepsilon$ ($k = 2, 3, 5, 6$), and

$$0 < \varepsilon' < \min \left\{ \frac{1}{6}, \frac{a_1^2 - 3}{6a_1^2}, \frac{b_1^3 - 3a_1}{6b_1^2} \right\}. \quad (30)$$

Clearly, H is invertible. Computing matrix DHD^{-1} , we obtain that

$$DHD^{-1} = \begin{pmatrix} b_1^{-2} & 0 & 0 & 0 & -a_1 b_1^{-3} d_1 d_5^{-1} & -a_1 b_1^{-3} d_1 d_6^{-1} \\ d_2 d_1^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & d_3 d_2^{-1} & 0 & 0 & 0 & 0 \\ 0 & -b_1 a_1^{-3} d_4 d_2^{-1} & -b_1 a_1^{-3} d_4 d_3^{-1} & a_1^{-2} & 0 & 0 \\ 0 & 0 & 0 & d_5 d_4^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_6 d_5^{-1} & 0 \end{pmatrix}. \quad (31)$$

From $d_1 > d_2 > d_3 > 0$ and $d_4 > d_5 > d_6 > 0$ we imply that

$$d_2 d_1^{-1} < 1, \quad d_3 d_2^{-1} < 1, \quad d_5 d_4^{-1} < 1, \quad d_6 d_5^{-1} < 1. \quad (32)$$

Furthermore,

$$\begin{aligned} & b_1^{-2} + a_1 b_1^{-3} d_1 d_5^{-1} + a_1 b_1^{-3} d_1 d_6^{-1} \\ &= b_1^{-2} \left(1 + \frac{a_1}{b_1} d_1 d_5^{-1} + \frac{a_1}{b_1} d_1 d_6^{-1} \right) \\ &< \frac{a_1}{b_1^3} \left(1 + \frac{1}{1-5\epsilon'} + \frac{1}{1-6\epsilon'} \right) < \frac{a_1}{b_1^3} \frac{3}{1-6\epsilon'} < 1, \\ & a_1^{-2} + b_1 a_1^{-3} d_4 d_2^{-1} + b_1 a_1^{-3} d_4 d_3^{-1} \\ &= a_1^{-2} \left(1 + \frac{b_1}{a_1} d_4 d_2^{-1} + \frac{b_1}{a_1} d_4 d_3^{-1} \right) \\ &< \frac{1}{a_1^2} \left(1 + \frac{1}{1-2\epsilon'} + \frac{1}{1-3\epsilon'} \right) < \frac{1}{a_1^2} \frac{3}{1-6\epsilon'} < 1. \end{aligned} \quad (33)$$

It is well known that H has the same eigenvalues as DHD^{-1} ; we have that

$$\begin{aligned} \max_{1 \leq i \leq 6} |\lambda_i'| &\leq \|DHD^{-1}\|_{\infty} \\ &= \max \{d_2 d_1^{-1}, d_3 d_2^{-1}, d_5 d_4^{-1}, d_6 d_5^{-1}, \\ &\quad b_1^{-2} + a_1 b_1^{-3} d_1 d_5^{-1} + a_1 b_1^{-3} d_1 d_6^{-1}, \\ &\quad a_1^{-2} + b_1 a_1^{-3} d_4 d_2^{-1} + b_1 a_1^{-3} d_4 d_3^{-1}\} < 1. \end{aligned} \quad (34)$$

This implies that the equilibrium (a_1, b_1) of (6) is locally asymptotically stable.

Similarly, we can prove that the equilibrium (a_2, b_2) is locally asymptotically stable. $\square$

4. Numerical Example

In order to illustrate the results of the previous sections and to support our theoretical discussions, we consider several interesting numerical examples in this section. These examples represent different types of qualitative behavior of solutions to nonlinear difference equations and system of nonlinear difference equations.

Example 1. Consider the following system of difference equations:

$$x_{n+1} = A + \frac{x_n}{y_{n-1} y_{n-2}}, \quad y_{n+1} = A + \frac{y_n}{x_{n-1} x_{n-2}}, \quad (35)$$

with $A = 2.4 > 2/\sqrt{3}$ and the initial values $x_{-i} = 1.2$, $y_{-i} = 0.5$ ($i = 0, 1, 2$). Then the solution (x_n, y_n) of system (35) is bounded and persists. And the system has a unique equilibrium $(x, y) = (2.7620, 2.7620)$, which is globally asymptotically stable (see Figure 1).

Let $A = 1.1$ and the initial values $x_{-i} = 2$, $y_{-i} = 1.2$ ($i = 0, 1, 2$). Then the solution (x_n, y_n) of system (35) is bounded and persists (see Figure 2).

FIGURE 1: The dynamics of system (35) at $A = 2.4$.

FIGURE 2: The dynamics of system (35) at $A = 1.1$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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